PHYSICAL REVIEW D VOLUME 5, NUMBER 4 15 FEBRUARY 1972

High-Energy Particles as External Sources*

David R. Harrington
Department of Physics, Rutgers, The State University, New Brunswick, New Jersey 08903
(Received 15 November 1971)

Using methods previously used to derive the relativistic eikonal approximation it is shown that high-energy particles are approximately equivalent to external moving point sources. This result is independent of the nature of the rest of the process involved, but holds only when the high-energy particle does not suffer significant changes in its four-momentum, nor changes in its internal state, during its interactions with the other particles.

I. INTRODUCTION

It has been known for some time that large-mass particles are approximately equivalent to static sources in quantum field theory: This fact is used implicitly, for example, in most treatments of the hydrogen atom.[1] A particle of large momentum is also quite "rigid" in the sense that its state is not easily changed significantly, and this feature has been used as the basis for the eikonal approximation to some high-energy processes in quantum field theory.[2-9] In this paper an approximation scheme which includes both the high-momentum and large-mass limits is discussed. It is shown that there may be situations where the effects of particles of high energy (i.e., particles with a large mass or a large momentum, or both) involved in some process may be approximately reproduced by external potentials which are static and spherically symmetric in the rest frames of the high-energy particles.[10]

The body of the paper begins, in Sec. II, with a review of the derivation of the eikonal approximation to the sum of crossed-ladder diagrams for two-body scattering. It is then pointed out that this result can be extended to a much wider class of processes: In Sec. III fairly arbitrary processes involving a single high-energy particle are considered, while Sec. IV discusses processes with two high-energy particles. The concluding Sec. V contains a summary of these results and suggests possible applications and generalizations.

II. THE EIKONAL APPROXIMATION

It has been shown in several different ways[2-9] that the sum of all arbitrarily crossed-ladder graphs in quantum field theory leads, in the high-energy small-angle limit, to a relativistic analog of the eikonal approximation. (It should be noted, however, that this result does not hold for all theories.[11-14] Roughly speaking it is valid only where the high-energy particles have no internal degrees of freedom,[15] and in particular where their composite nature is unimportant. This difficulty will be ignored in the remainder of this paper, so that the results obtained will not necessarily be accurate, or even applicable, in every situation.) The essential approximation in the derivation is the linearization of the denominators in the propagators of the high-energy particles. The graph shown in Fig. 1, for example, involves propagator denominators of the form

$$(q_i-k)^2-M^2+i\epsilon=-2q\cdot k+(q_f-q_i)\cdot k+k^2+i\epsilon\,,$$

where $q=\frac{1}{2}(q_f+q_i)$ is the average of the initial and final four-momenta of the high-energy q line. The linearization leading to the eikonal approximation simply drops the $(q_f-q_i)\cdot k+k^2$ terms, assuming that in the high-energy fixed-momentum-transfer limit

$$(q_i-k)^2-M^2+i\epsilon\approx-2q\cdot k+i\epsilon\,.$$

This approximation replaces the Feynman propagator

$$\Delta_F(x)=\int\frac{d^4p}{(2\pi)^4}\,\frac{e^{-ip\cdot x}}{p^2-M^2+i\epsilon}$$

by

$$\Delta_q(x)=\int\frac{d^4p}{(2\pi)^4}\,\frac{e^{-ip\cdot x}}{2q\cdot(p-q)+i\epsilon}$$
$$=\theta(x_0)\delta^3(\vec{r}-\vec{v}x_0)\,\frac{e^{-iq\cdot x}}{2iq_0}\,,$$

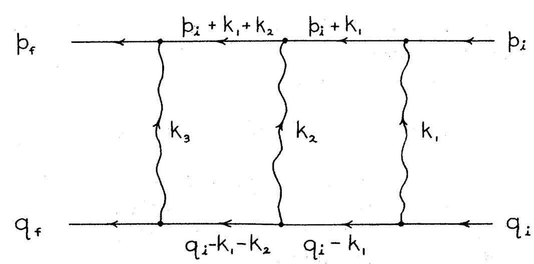

FIG. 1. One of the six diagrams contributing to the three-rung contribution to the two-particle scattering amplitude. The sum over all diagrams of this type can lead to the relativistic eikonal approximation in the high-energy small-angle limit.

