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[1]A. Böhm, (a) Phys. Rev. D 3, 367 (1971); (b) *ibid.* 3, 377 (1971); (c) Phys. Rev. Letters 23, 436 (1969). The notation of Refs. 1 and 2 is used. The present work is essentially an extension of the work reported in Ref. 1 and makes use of the results contained therein.

[2]A. Böhm, Phys. Rev. 158, 1408 (1967).

[3]A. Böhm and E. C. G. Sudarshan, Phys. Rev. 178, 2264 (1969); 182, 1918 (1969).

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[5]E.g., M. Gell-Mann, in *Strong and Weak Interactions: Present Problems*, edited by A. Zichichi (Academic, New York, 1967), p. 173; in *Hadrons and their Interactions*, edited by A. Zichichi (Academic, New York, 1967), p. 169. We use the mathematicians' notation for the SU(3) basis and our notation differs from the one conventionally used in physics literature. The connection is

$V_\mu^{\pm 1}$ corresponds to $\mathfrak{F}_{1\mu} \pm i\,\mathfrak{F}_{2\mu}$,

$A_\mu^{\pm 1}$ corresponds to $\mathfrak{F}_{1\mu}^5 \pm i\,\mathfrak{F}_{2\mu}^5$,

$V_\mu^{\pm 2}$ corresponds to $\mathfrak{F}_{4\mu} \pm i\,\mathfrak{F}_{5\mu}$,

$A_\mu^{\pm 2}$ corresponds to $\mathfrak{F}_{4\mu}^5 \pm i\,\mathfrak{F}_{5\mu}^5$,

etc.

[6]*Proof of (14c)*. From $[\Gamma_i, \Gamma_j] = -i S_{ij}$ and (9c) it follows that $[U_P \Gamma_i U_P^{-1}, U_P \Gamma_i U_P^{-1}] = -i S_{ij}$. If Γ_i has a definite transformation property under U (i.e., $U_P \Gamma_i U_P^{-1} = \eta \Gamma_i$), then it follows that $\eta^2 = 1$. Since $U^\dagger = U^{-1}$, it follows that $|\eta|^2 = 1$ and, consequently, that $\eta = +1$ or $\eta = -1$. The proofs are similar for (14a) and (14b).

[7]Haim Goldberg, Nuovo Cimento 60A, 509 (1969); E. P. Wigner, in *Group Theoretical Concepts and Methods in Elementary Particle Physics*, Lectures of the Istanbul Summer School of Theoretical Physics, edited by F. Gürsey (Gordon and Breach, New York, 1964), p. 37.

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Self-Consistent Quark Model*

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We construct a model in which the mass of the quark, two-quark, and quark-quark-antiquark states diverge while a quark and an antiquark and three quarks have finite-mass bound states. In this model the quarks are quark-meson bound states. All bound states are formed by a 4-fermion point interaction. For simplicity we use the static limit of the ladder approximation.

I. INTRODUCTION

The introduction of quarks into hadron physics[1] may have improved tremendously the understanding of strongly interacting particles. However, the interactions of the quarks themselves are not clearly understood through this concept. Indeed, no one (to the author's knowledge) has produced a consistent explanation of why no single-quark (q), two-quark, four-quark, or quark-quark-antiquark ($qq\bar{q}$) bound states have been observed, while quark-antiquark and three-quark bound states appear to abound in nature. It is the purpose of this paper to construct a model satisfying these experimental data; i.e., the existence or nonexistence of observable quarks and quark bound states. The most common explanation of the failure to observe a free quark in high-energy experiments is that the quark's physical mass is very high (more than ten times the mass of the proton), and therefore, it cannot be produced with the presently existing accelerators. This point of view will be adopted in this paper. The fact that no free quark has shown up in deep-ocean, moon rock, and numerous other samples can be accounted for by assuming a sufficiently high binding energy between a quark and an antiquark and between three quarks. If any free quarks ever existed at the beginning of the universe, they had time enough to meet their kin and form bound states with such large binding energy that they never disintegrated into free quarks. The question now is: What produces this high binding energy between, e.g., a quark and an antiquark? The simplest answer is the exchange of mesons (Fig. 1). The coupling constant G of quarks and mesons can always be chosen high enough as to account for any binding energy. However, if the exchanged particle is any of the known mesons, with a mass of a few hundred MeV, then the range of such forces will be comparable to the distances between the nucleons in a nucleus. In this case the quarks of different nucleons in the same nucleus will be able to interact so strongly as to make nuclear fusion and fission impossible, at least with the experimentally known energies. In order to avoid this difficulty we will assume that the binding of $q\bar{q}$ into mesons is due to a zero-range force [a 4-fermion point interaction (4-f.p.i.)] with respect to which contributions from meson exchanges are negligible.

From Fig. 2 we see that two quarks and an antiquark exchange a quark, using the 4-f.p.i., and could possibly be very strongly bound, e.g., into a quark. In a similar way, three quarks can exchange a quark, thus generating a short-range interaction between three quarks.

In order to simplify the initial presentation of this formalism, we will restrict ourselves to one quark only, with spin $\frac{1}{2}$, baryon number $\frac{1}{3}$, and all other quantum numbers 0. Furthermore, every particle and bound state will be taken to be in the lowest allowed energy and angular momentum state.

In Sec. II we define our basic 4-fermion point interaction. We show that it is possible, in the ladder approximation, to adjust the strength of the interaction in such a way that $q\bar{q}$ will be bound into a scalar meson while qq has no bound states.

In Sec. III, we will use the effective $q\bar{q}\mu$ interaction (where μ represents the meson) to show that $q\mu$ bind into q in the static limit of the ladder approximation.

In Sec. IV, we will show that qqq bind into baryons of spin $\frac{1}{2}$ and $\frac{3}{2}$ in our approximation.

In Sec. V, we will generalize our model to include three distinguishable quarks, of the same mass, and we will show that we obtain SU(3) (the Cutkosky relations), the additive and independent quark model, and the exclusion of exotic quarks and baryons.

In Sec. VI, we discuss the conclusions and prospects of this theory.

II. THE TWO-PARTICLE BOUND STATES

In order to simplify the presentation of this model and to be able to use some of the work of Nambu and Jona-Lasinio[2] (NJL) we choose as our basic interaction

$$g:(\bar{\psi}\psi\bar{\psi}\psi+\bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi):\ , \tag{1}$$

where : : represents the normal product.

