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CPT Invariance and Weak Interactions*

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The pure leptonic and semileptonic weak interactions, together with the possible violation of *CPT* invariance, are studied by explicitly constructing a model of weak interactions. At low energies, we have the Fermi effective Lagrangian. The divergences in the model are no worse than those in the well-known renormalizable theories. The model suggests that if *CPT* invariance is violated in the weak interactions, so is Lorentz invariance. Furthermore, *CPT* violation cannot be detected by measuring the difference of lifetime, mass, or magnetic moment between the usually observed particle and antiparticle. It should be tested by measuring the deviation of the particle lifetime at high energies. Experimentally, the lifetimes of the charged kaon and the muon appear to be shorter at higher energies, as predicted by the model. The model is in agreement with the known experiments and gives many specific predictions.

I. INTRODUCTION

Since the discovery of *CP* noninvariance, it has become clear that the experimental foundation for *CPT* and *T* invariances in weak interactions should be reexamined. Should weak interactions violate *CPT* invariance, probably they will also violate Lorentz invariance in some way, for these two invariance principles are intimately related. We note that *CPT* invariance is a sufficient but not necessary condition for the equality of the total lifetime and mass of particle and antiparticle. As we shall see later, the extremely small mass or lifetime difference between the usually observed particle and its antiparticle does not necessarily exclude a maximum *CPT* violation in weak interactions.

So far, the experimental tests of the possible *CPT* noninvariance by measuring the mass, the magnetic moment, or the total lifetime difference between particle and its antiparticle have failed; and there is no direct evidence for *T* noninvariance although *CP* violation has been established. On the other hand, some experimental evidences indicate a small decrease of the charged-kaon and the muon lifetimes at high energy. What do they mean? To inquire into the matter, we shall construct a model of weak interactions which violates *CPT* invariance without contradicting experiments and see what are the results. We take the viewpoint that the result indicated by a model is a possible one and might be suggestive. One of the interesting suggestions of the model is that *CPT* should be tested by measuring the lifetime of the particle at high energies.

It is desirable to have a model of weak interactions whose divergences are no worse than those of quantum electrodynamics,[1] so that we can "understand" the experimental absence of the higher-order weak processes such as the weakness of neutral leptonic decays, the smallness of the K_S^0-K_L^0 mass difference, etc. Furthermore, the universality of weak interactions is most naturally manifested by using the usual weak vector and axial-vector currents. Therefore, we would like to construct, based on the usual weak currents, a model of weak interactions by introducing a charged intermediate scalar boson S and a 4-vector operator to reduce the singular behavior of the weak propagation function at infinite momentum. If the 4-vector operator appears only in the interaction Lagrangian, it will not have the usual equation of motion, and therefore no usual dynamical manifestation of particles at all. The 4-vector operator is then interpreted physically as the operator of an entity with zero 4-momentum and spin one. This entity enables us to construct a "pseudorenormalizable" and "local interaction"[2] theory which does not have observable violation of causality and unitarity, and at the same time has no other choice but maximum *C*, *CP*, *CPT* noninvariances. In this paper, we shall only show that the theory is "pseudorenormalizable" in the sense that the divergences are no worse than those in the well-known renormalizable theory. Presumably, a finite number of counterterms in the Hamiltonian are sufficient to renormalize the theory. This will not be discussed here.

In the model, the amplitude for the first-order weak interactions is Lorentz-invariant even though *CPT* is violated. The usual observed second-order weak interactions apparently conserve *CP* and *CPT* invariances, and their S matrices look like the effective current-current Lagrangian in the

Fermi theory, so that the weak self-mass and the weak decay rate (due to the second-order weak interactions) of a particle and its antiparticle are the same. Thus, a careful measurement of the mass and lifetime differences between μ^- (π^-) and μ^+ (π^+) does not necessarily provide a definite proof for the extremely small CPT violation or CPT invariance in the weak interactions. However, the CPT noninvariance in the basic weak interactions will, roughly speaking, give rise to a non-Lorentz-invariant amplitude in the second-order weak interactions. The violation of Lorentz invariance in the second-order weak interactions, of course, does not imply that the speed of light is not constant nor $m \neq m_0/(1-v^2/c^2)^{1/2}$, and so on. Rather, it implies that an unstable particle (due to second-order weak interactions) at higher energy would have a shorter lifetime after the time dilatation of relativity theory has been taken into account. The lifetime of the kaon at different energies has been measured with great accuracy,[3] and shows some favor for our prediction. The relativistic time-dilatation effect has been carefully tested only at relatively low energies because the cosmic-ray data is not very accurate. Since it is the only kind of experiment which tests Lorentz invariance in the domain of weak interactions and things in the high-energy region should not be taken for granted, we must test it thoroughly.