where $\vec{v}=\vec{q}/q_0$, so that the q-line particle propagates only forward in time with velocity $\vec{v}$. In this approximation, therefore, the high-energy particle is completely impervious to outside influence, and it is reasonable to assume that it is equivalent to an external source. To obtain this result one must sum over the permutations of the points of attachment of the k lines to the q lines, making use of the identity[3]

$$\delta^4(q_f-q_i+k_1+\cdots+k_n)\sum_P \frac{1}{-2q\cdot k_{P1}+i\epsilon}\cdots\frac{1}{-2q\cdot(k_{P1}+\cdots+k_{P(n-1)})+i\epsilon} = 2q_0\delta^3(\vec{q}_f-\vec{q}_i+\vec{k}_1+\cdots+\vec{k}_n)\times(2\pi i)^{n-1}\prod_{j=1}^{n}\delta(2q\cdot k_j)\,.$$

This identity follows directly from the more obvious relation

$$\sum_P \theta(t_{Pn}-t_{P(n-1)})\cdots\theta(t_{P2}-t_{P1})=1$$

after Fourier transformation and some simple rearrangements and changes of variables. Its use eliminates the linearized q-line denominators from the integrand, and the resulting δ functions can be used to do the integrations over the time components of the k vectors. The end result is an integral which is equivalent to that for the scattering from an external potential, and at this stage a relativistic version of the potential-theory eikonal approximation can be used to obtain a compact expression for the scattering amplitude. The simplification made possible by linearizing the q-line denominators is a special case of more general results discussed below.

III. SINGLE HIGH-ENERGY PARTICLE

In this section we shall consider the contribution A_n to a process involving a single high-energy line which comes from all diagrams in which the high-energy particle (again represented by a "q line" in diagrams) interacts n times with the rest of the diagram. A typical diagram for $n=3$ is shown in Fig. 2: A_3 is the sum of this diagram and the $5=3!-1$ others obtained by permuting the points of attachment of the k lines to the q line. For general n we have

$$A_n=i^{n-1}\int\left(\prod_{j=1}^{n}\frac{d^4k_j}{(2\pi)^4}f_j(k_j)\right)(2\pi)^4\delta^4\left(q_f-q_i+\sum_{j=1}^{n}k_j\right)F_n(\{p\},\{k\})$$
$$\times\sum_P[(q-k_{P1})^2-M^2+i\epsilon]^{-1}\cdots[(q-k_{P1}-\cdots-k_{P(n-1)})^2-M^2+i\epsilon]^{-1}\,.$$

Here the $f_j(k_j)$ are the k-line propagators in momentum space, including the coupling to the q line, while F is the amplitude for the rest of the diagram, represented by the blob in Fig. 2.

In the limit in which the energy of the q-line particle becomes very large, with q_f-q_i remaining fixed, we assume that the q-line denominators can be linearized. This leads immediately to the approximate form

$$A_n\approx(-1)^{n-1}2q_0\int\left(\prod_{j=1}^{n}\frac{d^4k_j}{(2\pi)^4}f_j(k_j)\,2\pi\delta(2q\cdot k_j)\right)(2\pi)^3\delta^3\left(\vec{q}_f-\vec{q}_i+\sum_{j=1}^{n}\vec{k}_j\right)F_n(\{p\},\{k\})\,.$$

We want to compare this expression with the corresponding term B_n in the transition amplitude for the process F occurring in the presence of the external fields

$$\Phi_j(x)=\int\frac{d^4k}{(2\pi)^4}e^{ik\cdot x}a_j(k)$$

instead of in the presence of the q line, as indicated in Fig. 3:

$$B_n=\int\left(\prod_{j=1}^{n}\frac{d^4k_j}{(2\pi)^4}a_j(k_j)\right)(2\pi)^4\delta^4\left(P_f-P_i-\sum_{j=1}^{n}k_j\right)F_n(\{p\},\{k\})\,,$$

where $P_f=\sum p_f$ and $P_i=\sum p_i$. If we choose

$$a_j(k)=-2\pi\delta(2q\cdot k)f_j(k)\,,$$

then

$$B_n=(-1)^n\int\left(\prod_{j=1}^{n}\frac{d^4k_j}{(2\pi)^4}f_j(k_j)2\pi\delta(2q\cdot k_j)\right)(2\pi)^4\delta^4\left(P_f-P_i-\sum_{j=1}^{n}k_j\right)F_n(\{p\},\{k\})\,.$$