NJL[2] showed that the interaction (1), used in the ladder approximation, for the fermion-antifermion and fermion-fermion scattering amplitudes leads to the following conditions for the existence of bound states (poles in the scattering amplitudes):

(1) There is a scalar-meson bound state of $q\bar{q}$ at a mass m_S if

$$\frac{g}{4\pi^2}\int_{4m^2}^{\Lambda^2}\frac{(s-4m^2)(1-4m^2/s)^{1/2}}{s-m_S{}^2}\,ds=1, \tag{2}$$

where m is the quark mass.

(2) There is a pseudoscalar-meson bound state of $q\bar{q}$ at a mass m_P if

$$\frac{g}{4\pi^2}\int_{4m^2}^{\Lambda^2}\frac{s(1-4m^2/s)^{1/2}}{s-m_P{}^2}\,ds=1. \tag{3}$$

(3) There is a vector-meson bound state of $q\bar{q}$ at a mass m_V if

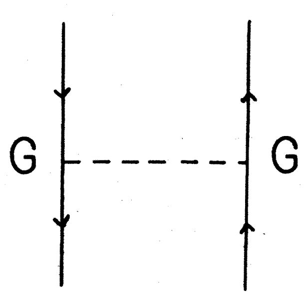

FIG. 1. One-meson exchange between a quark and an antiquark (the dashed line represents the meson, full lines represent quarks).

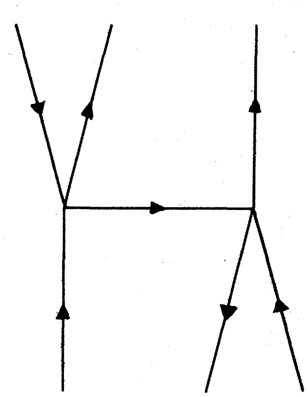

FIG. 2. One-quark exchange between two quarks and an antiquark.

$$-\frac{g}{4\pi^2}\frac{m_V^2}{3}\int_{4m^2}^{\Lambda^2}\frac{(1-4m^2/s)^{1/2}}{s-m_V^2}\left(1+\frac{2m^2}{s}\right)ds=1. \tag{4}$$

(4) There is an axial-vector-meson bound state of $q\bar{q}$ at a mass m_A if

$$\frac{g}{4\pi^2}\int_{4m^2}^{\Lambda^2}\frac{(1-4m^2/s)^{1/2}}{s-m_A^2}\left[\frac{m_A^2}{3}\left(1+\frac{2m^2}{s}\right)+\frac{m^2}{2}\right]ds=1. \tag{5}$$

(5) There is a qq, "diquark," bound state at a mass m_D if

$$\frac{gm^2}{2\pi^2}\int_{4m^2}^{\Lambda^2}\frac{(1-4m^2/s)^{1/2}}{s-m_D^2}\,ds=1. \tag{6}$$

These equations are obtained by summing over all bubble chains and identifying the sum, near its pole, with the corresponding bound state (Fig. 3). The vector and axial-vector interactions were obtained from the scalar and pseudoscalar interactions by using Fierz transformations which guarantee that all the bubbles are free of Dirac indices coming from the propagators (the two propagators of a bubble are contracted with each other at the two ends of the bubble, and never with the propagators of neighbor bubbles). Using the factorization theorem (Appendix A) we see that each term is the simple product of the bubbles in it, all bubbles in a chain having the same value. The theory considered by NJL possesses chiral symmetry from which they conclude that $m_P=0$ and all other masses are determined from this condition. In our theory, however, the total Hamiltonian is not necessarily chiral-invariant and there is no reason to fix m_P or any other mass *a priori*.

When the integrals in Eqs. (2) to (6) are worked out, they are functions of the dimensionless parameters, $4\pi^2/g\Lambda^2$, $2m/\Lambda$, m_S/Λ, m_P/Λ, m_V/Λ, m_A/Λ, and m_D/Λ. When we compute explicitly the integrals (2) to (6) as functions of the above parameters and try to satisfy Eqs. (2) to (6) with different values for the parameters, it becomes clear that there is no set of values of those parameters for which all five equations are satisfied. In particular, for $2m/\Lambda \le 10^{-2}$ there is a set of values for $4\pi^2/g\Lambda^2$, and m_S/Λ, for which (3) is satisfied but (4), (5), (6), and (7) are not. Thus, if for simplicity we impose the existence of a finite-mass scalar meson and $2m/\Lambda \le 10^{-2}$, then we will have no pseudoscalar, vector, axial-vector mesons, or a qq bound state. The above results were obtained through a numerical analysis of Eqs. (2) to (6) done on a computer.

In the same way as NJL, we then define the "phenomenological" coupling constant $g_{\bar{q}qS}$ between the quarks and the scalar meson as the residue at the pole of the scattering amplitude:

$$\frac{g_{\bar{q}qS}^{\;2}}{4\pi}=2\pi\left[\int_{4m^2}^{\Lambda^2}\frac{(s-4m^2)(1-4m^2/s)^{1/2}}{(s-m_S^{\,2})^2}ds\right]^{-1}. \tag{7}$$

III. THE $qq\bar{q}$ BOUND STATE

Having established in Sec. II that $q\bar{q}$ bind into a scalar meson and that there are no qq bound states, we can approximate the $qq\bar{q}$ scattering by a quark-meson scattering. The scattering amplitude of qS in the ladder approximation is graphically illustrated in Fig. 4 and leads to a rather complicated Bethe-Salpeter equation. Instead of trying to solve this equation exactly, we will limit ourselves in this paper to a preliminary study of its properties. In particular, we will consider a simple equation obtained from Fig. 3 after each of the exchanged-quark's propagators has been replaced by $1/m$. An intuitive, though by no means rigorous, justification of this prescription is that if the mass of the quark is as high as suggested by experimental evidence (and by this model as we shall see), it might be permissible to neglect the exchanged momentum with respect to the quark mass. The condition that a quark and a meson bind into a quark is then illustrated by Fig. 5, where the $\bar{q}qSS$ coupling constant is given by

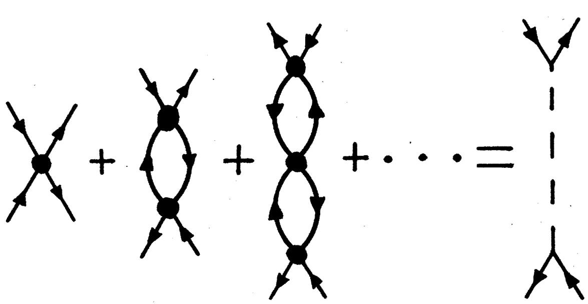

FIG. 3. The quark-antiquark bound-state condition in the ladder approximation for a 4-fermion point interaction.

