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Magnetic Quarks and Electric Quarks in Hadrons

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Dirac's magnetic monopoles are generalized to have aspects which are similar to the conventional quarks (electric quarks). Such magnetic monopoles are called magnetic quarks. This work assumes that a baryon consists of three solid bodies called electromagnetic quarks, and that an electromagnetic quark and an electromagnetic antiquark form a meson. Each of these electromagnetic quarks is considered to be composed of one electric quark and one magnetic antiquark. Such a speculation solves the difficulty in statistics faced by the paraquark model and allows the existence of anomalous charge conjugation parity C of mesonic states. New baryon mass relations and magnetic moments have been derived. Finally, a strong electric dipole moment is predicted to exist in a baryon state with nonzero electromagnetic-quark orbital angular momentum, $L \neq 0$.

I. INTRODUCTION

Even after the invention of the quark model[1] very little connection has been made between Dirac's[2] magnetic monopoles and the hadron structure. Dirac did not answer the question of whether a particle can carry both electric and magnetic charges. However, Schwinger suggested the existence of such dually charged particles, called dyons.[3,4] He replaced the quarks by dyons as the fundamental blocks of hadrons. We find that he had difficulty in explaining the electric dipole moment (EDM) and the magnetic dipole moment (MDM) of hadrons. We adopted Dirac's magnetic monopoles and the spirit of the quark model rather than Schwinger's dyons. Work related to this paper has appeared in the previous reports.[5-7] The plan of this paper is as follows.

Section II introduces the possibility of the existence of magnetic monopoles inside hadrons, and generalizes the magnetic monopoles to have aspects similar to the conventional quarks (electric quarks). The generalized magnetic monopoles are called magnetic quarks. It also describes the properties of electric quarks (Q_e) and magnetic quarks (Q_m). The Q_e's (fermions) and the Q_m's (bosons) are considered to be the same kind of particles only when the superstrong interaction is concerned. In Sec. III the electromagnetic quark (Q_{em}) is constructed from the Q_e and Q_m and its properties are described. In Sec. IV the baryon wave functions are expressed in terms of Q_{em}'s. In Sec. V baryon mass relations are derived from our new model and compared with those obtained from the paraquark model. In Sec. VI the EDM's and MDM's of baryons are predicted and also compared with the results of the paraquark model. In particular, we prove that this new model will give the baryon octet and the decuplet a zero EDM. Finally, in Sec. VII we briefly discuss the validity of this new model and some unsolved problems.

II. ELECTRIC QUARKS AND MAGNETIC QUARKS

The quark model of the hadron structure has been very successful in accounting for many of the properties of baryons and mesons. However, quarks (electric) have not yet been positively identified. Since McCusker and Cairns[8] claimed their discovery of quarks, many questions have been raised.[9] Another difficulty in the paraquark model is the parastatistics hypothesis.[10,11] The hadron experimental data favor a symmetric space wave function in the quark labels. The reason for this is that the calculated results[12] indicate that an antisymmetric space wave function will produce a node for the body form factor of the baryon, but the measured form factors show no evidence for a node.[13] Such a symmetric space wave function cannot be satisfied if quarks are fermions (if there is no other constituent except the three quarks). The paraquark model assumes that quarks are not fermions but parafermions which have spin $\frac{1}{2}$ and follow the symmetric requirement of the space wave function. Such a parastatistics hypothesis is an unattractive possibility, since it represents a drastic hypothesis which may raise more difficulties than it solves. A possible way to solve this difficulty is to assume that a baryon consists of some other kind of particles besides the three Q_e's. Such extra constituents can be magnetic monopoles. We will explore this feature directly.

Since Dirac's work on the magnetic monopole theory in 1931, several experiments[14] have been done to search for the monopoles. No positive results have been reported so far. The reason we have not observed quarks should have some connection with the reason we have not detected magnetic monopoles. To relate the magnetic monopoles to the quarks we assume that there also exist three different kinds of magnetic monopoles, called $\mathcal{P}'$, $\mathfrak{N}'$, and λ' magnetic quarks (Q_m's), in accordance with the conventional $\mathcal{P}$, $\mathfrak{N}$, and λ Q_e's. The three Q_m's carry fractional magnetic charges[15] $\frac{2}{3}$, $-\frac{1}{3}$, and $-\frac{1}{3}$. As mentioned before, the corresponding Q_e and Q_m can be treated as the same particle in different states, i.e., electric and magnetic states. The particle is called Q_e when it is in the electric state and called Q_m when it is in the magnetic state. The corresponding pairs, $\mathcal{P}$ and $\mathcal{P}'$, $\mathfrak{N}$ and $\mathfrak{N}'$, and λ and λ', should almost have equal masses. We should also have a set of quantum numbers, such as magnetic charge, magnetic isospin, magnetic strangeness, and magnetic baryon number, for the Q_m's. Here we assign the Q_m a zero spin and the Q_e a $\frac{1}{2}$ spin in accordance with the fact that baryons have MDM's but do not have EDM's.

In treating the Q_e's and Q_m's we may extend the quantities such as dipole moment, charge, isospin, strangeness, and baryon number from real number to complex number. We define

$$Q = Q^{(e)} + iQ^{(m)}, \tag{2.1}$$

$$I_z = I_z^{(e)} + iI_z^{(m)}, \tag{2.2}$$

$$S = S^{(e)} + iS^{(m)}, \tag{2.3}$$

$$B = B^{(e)} + iB^{(m)}, \tag{2.4}$$

and

$$\mu = \mu^{(e)} + i\mu^{(m)}, \tag{2.5}$$

where $\mu^{(m)}$ is the MDM and $\mu^{(e)}$ is the EDM. The properties of Q_e's and Q_m's then can be expressed as in Table I. Here we have the generalized Nishijima–Gell-Mann relation,

$$Q = I_z + \tfrac{1}{2}(B + S). \tag{2.6}$$

Obviously this equation contains

$$Q^{(e)} = I_z^{(e)} + \tfrac{1}{2}(B^{(e)} + S^{(e)}) \tag{2.7}$$

and

$$Q^{(m)} = I_z^{(m)} + \tfrac{1}{2}(B^{(m)} + S^{(m)}). \tag{2.8}$$

Introducing the Q_m can make Maxwell's equations symmetric. The generalized Maxwell equations will be of the form

$$\nabla \times \vec{H} - \frac{1}{c}\frac{\partial}{\partial t}\vec{E} = \frac{4\pi}{c}\vec{j}_e, \tag{2.9}$$

$$\nabla \cdot \vec{E} = 4\pi\rho_e, \tag{2.10}$$

$$-\nabla \times \vec{E} - \frac{1}{c}\frac{\partial}{\partial t}\vec{H} = \frac{4\pi}{c}\vec{j}_m, \tag{2.11}$$

$$\nabla \cdot \vec{H} = 4\pi\rho_m. \tag{2.12}$$

By introducing

$$\rho = \rho_e + i\rho_m,$$

$$\vec{j} = \vec{j}_e + i\vec{j}_m,$$

and

$$\vec{G} = \vec{E} + i\vec{H},$$

the Maxwell equations [(2.9)–(2.12)] can be simply rewritten as

$$\nabla \cdot \vec{G} = 4\pi\rho \tag{2.13}$$

and

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + i\nabla\times\right)\vec{G} = -\frac{4\pi}{c}\vec{j}. \tag{2.14}$$

III. ELECTROMAGNETIC QUARKS

As discussed in Sec. II, we cannot distinguish Q_e's from Q_m's if we turn off the electromagnetic interaction. Therefore, in accounting for the binding energy of a pair of quarks we should classify the pair as a particle-particle pair, a particle-antiparticle pair, or an antiparticle-antiparticle pair, rather than do it by their electromagnetic properties. The binding energy of a particle-particle pair should be almost equal to that of an antiparticle-antiparticle pair, and different from that

TABLE I. Properties of electric quarks and magnetic quarks and their counterparts.

Symbols[a]	Quarks	J^P	μ	Q	I_z	S	B
q	$\mathcal{P}$	$\frac{1}{2}^+$	$\frac{2}{3}i$	$\frac{2}{3}$	$\frac{1}{2}$	0	$\frac{1}{3}$
	$\mathfrak{N}$	$\frac{1}{2}^+$	$-\frac{1}{3}i$	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$\frac{1}{3}$
	λ	$\frac{1}{2}^+$	$-\frac{1}{3}i$	$-\frac{1}{3}$	0	-1	$\frac{1}{3}$
$\bar{q}$	$\bar{\mathcal{P}}$	$\frac{1}{2}^-$	$-\frac{2}{3}i$	$-\frac{2}{3}$	$-\frac{1}{2}$	0	$-\frac{1}{3}$
	$\bar{\mathfrak{N}}$	$\frac{1}{2}^-$	$\frac{1}{3}i$	$\frac{1}{3}$	$\frac{1}{2}$	0	$-\frac{1}{3}$
	$\bar{\lambda}$	$\frac{1}{2}^-$	$\frac{1}{3}i$	$\frac{1}{3}$	0	1	$-\frac{1}{3}$
q'	$\mathcal{P}'$	0^+	0	$\frac{2}{3}i$	$\frac{1}{2}i$	0	$\frac{1}{3}i$
	$\mathfrak{N}'$	0^+	0	$-\frac{1}{3}i$	$-\frac{1}{2}i$	0	$\frac{1}{3}i$
	λ'	0^+	0	$-\frac{1}{3}i$	0	$-i$	$\frac{1}{3}i$
$\bar{q}'$	$\bar{\mathcal{P}}'$	0^+	0	$-\frac{2}{3}i$	$-\frac{1}{2}i$	0	$-\frac{1}{3}i$
	$\bar{\mathfrak{N}}'$	0^+	0	$\frac{1}{3}i$	$\frac{1}{2}i$	0	$-\frac{1}{3}i$
	$\bar{\lambda}'$	0^+	0	$\frac{1}{3}i$	0	i	$-\frac{1}{3}i$

[a] q' and $\bar{q}'$ are magnetic quarks and magnetic antiquarks, respectively.

of a particle-antiparticle pair. This can be expressed by the equations

$$B(q,q)\simeq B(q,q')\simeq B(q',q')=a \qquad (3.1)$$

and

$$B(q,\overline{q})\simeq B(q,\overline{q}')\simeq B(\overline{q},q')\simeq B(q',\overline{q}')=b\,. \qquad (3.2)$$

Here the clusters qq, $q'q'$, $q\overline{q}$, and $q'\overline{q}'$ all carry integer spins, whereas qq', $q\overline{q}'$, and $\overline{q}q'$ all carry half-integer spins. From the paraquark model we know that the binding energy of $q\overline{q}$ is much greater[16] than that of qq; hence, that of $q\overline{q}'$ and $\overline{q}q'$ should be also much greater than that of qq'. We call $q\overline{q}'$ electromagnetic quark (Q_{em}), and $\overline{q}q'$ electromagnetic antiquark ($\overline{Q}_{em}$).

Like conventional mesonic states, both $q\overline{q}'$ and $\overline{q}q'$ form nonets. Many results obtained in the conventional $q\overline{q}$ quark model of mesons[17-20] can be applied to the $q\overline{q}'$ and $\overline{q}q'$ systems. However, care must be taken that $q\overline{q}$ is a boson, and $q\overline{q}'$ and $\overline{q}q'$ are fermions. J^P for $\overline{q}q'$ and $q\overline{q}'$ ground states are $\frac{1}{2}^-$ and $\frac{1}{2}^+$, respectively. There is no question that both $q\overline{q}'$ and $\overline{q}q'$ can be excited to higher states. In general, we have the parities $P=(-1)^l$ for $q\overline{q}'$ and $P=-(-1)^l$ for $\overline{q}q'$. Therefore, $q\overline{q}'$ can have

$$J^P=\tfrac{1}{2}^{\pm},\ \tfrac{3}{2}^{\pm},\ \tfrac{5}{2}^{\pm},\ \ldots \qquad (3.3)$$

and $\overline{q}q'$ can have (for the same l)

$$J^P=\tfrac{1}{2}^{\mp},\ \tfrac{3}{2}^{\mp},\ \tfrac{5}{2}^{\mp},\ \ldots\ . \qquad (3.4)$$

From this, the mesonic states $q\overline{q}'\overline{q}q'$ will have

$$C_n=\pm(-1)^{l+s}, \qquad (3.5)$$

$$P=\mp(-1)^l, \qquad (3.6)$$

and

$$C_nP=-(-1)^s\,. \qquad (3.7)$$

Here C_n (n for neutral) is the eigenvalue that C will have if applied to the neutral number of the multiplet. It is worthwhile to compare these results with those obtained from the conventional quark model, which has the following results[21]:

$$C_n=(-1)^{l+s}, \qquad (3.8)$$

$$P=-(-1)^l, \qquad (3.9)$$

and

$$C_nP=-(-1)^s \qquad (3.10)$$

It is apparent that our model can explain the abnormal C mesonic states,[21] which cannot be explained by the conventional $q\overline{q}$ quark model.

If we wish to describe the internal dynamic motions of $q\overline{q}'$ and $\overline{q}q'$ qualitatively, we must consider the interactions between q and $\overline{q}'$ and between $\overline{q}$ and q' as functions of their separation r. There are two kinds of interactions to be considered, namely, electromagnetic and superstrong attractive interactions. The superstrong force will be nominated for small r, and the electromagnetic force for large r. However, there is no doubt that q and $\overline{q}'$, and $\overline{q}$ and q', can form bound states. Even if we ignore the superstrong attractive force and take into account only the electromagnetic interaction, they still would form bound states.[22] Furthermore, Dirac[2] treated the magnetic monopoles as electromagnetic interacting particles in his first paper on this subject, but in his second paper he considered magnetic monopoles as possible constituents of protons. So, we think it is reasonable to generalize the magnetic monopoles to Q_m's.