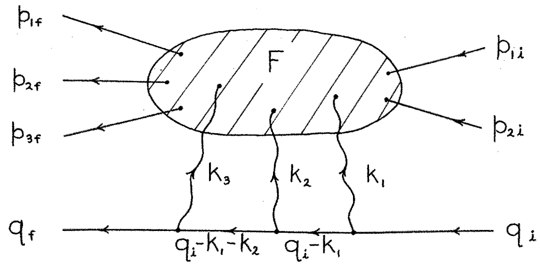

FIG. 2. One of the six diagrams contributing to the amplitude A_3 for the process $q_i + p_{1i} + p_{2i} \to q_f + p_{1f} + p_{2f} + p_{3f}$. The cross-hatched blob labeled F represents the complete amplitude for the sub-process $p_{1i} + p_{2i} + k_1 + k_2 + k_3 \to p_{1f} + p_{2f} + p_{3f}$ with the k lines off shell.

Making use of relations implied by the other δ functions, we have

$$\delta(P_{f0} - P_{i0} - \sum k_{j0}) = \delta(P_{f0} - P_{i0} - \vec{v} \cdot \sum \vec{k}_j)$$
$$= \delta(P_{f0} - P_{i0} - \vec{v} \cdot (\vec{P}_f - \vec{P}_i)) .$$

The energy δ function in this form can be taken outside the integral, giving

$$B_n = -2\pi\delta(2q \cdot (P_f - P_i))A_n .$$

The form chosen for a_i above implies that

$$\Phi_j(x) = -\int \frac{d^4k}{(2\pi)^4}\, e^{ik\cdot x}\, 2\pi\delta(2q\cdot k) f_j(k)$$
$$= -(2q_0)^{-1} \int \frac{d^3k}{(2\pi)^3}\, e^{-i\vec{k}\cdot(\vec{r} - \vec{v}t)} f(k)\Big|_{k_0 = \vec{v}\cdot\vec{k}} .$$

Assuming that the propagator f depends only on the invariant k^2, i.e., that $f(k) = \hat{f}(-k^2)$,

$$\Phi_j(x) = -(2q_0)^{-1} \int \frac{d^3k}{(2\pi)^3}\, e^{-i\vec{k}\cdot(\vec{r} - \vec{v}t)} \hat{f}_j(\vec{k}^2 - (\vec{v}\cdot\vec{k})^2) .$$

Taking $\vec{v}$ to define the z direction,

$$\Phi_j(x) = -(2M)^{-1}\, V_j\left(\left(b^2 + \frac{(z - vt)^2}{1 - v^2}\right)^{1/2}\right),$$

where

$$V_j(r) = \int \frac{d^3k}{(2\pi)^3}\, e^{-i\vec{k}\cdot\vec{r}} \hat{f}_j(\vec{k}^2) .$$

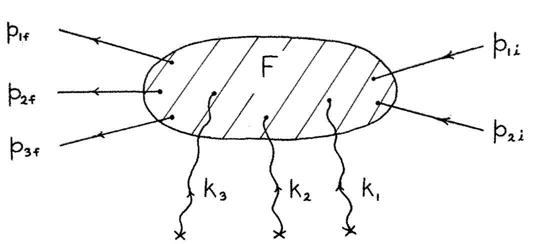

FIG. 3. The analog of Fig. 2 for the process $p_{1i} + p_{2i} \to p_{1f} + p_{2f} + p_{3f}$ in external potentials.

The interaction with the high-energy q-line particle is thus approximately equivalent to the interaction with the external potentials $\Phi_j(x)$ which are just the Lorentz transforms of potentials $V_j(r)$ which are static and spherically symmetric. The $\delta(2q \cdot (P_f - P_i))$ in the relation between B_n and A_n merely requires that energy is conserved in the frame where $\vec{q} = 0$ in which the external potentials are static. This equivalence of high-energy particles and moving external point sources producing simple potentials is more-or-less obvious for particles of large mass, and is contained implicitly in previous treatments of two-particle scattering in the ultrarelativistic limit. These two limiting cases are connected by the result obtained here, which is of course not limited to two-particle scattering since the process F is fairly arbitrary. It should be noted, however, that the q-line particle was assumed not to change significantly its internal state, nor its momentum or energy, while it was interacting with the other particles involved in the amplitude F. The result is thus limited to small-angle elastic or slightly inelastic scattering where the high-energy particle is "not very composite" and where there is a small probability for its spin or isospin state to change.