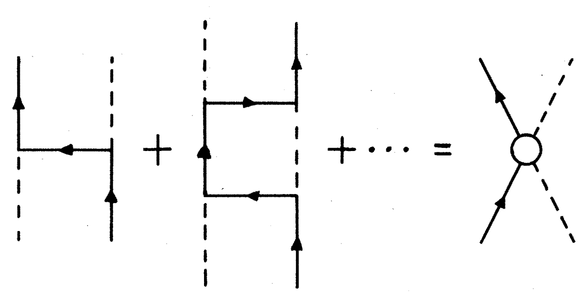

FIG. 4. The quark-meson scattering amplitude in the ladder approximation.

$$g_{q\bar{q}SS} = g_{q\bar{q}S}{}^2/m \;, \tag{8}$$

where $g_{q\bar{q}S}$ was defined in Sec. II.

If we call $A(p)$ the sum at the left-hand side of Fig. 5 and $B(p)$ the analytic expression of the qS bubble with $g_{q\bar{q}SS}$ attached to one end of the bubble, $A(p)$ is given by

$$[1 - B(p)][A(p) - g_{q\bar{q}SS}] = g_{q\bar{q}SS}B(p). \tag{9}$$

For $p^2 \approx m^2$, $B(p)$ can be written as

$$B(p) = (\not{p} + m)I(p) \tag{10}$$

and

$$I(p) = \frac{g_{q\bar{q}SS}}{\pi}\int_{(m+m_S)^2}^{\Lambda'^2} \frac{\rho_{qS}(s)}{s - p^2}\, ds \;, \tag{11}$$

where $\rho_{qS}(s)$ is given by (see, e.g., Nishijima, Ref. 3),

$$\rho_{qS}(s) = s^{-1}[s - (m - m_S)^2]^{1/2}[s - (m + m_S)^2]^{1/2} \tag{12}$$

and Λ' is a cutoff parameter related to Λ by the consistency requirement. Since any power of $B(p)$ will be proportional to $\not{p} + m$, we can write $A(p)$ as

$$A(p) = (\not{p} + m)K(p). \tag{13}$$

The condition that $q\bar{q}q$ bind to give a quark is that near $p^2 = m^2$ we have

$$A(p^2 \approx m^2) = g_{\bar{q}qS}{}^2(\not{p} + m)/(p^2 - m^2). \tag{14}$$

For $p^2 \approx m^2$, $g_{q\bar{q}SS}$ can be neglected with respect to $A(p)$ and the bound-state condition becomes

$$2mI(p^2 = m^2) = 1 \tag{15}$$

with a new expression for $g_{q\bar{q}S}$:

$$\frac{g_{\bar{q}qS}}{4\pi} = 2\pi\left[\int_{(m+m_S)^2}^{\Lambda^2} \frac{\rho_{qS}(s)}{(s - m_S{}^2)^2}\, ds\right]^{-1} . \tag{16}$$

We now require that the "new" value of $g_{q\bar{q}S}$ agree

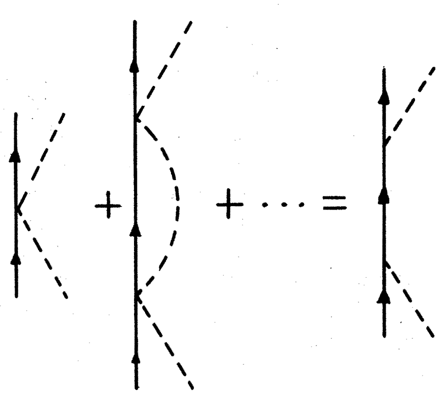

FIG. 5. The quark-meson bound-state condition in the static limit of the ladder approximation.

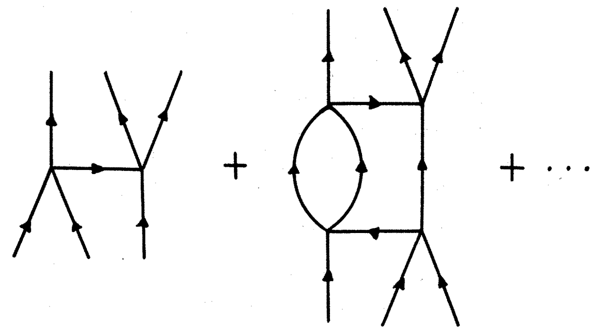

FIG. 6. The three-quark scattering amplitude in the ladder approximation.

with the previous one (this condition will relate the cutoff Λ' to the previous cutoff Λ) while the qS bound-state condition is satisfied. We thus get two new relations involving the parameters m/Λ and m_S/Λ. A numerical analysis of Eqs. (2), (7), (8), (15), and (16) determines the values of the parameters to be

$$m/\Lambda \approx 10^{-4}, \quad 1.1\times 10^{-5} \leqslant m_S/m \leqslant 1.1\times 10^{-4},$$
$$gm^2 \approx 0.25\times 10^{-3}, \quad g_{q\bar{q}S}{}^2 \approx 2.37,$$
$$\Lambda/\Lambda' = 0.9\times 10^{-4}.$$

We verify that for these values there is indeed only a scalar $q\bar{q}$ bound state and no qq bound state. The numbers thus obtained are quite reasonable, but are too crude for detailed comparison with experimental data because of our approximate treatment of the Bethe-Salpeter equation.