Now we should discuss the MDM and the EDM of Q_{em}'s and $\overline{Q}_{em}$'s. We know the ground state of $q\overline{q}'$ carries electric and magnetic charges and MDM. The electric charge and the MDM are contributed by q, and the magnetic charge by $\overline{q}'$. The electromagnetic interaction between q and $\overline{q}'$ will result in an increase or a decrease of the MDM of the Q_{em}. The total amount of change is mainly dependent on the strength of the intrinsic MDM of q and the magnetic charge of $\overline{q}'$. We will take this effect (which may be called the cooperative effect) into account in Sec. VI when we discuss the EDM and the MDM of baryons.

IV. WAVE FUNCTIONS

As mentioned before, the paraquark model has a serious difficulty in statistics because it ignores the existence of magnetic monopoles. In our model we do not have such problems. We assume that a baryon consists of three magnetic antiquarks ($\overline{Q}_m$'s) and three Q_e's. These Q_e's are identical to those assumed in the paraquark model. Therefore, a baryon and a meson can be expressed as $q_1q_2q_3\overline{q}_1'\overline{q}_2'\overline{q}_3'$ and $q\overline{q}'\overline{q}q'$, respectively. The physically observable hadrons lie in the lowest baryon or meson magnetic state which is a magnetic singlet and neutral. This implies that the observable baryons are in the form $q_1q_2q_3\overline{\mathcal{P}}'\overline{\mathfrak{N}}'\overline{\lambda}'$. Furthermore, as discussed in Sec. III, the binding energy between q and $\overline{q}'$ is much greater than that between q and q or that between $\overline{q}'$ and $\overline{q}'$. Therefore, $q_1q_2q_3\overline{\mathcal{P}}'\overline{\mathfrak{N}}'\overline{\lambda}'$ will form three clusters: $q_1\overline{\mathcal{P}}'$, $q_2\overline{\mathfrak{N}}'$, and $q_3\overline{\lambda}'$. Such clusters carrying fractional electric charges and magnetic charges are identified as Q_{em}'s. Each of the three Q_{em}'s can be considered as an entity as long as the kinetic energy of the Q_{em} is very small compared to its excitation energy. The internal structure of the Q_{em} is then irrelevant in accounting for the statistical model of the baryon. Denote the nonet Q_{em}'s as

$$\begin{pmatrix} \mathcal{P}_1 & \mathcal{P}_2 & \mathcal{P}_3 \\ \mathfrak{N}_1 & \mathfrak{N}_2 & \mathfrak{N}_3 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{pmatrix} = \begin{pmatrix} \mathcal{P}\bar{\mathcal{P}}' & \mathcal{P}\bar{\mathfrak{N}}' & \mathcal{P}\bar{\lambda}' \\ \mathfrak{N}\bar{\mathcal{P}}' & \mathfrak{N}\bar{\mathfrak{N}}' & \mathfrak{N}\bar{\lambda}' \\ \lambda\bar{\mathcal{P}}' & \lambda\bar{\mathfrak{N}}' & \lambda\bar{\lambda}' \end{pmatrix} \tag{4.1}$$

and the nonet $\overline{Q}_{em}$'s as

$$\begin{pmatrix} \bar{\mathcal{P}}_1 & \bar{\mathcal{P}}_2 & \bar{\mathcal{P}}_3 \\ \bar{\mathfrak{N}}_1 & \bar{\mathfrak{N}}_2 & \bar{\mathfrak{N}}_3 \\ \bar{\lambda}_1 & \bar{\lambda}_2 & \bar{\lambda}_3 \end{pmatrix} = \begin{pmatrix} \bar{\mathcal{P}}\mathcal{P}' & \bar{\mathcal{P}}\mathfrak{N}' & \bar{\mathcal{P}}\lambda' \\ \bar{\mathfrak{N}}\mathcal{P}' & \bar{\mathfrak{N}}\mathfrak{N}' & \bar{\mathfrak{N}}\lambda' \\ \bar{\lambda}\mathcal{P}' & \bar{\lambda}\mathfrak{N}' & \bar{\lambda}\lambda' \end{pmatrix}. \tag{4.2}$$

The baryon and meson wave functions then can be expressed by the 18 Q_{em}'s and $\overline{Q}_{em}$'s.

There are two possible Q_{em} models of baryons. We call them interchangeable and noninterchangeable models. The noninterchangeable model says that the three Q_e's and three $\overline{Q}_m$'s in a stable baryon are organized in a particular way to minimize the total energy of the baryon. In such a way each Q_e is combined in a pair with an appropriate $\overline{Q}_m$ and forms a certain Q_{em}. Therefore, the baryon consists of three fixed Q_{em}'s, say $\mathcal{P}_1\mathcal{P}_2\mathfrak{N}_3$. An interchange of just two Q_e's or just two $\overline{Q}_m$'s will change two of the three Q_{em}'s to some other kind, say $\mathcal{P}_3$, $\mathcal{P}_2$, $\mathfrak{N}_1$, which will have higher total baryon energy. The interchange of two $\mathcal{P}$ Q_e's, in this example, will not change the total baryon energy if these two $\mathcal{P}$ Q_e's have the same z component of spin. If the two $\mathcal{P}$ Q_e's do not have the same z component of spin, the total baryon energy may change because of the interaction of the magnetic moment of the $\mathcal{P}$ Q_e and the magnetic field of the $\overline{Q}_m$. If we interchange two Q_{em}'s (equivalent to an interchange of two Q_e's plus an interchange of two corresponding $\overline{Q}_m$'s), then the total baryon energy will not change. In this case, the three Q_e's can be treated as distinguishable particles. On the other hand, if we take a rough approximation treating the three Q_e's as identical particles, then we call this the interchangeable model. In other words, by introducing quantum numbers, spin, E-isospin, and M-isospin, the interchangeable model treats the three $\overline{Q}_m$'s as identical particles and the three Q_e's as another set of identical particles; therefore, the nine Q_{em}'s are also identical particles. The baryon wave function in this interchangeable model is symmetrized in the Q_e and $\overline{Q}_m$ labels as well as in the Q_{em} label. On the other hand, the noninterchangeable model only treats the nine Q_{em}'s as identical particles with different quantum numbers. This noninterchangeable model also considers the Q_{em} as an entity and an unchangeable solid body. Therefore, the baryon wave functions are only symmetrized in the Q_{em} label, not in the Q_e or Q_m label.

In the interchangeable Q_{em} model of baryons, a baryon wave function can be written in terms of Q_e's and $\overline{Q}_m$'s as well as in terms of Q_{em}'s. The Q_e wave function must be antisymmetric and the $\overline{Q}_m$ wave function must be symmetric, because Q_e's are fermions and $\overline{Q}_m$'s are bosons. The antisymmetric Q_e wave function can be decomposed into an antisymmetric space wave function and a symmetric combined wave function of spin and E-isospin, whereas the symmetric $\overline{Q}_m$ wave function contains an antisymmetric space wave function and an antisymmetric M-isospin wave function. An interchange of two Q_{em}'s is equivalent to an interchange of two Q_e's plus an interchange of two $\overline{Q}_m$'s; therefore, the baryon wave function must be antisymmetric with respect to Q_{em}'s. This antisymmetric Q_{em} wave function can be obtained from a combined antisymmetric wave function of spin, E-isospin, and M-isospin, and a symmetric Q_{em} space wave function. The symmetric Q_{em} space wave function can be decomposed into an antisymmetric Q_e space wave function and an antisymmetric $\overline{Q}_m$ space wave function. This argument can be illustrated by the following equations:

$$\Psi_{\text{antisym}}(Q_e, \overline{Q}_m) = \Psi_{\text{antisym}}(Q_e)\Psi_{\text{sym}}(\overline{Q}_m), \tag{4.3}$$

$$\Psi_{\text{antisym}}(Q_e) = \Psi_{\text{antisym}}(Q_e, \text{space}) \times \Psi_{\text{sym}}(Q_e, \text{spin}, E\text{-isospin}), \tag{4.4}$$

$$\Psi_{\text{sym}}(\overline{Q}_m) = \Psi_{\text{antisym}}(\overline{Q}_m, \text{space}) \times \Psi_{\text{antisym}}(\overline{Q}_m, M\text{-isospin}), \tag{4.5}$$

$$\Psi_{\text{sym}}(Q_{em}, \text{space}) = \Psi_{\text{antisym}}(Q_e, \text{space}) \times \Psi_{\text{antisym}}(\overline{Q}_m, \text{space}), \tag{4.6}$$

and

$$\Psi_{\text{antisym}}(Q_{em}) = \Psi_{\text{sym}}(Q_{em}, \text{space}) \times \Psi_{\text{antisym}}(Q_{em}, \text{spin}, E\text{-isospin}, M\text{-isospin}). \tag{4.7}$$

Here the space wave function of Q_{em}'s corresponds to the space wave function of conventional quarks, which is required experimentally[12,13] to be symmetric. Let $S(Q_e)$, $A(\overline{Q}_m)$, and $A(Q_{em})$, respectively, be the Q_e symmetrizing operator, the $\overline{Q}_m$ antisymmetrizing operator, and the Q_{em} antisymmetrizing operator. It is understood that these operators also normalize the wave function. Denote the Q_e unsymmetrized combined spin and E-isospin wave function by $f(Q_e)$ and the $\overline{Q}_m$ unantisymmetrized M-isospin wave function by $g(\overline{Q}_m)$; then from Eqs. (4.4) and (4.5) we obtain

$$\Psi_{\text{sym}}(Q_e, \text{spin}, E\text{-isospin}) = S(Q_e)f(Q_e), \tag{4.8}$$

$$\Psi_{\text{antisym}}(\overline{Q}_m, M\text{-isospin}) = A(\overline{Q}_m)g(\overline{Q}_m). \tag{4.9}$$

The interchangeable Q_{em} model of baryons has to satisfy Eqs. (4.3)–(4.7), and the noninterchangeable Q_{em} model of baryons has to satisfy Eq. (4.7) only. Therefore, we obtain the wave functions for interchangeable and noninterchangeable models as

$$\Psi_{\text{antisym}}(Q_{\text{em}},\ \text{interchangeable}) = \Psi_{\text{sym}}(Q_{\text{em}},\ \text{space}) S(Q_e) f(Q_e) A(\overline{Q}_m) g(\overline{Q}_m) \tag{4.10}$$

and

$$\begin{aligned}\Psi_{\text{antisym}}(Q_{\text{em}},\ \text{noninterchangeable}) &= \Psi_{\text{sym}}(Q_{\text{em}},\ \text{space}) \\ &\quad\times \Psi_{\text{antisym}}(Q_{\text{em}},\ \text{spin},\ E\text{-isospin},\ M\text{-isospin}) \\ &= \Psi_{\text{sym}}(Q_{\text{em}},\ \text{space}) A(Q_{\text{em}})[f(Q_e) g(\overline{Q}_m)]\,.\end{aligned} \tag{4.11}$$

It is clear that Eq. (4.10) satisfies the symmetrizing requirements of Q_e's, $\overline{Q}_m$'s and Q_{em}'s whereas Eq. (4.11) only satisfies the symmetrizing requirement of Q_{em}'s. Note that the paraquark model or the conventional quark model only uses the Q_e wave function, i.e.,

$$\Psi_{\text{sym}}(\text{paraquark}) = \Psi_{\text{sym}}(\text{space}) S(Q_e) f(Q_e)\,. \tag{4.12}$$

The paraquark wave functions, $f(Q_e)$ and $S(Q_e)f(Q_e)$, of the baryon octet and decuplet are given in the literature.[23,24] The $\overline{Q}_m$ wave functions, which are the same for all the baryon octet and decuplet, are

$$g(\overline{Q}_m) = (\overline{\mathcal{P}}'\overline{\mathcal{N}}' - \overline{\mathcal{N}}'\overline{\mathcal{P}}')\overline{\lambda}' \tag{4.13}$$

and

$$A(\overline{Q}_m) g(\overline{Q}_m) = 6^{-1/2}(\overline{\mathcal{P}}'\overline{\mathcal{N}}'\overline{\lambda}' + \overline{\mathcal{N}}'\overline{\lambda}'\overline{\mathcal{P}}' + \overline{\lambda}'\overline{\mathcal{P}}'\overline{\mathcal{N}}' - \overline{\mathcal{P}}'\overline{\lambda}'\overline{\mathcal{N}}' - \overline{\mathcal{N}}'\overline{\mathcal{P}}'\overline{\lambda}' - \overline{\lambda}'\overline{\mathcal{N}}'\overline{\mathcal{P}}')\,. \tag{4.14}$$