IV. TWO HIGH-ENERGY PARTICLES

The result obtained above for a single high-energy particle could be immediately generalized to an arbitrary number of such particles except for two complications: In general there will be no frame in which energy is conserved since the high-energy particles will not necessarily have a common rest frame, and furthermore it may be necessary to take into account the interaction of the high-energy particles with each other. In this section these complications will be studied in their simplest form by considering the amplitudes cor-

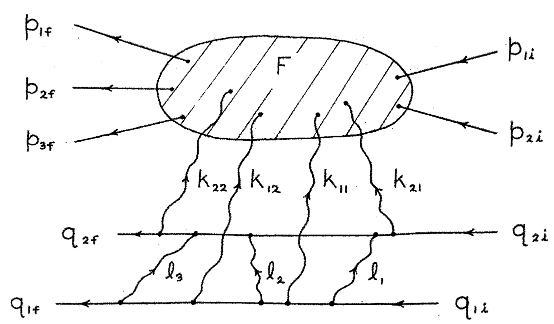

FIG. 4. One of the 2400 diagrams contributing to the amplitude A_{223} for the process $q_{1i} + q_{2i} + p_{1i} + p_{2i} \to q_{1f} + q_{2f} + p_{1f} + p_{2f} + p_{3f}$. The blob labled F represents the complete off-shell amplitude for the sub-process $p_{1i} + p_{2i} + k_{11} + k_{12} + k_{21} + k_{22} \to p_{1f} + p_{2f} + p_{3f}$.

responding to Feynman diagrams of the type illustrated in Fig. 4. To simplify the discussion we shall work in a frame where $\vec{q}_1=\frac{1}{2}(\vec{q}_{1f}+\vec{q}_{1i})$ and $\vec{q}_2=\frac{1}{2}(\vec{q}_{2f}+\vec{q}_{2i})$ are antiparallel, with $\vec{q}_1$ defining the z axis. The amplitude with n_1 k lines between the sub-diagram F and the q_1 line, n_2 between F and the q_2 line, and with m l lines between the two q lines, summed over all permutations of the points of attachment to the q lines, is

$$A_{n_1n_2m}=\frac{i^{m+n_1+n_2-2}}{m!}\int\left(\prod\frac{d^4k_1}{(2\pi)^4}f_1(k_1)\right)\left(\prod\frac{d^4k_2}{(2\pi)^4}f_2(k_2)\right)\left(\prod\frac{d^4l}{(2\pi)^4}g(l)\right)F_{n_1n_2}(\{p\},\{k\})$$
$$\times(2\pi)^4\delta^4\left(q_{1f}-q_{1i}+\sum_{\alpha=1}^{N_1}K_{1\alpha}\right)(2\pi)^4\delta^4\left(q_{2f}-q_{2i}+\sum_{\beta=1}^{N_2}K_{2\beta}\right)$$
$$\times\sum_{P_1}[(q_{1i}-K_{1P_11})^2-M_1{}^2+i\epsilon]^{-1}\cdots[(q_{1i}-K_{1P_11}-\cdots-K_{1P_1(N_1-1)})^2-M_1{}^2+i\epsilon]^{-1}$$
$$\times\sum_{P_2}[(q_{2i}-K_{2P_21})^2-M_2{}^2+i\epsilon]^{-1}\cdots[(q_{2i}-K_{2P_21}-\cdots-K_{2P_2(N_2-1)})^2-M_2{}^2+i\epsilon]^{-1}.$$

In this expression $N_1=n_1+m$ and $N_2=n_2+m$, and we have defined the new labels for the k and l lines:

$$\begin{array}{ll} K_{11}=k_{11}, & K_{21}=k_{21},\\ \cdots & \cdots\\ K_{1n_1}=k_{1n_1}, & K_{2n_2}=k_{2n_2},\\ K_{1(n_1+1)}=l_1, & K_{2(n_2+1)}=-l_1,\\ \cdots & \cdots\\ K_{1N_1}=l_m, & K_{2N_2}=-l_m. \end{array}$$

The factor of $1/m!$ compensates for the double counting in the permutations of the points of attachment of the l lines.