IV. THE qqq BOUND STATE

For the qqq scattering we consider the ladder approximation illustrated by Fig. 6. As in Sec. III we restrict ourselves to the study of the simpler graphs obtained from Fig. 6 by replacing the exchanged-quark's propagators by $1/m$, thus reducing Fig. 6 to Fig. 7 where the effective 6-fermion coupling constant G is given by

$$G = g^2/m. \tag{17}$$

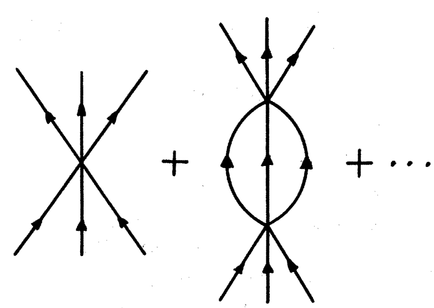

FIG. 7. The static limit of Fig. 6.

In order to satisfy Eq. (2) for $\Lambda\to\infty$ we must have $g\to 1/\Lambda^2$ as $\Lambda\to\infty$, and in order to satisfy (15) we must have $m\to\Lambda'$ (or Λ) as $\Lambda'\to\infty$ (which is consistent with the idea of a heavy quark). Therefore, Eq. (17) implies that $G\to 1/\Lambda^5$ as $\Lambda\to\infty$.

The sum $\mathcal{A}$ of the bubble chains for three quarks (Fig. 7) is given schematically by (see Appendix B)

$$(1-\mathcal{B})(\mathcal{A}-3!\overline{G})=3!\mathcal{B}\overline{G}\,, \tag{18}$$

where $\mathcal{B}$ is a tensor with six spinor indices representing the three-quark bubble and has the form

$$\begin{aligned}\mathcal{B}^{\alpha\beta\gamma}_{\delta\eta\epsilon}=3(2\pi)^{-12}\overline{G}^{\{\alpha'\beta'\gamma'\}}_{\delta\eta\epsilon}&\\ \times\{m^2(m\delta^{\alpha}_{\{\alpha'}\delta^{\beta}_{\beta'}\delta^{\gamma}_{\gamma'\}}+\not{p}^{\alpha}_{\{\alpha'}\delta^{\beta}_{\beta'}\delta^{\gamma}_{\gamma'\}})I_1&\\ +[m\delta^{\alpha}_{\{\alpha'}(\gamma_\mu)^{\beta}_{\beta'}(\gamma_\mu)^{\gamma}_{\gamma'\}}+\not{p}^{\alpha}_{\{\alpha'}(\gamma_\mu)^{\beta}_{\beta'}(\gamma_\mu)^{\gamma}_{\gamma'\}}]I_2\},&\end{aligned} \tag{19}$$

where the brackets denote symmetrization over the primed indices. I_1 and I_2 are given in Appendix B. $\overline{G}$ is given by

$$\overline{G}^{\alpha\beta\gamma}_{\delta\eta\epsilon}=G[\delta^{\alpha}_{\delta}\delta^{\beta}_{\eta}\delta^{\gamma}_{\epsilon}+\delta^{\alpha}_{\delta}(\gamma_5)^{\beta}_{\eta}(\gamma_5)^{\gamma}_{\epsilon}]\,, \tag{20}$$

where G is the 6-f.p.i. induced by the 4-f.p.i. Since the bubble $\mathcal{B}$ is made out of three quarks, it has spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ components, and so will the sum $\mathcal{A}$ and we can write

$$\mathcal{B}=\mathcal{B}^{(1/2)}+\mathcal{B}^{(3/2)}\,,$$

$$\mathcal{A}=\mathcal{A}^{(1/2)}+\mathcal{A}^{(3/2)}\,,$$

where $\mathcal{B}^{(1/2)}$, $\mathcal{B}^{(3/2)}$, $\mathcal{A}^{(1/2)}$, and $\mathcal{A}^{(3/2)}$ are the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ components of $\mathcal{B}$ and $\mathcal{A}$, respectively. Therefore, Eq. (18) can be decomposed into

$$(1-\mathcal{B}^{(1/2)})(\mathcal{A}^{(1/2)}-3!\overline{G})=3!\overline{G}\mathcal{B}^{(1/2)}\,, \tag{21a}$$

$$(1-\mathcal{B}^{(3/2)})(\mathcal{A}^{(3/2)}-3!\overline{G})=3!\overline{G}\mathcal{B}^{(3/2)}\,. \tag{21b}$$

Consider now the condition that three quarks in a spin-$\frac{1}{2}$ state are bound into a baryon N of mass M and three quarks in a spin-$\frac{3}{2}$ state are bound into a baryon N^* of mass M^*, and those baryons interact phenomenologically with the quarks according to

$$G_N q_\alpha C^{\alpha\beta}q_\beta q_\gamma \overline{N}^\gamma+\text{H.c.} \tag{22}$$

and

$$G_{N^*}q_\alpha(C\gamma_\mu)^{\alpha\beta}q_\beta q_\gamma(\overline{N}^*_\mu)^\gamma+\text{H.c.} \tag{23}$$

with symmetrization over spinor indices. C is the charge-conjugation matrix, q_α is the quark field, N_α is the spin-$\frac{1}{2}$ baryon field, and $N^*_{\mu\,\alpha}$ is the Rarita-Schwinger field for the spin-$\frac{3}{2}$ baryon. G_N and G_{N^*} are effective coupling constants to be determined. With these conditions, we can replace $\mathcal{A}^{(1/2)}$ and $\mathcal{A}^{(3/2)}$ near $p^2=M^2$ and $p^2=M^{*2}$, respectively, by

$$(\mathcal{A}^{(1/2)})^{\alpha\beta\gamma}_{\delta\eta\epsilon}(p^2\approx M^2)=G_N{}^2C^{\alpha}_{\delta}[(\not{p}+M)/(p^2-M^2)]^{\beta}_{\eta}(C^+)^{\gamma}_{\epsilon}\,, \tag{24}$$

$$(\mathcal{A}^{(3/2)})^{\alpha\beta\gamma}_{\delta\eta\epsilon}(p^2=M^{*2})=G_{N^*}{}^2(C\gamma_\mu)^{\alpha}_{\delta}\left[\frac{\not{p}+M^*}{p^2-M^{*2}}\left(g_{\mu\nu}-\frac{1}{3}\gamma_\mu\gamma_\nu-\frac{2p_\mu p_\nu}{3M^{*2}}+\frac{p_\mu\gamma_\nu-p_\nu\gamma_\mu}{3M^*}\right)\right]^{\beta}_{\eta}[(C\gamma_\nu)^+]^{\gamma}_{\epsilon}\,, \tag{25}$$