From Eqs. (4.10), (4.12), and (4.14) we obtain

$$\Psi_{\text{antisym}}(Q_{\text{em}},\ \text{interchangeable}) = \Psi_{\text{sym}}(\text{paraquark}) 6^{-1/2}(\overline{\mathcal{P}}'\overline{\mathcal{N}}'\overline{\lambda}' + \overline{\mathcal{N}}'\overline{\lambda}'\overline{\mathcal{P}}' + \overline{\lambda}'\overline{\mathcal{P}}'\overline{\mathcal{N}}' - \overline{\mathcal{P}}'\overline{\lambda}'\overline{\mathcal{N}}' - \overline{\mathcal{N}}'\overline{\mathcal{P}}'\overline{\lambda}' - \overline{\lambda}'\overline{\mathcal{N}}'\overline{\mathcal{P}}')\,, \tag{4.15}$$

$$= \Psi_{\text{sym}}(\text{paraquark}) 6^{-1/2}\{(\overline{\mathcal{P}}'\overline{\mathcal{N}}' - \overline{\mathcal{N}}'\overline{\mathcal{P}}')\overline{\lambda}' + [(\overline{\mathcal{N}}'\overline{\lambda}' - \overline{\lambda}'\overline{\mathcal{N}}')\overline{\mathcal{P}}' - (\overline{\mathcal{P}}'\overline{\lambda}' - \overline{\lambda}'\overline{\mathcal{P}}')\overline{\mathcal{N}}']\}\,, \tag{4.16}$$

whereas from Eq. (4.11) and the literature[23,24] we obtain the proton wave function of $J_z = \frac{1}{2}$ as

$$f(Q_e,\ p(+)) = [\mathcal{P}(+)\mathcal{N}(-) - \mathcal{P}(-)\mathcal{N}(+)]\mathcal{P}(+)\,, \tag{4.17}$$

and

$$\begin{aligned}\Psi_{\text{antisym}}&(Q_{\text{em}},\ \text{spin},\ E\text{-isospin},\ M\text{-isospin},\ p(+)) \\ &= A(Q_{\text{em}})[\mathcal{P}(+)\mathcal{N}(-) - \mathcal{P}(-)\mathcal{N}(+)]\mathcal{P}(+)(\overline{\mathcal{P}}'\overline{\mathcal{N}}' - \overline{\mathcal{N}}'\overline{\mathcal{P}}')\overline{\lambda}' \\ &= A(Q_{\text{em}})[\mathcal{P}_1(+)\mathcal{N}_2(-)\mathcal{P}_3(+) - \mathcal{P}_1(-)\mathcal{N}_2(+)\mathcal{P}_3(+) - \mathcal{P}_2(+)\mathcal{N}_1(-)\mathcal{P}_3(+) + \mathcal{P}_2(-)\mathcal{N}_1(+)\mathcal{P}_3(+)] \\ &= 24^{-1/2}[\mathcal{P}_1(+)\mathcal{N}_2(-)\mathcal{P}_3(+) - \mathcal{P}_1(-)\mathcal{N}_2(+)\mathcal{P}_3(+) - \mathcal{P}_2(+)\mathcal{N}_1(-)\mathcal{P}_3(+) + \mathcal{P}_2(-)\mathcal{N}_1(+)\mathcal{P}_3(+) \\ &\quad - \mathcal{P}_1(+)\mathcal{P}_3(+)\mathcal{N}_2(-) + \mathcal{P}_1(-)\mathcal{P}_3(+)\mathcal{N}_2(+) + \mathcal{P}_2(+)\mathcal{P}_3(+)\mathcal{N}_1(-) - \mathcal{P}_2(-)\mathcal{P}_3(+)\mathcal{N}_1(+) \\ &\quad + \mathcal{N}_2(-)\mathcal{P}_3(+)\mathcal{P}_1(+) - \mathcal{N}_2(+)\mathcal{P}_3(+)\mathcal{P}_1(-) - \mathcal{N}_1(-)\mathcal{P}_3(+)\mathcal{P}_2(+) + \mathcal{N}_1(+)\mathcal{P}_3(+)\mathcal{P}_2(-) \\ &\quad - \mathcal{N}_2(-)\mathcal{P}_1(+)\mathcal{P}_3(+) + \mathcal{N}_2(+)\mathcal{P}_1(-)\mathcal{P}_3(+) + \mathcal{N}_1(-)\mathcal{P}_2(+)\mathcal{P}_3(+) - \mathcal{N}_1(+)\mathcal{P}_2(-)\mathcal{P}_3(+) \\ &\quad + \mathcal{P}_3(+)\mathcal{P}_1(+)\mathcal{N}_2(-) - \mathcal{P}_3(+)\mathcal{P}_1(-)\mathcal{N}_2(+) - \mathcal{P}_3(+)\mathcal{P}_2(+)\mathcal{N}_1(-) + \mathcal{P}_3(+)\mathcal{P}_2(-)\mathcal{N}_1(+) \\ &\quad - \mathcal{P}_3(+)\mathcal{N}_2(-)\mathcal{P}_1(+) + \mathcal{P}_3(+)\mathcal{N}_2(+)\mathcal{P}_1(-) + \mathcal{P}_3(+)\mathcal{N}_1(-)\mathcal{P}_2(+) - \mathcal{P}_3(+)\mathcal{N}_1(+)\mathcal{P}_2(-)]\,.\end{aligned} \tag{4.18}$$

Here we combine the first $\overline{Q}_m$ with the first Q_e and the second $\overline{Q}_m$ with the second Q_e, and so on. So, the Q_e and the $\overline{Q}_m$ properties and the relevant permutation symmetries will still remain in the Q_{em} after the combination.

We may rewrite the wave function in such a way that the first and second Q_{em}'s are in eigenstates of spin, electric isospin, and magnetic isospin, as

$$\begin{aligned}\Psi_{\text{antisym}}&(Q_{\text{em}},\ \text{spin},\ E\text{-isospin},\ M\text{-isospin},\ p(+)) \\ &= 24^{-1/2}\{-[\mathcal{P}_1(+)\mathcal{P}_3(+)]\mathcal{N}_2(-) + [\mathcal{P}_3(+)\mathcal{P}_1(+)]\mathcal{N}_2(-) + [\mathcal{P}_2(+)\mathcal{P}_3(+)]\mathcal{N}_1(-) - [\mathcal{P}_3(+)\mathcal{P}_2(+)]\mathcal{N}_1(-) \\ &\quad - \tfrac{1}{2}[\mathcal{N}_2(+)\mathcal{P}_3(+) + \mathcal{P}_2(+)\mathcal{N}_3(+)]\mathcal{P}_1(-) - \tfrac{1}{2}[\mathcal{N}_2(+)\mathcal{P}_3(+) - \mathcal{P}_2(+)\mathcal{N}_3(+)]\mathcal{P}_1(-)\end{aligned}$$

$$
\begin{aligned}
&+\tfrac{1}{2}[\mathcal{P}_3(+)\mathcal{N}_2(+)+\mathcal{N}_3(+)\mathcal{P}_2(+)]\mathcal{P}_1(-)+\tfrac{1}{2}[\mathcal{P}_3(+)\mathcal{N}_2(+)-\mathcal{N}_3(+)\mathcal{P}_2(+)]\mathcal{P}_1(-)\\
&+\tfrac{1}{2}[\mathcal{N}_1(+)\mathcal{P}_3(+)+\mathcal{P}_1(+)\mathcal{N}_3(+)]\mathcal{P}_2(-)+\tfrac{1}{2}[\mathcal{N}_1(+)\mathcal{P}_3(+)-\mathcal{P}_1(+)\mathcal{N}_3(+)]\mathcal{P}_2(-)\\
&-\tfrac{1}{2}[\mathcal{P}_3(+)\mathcal{N}_1(+)+\mathcal{N}_3(+)\mathcal{P}_1(+)]\mathcal{P}_2(-)-\tfrac{1}{2}[\mathcal{P}_3(+)\mathcal{N}_1(+)-\mathcal{N}_3(+)\mathcal{P}_1(+)]\mathcal{P}_2(-)\\
&+\tfrac{1}{4}[\mathcal{N}_2(-)\mathcal{P}_3(+)+\mathcal{N}_2(+)\mathcal{P}_3(-)+\mathcal{P}_2(-)\mathcal{N}_3(+)+\mathcal{P}_2(+)\mathcal{N}_3(-)]\mathcal{P}_1(+)\\
&+\tfrac{1}{4}[\mathcal{N}_2(-)\mathcal{P}_3(+)+\mathcal{N}_2(+)\mathcal{P}_3(-)-\mathcal{P}_2(-)\mathcal{N}_3(+)-\mathcal{P}_2(+)\mathcal{N}_3(-)]\mathcal{P}_1(+)\\
&+\tfrac{1}{4}[\mathcal{P}_3(-)\mathcal{N}_2(+)-\mathcal{P}_3(+)\mathcal{N}_2(-)+\mathcal{N}_3(-)\mathcal{P}_2(+)-\mathcal{N}_3(+)\mathcal{P}_2(-)]\mathcal{P}_1(+)\\
&+\tfrac{1}{4}[\mathcal{P}_3(-)\mathcal{N}_2(+)-\mathcal{P}_3(+)\mathcal{N}_2(-)-\mathcal{N}_3(-)\mathcal{P}_2(+)+\mathcal{N}_3(+)\mathcal{P}_2(-)]\mathcal{P}_1(+)\\
&-\tfrac{1}{4}[\mathcal{N}_1(-)\mathcal{P}_3(+)+\mathcal{N}_1(+)\mathcal{P}_3(-)+\mathcal{P}_1(-)\mathcal{N}_3(+)+\mathcal{P}_1(+)\mathcal{N}_3(-)]\mathcal{P}_2(+)\\
&-\tfrac{1}{4}[\mathcal{N}_1(-)\mathcal{P}_3(+)+\mathcal{N}_1(+)\mathcal{P}_3(-)-\mathcal{P}_1(-)\mathcal{N}_3(+)-\mathcal{P}_1(+)\mathcal{N}_3(-)]\mathcal{P}_2(+)\\
&-\tfrac{1}{4}[\mathcal{P}_3(-)\mathcal{N}_1(+)-\mathcal{P}_3(+)\mathcal{N}_1(-)+\mathcal{N}_3(-)\mathcal{P}_1(+)-\mathcal{N}_3(+)\mathcal{P}_1(-)]\mathcal{P}_2(+)\\
&-\tfrac{1}{4}[\mathcal{P}_3(-)\mathcal{N}_1(+)-\mathcal{P}_3(+)\mathcal{N}_1(-)-\mathcal{N}_3(-)\mathcal{P}_1(+)+\mathcal{N}_3(+)\mathcal{P}_1(-)]\mathcal{P}_2(+)\\
&+\tfrac{1}{4}[\mathcal{N}_2(-)\mathcal{P}_3(+)-\mathcal{N}_2(+)\mathcal{P}_3(-)+\mathcal{P}_2(-)\mathcal{N}_3(+)-\mathcal{P}_2(+)\mathcal{N}_3(-)]\mathcal{P}_1(+)\\
&+\tfrac{1}{4}[\mathcal{N}_2(-)\mathcal{P}_3(+)-\mathcal{N}_2(+)\mathcal{P}_3(-)-\mathcal{P}_2(-)\mathcal{N}_3(+)+\mathcal{P}_2(+)\mathcal{N}_3(-)]\mathcal{P}_1(+)\\
&-\tfrac{1}{4}[\mathcal{P}_3(-)\mathcal{N}_2(+)+\mathcal{P}_3(+)\mathcal{N}_2(-)+\mathcal{N}_3(-)\mathcal{P}_2(+)+\mathcal{N}_3(+)\mathcal{P}_2(-)]\mathcal{P}_1(+)\\
&-\tfrac{1}{4}[\mathcal{P}_3(-)\mathcal{N}_2(+)+\mathcal{P}_3(+)\mathcal{N}_2(-)-\mathcal{N}_3(-)\mathcal{P}_2(+)-\mathcal{N}_3(+)\mathcal{P}_2(-)]\mathcal{P}_1(+)\\
&-\tfrac{1}{4}[\mathcal{N}_1(-)\mathcal{P}_3(+)-\mathcal{N}_1(+)\mathcal{P}_3(-)+\mathcal{P}_1(-)\mathcal{N}_3(+)-\mathcal{P}_1(+)\mathcal{N}_3(-)]\mathcal{P}_2(+)\\
&-\tfrac{1}{4}[\mathcal{N}_1(-)\mathcal{P}_3(+)-\mathcal{N}_1(+)\mathcal{P}_3(-)-\mathcal{P}_1(-)\mathcal{N}_3(+)+\mathcal{P}_1(+)\mathcal{N}_3(-)]\mathcal{P}_2(+)\\
&+\tfrac{1}{4}[\mathcal{P}_3(-)\mathcal{N}_1(+)+\mathcal{P}_3(+)\mathcal{N}_1(-)+\mathcal{N}_3(-)\mathcal{P}_1(+)+\mathcal{N}_3(+)\mathcal{P}_1(-)]\mathcal{P}_2(+)\\
&+\tfrac{1}{4}[\mathcal{P}_3(-)\mathcal{N}_1(+)+\mathcal{P}_3(+)\mathcal{N}_1(-)-\mathcal{N}_3(-)\mathcal{P}_1(+)-\mathcal{N}_3(+)\mathcal{P}_1(-)]\mathcal{P}_2(+)\\
&+[\mathcal{P}_1(+)\mathcal{N}_2(-)-\mathcal{P}_1(-)\mathcal{N}_2(+)+\mathcal{N}_2(+)\mathcal{P}_1(-)-\mathcal{N}_2(-)\mathcal{P}_1(+)\\
&\quad-\mathcal{P}_2(+)\mathcal{N}_1(-)+\mathcal{P}_2(-)\mathcal{N}_1(+)-\mathcal{N}_1(+)\mathcal{P}_2(-)+\mathcal{N}_1(-)\mathcal{P}_2(+)]\mathcal{P}_3(+)\\
&+\tfrac{1}{2}[\mathcal{P}_1(-)\mathcal{P}_3(+)+\mathcal{P}_1(+)\mathcal{P}_3(-)]\mathcal{N}_2(+)+\tfrac{1}{2}[\mathcal{P}_3(-)\mathcal{P}_1(+)-\mathcal{P}_3(+)\mathcal{P}_1(-)]\mathcal{N}_2(+)\\
&-\tfrac{1}{2}[\mathcal{P}_2(-)\mathcal{P}_3(+)+\mathcal{P}_2(+)\mathcal{P}_3(-)]\mathcal{N}_1(+)-\tfrac{1}{2}[\mathcal{P}_3(-)\mathcal{P}_2(+)-\mathcal{P}_3(+)\mathcal{P}_2(-)]\mathcal{N}_1(+)\\
&+\tfrac{1}{2}[\mathcal{P}_1(-)\mathcal{P}_3(+)-\mathcal{P}_1(+)\mathcal{P}_3(-)]\mathcal{N}_2(+)-\tfrac{1}{2}[\mathcal{P}_3(-)\mathcal{P}_1(+)+\mathcal{P}_3(+)\mathcal{P}_1(-)]\mathcal{N}_2(+)\\
&-\tfrac{1}{2}[\mathcal{P}_2(-)\mathcal{P}_3(+)-\mathcal{P}_2(+)\mathcal{P}_3(-)]\mathcal{N}_1(+)+\tfrac{1}{2}[\mathcal{P}_3(-)\mathcal{P}_2(+)+\mathcal{P}_3(+)\mathcal{P}_2(-)]\mathcal{N}_1(+)\}\,.
\end{aligned}
\tag{4.19}
$$