If we here again linearize the q-line denominators and use the identity introduced in Sec. II, we obtain

$$A_{n_1n_2m}\approx\frac{(-i)^m(-1)^{n_1+n_2}}{m!}\int\left(\prod\frac{d^4k_1}{(2\pi)^4}2\pi\delta(2q_1\cdot k_1)f_1(k_1)\right)\left(\prod\frac{d^4k_2}{(2\pi)^4}2\pi\delta(2q_2\cdot k_2)f_2(k_2)\right)F_{n_1n_2}(\{p\},\{k\})$$
$$\times(2\pi)^3\delta^3\left(\vec{q}_{1f}-\vec{q}_{1i}+\sum\vec{K}_1\right)(2\pi)^3\delta^3\left(\vec{q}_{2f}-\vec{q}_{2i}+\sum\vec{K}_2\right)\left(\prod\frac{d^4l}{(2\pi)^4}2\pi\delta(2q_1\cdot l)2\pi\delta(2q_2\cdot l)g(l)\right).$$

The identity

$$\delta(x)\delta(y)=\delta(x+y)\delta(c_1x-c_2y),$$

where $c_1+c_2=1$, can be used to transform the δ functions:

$$(2\pi)^3\delta^3(\vec{q}_{1f}-\vec{q}_{1i}+\sum\vec{K}_1)(2\pi)^3\delta^3(\vec{q}_{2f}-\vec{q}_{2i}+\sum\vec{K}_2)=(2\pi)^3\delta^3(\vec{P}_f-\vec{P}_i-\sum\vec{k}_1-\sum\vec{k}_2)(2\pi)^3\delta^3(\vec{\delta}+\vec{\Delta}+\sum\vec{l}),$$

where

$$\vec{\delta}=c_1(\vec{q}_{1f}-\vec{q}_{1i})-c_2(\vec{q}_{2f}-\vec{q}_{2i})$$

and

$$\vec{\Delta}=c_1(\sum\vec{k}_1)-c_2(\sum\vec{k}_2).$$

For reasons that will become clear below the coefficients c_1 and c_2 will be chosen as

$$c_1=v_1/(v_1+v_2)\quad\text{and}\quad c_2=v_2/(v_1+v_2),$$

where $v_i=|\vec{q}_i|/q_{i0}$. With this transformation we have

$$A_{n_1n_2m}=(-1)^{n_1+n_2}\int\left(\prod\frac{d^4k_1}{(2\pi)^4}2\pi\delta(2q_1\cdot k_1)f_1(k_1)\right)\left(\prod\frac{d^4k_2}{(2\pi)^4}2\pi\delta(2q_2\cdot k_2)f_2(k_2)\right)F_{n_1n_2}(\{p\},\{k\})$$
$$\times(2\pi)^3\delta^3(\vec{P}_f-\vec{P}_i-\sum\vec{k}_1-\sum\vec{k}_2)$$
$$\times\left[\frac{(-i)^m}{m!}\int\left(\prod\frac{d^4l}{(2\pi)^4}2\pi\delta(2q_1\cdot l)2\pi\delta(2q_2\cdot l)g(l)\right)(2\pi)^3\delta^3(\vec{\delta}+\vec{\Delta}+\sum\vec{l})\right].$$

The factor in large square brackets can be simplified by using the integral representations

$$(2\pi)^3\delta^3(\vec{\delta}+\vec{\Delta}+\sum\vec{l}) = \int d^3r\, \exp[-i\vec{r}\cdot(\vec{\delta}+\vec{\Delta}+\sum\vec{l})],$$

$$2\pi\delta(2q\cdot l) = \int d\tau \exp(-i2q\cdot l\tau),$$

and

$$g(l) = \int d^4x\, e^{il\cdot x} D(x),$$

doing the integrations over l_i, and then using the resulting δ functions to do the integrations over the x's. The term in large square brackets then becomes

$$2\pi\delta(\delta_z+\Delta_z)\int d^2b\, e^{-i(\vec{\delta}+\vec{\Delta})\cdot\vec{b}}[i\chi(b)]^m/m!,$$

where

$$\chi(b) = -\int d\tau_1 d\tau_2 D(x+2q_1\tau_1+2q_2\tau_2)$$

is the eikonal function for the interaction between the two q-line particles. As indicated, χ depends only on the magnitude of $\vec{b}$, the projection of the space component of the four-vector x on the x-y plane. Summing over all m, we find

$$A_{n_1n_2} = \sum_{m=0}^{\infty} A_{n_1n_2m} = (-1)^{n_1+n_2}\int d^2b\, e^{-i\vec{\delta}\cdot\vec{b}}\, e^{i\chi(b)}\int\left(\prod\frac{d^4k_1}{(2\pi)^4}\, 2\pi\delta(2q_1\cdot k_1)f_1(k_1)\right)\left(\prod\frac{d^4k_2}{(2\pi)^4}\, 2\pi\delta(2q_2\cdot k_2)f_2(k_2)\right)$$
$$\times(2\pi)^3\delta^3(\vec{P}_f-\vec{P}_i-\sum\vec{k}_1-\sum\vec{k}_2)2\pi\delta(\Delta_z+\delta_z)\, e^{-i\vec{\Delta}\cdot\vec{b}}F_{n_1n_2}(\{p\},\{k\}).$$