and we can neglect the term $\overline{G}\,3!$ with respect to $\mathcal{A}^{(1/2)}$ and $\mathcal{A}^{(3/2)}$ near $p^2=M^2$ and $(M^*)^2$, respectively. In order to obtain $\mathcal{B}^{(1/2)}$, we use $(M+\not{p})/2M$ to project the N state out of $\mathcal{B}$. Since we know that only the term proportional to I_1 will contribute to the spin-$\frac{1}{2}$ state, we get

$$\begin{aligned}(\mathcal{B}^{(1/2)})^{\alpha\beta\gamma}_{\delta\eta\epsilon}&=\tfrac{3}{2}(2\pi)^{-12}m^2I_1\overline{G}^{\alpha\beta\gamma}_{\{\delta'\eta'\epsilon'\}}[(m+M)\delta^{\{\delta'}_{\delta}\delta^{\eta'}_{\eta}\delta^{\epsilon'\}}_{\epsilon}+(1+m/M)\not{p}^{\{\delta'}_{\delta}\delta^{\eta'}_{\eta}\delta^{\epsilon'\}}_{\epsilon}]\\ &\approx\tfrac{3}{2}(2\pi)^{-12}m^2I_1\overline{G}^{\alpha\beta\gamma}_{\{\delta'\eta'\epsilon'\}}(m/M)(M\delta^{\{\delta'}_{\delta}\delta^{\eta'}_{\eta}\delta^{\epsilon'\}}_{\epsilon}+\not{p}^{\{\delta'}_{\delta}\delta^{\eta'}_{\eta}\delta^{\epsilon'\}}_{\epsilon})\,,\end{aligned} \tag{26}$$

where we assumed $M/m\approx 0$. $\mathcal{B}^{(3/2)}$ will be given by $\mathcal{B}-\mathcal{B}^{(1/2)}$. Inserting (24) and (26) in (21a), for $p^2\approx M^2$, we see that the left-hand side has a pole at $p^2=M^2$ that must be balanced by a zero at $p^2=M^2$:

$$(\not{p}+M)[1-(2\pi)^{-12}18m^3I_1(p^2=M^2)\overline{G}]=0 \tag{27}$$

or

$$G\sim[I(p^2=M^2)m^3]^{-1}\sim\Lambda^{-5}, \tag{28}$$

which is implied by the $q\overline{q}$ and qS bound-state condition as shown in the beginning of this section. Therefore, the condition for three quarks in a spin-$\frac{1}{2}$ state to have a bound state with a finite mass M to be determined from (27) can be satisfied. Simply by looking at the dimensions of the terms in Eqs. (21a), (21b), and (25) we see that the bound-state condition for three quarks in a spin-$\frac{3}{2}$ state reduces also to $G\sim\Lambda^{-5}$, while the precise equation will be quite complicated and will depend on M^*. It is not necessary for our purpose to write down this equation since we are not interested in the explicit value of M^* but merely in its existence. We only argue here that our model has spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ qqq bound states, but all we might be able to compute is M/Λ'', M^*/Λ'', $G_N\Lambda''^2$, and $G_{N^*}\Lambda''^2$. Here Λ'' is the cutoff parameter in the integrals I_1 and I_2. Thus, we are not able to compute M or M^* but only M/M^*, G_NM^2, and $G_{N^*}M^2$. Elaborate numerical calculations might allow us to determine those quantities. But even if our approximations are well justified (up to, e.g., 1%), the tremendous mass of the quark can imply an error of several times the mass of the bound states in the computed masses M and M^* and render these values meaningless. Therefore, we do not undertake any numerical calculations here.

G_N and G_{N^*} can be determined from the residues at the poles of Eqs. (21a) and (21b), respectively.

V. SU(3)

A. The Additive Quark Model

When we generalize our model to include three distinguishable quarks (but degenerate in mass) we have to rewrite the basic 4-f.p.i. in such a way that any pair $q_i\overline{q}_j$ $(i,j=1,2,3)$ will be bound. For simplicity we will discuss here only the $q\overline{q}q\overline{q}$ term of the interaction. (The same discussion will apply trivially to the $\overline{q}\gamma_5 q\overline{q}\gamma_5 q$ term.) In the present discussion q represents

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix},$$

where each q_i $(i=1,2,3)$ is a Dirac spinor. If the interaction is given by

$$g:\overline{q}q\overline{q}q:=g:(\overline{q}_1q_1\overline{q}_1q_1+\overline{q}_2q_2\overline{q}_2q_2+\overline{q}_3q_3\overline{q}_3q_3+2\overline{q}_1q_1\overline{q}_2q_2+2\overline{q}_1q_1\overline{q}_3q_3+2\overline{q}_2q_2\overline{q}_3q_3):\ ,$$

it can be seen that the bound-state condition of $\overline{q}_iq_j$ in our approximation is $2gJ(p^2=\mu^2)=1$ for $i\neq j$ or $8gJ(p^2=\mu^2)=1$ for $i=j$, where $J(p^2)$ is the analytic expression of the q_iq_j bubble, p is the total momentum, and μ is the mass of the bound state. The factors 2 and 8 come from the different contractions prescribed by Wick's theorem and from the conservation of isospin and hypercharge. (E.g., only $q_1\overline{q}_2$ bubbles contribute to $q_1\overline{q}_2$ scattering, but $q_1\overline{q}_1$, $q_2\overline{q}_2$, and $q_3\overline{q}_3$ bubbles contribute to $q_1\overline{q}_1$ scattering.) Notice that in this section we do not prove that our model implies SU(3), but we simply show that our formalism can be generalized to include SU(3).