The wave functions for the remaining states of the spin-$\frac{1}{2}$ octet can be obtained by spin reflection, electric isospin reflection, or electric u-spin reflection. Σ^0 and Λ states are special; however, their Q_e wave functions are well known.[23,24] They are

$$f(Q_e,\Lambda,\Sigma)=[\mathcal{P}(+)\lambda(-)-\mathcal{P}(-)\lambda(+)]\mathcal{N}(+)\pm[\mathcal{N}(+)\lambda(-)-\mathcal{N}(-)\lambda(+)]\mathcal{P}(+)\,, \tag{4.20}$$

where the upper sign gives electric isospin $I^{(e)}=1$ and the lower sign gives $I^{(e)}=0$. Therefore, we obtain

$$
\begin{aligned}
&\Psi_{\text{antisym}}(Q_{\text{em}},\ \text{spin},\ E\text{-isospin},\ M\text{-isospin},\ \Lambda(+))\\
&\quad=A(Q_{\text{em}})[\mathcal{P}(+)\mathcal{N}(-)-\mathcal{P}(-)\mathcal{N}(+)]\,\lambda(+)(\bar{\mathcal{P}}'\bar{\mathcal{N}}'-\bar{\mathcal{N}}'\bar{\mathcal{P}}')\bar{\lambda}'\\
&\quad=A(Q_{\text{em}})[\mathcal{P}_1(+)\mathcal{N}_2(-)\lambda_3(+)-\mathcal{P}_1(-)\mathcal{N}_2(+)\lambda_3(+)-\mathcal{P}_2(+)\mathcal{N}_1(-)\lambda_3(+)+\mathcal{P}_2(-)\mathcal{N}_1(+)\lambda_3(+)]\\
&\quad=24^{-1/2}\{[\mathcal{P}_1(+)\mathcal{N}_2(-)\lambda_3(+)-\mathcal{P}_1(-)\mathcal{N}_2(+)\lambda_3(+)]-[\mathcal{P}_2(+)\mathcal{N}_1(-)\lambda_3(+)-\mathcal{P}_2(-)\mathcal{N}_1(+)\lambda_3(+)]\\
&\qquad-[\mathcal{P}_1(+)\lambda_3(+)\mathcal{N}_2(-)-\mathcal{P}_1(-)\lambda_3(+)\mathcal{N}_2(+)]+[\mathcal{P}_2(+)\lambda_3(+)\mathcal{N}_1(-)-\mathcal{P}_2(-)\lambda_3(+)\mathcal{N}_1(+)]\\
&\qquad+[\mathcal{N}_2(-)\lambda_3(+)\mathcal{P}_1(+)-\mathcal{N}_2(+)\lambda_3(+)\mathcal{P}_1(-)]-[\mathcal{N}_1(-)\lambda_3(+)\mathcal{P}_2(+)-\mathcal{N}_1(+)\lambda_3(+)\mathcal{P}_2(-)]\\
&\qquad-[\mathcal{N}_2(-)\mathcal{P}_1(+)\lambda_3(+)-\mathcal{N}_2(+)\mathcal{P}_1(-)\lambda_3(+)]+[\mathcal{N}_1(-)\mathcal{P}_2(+)\lambda_3(+)-\mathcal{N}_1(+)\mathcal{P}_2(-)\lambda_3(+)]
\end{aligned}
$$

$$\begin{aligned}
&+[\lambda_3(+)\mathcal{P}_1(+)\mathcal{N}_2(-) - \lambda_3(+)\mathcal{P}_1(-)\mathcal{N}_2(+)] - [\lambda_3(+)\mathcal{P}_2(+)\mathcal{N}_1(-) - \lambda_3(+)\mathcal{P}_2(-)\mathcal{N}_1(+)]\\
&-[\lambda_3(+)\mathcal{N}_2(-)\mathcal{P}_1(+) - \lambda_3(+)\mathcal{N}_2(+)\mathcal{P}_1(-)] + [\lambda_3(+)\mathcal{N}_1(-)\mathcal{P}_2(+) - \lambda_3(+)\mathcal{N}_1(+)\mathcal{P}_2(+)]\}\\
= 24^{-1/2}\{&[\mathcal{P}_1(+)\mathcal{N}_2(-) - \mathcal{P}_1(-)\mathcal{N}_2(+) - \mathcal{N}_2(-)\mathcal{P}_1(+) + \mathcal{N}_2(+)\mathcal{P}_1(-) - \mathcal{P}_2(+)\mathcal{N}_1(-)\\
&+\mathcal{P}_2(-)\mathcal{N}_1(+) + \mathcal{N}_1(-)\mathcal{P}_2(+) - \mathcal{N}_1(+)\mathcal{P}_2(-)]\lambda_3(+) + [\lambda_3(+)\mathcal{P}_1(+)]\mathcal{N}_2(-)\\
&-[\lambda_3(+)\mathcal{P}_2(+)]\mathcal{N}_1(-) + [\lambda_3(+)\mathcal{N}_2(+)]\mathcal{P}_1(-) - [\lambda_3(+)\mathcal{N}_1(+)]\mathcal{P}_2(-)\\
&-[\mathcal{N}_2(+)\lambda_3(+)]\mathcal{P}_1(-) + [\mathcal{N}_1(+)\lambda_3(+)]\mathcal{P}_2(-) - [\mathcal{P}_1(+)\lambda_3(+)]\mathcal{N}_2(-)\\
&+[\mathcal{P}_2(+)\lambda_3(+)]\mathcal{N}_1(-) + \tfrac{1}{2}[\mathcal{N}_2(-)\lambda_3(+) + \mathcal{N}_2(+)\lambda_3(-)]\mathcal{P}_1(+)\\
&-\tfrac{1}{2}[\mathcal{N}_1(-)\lambda_3(+) + \mathcal{N}_1(+)\lambda_3(-)]\mathcal{P}_2(+) + \tfrac{1}{2}[\mathcal{N}_2(-)\lambda_3(+) - \mathcal{N}_2(+)\lambda_3(-)]\mathcal{P}_1(+)\\
&-\tfrac{1}{2}[\mathcal{N}_1(-)\lambda_3(+) - \mathcal{N}_1(+)\lambda_3(-)]\mathcal{P}_2(+) - \tfrac{1}{2}[\lambda_3(+)\mathcal{N}_2(-) + \lambda_3(-)\mathcal{N}_2(+)]\mathcal{P}_1(+)\\
&+\tfrac{1}{2}[\lambda_3(+)\mathcal{N}_1(-) + \lambda_3(-)\mathcal{N}_1(+)]\mathcal{P}_2(+) - \tfrac{1}{2}[\lambda_3(+)\mathcal{N}_2(-) - \lambda_3(-)\mathcal{N}_2(+)]\mathcal{P}_1(+)\\
&+\tfrac{1}{2}[\lambda_3(+)\mathcal{N}_1(-) - \lambda_3(-)\mathcal{N}_1(+)]\mathcal{P}_2(+) - \tfrac{1}{2}[\lambda_3(+)\mathcal{P}_1(-) + \lambda_3(-)\mathcal{P}_1(+)]\mathcal{N}_2(+)\\
&+\tfrac{1}{2}[\lambda_3(+)\mathcal{P}_2(-) + \lambda_3(-)\mathcal{P}_2(+)]\mathcal{N}_1(+) - \tfrac{1}{2}[\lambda_3(+)\mathcal{P}_1(-) - \lambda_3(-)\mathcal{P}_1(+)]\mathcal{N}_2(+)\\
&+\tfrac{1}{2}[\lambda_3(+)\mathcal{P}_2(-) - \lambda_3(-)\mathcal{P}_2(+)]\mathcal{N}_1(+) + \tfrac{1}{2}[\mathcal{P}_1(-)\lambda_3(+) + \mathcal{P}_1(+)\lambda_3(-)]\mathcal{N}_2(+)\\
&-\tfrac{1}{2}[\mathcal{P}_2(-)\lambda_3(+) + \mathcal{P}_2(+)\lambda_3(-)]\mathcal{N}_1(+) + \tfrac{1}{2}[\mathcal{P}_1(-)\lambda_3(+) - \mathcal{P}_1(+)\lambda_3(-)]\mathcal{N}_2(+)\\
&-\tfrac{1}{2}[\mathcal{P}_2(-)\lambda_3(+) - \mathcal{P}_2(+)\lambda_3(-)]\mathcal{N}_1(+)\} \qquad (4.21)
\end{aligned}$$

and

$\Psi_{\text{antisym}}(Q_{\text{em}}$, spin, E-isospin, M-isospin, $\Sigma^0(+))$

$$\begin{aligned}
= 72^{-1/2}[&2\mathcal{P}_1(+)\mathcal{N}_2(+)\lambda_3(-) - \mathcal{P}_1(+)\mathcal{N}_2(-)\lambda_3(+) - \mathcal{P}_1(-)\mathcal{N}_2(+)\lambda_3(+)\\
&-2\mathcal{P}_1(+)\lambda_3(-)\mathcal{N}_2(+) + \mathcal{P}_1(+)\lambda_3(+)\mathcal{N}_2(-) + \mathcal{P}_1(-)\lambda_3(+)\mathcal{N}_2(+)\\
&+2\mathcal{N}_2(+)\lambda_3(-)\mathcal{P}_1(+) - \mathcal{N}_2(-)\lambda_3(+)\mathcal{P}_1(+) - \mathcal{N}_2(+)\lambda_3(+)\mathcal{P}_1(-)\\
&-2\mathcal{N}_2(+)\mathcal{P}_1(+)\lambda_3(-) + \mathcal{N}_2(-)\mathcal{P}_1(+)\lambda_3(+) + \mathcal{N}_2(+)\mathcal{P}_1(-)\lambda_3(+)\\
&+2\lambda_3(-)\mathcal{P}_1(+)\mathcal{N}_2(+) - \lambda_3(+)\mathcal{P}_1(+)\mathcal{N}_2(-) - \lambda_3(+)\mathcal{P}_1(-)\mathcal{N}_2(+)\\
&-2\lambda_3(-)\mathcal{N}_2(+)\mathcal{P}_1(+) + \lambda_3(+)\mathcal{N}_2(-)\mathcal{P}_1(+) + \lambda_3(+)\mathcal{N}_2(+)\mathcal{P}_1(-)\\
&-2\mathcal{P}_2(+)\mathcal{N}_1(+)\lambda_3(-) + \mathcal{P}_2(+)\mathcal{N}_1(-)\lambda_3(+) + \mathcal{P}_2(-)\mathcal{N}_1(+)\lambda_3(+)\\
&+2\mathcal{P}_2(+)\lambda_3(-)\mathcal{N}_1(+) - \mathcal{P}_2(+)\lambda_3(+)\mathcal{N}_1(-) - \mathcal{P}_2(-)\lambda_3(+)\mathcal{N}_1(+)\\
&-2\mathcal{N}_1(+)\lambda_3(-)\mathcal{P}_2(+) + \mathcal{N}_1(-)\lambda_3(+)\mathcal{P}_2(+) + \mathcal{N}_1(+)\lambda_3(+)\mathcal{P}_2(-)\\
&+2\mathcal{N}_1(+)\mathcal{P}_2(+)\lambda_3(-) - \mathcal{N}_1(-)\mathcal{P}_2(+)\lambda_3(+) - \mathcal{N}_1(+)\mathcal{P}_2(-)\lambda_3(-)\\
&-2\lambda_3(-)\mathcal{P}_2(+)\mathcal{N}_1(+) + \lambda_3(+)\mathcal{P}_2(+)\mathcal{N}_1(-) + \lambda_3(+)\mathcal{P}_2(-)\mathcal{N}_1(+)\\
&+2\lambda_3(-)\mathcal{N}_1(+)\mathcal{P}_2(+) - \lambda_3(+)\mathcal{N}_1(-)\mathcal{P}_2(+) - \lambda_3(+)\mathcal{N}_1(+)\mathcal{P}_2(-)]\\
= 72^{-1/2}\{&2[\mathcal{P}_1(+)\mathcal{N}_2(+) - \mathcal{P}_2(+)\mathcal{N}_1(+) - \mathcal{N}_2(+)\mathcal{P}_1(+) + \mathcal{N}_1(+)\mathcal{P}_2(+)]\lambda_3(-)\\
&-2[\mathcal{P}_1(+)\lambda_3(-) - \lambda_3(-)\mathcal{P}_1(+) - \mathcal{P}_1(-)\lambda_3(+) + \lambda_3(+)\mathcal{P}_1(-)]\mathcal{N}_2(+)\\
&-\tfrac{1}{2}[-\mathcal{P}_1(+)\lambda_3(-) + \lambda_3(-)\mathcal{P}_1(+) + \mathcal{P}_1(-)\lambda_3(+) - \lambda_3(+)\mathcal{P}_1(-)]\mathcal{N}_2(+)\\
&-\tfrac{1}{2}[\mathcal{P}_1(+)\lambda_3(-) - \lambda_3(-)\mathcal{P}_1(+) + \mathcal{P}_1(-)\lambda_3(+) - \lambda_3(+)\mathcal{P}_1(-)]\mathcal{N}_2(+)\\
&+2[\mathcal{P}_2(+)\lambda_3(-) - \lambda_3(-)\mathcal{P}_2(+) - \mathcal{P}_2(-)\lambda_3(+) + \lambda_3(+)\mathcal{P}_2(-)]\mathcal{N}_1(+)\\
&+\tfrac{1}{2}[-\mathcal{P}_2(+)\lambda_3(-) + \lambda_3(-)\mathcal{P}_2(+) + \mathcal{P}_2(-)\lambda_3(+) - \lambda_3(+)\mathcal{P}_2(-)]\mathcal{N}_1(+)\\
&+\tfrac{1}{2}[\mathcal{P}_2(+)\lambda_3(-) - \lambda_3(-)\mathcal{P}_2(+) + \mathcal{P}_2(-)\lambda_3(+) - \lambda_3(+)\mathcal{P}_2(-)]\mathcal{N}_1(+)\\
&-2[\mathcal{N}_1(+)\lambda_3(-) - \lambda_3(-)\mathcal{N}_1(+) - \mathcal{N}_1(-)\lambda_3(+) + \lambda_3(+)\mathcal{N}_1(-)]\mathcal{P}_2(+)\\
&-\tfrac{1}{2}[-\mathcal{N}_1(+)\lambda_3(-) + \lambda_3(-)\mathcal{N}_1(+) - \mathcal{N}_1(-)\lambda_3(+) + \lambda_3(+)\mathcal{N}_1(-)]\mathcal{P}_2(+)
\end{aligned}$$