The model agrees with all known experiments and gives many specific predictions as we shall see later. In the following, we shall discuss mainly the pure leptonic and semileptonic weak interactions. We shall call the zero 4-momentum quantum an "aoraton" from the Greek word meaning "invisible."

II. THE ZERO-MOMENTUM AORATON

We shall introduce a 4-vector operator h_α, together with a charged scalar intermediate boson S, to reduce the singular behavior of the weak propagation function at infinite momentum and, at the same time, to have the vector nature of the weak interactions. This is possible if and only if the "h_α operator" is interpreted physically as a "zero 4-momentum vector operator." More exactly, this new physical entity h (the aoraton) does not carry electric charge, energy, momentum, or mass; however, it does carry one unit of spin angular momentum. According to the uncertainty principle, the space-time position of the aoraton is completely unknown, so that the aoraton itself is not directly observable.

The aoraton should appear only in the interaction Lagrangian, and it must not have a "free Lagrangian" because of the zero–4-momentum character. Thus, a free aoraton does not have the usual equation of motion. The very fact that the aoraton does not obey the usual equation of motion means that one should not interpret the corresponding field variables in a mechanical language of conjugate coordinates and momenta. Obviously, since there is no corresponding classical field for the aoraton, the conventional quantization of field can not be applied to this h of zero 4-momentum. However, we may still assume that there is zero-momentum and spin-1 quantum h associated with the vector operator h_α.

We shall apply the Gupta-Bleuler formalism[4] for the aoraton h. Suppose the "field operator" h_α and its Hermitian conjugate $h_\alpha^\dagger$ could be written in terms of the annihilation operator a_m and the creation operator $a_m^\dagger$ of the h quanta as follows:

$$\begin{aligned} h_\alpha &= \sum_{m=1}^{4} (a_m + \chi a_m^\dagger) e_\alpha^{(m)} , \\ h_\alpha^\dagger &= \sum_{m=1}^{4} (a_m^\dagger + \chi a_m) e_\alpha^{(m)*} , \\ a_4^\dagger &= (i a_0)^\dagger = -i a_0^\dagger , \end{aligned} \tag{1}$$

where an asterisk denotes the complex conjugate and $e_\alpha^{(m)}$ forms a set of four mutually orthogonal unit vectors ($m=1,2,3,4$ labels the vectors). The deviation of the real parameter χ ($0 \leq \chi \leq 1$) from unity characterizes the amount of C, CP, and CPT violations of the weak interactions due to the zero-momentum aoraton. Since the aoraton is not observable because of its zero-momentum character, $\chi \neq 1$ may occur without obviously contradicting the known experimental facts as we shall see later.

If χ is equal to unity, then the model will satisfy CPT invariance and Lorentz invariance (cf. Sec. III). Thus we see that the existence of such a nonlocal object, the aoraton, does not necessarily lead to the violation of Lorentz invariance. However, since the aoraton is practically not observable, we have to interpret, for example, the observed $\mu^- \to e\bar{\nu}_e\nu_\mu$ decay as a mixture of $\mu^- \to e^-\bar{\nu}_e\nu_\mu$ and $\mu^- \to e^-\bar{\nu}_e\nu_\mu hh$ in our model. This will obviously contradict the experimental data of the muon decay. Nevertheless, if $\chi = 0$, then the amplitude of the decay $\mu^- \to e^-\bar{\nu}_e\nu_\mu hh$ vanishes and everything will be consistent with experiment (cf. Sec. III). Thus, there is no choice; we must have the maximum (or nearly maximum) C, CP, and CPT noninvariances in the weak interactions due to the aoraton. Therefore, we shall assume $\chi = 0$ throughout the paper.