We see that with a simple generalization of the basic 4-f.p.i. it is impossible to have $q_i\overline{q}_j$ bound states both for $i=j$ and $i\neq j$. In order to have the same bound-state condition for the case $i=j$ and for the case $i\neq j$, we must choose as our basic interaction

$$g:[\overline{q}q\overline{q}q-\tfrac{3}{2}(\overline{q}\Lambda_3q\overline{q}\Lambda_3q+\overline{q}\Lambda_8q\overline{q}\Lambda_8q)]:, \qquad (29)$$

where Λ_3 and Λ_8 are given by

$$\Lambda_3=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix},\quad \Lambda_8=\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&2\end{pmatrix}.$$

With this interaction the bound-state condition for $\overline{q}_iq_j$ is $4gJ(p^2=\mu^2)=1$, whether $i=j$ or $i\neq j$, thus we can choose (29) (plus its pseudoscalar counterpart) as our basic interaction, and all the results previously obtained for a single quark will remain valid. We notice that (29) commutes with Λ_3 and Λ_8, therefore the isospin and hypercharge will be conserved as expected. Therefore, the SU(3) quantum numbers of a bound state of quarks will be the sum of the SU(3) quantum numbers of the composing quarks. We thus see that the present generalization of our formalism is compatible with the additive quark model.

B. The Cutkosky Relations

We will examine the quark-meson bound-state condition in the ladder approximation, assuming that we have three distinguishable quarks of the same mass m.[4] Before reducing the relevant graphs into bubble chains according to the prescription of Sec. III, we will stop the reduction process at the ladder level and only later will we replace the exchanged quark propagators by $1/m$. In the ladder approximation, the bound-state condition is given by the Bethe-Salpeter equation:

$$g_{ij}^{\alpha}=\sum_{\beta kl}g_{kl}^{\alpha}g_{li}^{\beta}g_{kj}^{\beta}\int S_lS_k\Delta_\beta\,, \qquad (30)$$

which is illustrated by Fig. 8.

In (30) S_l represents the corresponding quark propagator, Δ_β the meson propagator. The integration is over internal momentum. g_{ij}^{α} are the $\overline{q}q\mu$ coupling constants which are obtained from the $q\overline{q}$ bound-state condition. The indices i, j, k, and l are quark indices and take the values 1, 2, and 3; α and β are meson indices and take the values 1–8 or 1.

Lurié and Macfarlane[5] have shown that a theory with a 4-fermion point interaction in which $q\overline{q}$ bind into a meson is equivalent to a Yukawa theory in which the renormalization constant for the meson propagator is zero. Kaus and Zachariasen[6] have shown that in this case we have

$$J'(\mu^2)\sum_{ij}g_{ij}^{\alpha}g_{ij}^{\beta}=\delta^{\alpha\beta}\,, \qquad (31)$$

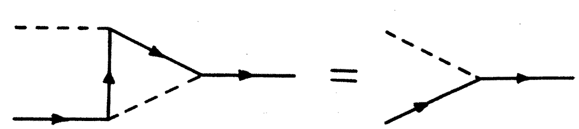

FIG. 8. The quark-meson bound-state condition (Bethe-Salpeter equation) in the ladder approximation.

where

$$J'(\mu^2)=\frac{\partial}{\partial p^2}J(p^2)\bigg|_{p^2=\mu^2} \tag{32}$$

and $J(p^2)$ is the $q\bar{q}$ bubble. We reserve to a forthcoming publication the discussion of the validity and the significance of all $Z=0$ conditions in our model.

Equations (30) and (31) are precisely the Cutkosky[4] relations which imply that g_{ij}^{α} are proportional to the structure constants of SU(3).

C. The Exclusion of Exotic Quarks

Let us consider again the $q\mu$ bound-state condition in the ladder approximation. Consider first the scattering amplitude of $\mathcal{P}$ (where $\mathcal{P}$ is the "proton" quark) and π^0. At each q exchange in the ladder approximation we have the two possibilities illustrated by Figs. 9 and 10, where $\mathfrak{N}$ is the "neutron" quark.

From Fig. 9 we see that the effective $\mathcal{P}\bar{\mathcal{P}}\pi^0\pi^0$ coupling constant in the ladder approximation, where the exchanged q propagators have been replaced by $1/m$, is given by $g_{\mathcal{P}\bar{\mathcal{P}}\pi^0}{}^2/m=\bar{g}^2/2m$, where $\bar{g}$ is the $\bar{\mathcal{P}}\mathfrak{N}\pi^+$ coupling constant. Also from Fig. 10 we see that the $\bar{\mathcal{P}}\mathfrak{N}\pi^+\pi^0$ coupling constant is $g_{\bar{\mathcal{P}}\mathfrak{N}\pi}+g_{\bar{\mathfrak{N}}\mathfrak{N}\pi^0}/m=\bar{g}^2/m\sqrt{2}$. Therefore, the sum of $\pi^0\mathcal{P}$ bubble chains will be

$$G'/[1-G'J_{q\mu}(p^2)]\,, \tag{33}$$

where

$$G'=\bar{g}^2(1+\sqrt{2}\,)/2m \tag{34}$$

and $J_{q\mu}(p^2)$ is the analytic expression of the quark-meson bubble. The bound-state condition will be

$$[(1+\sqrt{2}\,)/2m]\bar{g}^2J_{q\mu}(p^2=m^2)=1. \tag{35}$$

In the same way we can see that the $\mathfrak{N}\pi^0$ bound-state condition is precisely Eq. (35). Consider now the $\mathcal{P}\pi^-$ scattering amplitude. It is easily seen that at the first $\mathcal{P}\pi^-$ interaction the system will transform into $\mathfrak{N}\pi^0$. Since $\mathfrak{N}\pi^0$ has a bound state, so will $\mathcal{P}\pi^-$, but with a different residue at the pole. All the $q\mu$ bound states considered so far are regular quarks. Consider now the $\mathcal{P}\pi^+$ scattering amplitude. It is easily seen that the only interaction which will contribute is the one illustrated by Fig. 11 which determines the $\bar{\mathcal{P}}\mathcal{P}\pi^+\pi^-$ coupling constant to be $\bar{g}^2/m$. The $\mathcal{P}\pi^+$ bound-state condition is thus