$$
\begin{aligned}
&-\tfrac{1}{2}[\mathcal{N}_1(+)\lambda_3(-)-\lambda_3(-)\mathcal{N}_1(+)-\mathcal{N}_1(-)\lambda_3(+)+\lambda_3(+)\mathcal{N}_1(-)]\mathcal{P}_2(+)\\
&+2[\mathcal{N}_2(+)\lambda_3(-)-\lambda_3(-)\mathcal{N}_2(+)-\mathcal{N}_2(-)\lambda_3(+)+\lambda_3(+)\mathcal{N}_2(-)]\mathcal{P}_1(+)\\
&+\tfrac{1}{2}[-\mathcal{N}_2(+)\lambda_3(-)+\lambda_3(-)\mathcal{N}_2(+)-\mathcal{N}_2(-)\lambda_3(+)+\lambda_3(+)\mathcal{N}_2(-)]\mathcal{P}_1(+)\\
&+\tfrac{1}{2}[\mathcal{N}_2(+)\lambda_3(-)-\lambda_3(-)\mathcal{N}_2(+)-\mathcal{N}_2(-)\lambda_3(+)+\lambda_3(+)\mathcal{N}_2(-)]\mathcal{P}_1(+)\\
&+[-\mathcal{P}_1(+)\mathcal{N}_2(-)-\mathcal{P}_1(-)\mathcal{N}_2(+)+\mathcal{P}_2(+)\mathcal{N}_1(-)+\mathcal{P}_2(-)\mathcal{N}_1(+)+\mathcal{N}_2(-)\mathcal{P}_1(+)\\
&\quad+\mathcal{N}_2(+)\mathcal{P}_1(-)-\mathcal{N}_1(-)\mathcal{P}_2(+)-\mathcal{N}_1(+)\mathcal{P}_2(-)]\lambda_3(+)\\
&+[\mathcal{P}_1(+)\lambda_3(+)]\mathcal{N}_2(-)-[\lambda_3(+)\mathcal{P}_1(+)]\mathcal{N}_2(-)-[\mathcal{P}_2(+)\lambda_3(+)]\mathcal{N}_1(-)\\
&+[\lambda_3(+)\mathcal{P}_2(+)]\mathcal{N}_1(-)-[\mathcal{N}_2(+)\lambda_3(+)]\mathcal{P}_1(-)+[\lambda_3(+)\mathcal{N}_2(+)]\mathcal{P}_1(-)\\
&+[\mathcal{N}_1(+)\lambda_3(+)]\mathcal{P}_2(-)-[\lambda_3(+)\mathcal{N}_1(+)]\mathcal{P}_2(-)\}\,. \qquad (4.22)
\end{aligned}
$$

We may extend this work to the baryon decuplet. The standard Q_e wave functions for the 40 baryon decuplet states are found in the literature,[23,24] from which we can construct the wave functions in the Q_{em} model of baryons.

V. MASS RELATIONS

In the three-Q_{em} models, the internal contribution to the baryon mass is mainly the following: one-Q_{em} effect, pairing effect, and three-Q_{em} effect. However, since the three-Q_{em} effect is less important, we will neglect it. There are three Q_{em}'s in a baryon, and hence for each Q_{em} configuration there are three different pairing interactions which must be summed. A baryon wave function is expressed in terms of linear combinations of distinct Q_{em} configurations, and we must sum over those as well. Since the baryon wave function is antisymmetrized with respect to the three constituents, the Q_{em}'s, it is sufficient to calculate the pairing energy due to the interactions of the first and the second Q_{em}'s. If we multiply the result by three, then we obtain the total interaction energy. Therefore, assuming the conservation of spin, E-isospin, and M-isospin, from Eq. (4.19) we obtain, by inspection, the proton mass as

$$
\begin{aligned}
p=\;&\tfrac{1}{2}[m(\mathcal{P}_1)+m(\mathcal{N}_2)+m(\mathcal{P}_3)+m(\mathcal{P}_2)+m(\mathcal{N}_1)+m(\mathcal{P}_3)]\\
&+3\times\tfrac{1}{24}[V(\mathcal{P}_1,\mathcal{P}_3,1,1,\tfrac{1}{2})+V(\mathcal{P}_3,\mathcal{P}_1,1,1,\tfrac{1}{2})\\
&\quad+V(\mathcal{P}_2,\mathcal{P}_3,1,1,\tfrac{1}{2})+V(\mathcal{P}_3,\mathcal{P}_2,1,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{4}V(\mathcal{N}_2,\mathcal{P}_3,1,1,\tfrac{1}{2})+\tfrac{1}{4}V(\mathcal{P}_2,\mathcal{N}_3,1,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{4}V(\mathcal{N}_2,\mathcal{P}_3,1,0,\tfrac{1}{2})+\tfrac{1}{4}V(\mathcal{P}_2,\mathcal{N}_3,1,0,\tfrac{1}{2})\\
&\quad+\tfrac{1}{4}V(\mathcal{P}_3,\mathcal{N}_2,1,1,\tfrac{1}{2})+\tfrac{1}{4}V(\mathcal{N}_3,\mathcal{P}_2,1,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{4}V(\mathcal{P}_3,\mathcal{N}_2,1,0,\tfrac{1}{2})+\tfrac{1}{4}V(\mathcal{N}_3,\mathcal{P}_2,1,0,\tfrac{1}{2})\\
&\quad+\tfrac{1}{4}V(\mathcal{N}_1,\mathcal{P}_3,1,1,\tfrac{1}{2})+\tfrac{1}{4}V(\mathcal{P}_1,\mathcal{N}_3,1,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{4}V(\mathcal{N}_1,\mathcal{P}_3,1,0,\tfrac{1}{2})+\tfrac{1}{4}V(\mathcal{P}_1,\mathcal{N}_3,1,0,\tfrac{1}{2})\\
&\quad+\tfrac{1}{4}V(\mathcal{P}_3,\mathcal{N}_1,1,1,\tfrac{1}{2})+\tfrac{1}{4}V(\mathcal{N}_3,\mathcal{P}_1,1,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{4}V(\mathcal{P}_3,\mathcal{N}_1,1,0,\tfrac{1}{2})+\tfrac{1}{4}V(\mathcal{N}_3,\mathcal{P}_1,1,0,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{N}_2,\mathcal{P}_3,1,1,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{P}_2,\mathcal{N}_3,1,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{N}_2,\mathcal{P}_3,1,0,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{P}_2,\mathcal{N}_3,1,0,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{P}_3,\mathcal{N}_2,0,1,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{N}_3,\mathcal{P}_2,0,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{P}_3,\mathcal{N}_2,0,0,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{N}_3,\mathcal{P}_2,0,0,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{N}_1,\mathcal{P}_3,1,1,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{P}_1,\mathcal{N}_3,1,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{N}_1,\mathcal{P}_3,1,0,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{P}_1,\mathcal{N}_3,1,0,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{P}_3,\mathcal{N}_1,0,1,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{N}_3,\mathcal{P}_1,0,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{P}_3,\mathcal{N}_1,0,0,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{N}_3,\mathcal{P}_1,0,0,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{N}_2,\mathcal{P}_3,0,1,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{P}_2,\mathcal{N}_3,0,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{N}_2,\mathcal{P}_3,0,0,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{P}_2,\mathcal{N}_3,0,0,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{P}_3,\mathcal{N}_2,1,1,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{N}_3,\mathcal{P}_2,1,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{P}_3,\mathcal{N}_2,1,0,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{N}_3,\mathcal{P}_2,1,0,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{N}_1,\mathcal{P}_3,0,1,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{P}_1,\mathcal{N}_3,0,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{N}_1,\mathcal{P}_3,0,0,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{P}_1,\mathcal{N}_3,0,0,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{P}_3,\mathcal{N}_1,1,1,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{N}_3,\mathcal{P}_1,1,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{8}V(\mathcal{P}_3,\mathcal{N}_1,1,0,\tfrac{1}{2})+\tfrac{1}{8}V(\mathcal{N}_3,\mathcal{P}_1,1,0,\tfrac{1}{2})\\
&\quad+4V(\mathcal{P}_1,\mathcal{N}_2,0,0,0)+4V(\mathcal{P}_2,\mathcal{N}_1,0,0,0)\\
&\quad+\tfrac{1}{4}V(\mathcal{P}_1,\mathcal{P}_3,1,1,\tfrac{1}{2})+\tfrac{1}{2}V(\mathcal{P}_3,\mathcal{P}_1,0,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{2}V(\mathcal{P}_2,\mathcal{P}_3,1,1,\tfrac{1}{2})+\tfrac{1}{2}V(\mathcal{P}_3,\mathcal{P}_2,0,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{2}V(\mathcal{P}_1,\mathcal{P}_3,0,1,\tfrac{1}{2})+\tfrac{1}{2}V(\mathcal{P}_3,\mathcal{P}_1,1,1,\tfrac{1}{2})\\
&\quad+\tfrac{1}{2}V(\mathcal{P}_2,\mathcal{P}_3,0,1,\tfrac{1}{2})+\tfrac{1}{2}V(\mathcal{P}_3,\mathcal{P}_2,1,1,\tfrac{1}{2})]\,, \qquad (5.1)
\end{aligned}
$$

where $V(\mathcal{P}_1, \mathcal{P}_2, 1, 1, 0)$ represents the pairing energy between $\mathcal{P}_1$ and $\mathcal{P}_2$ Q_{em}'s, and the third, fourth, and fifth indices refer, respectively, to the spin, electric isospin, and magnetic isospin of the two-Q_{em} system.