The aoraton has zero momentum, so that the choice of the unit vectors $e_\alpha^{(m)}$ is arbitrary and it is impossible to define the longitudinal and the transverse polarizations. Yet, the timelike polarization can still be defined in the usual way. The spin-one part of the aoraton has three stationary states, which can be transformed into each other by rotations of the coordinate system. We choose $e_4^{(1,2,3)}$ to be imaginary and $e_4^{(4)}$ real, so that we have

$$\sum_{m=1}^{4} e_\alpha^{(m)} e_\beta^{(m)} = \delta_{\alpha\beta} \quad (\alpha, \beta = 1, 2, 3, 4). \tag{2}$$

Let us define

$$\begin{aligned} a_4^\star &= -\eta a_4^\dagger \eta, \quad a_l^\star = \eta a_l^\dagger \eta \quad (l = 1, 2, 3), \\ h_4^\star &= -\eta h_4^\dagger \eta, \quad h_\lambda^\star = \eta h_\lambda^\dagger \eta \quad (\lambda = 1, 2, 3), \end{aligned} \tag{3}$$

where the metric operator η is chosen to satisfy $\eta^\dagger = \eta$, $\eta^2 = 1$, and

$$[\eta, a_4]_+ = 0, \quad [\eta, a_l] = 0 \quad (l = 1, 2, 3). \tag{4}$$

Assume that $a_m^\star$ and a_m satisfy the commutation relations

$$[a_m, a_n] = [a_m^\star, a_n^\star] = 0, \tag{5}$$

$$[a_m, a_n^\star] = \delta_{mn} \quad (m, n = 1, 2, 3, 4), \tag{6}$$

so that we have

$$[h_\alpha, h_\beta^\star] = \delta_{\alpha\beta}. \tag{7}$$

The relations in (4) are satisfied if η is the diagonal operator given by

$$\langle \cdots n_m \cdots | \eta | \cdots n_m' \cdots \rangle = (-1)^{n_4} \prod_{m=1}^{4} \delta_{n_m n_m'}, \tag{8}$$

where $n_4 = a_0^\dagger a_0$ is the total number of timelike aoratons in the state $| \cdots n_m \cdots \rangle$.

We have seen that a free aoraton is, by definition, independent of space-time, so that it satisfies

$$\frac{\partial h_\alpha}{\partial x_\mu} = 0, \quad \frac{\partial h_\alpha^\star}{\partial x_\mu} = 0 \quad (\mu = 1, 2, 3, 4; \quad x_4 = it). \tag{9}$$

In the c-number theory we use, as usual, Noether's theorem to find the conserved energy-momentum 4-vector. The total energy and the total momentum of the system do not involve h if the aoraton is not coupled with other particles. In the q-number theory, if we identify P_μ in the unitary operator $U(x) = \exp(-ix_\mu P_\mu)$, which generates the coordinate displacement, with the conserved energy-momentum 4-vector of the system, we would have

$$[P_\lambda, h_\alpha] = [P_\lambda, h_\alpha^\star] = 0 \quad (\lambda = 1, 2, 3), \tag{10}$$

$$[H, h_\alpha] = [H, h_\alpha^\star] = 0 \quad (H = P_4/i), \tag{11}$$

which are consistent with (9). Moreover, we can define the spin angular momentum operator of the aoraton directly in analogy with the usual vector meson at rest.

The aoraton of physical interest is that coupled with ordinary particles at a certain space-time point x in the interaction Lagrangian density. That is, an aoraton must be created or annihilated at a certain point together with the creation or annihilation of ordinary particles. Therefore, a coupled aoraton may be denoted symbolically by $h_\alpha(x)$ and considered as localized at $(\vec{x}, x_0)$.[2] Aoratons belonging to the same point x cannot be distinguished. Thus, the Bose statistics should be applied to the aoraton connected with one or several field operators at the same point.

Since the aoraton and its interaction are different from an ordinary particle in several respects, one should not take for granted that the properties of conventional fields still hold for the aoraton. Take causality, for example. From causality arguments one might expect that the aoraton $h_\alpha(x)$, which couples with another particle at x, and the aoraton $h_\beta(y)$ commute when $x-y$ is spacelike and believe that the aoraton cannot propagate faster than the speed of light. A similar argument has been applied to the "spurion".[2] However, this is not reliable because the aoraton (or the spurion) does not have the usual wave equation and therefore no local propagation properties. The requirement

$$[h_\alpha(x), h_\beta^\star(y)] = 0 \tag{12}$$

when $x-y$ is spacelike, is incompatible with (7). Thus, we see that microscopic causality is in principle violated by the aoraton in our framework. Nevertheless, this is not disturbing. It is "practically impossible" to have observable violation of causality due to the aoraton, since the aoraton interacts only weakly and it is not measurable in the same sense as the electric and magnetic fields are measurable. Moreover, it is clear that one can never use the aoraton to carry a signal to show the violation of the "classical causality principle," so that there will be no observable violation of causality in the theory. We also note that the usual connection between spin and statistics for an ordinary particle no longer holds for the aoraton. Thus, the aoraton obeying the Bose statistics is an independent assumption. Furthermore, the usual particles with zero rest mass travel with the velocity of light and have only two directions of polarization no matter what their spin is. These are obviously no longer true for the aoraton.