$$(\bar{g}^2/m)J_{q\mu}(p^2=m^2)=1. \tag{36}$$

But since (35) is satisfied, (36) cannot be satisfied and we cannot have any $\mathcal{P}\pi^+$ bound state. An identical argument goes for all systems with exotic quark quantum numbers. It is also seen from (36) that the $\mathcal{P}\pi^0$ interaction is stronger by a factor of $(1+\sqrt{2})/2$ than the $\mathcal{P}\pi^+$ interaction, explaining why $\mathcal{P}\pi^+$ is not a bound state. It can be argued that the above argument is incorrect since (36) might be satisfied if the mass of the $\mathcal{P}\pi^+$ bound state is different from m (the regular quark mass). However, since the $\mathcal{P}\pi^+$ coupling is weaker then the $\mathcal{P}\pi^0$ coupling, $\mathcal{P}\pi^+$ cannot be bound stronger than $\mathcal{P}\pi^0$. Therefore, if a $\mathcal{P}\pi^+$ bound state exists, it is heavier than the quark. However, if $\mathcal{P}\pi^+$ can form a true bound state, the mass of the bound state should be less then the sum of the rest masses of $\mathcal{P}$ and π^+, i.e., less then $m(1+10^{-5})$ (see Sec. III). Obviously, it is very unlikely that a decrease of $\bar{g}$ in Eq. (36) by $0.176\bar{g}$ will cause an increase of m, in the same equation, of only $10^{-5}m$. We therefore conclude that there are no stable exotic quarks in the bubble-chain approximation, but there is a possibility of having highly unstable exotic quark resonances. In what follows we will ignore those possible resonances.

D. Exclusion of Exotic Baryons

So far we have avoided the question of possible bound states of four and five quarks. In principle, we can write a Bethe-Salpeter equation for the scattering amplitude of four quarks, in some generalized ladder approximation, using our initial 4-f.p.i. and investigate under what conditions poles

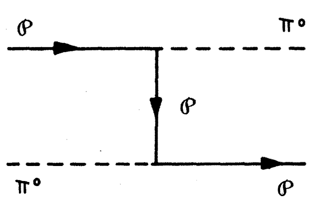

FIG. 9. The effective $\mathcal{P}\bar{\mathcal{P}}\pi^0\pi^0$ coupling constant.

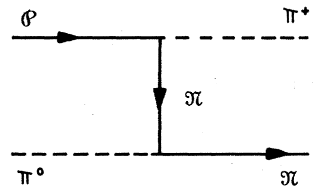

FIG. 10. The effective $\mathfrak{N}\bar{\mathcal{P}}\pi^+\pi^0$ coupling constant.

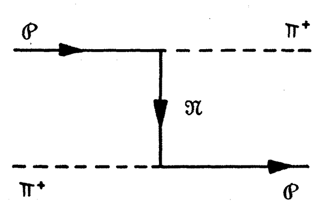

FIG. 11. The effective $\mathcal{P}\bar{\mathcal{P}}\pi^+\pi^-$ coupling constant.

develop in this scattering amplitude. This program, however, is too difficult to be considered seriously. We can instead try an easy explanation for the absence of four-quark bound states: Assume that the quarks obey an exclusion principle which forbids four quarks (or antiquarks) to propagate through the same point in space-time. Because the interactions between quarks (except for meson exchanges which we neglect here) have very short range in our theory, four quarks simply cannot interact. This "explanation" for the absence of four-quark bound states is of course more a restatement of the fact rather than the answer to the question. However, this assumption, in our formalism, will have the extra benefit of explaining the absence of exotic baryons. Indeed, as we have no exotic quarks in our formalism, the only way to produce exotic [e.g., in the $\underline{10}$ and $\underline{27}$ representations of SU(3)] baryons is to consider at least four quarks and one antiquark. It is easily seen, using the new exclusion principle, that the only acceptable graphs for $qqqq\bar{q}$ will be a nucleon (nonexotic) and a meson (nonexotic) propagating without interacting; or, if the appropriate quarks are available, three quarks (nonexotic) which will bind into a nonexotic baryon.

A similar argument holds for mesons. The only possibility to create exotic mesons is to start with two $q\bar{q}$ pairs. We then see that in the bubble-chain approximation we can have either two noninteracting mesons (nonexotic) or q and $\bar{q}$ which will combine into one nonexotic meson.

VI. CONCLUSIONS AND PROSPECTS

Traditionally, particles are divided into two classes: elementary and composite. In bootstrap theories, however, democracy is installed among the particles and all are composite. The theories based on quarks take a slightly different view: All observable particles are composite of quarks, but quarks form an aristocracy of truly elementary particles, thereby breaking the democracy. Our point of view is of partially conserved democracy: All particles are composite, including quarks; the quarks, however, conserve some of their aristocratic character since they are made only out of themselves and do not depend on other particles for their existence. All other particles are made out of quarks. Those particles can still be divided into two classes: those which are "essentially" bound states of quarks (lowest-lying bound states, like those discussed in this note), and those which can be considered as bound states of other observable particles (e.g., a vector meson might be composed of two spin-zero mesons). The rule of partially conserved democracy can thus be stated: "All particles are composite, some more, and some less." The advantage of this theory is that it contains the other three as different descriptions, with different points of emphasis, of the same situation. Indeed, it accepts the bootstrap-type philosophy that all particles are composite, it contains the original quark idea since all observable particles are made out of quarks (in our case, this is true, even for the quarks), and it should allow a phenomenological description where some particles can be considered as composed of some other observable particles. We emphasize however, that even though all particles are composite in our model, this is not a real bootstrap because not all parameters are determined by the self-consistency condition; e.g., the strength (g) of the initial 4-f.p.i. is a free parameter which we adjusted arbitrarily to ensure that $q\bar{q}$ bind into a scalar meson with no other $q\bar{q}$ nor qq bound states.

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APPENDIX A: THE FACTORIZATION THEOREM

Consider a $2n$-fermion point-interaction Hamiltonian. (For our purpose the cases $2n=4$ and 6 are sufficient, but we prefer to state the theorem in a more general form.) Consider a Feynman graph in momentum space, due to this interaction, which has a simple vertex such that if we cut the graph at this simple vertex, then the graph falls apart into two disconnected graphs. We will call those disconnected graphs A and B (see Fig. 12).

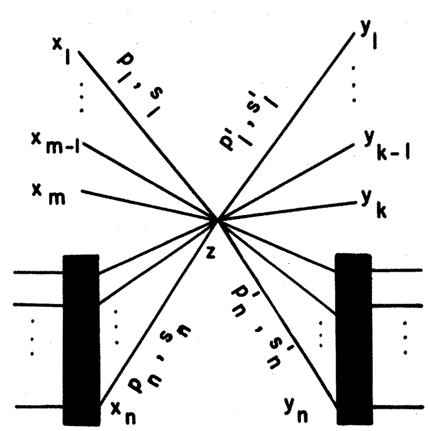

FIG. 12. A Feynman graph in momentum space with a factorizable vertex. The black boxes represent arbitrary Feynman graphs.