By the first approximation each Q_{em} mass and Q_{em} pairing energy of Eq. (5.1) can be decomposed into three and four terms, respectively. For example,

$$m(\mathcal{P}_1) = m(\mathcal{P}, \bar{\mathcal{P}}') = m(\mathcal{P}) + m(\bar{\mathcal{P}}') + V(\mathcal{P}, \bar{\mathcal{P}}') \tag{5.2}$$

and

$$\begin{aligned} V(\mathcal{P}_1, \mathcal{N}_2) &= V(\mathcal{P}\bar{\mathcal{P}}', \mathcal{N}\bar{\mathcal{N}}') \\ &= V(\mathcal{P}, \mathcal{N}) + V(\mathcal{P}, \bar{\mathcal{N}}') + V(\bar{\mathcal{P}}', \mathcal{N}) + V(\bar{\mathcal{P}}', \bar{\mathcal{N}}')\,. \end{aligned} \tag{5.3}$$

From Eqs. (5.2) and (5.3) we obtain

$$V(\mathcal{P}_1, \mathcal{N}_2) = V(\mathcal{N}_2, \mathcal{P}_1) = V(\mathcal{P}_2, \mathcal{N}_1) = V(\mathcal{N}_1, \mathcal{P}_2)\,. \tag{5.4}$$

Let

$$m'(q) = m(q) + V(q, \bar{\mathcal{P}}') + V(q, \bar{\mathcal{N}}') + V(q, \bar{\lambda}') \tag{5.5}$$

and

$$\begin{aligned} A = {} & m(\bar{\mathcal{P}}') + m(\bar{\mathcal{N}}') + m(\bar{\lambda}') + V(\bar{\mathcal{P}}', \bar{\mathcal{N}}') \\ & + V(\bar{\mathcal{N}}', \bar{\lambda}') + V(\bar{\lambda}', \bar{\mathcal{P}}'), \end{aligned} \tag{5.6}$$

and, assuming that the pairing energies are independent of electric isospin and magnetic isospin, we obtain

$$\begin{aligned} p \approx {} & 2m'(\mathcal{P}) + m'(\mathcal{N}) \\ & + \tfrac{1}{8}[2V(\mathcal{P}_1, \mathcal{P}_3, 1) + 2V(\mathcal{P}_2, \mathcal{P}_3, 1) \\ & + 2V(\mathcal{P}_3, \mathcal{N}_2, 1) + 2V(\mathcal{P}_3, \mathcal{N}_1, 1) \\ & + V(\mathcal{P}_3, \mathcal{N}_2, 1) + V(\mathcal{P}_3, \mathcal{N}_1, 1) + V(\mathcal{P}_3, \mathcal{N}_2, 0) \\ & + V(\mathcal{P}_3, \mathcal{N}_1, 0) + 4V(\mathcal{P}_1, \mathcal{N}_2, 0) + 4V(\mathcal{P}_2, \mathcal{N}_1, 0) \\ & + V(\mathcal{P}_1, \mathcal{P}_3, 1) + V(\mathcal{P}_2, \mathcal{P}_3, 1) \\ & + V(\mathcal{P}_1, \mathcal{P}_3, 0) + V(\mathcal{P}_2, \mathcal{P}_3, 0)] \\ = {} & 2m'(\mathcal{P}) + m'(\mathcal{N}) + \tfrac{3}{4}V(\mathcal{P}, \mathcal{P}, 1) + \tfrac{3}{4}V(\mathcal{P}, \mathcal{N}, 1) \\ & + \tfrac{1}{4}V(\mathcal{P}, \mathcal{P}, 0) + \tfrac{5}{4}V(\mathcal{P}, \mathcal{N}, 0) + A\,. \end{aligned} \tag{5.7}$$

By the same procedure, from Eqs. (4.21) and (4.22) we obtain

$$\begin{aligned} \Lambda = {} & \tfrac{1}{2}[m(\mathcal{P}_1) + m(\mathcal{N}_2) + m(\lambda_3) + m(\mathcal{P}_2) + m(\mathcal{N}_1) + m(\lambda_3)] \\ & + 3 \times \tfrac{1}{24}[4V(\mathcal{P}_1, \mathcal{N}_2, 0, 0) + 4V(\mathcal{P}_2, \mathcal{N}_1, 0, 0, 0) \\ & + V(\mathcal{P}_1, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) + V(\mathcal{P}_2, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + V(\mathcal{N}_2, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) + V(\mathcal{N}_1, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + V(\mathcal{N}_2, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) + V(\mathcal{N}_1, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + V(\mathcal{P}_1, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) + V(\mathcal{P}_2, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + \tfrac{1}{2}V(\mathcal{N}_2, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) + \tfrac{1}{2}V(\mathcal{N}_1, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + \tfrac{1}{2}V(\mathcal{N}_2, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2}) + \tfrac{1}{2}V(\mathcal{N}_1, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + \tfrac{1}{2}V(\mathcal{N}_2, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) + \tfrac{1}{2}V(\mathcal{N}_1, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + \tfrac{1}{2}V(\mathcal{N}_2, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2}) + \tfrac{1}{2}V(\mathcal{N}_1, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + \tfrac{1}{2}V(\mathcal{P}_1, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) + \tfrac{1}{2}V(\mathcal{P}_2, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + \tfrac{1}{2}V(\mathcal{P}_1, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2}) + \tfrac{1}{2}V(\mathcal{P}_2, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + \tfrac{1}{2}V(\mathcal{P}_1, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) + \tfrac{1}{2}V(\mathcal{P}_2, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + \tfrac{1}{2}V(\mathcal{P}_1, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2}) + \tfrac{1}{2}V(\mathcal{P}_2, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2})] \\ \approx {} & m'(\mathcal{P}) + m'(\mathcal{N}) + m'(\lambda) + \tfrac{3}{4}V(\mathcal{N}, \lambda, 1) + \tfrac{3}{4}V(\lambda, \mathcal{P}, 1) \\ & + V(\mathcal{P}, \mathcal{N}, 0) + \tfrac{1}{4}V(\mathcal{N}, \lambda, 0) + \tfrac{1}{4}V(\lambda, \mathcal{P}, 0) + A \end{aligned} \tag{5.8}$$

and

$$\begin{aligned} \Sigma^0 = {} & \tfrac{1}{2}[m(\mathcal{P}_1) + m(\mathcal{N}_2) + m(\lambda_3) + m(\mathcal{P}_2) + m(\mathcal{N}_1) + m(\lambda_3)] \\ & + \tfrac{3}{72} \times 4[2V(\mathcal{P}_1, \mathcal{N}_2, 1, 1, 0) + 2V(\mathcal{P}_2, \mathcal{N}_1, 1, 1, 0) \\ & + 8V(\mathcal{P}_1, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2}) + V(\mathcal{P}_1, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + V(\mathcal{P}_1, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) + 8V(\mathcal{P}_2, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + V(\mathcal{P}_2, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2}) + V(\mathcal{P}_2, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + 8V(\mathcal{N}_1, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2}) + V(\mathcal{N}_1, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + V(\mathcal{N}_1, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) + 8V(\mathcal{N}_2, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + V(\mathcal{N}_2, \lambda_3, 0, \tfrac{1}{2}, \tfrac{1}{2}) + V(\mathcal{N}_2, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + 4V(\mathcal{P}_1, \mathcal{N}_2, 1, 1, 0) + 4V(\mathcal{P}_2, \mathcal{N}_1, 1, 1, 0) \\ & + V(\mathcal{P}_1, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) + V(\lambda_3, \mathcal{P}_1, 1, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + V(\mathcal{P}_2, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) + V(\lambda_3, \mathcal{P}_2, 1, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + V(\mathcal{N}_2, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) + V(\lambda_3, \mathcal{N}_2, 1, \tfrac{1}{2}, \tfrac{1}{2}) \\ & + V(\mathcal{N}_1, \lambda_3, 1, \tfrac{1}{2}, \tfrac{1}{2}) + V(\lambda_3, \mathcal{N}_1, 1, \tfrac{1}{2}, \tfrac{1}{2})] \\ \approx {} & m'(\mathcal{P}) + m'(\mathcal{N}) + m'(\lambda) + V(\mathcal{P}, \mathcal{N}, 1) + \tfrac{1}{4}V(\mathcal{P}, \lambda, 1) \\ & + \tfrac{1}{4}V(\mathcal{N}, \lambda, 1) + \tfrac{3}{4}V(\mathcal{P}, \lambda, 0) + \tfrac{3}{4}V(\mathcal{N}, \lambda, 0) + A\,. \end{aligned} \tag{5.9}$$

Therefore, for the noninterchangeable Q_{em} model we obtain the baryon mass expressions as

$$\begin{aligned} p = {} & 2m'(\mathcal{P}) + m'(\mathcal{N}) + \tfrac{3}{4}V(\mathcal{P}, \mathcal{P}, 1) + \tfrac{3}{4}V(\mathcal{P}, \mathcal{N}, 1) \\ & + \tfrac{1}{4}V(\mathcal{P}, \mathcal{P}, 0) + \tfrac{5}{4}V(\mathcal{P}, \mathcal{N}, 0) + A, \end{aligned} \tag{5.10}$$

$$\begin{aligned} n = {} & m'(\mathcal{P}) + 2m'(\mathcal{N}) + \tfrac{3}{4}V(\mathcal{N}, \mathcal{N}, 1) + \tfrac{3}{4}V(\mathcal{P}, \mathcal{N}, 1) \\ & + \tfrac{1}{4}V(\mathcal{N}, \mathcal{N}, 0) + \tfrac{5}{4}V(\mathcal{P}, \mathcal{N}, 0) + A, \end{aligned} \tag{5.11}$$

$$\begin{aligned} \Lambda = {} & m'(\mathcal{P}) + m'(\mathcal{N}) + m'(\lambda) + \tfrac{3}{4}V(\mathcal{N}, \lambda, 1) + \tfrac{3}{4}V(\lambda, \mathcal{P}, 1) \\ & + V(\mathcal{P}, \mathcal{N}, 0) + \tfrac{1}{4}V(\mathcal{N}, \lambda, 0) + \tfrac{1}{4}V(\lambda, \mathcal{P}, 0) + A, \end{aligned} \tag{5.12}$$

$$\begin{aligned} \Sigma^+ = {} & 2m'(\mathcal{P}) + m'(\lambda) + \tfrac{3}{4}V(\mathcal{P}, \mathcal{P}, 1) + \tfrac{3}{4}V(\lambda, \mathcal{P}, 1) \\ & + \tfrac{1}{4}V(\mathcal{P}, \mathcal{P}, 0) + \tfrac{5}{4}V(\mathcal{P}, \lambda, 0) + A, \end{aligned} \tag{5.13}$$

$$\begin{aligned} \Sigma^0 = {} & m'(\mathcal{P}) + m'(\mathcal{N}) + m'(\lambda) + V(\mathcal{P}, \mathcal{N}, 1) + \tfrac{1}{4}V(\mathcal{N}, \lambda, 1) \\ & + \tfrac{1}{4}V(\lambda, \mathcal{P}, 1) + \tfrac{3}{4}V(\mathcal{N}, \lambda, 0) + \tfrac{3}{4}V(\lambda, \mathcal{P}, 0) + A, \end{aligned} \tag{5.14}$$

$$\begin{aligned} \Sigma^- = {} & 2m'(\mathcal{N}) + m'(\lambda) + \tfrac{3}{4}V(\mathcal{N}, \mathcal{N}, 1) + \tfrac{3}{4}V(\mathcal{N}, \lambda, 1) \\ & + \tfrac{1}{4}V(\mathcal{N}, \mathcal{N}, 0) + \tfrac{5}{4}V(\mathcal{N}, \lambda, 0) + A, \end{aligned} \tag{5.15}$$

$$\Xi^0 = m'(\mathcal{P}) + 2m'(\lambda) + \tfrac{3}{4}V(\lambda, \lambda, 1) + \tfrac{3}{4}V(\lambda, \mathcal{P}, 1) + \tfrac{1}{4}V(\lambda, \lambda, 0) + \tfrac{5}{4}V(\lambda, \mathcal{P}, 0) + A, \quad (5.16)$$

$$\Xi^- = m'(\mathcal{N}) + 2m'(\lambda) + \tfrac{3}{4}V(\lambda, \lambda, 1) + \tfrac{3}{4}V(\lambda, \mathcal{N}, 1) + \tfrac{1}{4}V(\lambda, \lambda, 0) + \tfrac{5}{4}V(\mathcal{N}, \lambda, 0) + A, \quad (5.17)$$

$$\Delta^{*++} = 3m'(\mathcal{P}) + 3V(\mathcal{P}, \mathcal{P}, 1) + A, \quad (5.18)$$

$$\Delta^{*+} = 2m'(\mathcal{P}) + m'(\mathcal{N}) + V(\mathcal{P}, \mathcal{P}, 1) + 2V(\mathcal{P}, \mathcal{N}, 1) + A, \quad (5.19)$$

$$\Delta^{*0} = m'(\mathcal{P}) + 2m'(\mathcal{N}) + V(\mathcal{N}, \mathcal{N}, 1) + 2V(\mathcal{P}, \mathcal{N}, 1) + A, \quad (5.20)$$

$$\Delta^{*-} = 3m'(\mathcal{N}) + 3V(\mathcal{N}, \mathcal{N}, 1) + A, \quad (5.21)$$

$$\Sigma^{*+} = 2m'(\mathcal{P}) + m'(\lambda) + V(\mathcal{P}, \mathcal{P}, 1) + 2V(\lambda, \mathcal{P}, 1) + A, \quad (5.22)$$

$$\Sigma^{*0} = m'(\mathcal{P}) + m'(\mathcal{N}) + m'(\lambda) + V(\mathcal{P}, \mathcal{N}, 1) + V(\mathcal{N}, \lambda, 1) + V(\lambda, \mathcal{P}, 1) + A, \quad (5.23)$$

$$\Sigma^{*-} = 2m'(\mathcal{N}) + m'(\lambda) + V(\mathcal{N}, \mathcal{N}, 1) + 2V(\mathcal{N}, \lambda, 1) + A, \quad (5.24)$$

$$\Xi^{*0} = m'(\mathcal{P}) + 2m'(\lambda) + V(\lambda, \lambda, 1) + 2V(\lambda, \mathcal{P}, 1) + A, \quad (5.25)$$

$$\Xi^{*-} = m'(\mathcal{N}) + 2m'(\lambda) + V(\lambda, \lambda, 1) + 2V(\mathcal{N}, \lambda, 1) + A, \quad (5.26)$$

$$\Omega^- = 3m'(\lambda) + 3V(\lambda, \lambda, 1) + A. \quad (5.27)$$

Among Eqs. (5.10)–(5.27) there are only 12 independent ones, six each from the octet and from the decuplet. Therefore, the 12 V values can be obtained from Eqs. (5.10)–(5.27) in terms of A, 12 chosen baryon masses, and the three reduced Q_e masses. The 12 baryon masses are chosen as p, n, Λ, Σ^+, Σ^-, Ξ^0, Δ^{*++}, Δ^{*0}, Δ^{*-}, Σ^{*+}, Σ^{*-}, and Ξ^{*0}. The results in terms of MeV are