One might think that an "empty" space without aoratons cannot be distinguished from a space in which there are aoratons with zero 4-momentum, and that any number of aoratons can be added to a "physical" state without changing anything. This is not true in our framework because the aoraton is assumed to be coupled with the ordinary particle in a definite way (cf. Sec. III). For example, the positive intermediate boson S^+ will never decay in an empty space; however, if one adds an aoraton into the state $|S^+\rangle$, then the S^+ boson will decay:

$$|S^+, h\rangle \to |\mu^+, \nu_\mu\rangle \,. \tag{13}$$

In this sense, one can distinguish an "empty" space and a space with aoratons. Therefore, it would be reasonable to define the vacuum $|0\rangle$ in our framework as a state without energy, momentum, angular momentum and without any aoraton in it.

Given the total Hamiltonian whose interaction part involves the aoraton, the state vector and the operator of the Heisenberg representation and of the interaction representation are obtained from the corresponding quantities in the Schrödinger representation in the usual way. Thus, when the aoraton couples with ordinary particles in a Hamiltonian, the coupled aoraton in the Heisenberg picture is no longer independent of time but obeys, like an ordinary field operator, the Heisenberg equation of motion. In the calculation of the S matrix, we shall use the interaction representation in which the aoraton operator will be independent of space-time.

III. LAGRANGIAN

Let us consider a charged boson S to interact with the weak current $J_\alpha(x)$ in the following way:

$$\mathcal{L}_{\rm int}(x) = gJ_\alpha(x)S^\dagger(x)h^\star_\alpha(x) + gh_\alpha(x)S(x)J^\star_\alpha(x)\,, \tag{14}$$

and the Lagrangian density of the system is

$$\mathcal{L}(x) = -\partial_\alpha S(x)\partial_\alpha S^\dagger(x) - m_S{}^2 S(x)S^\dagger(x) + \mathcal{L}_{\rm int}(x)\,, \tag{15}$$

$S^\dagger(x)$ is the Hermitian conjugate of $S(x)$, and $J^\star_\alpha$ is defined as

$$J^\star_4(x) = -\eta J^\dagger_4(x)\eta\,, \quad J^\star_\lambda(x) = \eta J^\dagger_\lambda(x)\eta \quad (\lambda = 1, 2, 3). \tag{16}$$

The weak current $J_\alpha(x)$ is given by

$$\begin{aligned} &J_\alpha(x) = J^{(h)}_\alpha(x) + J^{(l)}_\alpha(x) \\ &J^{(l)}_\alpha(x) = i\,\bar{e}(x)\gamma_\alpha(1+\gamma_5)\nu_e(x) + i\,\bar{\mu}(x)\gamma_\alpha(1+\gamma_5)\nu_\mu(x)\,, \end{aligned} \tag{17}$$

where $J^{(h)}_\alpha(x)$ is the hadronic current and $J^{(l)}_\alpha(x)$ is the leptonic current. The γ's are the Hermitian Dirac matrices.

To quantize the S-boson field, one can follow the canonical method of quantization with the Lagrangian density (15). We define the canonical momenta conjugate to $S(x)$ and $S^\dagger(x)$ as

$$\pi(x) = \frac{\partial\mathcal{L}(x)}{\partial(\partial S(x)/\partial t)}\,, \quad \pi^\dagger(x) = \frac{\partial\mathcal{L}(x)}{\partial(\partial S^\dagger(x)/\partial t)}\,, \tag{18}$$

respectively. The total Hamiltonian density is

$$\begin{aligned}\mathcal{H}(x) = {}&\pi(x)\pi^\dagger(x) + \vec{\nabla}S(x)\cdot\vec{\nabla}S^\dagger(x) + m_S{}^2 S(x)S^\dagger(x) \\ &-gJ_\alpha(x)S^\dagger(x)h^\star_\alpha(x) - gh_\alpha(x)S(x)J^\star_\alpha(x)\,. \end{aligned} \tag{19}$$