Now consider the corresponding term in the amplitude $B_{n_1n_2}$ for the process F to take place in the presence of the external potentials

$$\Phi_{1j}(x) = \int\frac{d^4k}{(2\pi)^4}\, e^{ik\cdot x} a_{1j}(k)$$

and

$$\Phi_{2j}(x) = \int\frac{d^4k}{(2\pi)^4}\, e^{ik\cdot x} a_{2j}(k):$$

$$B_{n_1n_2} = \int\left(\prod\frac{d^4k_1}{(2\pi)^4}\, a_1(k_1)\right)\left(\prod\frac{d^4k_2}{(2\pi)^4}\, a_2(k_2)\right)(2\pi)^4\delta^4(P_f-P_i-\sum k_1-\sum k_2)F_{n_1n_2}(\{p\},\{k\}).$$

If we choose

$$a_{1j}(k_{1j}) = -2\pi\delta(2q_1\cdot k_{1j})f_{1j}(k_{1j})\, e^{-ic_1\vec{k}_{1j}\cdot\vec{b}}$$

and

$$a_{2j}(k_{2j}) = -2\pi\delta(2q_2\cdot k_{2j})f_{2j}(k_{2j})\, e^{ic_2\vec{k}_{2j}\cdot\vec{b}},$$

then, using the definition of $\vec{\Delta}$ above,

$$B_{n_1n_2}(\vec{b}) = (-1)^{n_1+n_2}\int\left(\prod\frac{d^4k_1}{(2\pi)^4}\, 2\pi\delta(2q_1\cdot k_1)f_1(k_1)\right)\left(\prod\frac{d^4k_2}{(2\pi)^4}2\pi\delta(2q_2\cdot k_2)f_2(k_2)\right)$$
$$\times(2\pi)^4\delta^4(P_f-P_i-\sum k_1-\sum k_2)\, e^{-i\vec{\Delta}\cdot\vec{b}}F_{n_1n_2}(\{p\},\{k\}).$$

The other δ functions can be used to transform the energy δ function:

$$\begin{aligned}\delta(P_{f0}-P_{i0}-\sum k_{10}-\sum k_{20}) &= \delta(P_{f0}-P_{i0}-\vec{v}_1\cdot\sum\vec{k}_1-\vec{v}_2\cdot\sum\vec{k}_2)\\ &= (v_1+v_2)^{-1}\delta((v_1+v_2)^{-1}(P_{f0}-P_{i0})-\Delta_z)\\ &= (v_1+v_2)^{-1}\delta(\delta_z+\Delta_z),\end{aligned}$$

where the last step follows because, from energy conservation in the amplitude A, we can identify $P_{f0}-P_{i0}$ with

$$\begin{aligned}q_{1i0}+q_{2i0}-q_{1f0}-q_{2f0} &= v_1(q_{1iz}-q_{1fz})-v_2(q_{2iz}-q_{2fz})\\ &= -(v_1+v_2)\delta_z.\end{aligned}$$

(Note that our particular choice of the c_i's was used at this point.) This result gives

$$B_{n_1n_2}(\vec{b}) = (-1)^{n_1+n_2}(v_1+v_2)^{-1}\int\left(\prod\frac{d^4k_1}{(2\pi)^4}2\pi\delta(2q_1\cdot k_1)f_1(k_1)\right)\left(\prod\frac{d^4k_2}{(2\pi)^4}2\pi\delta(2q_2\cdot k_2)f_2(k_2)\right)2\pi\delta(\delta_z+\Delta_z)$$
$$\times(2\pi)^3\delta^3(\vec{P}_f-\vec{P}_i-\sum\vec{k}_1-\sum\vec{k}_2)e^{-i\vec{\Delta}\cdot\vec{b}}F_{n_1n_2}(\{p\},\{k\})$$

and, comparing with the expression for $A_{n_1n_2}$ above, we find that

$$A_{n_1n_2} = (v_1+v_2)\int d^2b\, e^{-i\vec{\delta}\cdot\vec{b}}\, e^{i\chi(b)}B_{n_1n_2}(\vec{b})\,.$$