$$V(\mathcal{P}, \mathcal{P}, 1) = -m'(\mathcal{P}) + \tfrac{1}{3}\Delta^{*++} - \tfrac{1}{3}A = -m'(\mathcal{P}) - \tfrac{1}{3}A + 412.0, \quad (5.28)$$

$$V(\mathcal{N}, \mathcal{N}, 1) = -m'(\mathcal{N}) + \tfrac{1}{3}\Delta^{*-} - \tfrac{1}{3}A = -m'(\mathcal{N}) - \tfrac{1}{3}A + 413.5, \quad (5.29)$$

$$V(\lambda, \lambda, 1) = -m'(\lambda) + \tfrac{1}{3}\Delta^{*++} - \Sigma^{*+} + \Xi^{*0} - \tfrac{1}{3}A = -m'(\lambda) - \tfrac{1}{3}A + 558.0, \quad (5.30)$$

$$V(\mathcal{P}, \mathcal{N}, 1) = -\tfrac{1}{2}[m'(\mathcal{P}) + m'(\mathcal{N})] + \tfrac{1}{2}\Delta^{*0} - \tfrac{1}{6}\Delta^{*-} - \tfrac{1}{3}A = -\tfrac{1}{2}[m'(\mathcal{P}) + m'(\lambda)] - \tfrac{1}{3}A + 412.2, \quad (5.31)$$

$$V(\mathcal{N}, \lambda, 1) = -\tfrac{1}{2}[m'(\mathcal{N}) + m'(\lambda)] - \tfrac{1}{6}\Delta^{*-} + \tfrac{1}{2}\Sigma^{*-} - \tfrac{1}{3}A = -\tfrac{1}{2}[m'(\mathcal{N}) + m'(\lambda)] - \tfrac{1}{3}A + 487.3, \quad (5.32)$$

$$V(\lambda, \mathcal{P}, 1) = -\tfrac{1}{2}[m'(\lambda) + m'(\mathcal{P})] - \tfrac{1}{6}\Delta^{*++} + \tfrac{1}{2}\Sigma^{*+} - \tfrac{1}{3}A = -\tfrac{1}{2}[m'(\lambda) + m'(\mathcal{P})] - \tfrac{1}{3}A + 485.0, \quad (5.33)$$

$$V(\mathcal{P}, \mathcal{P}, 0) = -m'(\mathcal{P}) + \tfrac{10}{3}p - \tfrac{2}{3}n - \tfrac{10}{3}\Lambda + \tfrac{2}{3}\Sigma^+ + \tfrac{2}{3}\Sigma^- - \tfrac{4}{3}\Delta^{*++} - \Delta^{*0} + \Sigma^{*+} + \Sigma^{*-} - \tfrac{1}{3}A = -m'(\mathcal{P}) - \tfrac{1}{3}A + 258.2, \quad (5.34)$$

$$V(\mathcal{N}, \mathcal{N}, 0) = -m'(\mathcal{N}) - \tfrac{2}{3}p + \tfrac{10}{3}n - \tfrac{10}{3}\Lambda + \tfrac{2}{3}\Sigma^+ + \tfrac{2}{3}\Sigma^- - \tfrac{1}{3}\Delta^{*++} - \Delta^{*0} - \Delta^{*-} + \Sigma^{*+} + \Sigma^{*-} - \tfrac{1}{3}A = -m'(\mathcal{N}) - \tfrac{1}{3}A + 255.5, \quad (5.35)$$

$$V(\lambda, \lambda, 0) = -m'(\lambda) + \tfrac{10}{3}p - \tfrac{2}{3}n - \tfrac{10}{3}\Lambda - \tfrac{10}{3}\Sigma^+ + \tfrac{2}{3}\Sigma^- + 4\Xi^0 - \tfrac{4}{3}\Delta^{*++} - \Delta^{*0} + 4\Sigma^{*+} + \Sigma^{*-} - 3\Xi^{*0} - \tfrac{1}{3}A = -m'(\lambda) - \tfrac{1}{3}A + 321.7, \quad (5.36)$$

$$V(\mathcal{P}, \mathcal{N}, 0) = -\tfrac{1}{2}[m'(\mathcal{P}) + m'(\mathcal{N})] + \tfrac{2}{15}p + \tfrac{2}{15}n + \tfrac{2}{3}\Lambda - \tfrac{2}{15}\Sigma^+ - \tfrac{2}{15}\Sigma^- + \tfrac{1}{15}\Delta^{*++} - \tfrac{1}{10}\Delta^{*0} + \tfrac{1}{10}\Delta^{*-} - \tfrac{1}{5}\Sigma^{*+} - \tfrac{1}{5}\Sigma^{*-} - \tfrac{1}{3}A = -\tfrac{1}{2}[m'(\mathcal{P}) + m'(\mathcal{N})] - \tfrac{1}{3}A + 205.2, \quad (5.37)$$

$$V(\mathcal{N}, \lambda, 0) = -\tfrac{1}{2}[m'(\mathcal{N}) + m'(\lambda)] + \tfrac{2}{15}p - \tfrac{2}{3}n + \tfrac{2}{3}\Lambda - \tfrac{2}{15}\Sigma^+ + \tfrac{2}{3}\Sigma^- + \tfrac{1}{15}\Delta^{*++} + \tfrac{1}{5}\Delta^{*0} + \tfrac{1}{10}\Delta^{*-} - \tfrac{1}{5}\Sigma^{*+} - \tfrac{1}{2}\Sigma^{*-} - \tfrac{1}{3}A = -\tfrac{1}{2}[m'(\mathcal{N}) + m'(\lambda)] - \tfrac{1}{3}A + 366.6, \quad (5.38)$$

$$V(\lambda, \mathcal{P}, 0) = -\tfrac{1}{2}[m'(\lambda) + m'(\mathcal{P})] - \tfrac{2}{3}p + \tfrac{2}{15}n + \tfrac{2}{3}\Lambda + \tfrac{2}{3}\Sigma^+ - \tfrac{2}{15}\Sigma^- + \tfrac{1}{6}\Delta^{*++} + \tfrac{1}{5}\Delta^{*0} - \tfrac{1}{2}\Sigma^{*+} - \tfrac{1}{5}\Sigma^{*-} - \tfrac{1}{3}A = -\tfrac{1}{2}[m'(\lambda) + m'(\mathcal{P})] - \tfrac{1}{3}A + 361.4. \quad (5.39)$$

These results indicate that the triplet–spin-interaction potentials are larger than the singlet–spin-interaction potentials for the same pair of Q_e's. This also means that the singlet binding energy is higher than that of the triplet for the same pair of Q_e's. Putting the V values from Eqs. (5.28)–(5.39) into Eqs. (5.14), (5.17), (5.19), (5.23), (5.26), and (5.27) yields six relations between the baryon masses. These relations are

$$\Delta^{*++} - \Delta^{*-} = 3(\Delta^{*+} - \Delta^{*0}), \quad (5.40)$$

$$\Delta^{*+} - \Delta^{*0} = \Sigma^{*+} - \Sigma^{*-} - \Xi^{*0} + \Xi^{*-}, \quad (5.41)$$

$$\Delta^{*+} + \Delta^{*-} - 2\Delta^{*0} = \Sigma^{*+} + \Sigma^{*-} - 2\Sigma^{*0}, \quad (5.42)$$

$$\Omega^- - \Delta^{*-} = 3(\Xi^{*-} - \Sigma^{*-}), \quad (5.43)$$

$$p - n = \Sigma^+ - \Sigma^- - \Xi^0 + \Xi^-, \quad (5.44)$$

$$p + n = \tfrac{5}{2}(\Lambda - \Sigma^0) + (\Sigma^+ + \Sigma^-) - (\Sigma^{*+} + \Sigma^{*-}) + \tfrac{1}{3}(\Delta^{*++} - \Delta^{*-}) + 2\Delta^{*0}. \quad (5.45)$$

The first five relations, (5.40)–(5.44), are identical to those derived from the paraquark model[24] or from the interchangeable Q_{em} model. The equation corresponding to Eq. (5.45) obtained from either of these models is

$$p + n = 3\Lambda - (\Xi^0 + \Xi^-) - \tfrac{1}{2}(\Xi^{*0} + \Xi^{*-}) + \tfrac{1}{3}(\Sigma^+ + \Sigma^0 + \Sigma^- - \Sigma^{*+} - \Sigma^{*0} - \Sigma^{*-}) + \tfrac{1}{4}(\Delta^{*++} + \Delta^{*+} + \Delta^{*0} + \Delta^{*-}) + \Omega^-. \quad (5.46)$$

Substitution of the values for the baryon masses[25] into Eqs. (5.45) and (5.46) shows that the left- and right-hand sides of these expressions are:

Left of (5.45) and (5.46) = 1878 MeV,

Right of (5.45) = 1896 MeV,

Right of (5.46) = 1932 MeV.

Here the mass of Δ^{*+} is taken as the average of Δ^{*++} and Δ^{*0} without significant error, because the difference between Δ^{*++} and Δ^{*0} is very small, viz., 0.45 ± 0.85 MeV. Clearly, our expression in the interchangeable Q_{em} model shows closer agreement to experimental results than the other. In the paraquark-model derivations the strong and the electromagnetic symmetry-breaking interactions have been taken into account. Thus the discrepancy of about 54 MeV is difficult to explain.

Note that from either the paraquark model or the interchangeable Q_{em} model we can obtain nine mass relations, but we can only get six mass relations from the noninterchangeable Q_{em} model. It is worth examining how we can get the three extra relations. If we assume

$$V(\mathcal{P}, \mathcal{P}, 1) - V(\mathcal{P}, \mathcal{P}, 0) = V(\mathcal{N}, \mathcal{N}, 1) - V(\mathcal{N}, \mathcal{N}, 0), \quad (5.47)$$

then from Eqs. (5.28), (5.29), (5.34), and (5.35) we obtain

$$n - p = \Delta^{*0} - \Delta^{*+}. \quad (5.48)$$

Again, if we assume

$$V(\mathcal{N}, \mathcal{N}, 1) - V(\mathcal{N}, \mathcal{N}, 0) = V(\lambda, \lambda, 1) - V(\lambda, \lambda, 0), \quad (5.49)$$

then from Eqs. (5.29), (5.35), (5.30), and (5.36) we obtain

$$\Xi^{*-} - \Xi^- = \Sigma^{*-} - \Sigma^-. \quad (5.50)$$

Once again, if we assume

$$V(\mathcal{P}, \mathcal{P}, 1) - V(\mathcal{P}, \mathcal{P}, 0) + V(\mathcal{N}, \mathcal{N}, 1) - V(\mathcal{N}, \mathcal{N}, 0) = V(\mathcal{P}, \lambda, 1) - V(\mathcal{P}, \lambda, 0) + V(\mathcal{N}, \lambda, 1) - V(\mathcal{N}, \lambda, 0), \quad (5.51)$$

then from Eqs. (5.28), (5.29), (5.32)–(5.35), (5.38), and (5.39) we obtain

$$\Sigma^{*+} - 2\Sigma^{*0} + \Sigma^{*-} = \Sigma^+ - 2\Sigma^0 + \Sigma^-. \quad (5.52)$$

The above three relations, (5.48), (5.50), and (5.52), are the three extra relations obtained in the paraquark model. Equation (5.47) is an electric isospin reflection between $\mathcal{P}$ and $\mathcal{N}$ Q_e's, which should be a good approximation. In fact, relation (5.48) does fit experimental values well. Again, Eq. (5.49) is the electric u-spin reflection between $\mathcal{N}$ and λ Q_e's; this should not be a good approximation. Equation (5.50) indeed does not fit the experimental results very well. Finally, Eq. (5.51) is the electric v-spin and u-spin reflections between $\mathcal{P}$ and λ and $\mathcal{N}$ and λ Q_e's, respectively. This should be poorer than the electric isospin reflection between $\mathcal{P}$ and $\mathcal{N}$ Q_e's. To see this, put

$$V(\mathcal{P}, \lambda, 1) - V(\mathcal{P}, \lambda, 0) = V(\mathcal{N}, \lambda, 1) - V(\mathcal{N}, \lambda, 0). \quad (5.53)$$

Then from Eqs. (5.33), (5.39), (5.32), and (5.38) we obtain

$$(\Sigma^- - \Sigma^+) - (n - p) = (\Sigma^{*-} - \Sigma^{*+}) - (\Delta^{*0} - \Delta^{*+}). \quad (5.54)$$

Experimentally, Eq. (5.54) is better than Eq. (5.52).

Clearly, these three extra mass relations obtained from the paraquark model were derived under the above less adequate assumptions. That is why some of these relations, including Eq. (5.46), do not fit the experimental data well. It seemed that the paraquark model did not involve any such assumptions in the derivation of these relations, but, indeed, these assumptions were already made without attention when we employed the paraquark model. In our derivation of the six mass relations from the noninterchangeable Q_{em} model we did not make any like assumptions. This accounts for the fact that we only obtained six relations from the noninterchangeable model. Consequently, the noninterchangeable model is more feasible than the paraquark model or the interchangeable model in deriving the baryon mass relations.

VI. ELECTRIC AND MAGNETIC DIPOLE MOMENTS OF BARYONS

As we know, the magnetic dipole moment (MDM) of a nucleus is mainly contributed by the intrinsic MDM's of the nucleons inside and their orbital angular momenta. The relativistic corrections to the MDM's of individual nucleons,[26,27] the cooperative effects,[27] and the influence of spin-orbit coupling are very small in accounting for the MDM of the nucleus. However, the situation is different for calculating the MDM of a baryon instead of a nucleus. In the three-Q_{em} model the three Q_{em}'s ($q_1\overline{\mathcal{P}}'$, $q_2\overline{\mathfrak{N}}'$, and $q_3\overline{\lambda}'$) of a baryon are very much closer to each other and stronger in interacting with each other than the nucleons of a nucleus. Also, as discussed in Sec. II, the interaction between q and $\overline{q}'$ in a Q_{em} will change the strength of the MDM of the Q_{em}. Therefore, the MDM of a baryon contributed by the cooperative effects[27-29] (or the so-called exchange MDM's) may be comparable to that contributed by the intrinsic MDM's of the three Q_{em}'s.