Since the independent field variables are the same as those in the free-field theories, we adopt the same equal-time canonical commutation relations

$$\begin{aligned} &[\pi(x),\ S(x')]_{t=t'} = -i\,\delta^{(3)}(\vec{x}-\vec{x}')\,, \\ &[\pi^\dagger(x),\ S^\dagger(x')]_{t=t'} = -i\delta^{(3)}(\vec{x}-\vec{x}')\,, \end{aligned} \tag{20}$$

and let all other commutators vanish:

$$\begin{aligned} &[S(x),\ S^\dagger(x')]_{t=t'} = [\pi(x),\ \pi^\dagger(x')]_{t=t'} = 0\,, \\ &[S(x),\ h_\alpha(x')]_{t=t'} = [S(x),\ h^\star_\alpha(x')]_{t=t'} = 0\,, \quad \text{etc.} \end{aligned} \tag{21}$$

IV. TRANSITION PROBABILITY

Since we have used the indefinite metric, the interaction Hamiltonian is not Hermitian but pseudo-Hermitian:

$$\begin{aligned} &H^\star_{\rm int} = \eta H^\dagger_{\rm int}\eta = H_{\rm int}, \\ &H_{\rm int} = \int d\vec{x}\,\mathcal{H}_{\rm int}(\vec{x}, t)\,. \end{aligned} \tag{22}$$

The total transition probability for a state $|i\rangle$ going over into any state $|f\rangle$ is 1:

$$\sum_f W_{fi} \equiv \sum_f |\langle f|\eta S|i\rangle|^2 N_f{}^{-1}N_i{}^{-1} = 1\,,$$

where S denotes the S matrix and η denotes the metric operator; $N_f = \langle f|\eta|f\rangle$ and $N_i = \langle i|\eta|i\rangle$ are, respectively, the norms of the states $|f\rangle$ and $|i\rangle$.[5] We note that a process involving a "timelike aoraton" in $|f\rangle$ or $|i\rangle$ would have negative W_{fi}, which makes no sense. However, the present theory is physically meaningful because the aoraton itself is not directly observable and, moreover, any "physically observable process" has positive transition probability. The reasons are that the polarization of the aoraton can never be directly observed and the timelike aoraton can never be separated experimentally from the spacelike aoraton. Therefore, we must always sum over the spacelike and the timelike aoraton states whenever an aoraton is emitted in a process, e.g., $S^- \to \mu^-\nu_\mu h$.[6] In this way, we have a positive probability for the "physically observable processes."

For clarity, let us consider a first-order weak process $S^- \to \mu^-(p) + \bar{\nu}_\mu(q) + h$. Suppose a timelike aoraton is emitted; according to (19), the transition probability involves a factor $(-\vec{p}\cdot\vec{q}-p_0q_0)$, which is not Lorentz-invariant and will lead to a negative probability. The corresponding factor for the spacelike aoraton is $(3p_0q_0-\vec{p}\cdot\vec{q})$. When we sum over the spacelike aoraton and timelike aoraton, we get $(-2p_\alpha q_\alpha)$ for the corresponding factor in the transition probability, which is positive and Lorentz-invariant. In some higher-order processes, a virtual S boson may be absorbed at a certain vertex and an aoraton and other particles (e.g., $\mu^-\bar{\nu}_\mu$ or $\pi^-\pi^0$) are emitted in the final state. The matrix element will contain among other things a factor $Z_\alpha e_\alpha^{(m)}$ where Z_α consists of the momentum q_μ or the γ matrix contributed by the weak current associated with the aoraton. After summing over all polarizations of the aoraton in the transition probability, we have, among other things,

$$4(1+aa^*)(p_\alpha q_\beta - p\cdot q\delta_{\alpha\beta}+q_\alpha p_\beta)\delta_{\alpha\beta}\equiv X, \tag{23}$$

when the weak current associated with the aoraton takes the form

$$i\bar{u}(\vec{p})\gamma_\alpha(1+a\gamma_5)u'(\vec{q}).$$

And we have, among other things,

$$(p_\alpha - q_\alpha)(p_\beta - q_\beta)\delta_{\alpha\beta}\equiv Y, \tag{24}$$

when the weak current associated with aoraton takes the form $\pi^0\partial_\alpha\pi^+ - \pi^+\partial_\alpha\pi^0$, where p_α and q_α are the momenta associated with the fields $\pi^+(x)$ and $\pi^0(x)$, respectively. Both X and Y are positive because the 4-momentum of the external particles is either timelike or lightlike, so that the transition probability will be positive. [We have set $m(\pi^+)=m(\pi^0)$ for simplicity.]