The forms chosen for the a_{1j} and a_{2j} imply that

$$\Phi_{1j}(x) = -(2M_1)^{-1}V_{1j}\left(\left((\vec{r}_\perp + c_1\vec{b})^2 + \frac{(z-v_1t)^2}{1-v_1^{\,2}}\right)^{1/2}\right)$$

and

$$\Phi_{2j}(x) = -(2M_2)^{-1}V_{2j}\left(\left((\vec{r}_\perp - c_2\vec{b})^2 + \frac{(z+v_2t)^2}{1-v_2^{\,2}}\right)^{1/2}\right),$$

where, as in Sec. III,

$$V(r) = \int\frac{d^3k}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{r}}\hat{f}(\vec{k}^2)\,.$$

The amplitude A, involving two high-energy particles, can thus be approximately obtained from the amplitude B for the same process to occur in the presence of two sets of external potentials, Φ_1 and Φ_2, which are static and spherically symmetric in the rest frames of the two high-energy particles, but which in the frame under consideration are Lorentz-transformed so as to move with the corresponding particles. In this frame the potential centers move in the $\pm z$ directions and have relative impact parameter $\vec{b}$. The amplitude B for this configuration is then multiplied by the function $\exp[i\chi(b)]$ which accounts for the interaction between the two high-energy particles, by a factor $(v_1+v_2)\exp(-i\vec{\delta}\cdot\vec{b})$, and then integrated over the entire $\vec{b}$ plane to give the amplitude A.

V. CONCLUSION

It was shown above that under certain conditions a high-energy particle may be approximately equivalent to an external moving point source: Its influence on other particles is nearly the same as that of external potentials which are static and spherically symmetric in the average rest frame of the high-energy particle. The reason for this is that a high-energy particle without internal degrees of freedom is very little affected by its interactions, and the "feedback" it receives from the other particles can be almost ignored.[16] This "as given" nature is just the identifying feature which distinguishes external potentials or sources from their dynamically determined counterparts.

This observation, or generalizations of it, may prove useful in a variety of calculations. The simple case of two interacting high-energy particles is well known,[2-9] and has been extended to a restricted class of inelastic two-body collisions.[10] It should also be possible to develop approximate expressions for the absorptive corrections to productions reactions, at least in restricted kinematical regions.[17]

The results discussed above apply only when the high-energy particle changes neither its four-momentum significantly, nor its internal state, either virtually in its intermediate states or between the real initial and final states. It is probable, however, that they can be generalized, along the lines of some earlier work,[6,9] to include processes with a single "strongly inelastic" vertex where the high-energy particle's state is changed significantly. This would considerably broaden the class of processes which could be treated, although almost certainly at the expense of complicating the formulas.

ACKNOWLEDGMENT

The author would like to thank Professor G. Lüders for his hospitality at the Institute for Theoretical Physics at the University of Göttingen where a large part of this work was completed.

*Research supported in part by NSF Grant No. GP-28025.

[1]See, for example, the papers by F. Gross, Phys. Rev. 186, 1448 (1969), and by C. Fronsdal and R. W. Huff, Phys. Rev. D 3, 933 (1971), and other references given there.

[2]R. Torgerson, Phys. Rev. 143, 1194 (1966).

[3]H. Cheng and T. T. Wu, Phys. Rev. 186, 1611 (1969).

[4]S.-J. Chang and S. Ma, Phys. Rev. Letters 22, 1334 (1969).

[5]H. D. I. Abarbanel and C. I. Itzykson, Phys. Rev. Letters 23, 53 (1969).

[6]M. Lévy and J. Sucher, Phys. Rev. 186, 1656 (1969).

[7]F. Englert, P. Nicoletopoulos, R. Brout, and C. Truffin, Nuovo Cimento 64, 561 (1969).