From the baryon wave functions in Sec. IV we can calculate the MDM's of the baryon octet. Let $\mu(\mathcal{P}_1)$ and $\mu(\mathcal{P})$ be the intrinsic MDM's of Q_{em} $\mathcal{P}_1$ and Q_e $\mathcal{P}$, respectively. The relation between $\mu(\mathcal{P}_1)$ and $\mu(\mathcal{P})$ is still unknown. However, if the Q_e's are structureless particles, then we have

$$\mu(\mathcal{P}) = -2\mu(\mathfrak{N}) = -2\mu(\lambda)\,. \tag{6.1}$$

It seems to be a reasonable approximation to assume that

$$\mu(Q_{em}) = \mu(Q_e) + \mu_x\,. \tag{6.2}$$

Here μ_x may be called a cooperative MDM which can be positive or negative or zero.

Now we discuss μ_x. In nuclear physics the exchange MDM of a nucleus[28] can be expressed as

$$\mu_x = \frac{i}{2\hbar c}\sum_k \sum_{j<k}(e_k - e_j)\int(\psi, J_{jk}P_{jk}\psi) \times(\vec{r}_k\times\vec{r}_j)\,d\tau_1 d\tau_2\ldots d\tau_n\,, \tag{6.3}$$

where n is the number of nucleons in the nucleus; $j, k = 1, 2, \ldots, n$; e_k and e_j are the electric charges of the kth and jth nucleons; ψ is the wave function; J_{jk} is the potential function between the j and k nucleons; P_{jk} is the exchange operator; and $\vec{r}_k$ and $\vec{r}_j$ are the position vectors of the two nucleons involved.

From Eq. (6.3) we obtain the following: The exchange MDM's of conjugate pairs of nuclei (i.e., those that can be obtained from one another by interchange of proton and neutron) are equal in magnitude and opposite in sign.[28] It also follows that the exchange MDM's of self-conjugate nuclei vanish.

To apply the above facts about the exchange MDM's to the case of baryons, instead of nuclei, we have to make a little change, because there are three different kinds of Q_e's instead of two kinds of nucleons. If a baryon is made up of only two different kinds of Q_e's then the application is straightforward. However, we have to keep in mind that $\mathcal{P}\mathfrak{N}$ pairs are almost the same as $\mathcal{P}\lambda$ pairs because both $\mathfrak{N}$ and λ carry the same electric charge. It also follows that if a baryon is made up of two kinds of Q_e's, $\mathfrak{N}$ and λ, then there is no exchange MDM, since both $\mathfrak{N}$ and λ carry the same electric charge. Obviously, if a baryon is made up of only one kind of Q_e then the exchange MDM is also zero. Finally, if a baryon is made up of three different kinds of Q_e's, $\mathcal{P}$, $\mathfrak{N}$, and λ, then the exchange MDM will also vanish, because the three Q_m's in a baryon do not produce an exchange EDM (if there was an exchange EDM in a baryon, then it would be difficult to explain why the baryon does not carry an EDM). The vanishing may be due to the fact that the baryon is self-conjugate (i.e., the baryon remains the same if we interchange $\mathcal{P}$ and $\mathfrak{N}$, $\mathfrak{N}$ and λ, λ and $\mathcal{P}$, $\overline{\mathcal{P}}'$ and $\overline{\mathfrak{N}}'$, $\overline{\mathfrak{N}}'$ and $\overline{\lambda}'$, and $\overline{\lambda}'$ and $\overline{\mathcal{P}}'$). From Eq. (6.2) and the wave functions in Sec. IV, we can express the MDM's of the baryon octet in the noninterchangeable Q_{em} model as

$$\mu(p) = \mu(\mathcal{P}) + \mu_x\,, \tag{6.4}$$

$$\mu(n) = \mu(\mathfrak{N}) - \mu_x\,, \tag{6.5}$$

$$\mu(\Lambda) = \mu(\lambda)\,, \tag{6.6}$$

$$\mu(\Sigma^+) = \mu(\mathcal{P}) + \mu_x\,, \tag{6.7}$$

$$\mu(\Sigma^0) = \tfrac{2}{3}\mu(\mathcal{P}) + \tfrac{2}{3}\mu(\mathfrak{N}) - \tfrac{1}{3}\mu(\lambda)\,, \tag{6.8}$$

$$\mu(\Sigma^-) = \mu(\mathfrak{N})\,, \tag{6.9}$$

$$\mu(\Xi^0) = \mu(\lambda) - \mu_x\,, \tag{6.10}$$

$$\mu(\Xi^-) = \mu(\lambda)\,. \tag{6.11}$$

From Eqs. (6.1), (6.4), and (6.5) we obtain

$$\mu(\mathcal{P}) = 2[\mu(p) + \mu(n)] = 1.760\mu_N\,, \tag{6.12}$$

$$\mu(\lambda) = \mu(\mathfrak{N}) = -[\mu(p) + \mu(n)] = -0.880\mu_N\,, \tag{6.13}$$

$$\mu_x = -[\mu(p) + 2\mu(n)] = 1.033\mu_N\,. \tag{6.14}$$

It is clear that we can express the MDM's of the rest of the baryon octet in terms of $\mu(p)$ and $\mu(n)$ as

$$\mu(\Sigma^+) = \mu(p) = 2.793\mu_N\,, \tag{6.15}$$

$$\mu(\Lambda) = \mu(\Sigma^-) = \mu(\Xi^-) = -[\mu(p) + \mu(n)] = -0.880\mu_N\,, \tag{6.16}$$

$$\mu(\Xi^0) = \mu(n) = -1.913\mu_N\,, \tag{6.17}$$

$$\mu(\Sigma^0) = [\mu(p) + \mu(n)] = 0.880\mu_N\,. \tag{6.18}$$

The results of the paraquark model[23] and the interchangeable Q_{em} model are

$$\mu(\Lambda) = \mu(\Sigma^-) = \mu(\Xi^-) = -0.931\mu_N\,,$$

$\mu(\Sigma^+) = \mu(p) = 2.793\,\mu_N$,

$\mu(\Sigma^0) = 0.53\,\mu_N$,

$\mu(\Xi^0) = -1.863\,\mu_N$.

The experimental data[25] are $\mu(\Lambda) = (-0.73 \pm 1.6)\mu_N$ and $\mu(\Sigma^+) = (2.57 \pm 0.52)\mu_N$. We can see that the predictions from the noninterchangeable or interchangeable model and the paraquark model are almost the same. Note that one may also introduce the idea of exchange MDM into the calculation of the MDM of baryons from the paraquark and interchangeable $Q_{\rm em}$ models. The experimental data show that the exchange MDM for these two models is almost zero, which is expected.

Next, we will prove that this model predicts a zero EDM for baryons in an S state. One difficulty of introducing magnetic monopoles into the hadron structure is in solving the problem of the EDM and MDM of baryons. We know that a rotating electric charge can produce an MDM, whereas a rotating magnetic charge can produce an EDM. If there is any magnetic monopole inside a baryon at all, why do protons and neutrons not carry EDM's?[30] It is not a matter of T and P violations. A particle carrying an EDM and MDM will violate only P, but not T, if the EDM is induced by its moving magnetic charge. The answer to this question can be that the three Q_m's ($\overline{\mathcal{P}}'$, $\overline{\mathfrak{N}}'$, and $\overline{\lambda}'$) in a baryon are in an $L=0$ and $S=0$ state. We may employ the same formula used to calculate the MDM of a baryon to calculate its EDM. However, we must change electric charges to negative magnetic charges and keep in mind that Q_m's do not carry intrinsic EDM's, i.e.,

$$\mu^{(e)}(\overline{\mathcal{P}}') = \mu^{(e)}(\overline{\mathfrak{N}}') = \mu^{(e)}(\overline{\lambda}') = 0.$$

The EDM $\mu^{(e)}$ produced by a Q_m carrying magnetic charge Q' in a state L, S can be rewritten[26] as follows:

(i) In the Newton approximation,

$$\mu^{(e)} = (L+2S)\mu_0 + \delta. \tag{6.19}$$

(ii) In the $v \ll c$ approximation,

$$\mu^{(e)} = (L+2S)\mu_0 + \delta - \left(\mu_0 \frac{(L+2S)^2}{L+3S} + \frac{L+2S}{L+3S}\delta\right)\overline{T}. \tag{6.20}$$

(iii) In the extreme-relativistic approximation,

$$\mu^{(e)} = \frac{1}{2(L+3S)}[(L+2S)\mu_0 + \delta], \tag{6.21}$$

where $\mu_0 = -(Q'\hbar)/(2mc)$, δ is the supplementary EDM, and $\overline{T}$ is the average kinetic energy of the Q_m. For the baryon octet and decuplet, $L=0$, $S=0$, and $\delta=0$, because Q_m's are bosons and structureless. We should note that the exchange EDM's of existing baryons are zero, since each baryon consists of $\overline{\mathcal{P}}'$, $\overline{\mathfrak{N}}'$, and $\overline{\lambda}'$ Q_m's, which make the baryons magnetically self-conjugate. Therefore, EDM is zero for the baryon octet and decuplet. However, for highly excited states, $L \neq 0$, a strong EDM is expected to exist. The strength of such an EDM will be in the order of $10^{-12} e$ cm. The existence of such an EDM may be strong evidence of the existence of the Q_m's. Experimental investigations are very much encouraged.

VII. SUMMARY AND DISCUSSION

Quantum theory allows the existence of magnetic monopoles. The purpose of this paper is to introduce such monopoles into the hadron structure and to solve some of the difficulties faced by the paraquark model. To this end, the magnetic monopoles are generalized to match the conventional quarks (electric quarks) $\mathcal{P}$, $\mathfrak{N}$, and λ; i.e., there are also three kinds of magnetic monopoles called $\mathcal{P}'$, $\mathfrak{N}'$, and λ' magnetic quarks (Q_m's), and their counterpart $\overline{Q}_m$'s also exist. The Q_m's, as required, are all bosons. Electric quarks (Q_e's) and Q_m's are all superstrong-interaction particles, and are considered to be the same kind of particles as far as the superstrong interaction is concerned. The magnetic clusters, $\mathcal{P}'\overline{\mathcal{P}}' + \mathfrak{N}'\overline{\mathfrak{N}}' + \lambda'\overline{\lambda}'$ and $\overline{\mathcal{P}}'\,\overline{\mathfrak{N}}'\,\overline{\lambda}'$, carrying zero net magnetic charges, become the fundamental blocks of the existing mesons and baryons, respectively. An Q_e and an $\overline{Q}_m$ pair is called an electromagnetic quark ($Q_{\rm em}$). Three $Q_{\rm em}$'s, $q_1\overline{\mathcal{P}}'$, $q_2\overline{\mathfrak{N}}'$, and $q_3\overline{\lambda}'$, replacing the traditional three quarks q_1, q_2, and q_3, form a baryon.

The three-$Q_{\rm em}$ model can be divided into two possible submodels called interchangeable and noninterchangeable $Q_{\rm em}$ models. The interchangeable model allows the three Q_e's or Q_m's to interchange, whereas the noninterchangeable model does not. The interchangeable model will have the same predictions in baryon mass relations and magnetic dipole moments as the paraquark model. However, the paraquark model has statistical difficulty, which the interchangeable $Q_{\rm em}$ model does not have. The noninterchangeable $Q_{\rm em}$ model predicts better baryon mass relations than the other two models. The three-$Q_{\rm em}$ model, noninterchangeable or interchangeable, also predicts the existence of a strong electric dipole moment (EDM) in a baryon state with nonzero $Q_{\rm em}$ angular orbital momentum. The strength is in the order of $10^{-12}\,e$ cm. An experimental investigation is strongly recommended.

There remains to be discussed the question of why Q_e's carry spin $\frac{1}{2}$ and Q_m's carry spin 0. This seems to break the symmetric property between electricity and magnetism, just as Q_m's hold much stronger charges than Q_e's. The Q_e and Q_m differ only in charge and spin. Maybe there is some relationship between their charges and spins.

In calculating the baryon mass relations in the noninterchangeable Q_{em} model, we have expressed the Q_{em} pairing energies as a function of the spin, electric isospin, and magnetic isospin of the pairing system. For a unique solution there are too many different pairing energies or too many unknowns in the mass expressions. In order to make the problem solvable we have ignored the electric-isospin and magnetic-isospin dependence of the pairing energies. We then have expressed the Q_{em} pairing energies in terms of Q_e pairing energies plus a constant. In this approximation the existence of $\overline{Q}_m$'s only serves as a label marker to distinguish the Q_e's. We may get the same baryon mass expressions if we assume that the baryon consists of three distinguishable Q_e's (even when they are in the same spin and electric-isospin states) and completely symmetrize the wave function in the Q_e labels.

Finally, in calculating the EDM and the MDM of the baryon we have only considered the relativistic effect and the electric and magnetic exchange moments. The influence of spin-orbit coupling and recoil effect is not treated. The influence of such extra effects is negligible in accounting for the MDM of baryons; however, whether or not these effects will produce an observable nonzero EDM of the baryon is unknown. The author hopes to return to these questions later.

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