In general, since the model does not have local gauge invariance or a conserved current, we are unable to give a general proof of the non-negative character of the transition probabilities. Yet we observed that the possible negative transition probabilities exist only in the processes in which the initial state and/or final state involve the aoratons. There is no problem when the aoratons appear in the intermediate steps of a higher-order process, because we always sum over all polarization indices. We have examined several specific cases and found that the negative norm of the timelike aoraton does not give rise to any trouble because it is not directly and separately observable. Presumably, this is true in all cases and therefore the pseudo-Hermitian H_{int} is effectively a Hermitian interaction Hamiltonian.

V. SOME SIMPLE RESULTS

In the present framework, the weak processes can be classified into the following four classes:

(A) The initial state $|i\rangle$ and the final state $|f\rangle$ do not involve the aoraton, e.g., $n\to pe^-\bar{\nu}$.

(B) The final state involves the aoraton, e.g., $S^-\to\mu^-\bar{\nu}_\mu h$.

(C) The initial state involves the aoraton, e.g., $S^+h\to\mu^+\nu_\mu$.

(D) Both $|i\rangle$ and $|f\rangle$ involve the aoraton, e.g., $nh\to phe^-\bar{\nu}_e$.

Since the aoraton is impossible to handle in the laboratory, the processes in classes (C) and (D) are practically impossible to detect. We note that the aoratons in the universe can only be created in the process which creates the S boson and in the decay process of an S boson. Thus, if the S-boson mass is very large, then the chance for the processes in classes (C) and (D) to occur and to be detected in nature is very small. Therefore, only those processes in (A) and (B) are of interest to us, for they are observable. Here, we shall only consider the observed processes in (A).

(1) Let us consider the usual weak-interaction processes. They are all second-order weak interactions in g. The aoraton and the S boson are emitted from one vertex and absorbed into the other vertex. Thus, we have to calculate the S boson and the aoraton exchange term to the lowest order. Instead of two weak currents being coupled at a point, we have the following second-order effective interaction:

$$S^{(2)}=i\int\mathfrak{L}_{\text{eff}}(x)d^4x$$
$$=-ig^2\int J_\alpha^\star(x)J_\beta(y)\Delta_{\alpha\beta}(x-y)\theta(x_0-y_0)d^4y\,d^4x, \tag{25}$$

where

$$\Delta_{\alpha\beta}(x-y)=\int\frac{\delta_{\alpha\beta}}{q^2+m_S^2}e^{iq\cdot(x-y)}\frac{d^4q}{(2\pi)^4}. \tag{26}$$

The effective Lagrangian in (25) can also be written as

$$\mathfrak{L}_{\text{eff}}(x)=-g^2\int J_\alpha^\star(x)J_\alpha(y)\frac{1}{q^2+m_S^2}\times\left(\frac{1}{2}+\frac{q_0}{2(\vec{q}^2+m_S^2)^{1/2}}\right)e^{iq\cdot(x-y)}\frac{d^4q}{(2\pi)^4}d^4y. \tag{27}$$

For the low-energy processes with momentum transfer very much smaller than m_S, i.e., $-q^2\ll m_S^2$, we have effectively the Fermi interaction with

$$g^2/(2m_S^2)=G/\sqrt{2}. \tag{28}$$

(2) The high-momentum behavior of the "effec-

tive weak propagation function" $\Delta^h_{\alpha\beta}(q)$ is less singular than that given by the intermediate W boson or the Fermi theory, to wit,

$$\Delta^h_{\alpha\beta}(q) = \left(\frac{-i\delta_{\alpha\beta}}{2}\right)\left(1+\frac{q_0}{(\vec{q}^{\,2}+m_S^{\,2})^{1/2}}\right)\Big/(q^2+m_S^{\,2})$$

$$\propto \delta_{\alpha\beta}q^{-2} \quad (q_\alpha \to \infty)\,. \qquad (29)$$

Thus, only logarithmically divergent integrals appear in the higher-order amplitudes, and the model is "pseudorenormalizable."