[8]H. M. Fried and T. K. Gaisser, Phys. Rev. 179, 1491

(1969).
[9]R. Blankenbecler and R. L. Sugar, Phys. Rev. D 2, 3024 (1970).
[10]A special case of this result has previously been obtained by the author: Phys. Rev. D 4, 840 (1971).
[11]G. Tiktopoulos and S. B. Treiman, Phys. Rev. D 2, 805 (1970).
[12]I. J. Muzinich, G. Tiktopoulos, and S. B. Treiman, Phys. Rev. D 3, 1041 (1971).
[13]H. Cheng and T. T. Wu, Phys. Rev. D 3, 2394 (1971).
[14]H. Cheng and T. T. Wu, Phys. Rev. D 3, 2397 (1971).
[15]See, however, the recent discussion by S. Weinberg, Phys. Letters 37B, 494 (1971).
[16]A related analysis for nonrelativistic potential scattering has been given by E. A. Remler, Ann. Phys. (N. Y.) 67, 114 (1971).
[17]A preliminary calculation of this type was made some time ago by A. M. Saperstein and E. Shrauner, Phys. Rev. 163, 1559 (1967).

PHYSICAL REVIEW D VOLUME 5, NUMBER 4 15 FEBRUARY 1972

Consistency of Current-Algebra Results in the Presence of Electromagnetism*

M. A. González†
Department of Physics, University of Guelph, Guelph, Ontario, Canada
and Department of Physics, University of Washington, Seattle, Washington 98105

(Received 14 October 1971)

We demonstrate in this paper that despite the appearance of dynamical terms in the current-commutation relations as a result of the electromagnetic interaction, one can still use the algebra of currents to derive low-energy results.

INTRODUCTION

It is generally assumed that the algebra of the currents is preserved even in the presence of symmetry-breaking effects. However, when electromagnetic fields are included, two effects combine to make their appearance felt in the commutation relations: (1) The general principles of quantum theory require that gradient terms be included in the commutator of time and space components, and (2) gauge invariance then effectively modifies these commutation relations through the substitution $\partial_\mu \to \partial_\mu - ieA_\mu$. These terms can be explicitly calculated in several models, e.g., spin-zero electrodynamics, the algebra of fields,[1] etc.

In this paper we show that the inclusion of these new dynamical terms does not affect the general results obtained from current algebra by means of low-energy theorems, and that one can still make model-independent calculations provided that one is careful with relativity and gauge invariance. Furthermore, we show that the appearance of noncanonical terms is in fact necessary if one is to avoid certain paradoxes.[2]

A LOW-ENERGY THEOREM

Consider the one-photon–vacuum matrix element of the vector-vector current correlation function $iT^*(V_+^\mu(x)V_-^\nu(0))$, namely,

$$T_{+-}^{\mu\nu}(k,q)=\int dx\, e^{-iq\cdot x} i\langle\gamma(k)|T^*(V_+^\mu(x)V_-^\nu(0))|0\rangle, \tag{1}$$

where the T^* product is the covariant time-ordered product[3]

$$iT^*(\ )=iT(\ )+\rho_{+-}^{\mu\nu}\delta^4(x)$$

and $\rho_{+-}^{\mu\nu}$ is the coefficient of the gradient term in the commutator

$$[V_+^0(x),\ V_-^\mu(0)]\delta(x^0)=2V_3^\mu(x)\delta^4(x) + i(\partial_\lambda+ieA_\lambda)\rho_{+-}^{\lambda\mu}(x)\delta^4(x), \tag{2}$$

with $\rho_{+-}^{\mu\nu}=\rho_{-+}^{\nu\mu}$ and $\rho_{+-}^{0\mu}=0$. The explicit appearance of A_λ in the commutator is required by gauge invariance.[4-6] Calculating the divergence of (1) in the standard way, one obtains

$$iq_\mu T_{+-}^{\mu\nu}=\int dx\, e^{-iq\cdot x} i\langle\gamma|T(D_+(x)V_-^\nu(0))|0\rangle - ie\langle\gamma|A_\lambda\rho_{+-}^{\lambda\nu}|0\rangle+2i\langle\gamma|V_3^\nu(0)|0\rangle. \tag{3}$$

One can view Eq. (3) as a way of calculating the matrix element $\langle\gamma|V_3^\mu(0)|0\rangle$. On the other hand, one can show[7] on the basis of Lorentz and gauge invariance alone that the matrix element of a gauge-invariant four-vector field between the vacuum and the one-photon state must vanish, i.e.,

$$\langle\gamma(\vec{k})|J^\mu(0)|0\rangle=0 \tag{4}$$

whether J^μ is conserved or not. The inclusion of