(3) The aoraton must be emitted and reabsorbed in the observable even-order processes, so that the final state does not involve the aoraton and the maximum CPT violation will not appear in the observable even-order processes. Therefore, the effects of C, CP, and CPT violations due to the aoraton *cannot* be detected by measuring the mass or the total lifetime difference between the "usually observed" particle and its antiparticle.

(4) The CPT violation of the basic weak interactions due to the aoraton will show up in the decay rate and the angular distribution of the second-order weak processes. For example, the decay rate of $\pi\to\mu\nu$ (or $K\to\mu\nu$) will vary, besides the variation due to the relativistic time dilatation, with the energy of the pion (or the kaon) in the laboratory. From the effective Lagrangian (27), we have

$$\frac{\tau^{-1}(E_1)}{\tau^{-1}(E_2)} = \left(\frac{1+E_1/(\vec{p}_1^{\,2}+m_S^{\,2})^{1/2}}{1+E_2/(\vec{p}_2^{\,2}+m_S^{\,2})^{1/2}}\right)^2 \qquad (30)$$

for $\pi\to\mu\nu$ (or $K\to\mu\nu$) due to the CPT violation of the aoraton, where E and $\vec{p}$ are, respectively, the laboratory energy and momentum of the pion (or the kaon).

It follows from (30) that if one measures the kaon lifetime at different momenta in the laboratory, one should find that the higher-momentum kaon has the shorter lifetime after the relativistic time-dilatation effect has been taken into account. Experimentally, the charged-kaon lifetime has been measured with great accuracy[7]:

$$\left.\begin{aligned} &p_K = 0.475\ \text{GeV},\\ &\tau(K^+) = (12.443\pm0.038)\times10^{-9}\ \text{sec}, \end{aligned}\right\}\ [\text{Ref. 7(a)}]$$

$$\left.\begin{aligned} &p_K = 1.6\ \text{and}\ 2.0\ \text{GeV},\\ &\tau(K^+) = (12.265\pm0.036)\times10^{-9}\ \text{sec}, \end{aligned}\right\}\ [\text{Ref. 7(b)}] \qquad (31)$$

$$\left.\begin{aligned} &p_K = 3.0\ \text{GeV},\\ &\tau(K^+) = (12.21\pm0.11)\times10^{-9}\ \text{sec}\,. \end{aligned}\right\}\ [\text{Ref. 7(c)}]$$

These results are in agreement with our prediction. The energy dependence of the pion lifetime is experimentally not yet clear. Therefore, in order to test the prediction (30) the experiments of kaon, pion, and muon lifetimes at higher energies are needed.

(5) The S-boson mass can be roughly estimated from the data in (31):

$$m_S \sim 200\ \text{GeV}\,. \qquad (32)$$

With such a large S-boson mass, the chance of detecting the processes in classes (C) and (D) is indeed extremely small.

On the other hand, in a precision measurement of the anomalous magnetic moment of the muon[8] at muon momentum 1.28 GeV, the lifetime of the muon is found to be smaller[9] than that measured at zero momentum by $(1.1\pm0.1)\%$. This gives a more reliable S-boson mass in our framework:

$$m_S = 144\pm14\ \text{GeV}\,. \qquad (33)$$

(6) The interaction Hamiltonian densities $\mathcal{H}_{\text{int}}(x)$ and $\mathcal{H}_{\text{int}}(y)$ do not commute when $(x-y)^2$ is spacelike. This noncommutativity comes from the noncommutativity of the aoraton operator as shown in (7). In a sense, such a Hamiltonian density is "nonlocal." It is known that such kinds of nonlocality will lead to a breakdown of Lorentz invariance in the standard perturbation theory.

(7) The anomalous magnetic moments μ_a for both positively and negatively charged leptons, due to the virtual S boson and aoraton, are the same. We estimated $\mu_a \sim 10^{-9}(e/2m_\mu)$ for the muon; it is too small to be detected.

VI. CONCLUDING REMARKS

Although the aoraton does not carry energy, momentum, etc., it does carry one unit of spin angular momentum. Thus, even though it is not directly observable, it does have some observable effects in the weak processes whenever it is created in the final state. In this sense, the aoraton is, like the usual particle, a real physical entity; and it is therefore basically different from the so-called "spurion."

We assume neither that the vacuum be a "sea" of infinitely many aoratons nor the interaction of the particle with the aoraton sea as a whole. Thus, our approach has little to do with that of the field theories with an asymmetrical vacuum or the Goldstone theorem.[10] Yet the concept of vacuum expectation value becomes somewhat problematic when the vacuum is degenerate; the situation in a spurion formalism has been discussed, for instance, by Dürr and Heisenberg.[2] The problem still remains open.

Within the present framework, we must set $\chi = 0$ in (1) in order to be consistent with experiment; it follows that both the *CPT* and the Lorentz invariances are violated. It is apparently not possible to have *CPT* noninvariance and Lorentz invariance at the same time without losing some form of unitarity or contradicting the known experiments.

Now, we have seen that a maximum *CPT* violation in the weak interactions is possible without contradicting experiment, although a direct detection of such *CPT* violation by measuring the lifetime difference between particle and antiparticle is impossible. However, the associated nonrelativistic effect of the kaon, the pion, and the muon lifetimes can certainly be detected with the present experimental technique even if $m_S \approx 200$ GeV. We feel that the possible small deviation of the kaon and the muon lifetimes at relatively high energies deserves to have further experimental study rather than to be simply ignored. Further implications of the model will be reported later.

Note added. Since the aoraton h is unobservable, the observed μ decay is in general a mixture of $\mu \rightarrow e\nu\nu$ and $\mu h \rightarrow e\nu\nu h$. It can be shown that the spectrum of the electron coming from $\mu h \rightarrow e\nu\nu h$ is quite different from that of the electron coming from $\mu \rightarrow e\nu\nu$. The observed electron spectrum from μ decay provides a way to estimate the density of the aoraton in space. But it shows no evidence for the presence of $\mu h \rightarrow e\nu\nu h$. Therefore, the processes of class (D) are practically negligible.

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[1]W. Kummer and G. Segrè, Nucl. Phys. 64, 585 (1965); Y. Janikawa, Phys. Rev. 108, 1615 (1967); Y. Janikawa and S. Nakamura, Progr. Theoret. Phys. (Kyoto) Suppl. 37–38, 306 (1966); T. D. Lee and G. C. Wick, Nucl. Phys. B9, 209 (1969).

[2]H. P. Dürr and W. Heisenberg, Nuovo Cimento 37, 1446 (1965); H. P. Dürr, in *Proceedings of the 1967 International Conference on Particles and Fields,* edited by C. R. Hagen *et al.* (Interscience, New York, 1967), p. 328. According to these authors, those spurions which are not explicitly involved with localized states may be referred to as "not localized" or free spurions, while the coupled spurions however can be considered as localized. The same consideration holds for the aoraton.

[3]Particle Data Group, Rev. Mod. Phys. 42, 87 (1970).

[4]See, for example, F. Mandl, *Introduction to Quantum Field Theory* (Interscience, New York, 1959).

[5]W. Pauli, Rev. Mod. Phys. 15, 175 (1943).

[6]Within our framework, it is impossible to have $h_4 = a_4 e_4^{(4)}$ and $h_k = 0$ $(k = 1, 2, 3)$ by choosing a particular reference frame and measuring the momenta and the polarizations of μ and ν_μ, because even in this case both the timelike and the spacelike (at least one of the components) aoraton states would contribute to the amplitude. We note that the separation of the timelike aoraton from the spacelike aoraton has experimentally no operational meaning.

[7](a) V. L. Fitch, C. A. Quarles, and H. C. Wilkins, Phys. Rev. 140, B1088 (1965); (b) F. Lobkowicz, A. C. Melissinos, Y. Nagashima, S. Tewksburg, H. von Briesen, Jr., and J. D. Fox, Phys. Rev. Letters 17, 548 (1966); (c) W. T. Ford, A. Lemonick, U. Nauenberg, and P. A. Piroué, *ibid.* 18, 1214 (1967).

[8]E. Picasso (private communication). See also J. Bailey and E. Picasso, in *Progress In Nuclear Physics,* edited by D. M. Brink and J. H. Mulvey (Pergamon, Oxford, England, 1970), Vol. 12, p. 62.

[9]The shorter measured lifetime of the muon is ascribed to a slow loss of the muons due to imperfections in the magnetic field. However, the deviation is only *partly* explained by losses of the muons in the storage ring. The author would like to thank G. von Bochmann for a discussion of this point.

[10]See, for example, Ref. 2 and J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962).