Then the factorization theorem tells us the following: (1) The analytic expression in momentum space of the above-considered graph will be the algebraic product of the analytic expressions of the two graphs A and B. By algebraic product we mean that if in the original graph l fermion propagators from A are contracted (through their Dirac indices) with l fermion propagators from B at the considered point, then the product of A and B will consist of l contractions over Dirac indices. (2) Each of the two graphs A and B will be independent of the relative momenta of the fermion propagators of the other graph. To prove this theorem we use the notation and conventions of Bjorken and Drell.[7] We consider the most general graph that satisfies the conditions stated above (Fig. 12) where external as well as internal fermion lines are connected at the considered vertex. We notice that a graph which satisfies the topological conditions of this theorem in momentum space also satisfies them in coordinate space (but the factorization holds for momentum space only). We will write the analytic expression corresponding to Fig. 12 in coordinate space; then we will make a Fourier transformation into momentum space and show that the result agrees with the theorem. The analytic expression of Fig. 12 will include an integral over z (which is the considered simple vertex). The functions of z in the integral are the following: (a) From each external line such as x_1z we get (after applying the appropriate Dirac operator)

$$(m/E_1)^{1/2}(2\pi)^{3/2}u(p_1,\, s_1)e^{ix_1\cdot p_1}\delta(z-x_1).$$

Because we also integrate over $x_1,\, x_2, \ldots,\, x_m,\, y_1,\, \ldots,\, y_k$, from all external lines we will get

$$(m^{m+k}/E_1\cdots E_m E_1'\cdots E_k')^{1/2}(2\pi)^{-3(m+k)/2}u(p_1,\, s_1)\cdots u(p_m,\, s_m)$$
$$\times\,\bar{u}(p_1',\, s_1')\cdots\bar{u}(p_k',\, s_k')\exp[iz(p_1+\cdots+p_m-p_1'-\cdots-p_k')]\,.$$

(b) From every internal line such as x_nz we get a factor $iS_F(z-x_n)$. When we write each $S_F(z-x_i)$ as a Fourier transformation of $S(q_i)$ we get (from internal and external lines)

$$i^{2n-m-k}(2\pi)^{-3(m+k)/2}m^{(m+k)/2}(E_1\cdots E_m E_1'\cdots E_k')^{-1/2}u(p_1,\, s_1)\cdots u(p_m,\, s_m)\bar{u}(p_1',\, s_1')\cdots\bar{u}(p_k',\, s_k')$$
$$\times\int dq_1\cdots dq_{2n-m-k}dz\,(2\pi)^{-(2n-m-k)}\exp[iz(p_1+\cdots+p_m-p_1'-\cdots-p_k')]\exp[-iq_1(z-x_{m+1})]\,\exp[-iq_2(z-x_{m+2})]\cdots$$
$$\times\exp[-iq_{2n-m-k}(y_n-z)]S(q_1)S(q_2)\cdots S(q_{2n-m-k})\,.$$

Integration over z gives

$$\delta\Bigg(\sum_{i=1}^{m}p_i-\sum_{i=1}^{k}p_i'+\sum_{i=1}^{n-m}q_i-\sum_{i=n-m+1}^{2n-m-k}q_i\Bigg).$$

If the time axis in Fig. 12 is horizontal, then conservation of total 4-momentum implies that the sum of 4-momenta carried by fermion lines connected to the point z of graph A is equal to the sum of 4-momenta carried by fermion lines connected to the point z of the graph B and is equal to the total 4-momentum of the system. (If the time axis in Fig. 12 is not horizontal, the proof of the theorem for arbitrary n becomes quite complicated and will not be given here.) Thus, for the special case that we consider in this proof (notice that whenever we use this theorem in this paper, the special situation considered for the proof is indeed satisfied) we have

$$\delta(p_1+\cdots+p_m+q_1+\cdots+q_{n-m}-p_{\text{total}})\delta(p_1'+\cdots+p_k'+q_{n-m+1}+\cdots+q_{2n-m-k}-p_{\text{total}})\,.$$

This can be obtained by performing all the integrations corresponding to the considered graph (including whatever might be inside the black boxes of Fig. 12) and using the δ function that we got from the integration over z. Thus we see that the integrand of the integrals over the q's is the product of a function of q_1 to q_{n-m} and a function of q_{n-m+1} to q_{2n-m-k} and hence the integration factorizes. It is easily seen that the first (second) factor corresponds to integration over the internal momenta of graph A (B). Notice that the contributions from the external lines already appear as simple factors in front of the integral. Now it remains to show that the numerical factors attached to the graph also factorize.

Each Feynman graph of order r contains a factor $(r!)^{-1}$. This factor results from the conversion of r time-ordered (convolution) integrals into r regular integrals. When the time-ordered integral over r variables breaks into two time-ordered integrals over k and r-k variables (we have just shown that it does), $(r!)^{-1}$ is simply replaced by $(k!)^{-1}[(r-k)!]^{-1}$ when we go over to regular integrals. Therefore, for the "bubble chain" every bubble will contribute a factor $1/2!$ So, a chain of h bubbles will have a factor $(2!)^{-h}$ and not $1/(h+1)!$ ($h+1$ is the order in the coupling constant of the considered contribution). The symmetry factor resulting from counting all possible contractions at a vertex in a bubble chain is $n!$ at each vertex if we have a $2n$ point interaction. If we factor out of the series $n!$ then every bubble will contribute one factor $n!$. We finally notice that the theorem can be generalized to point interaction by any number of fermions and mesons.

APPENDIX B

The three-quark bubble is given by

$$\mathcal{B}^{\alpha\beta\gamma}_{\delta\eta\epsilon}(p)=3\overline{G}^{\{\delta'\eta'\epsilon'\}}_{\delta\eta\epsilon}\int^{\Lambda^2}\frac{1}{(2\pi)^{12}}d^4q_1d^4q_2d^4q_3\delta^4(q_1+q_2+q_3-p)\left(\frac{\not{q}_2+m}{q_1^{\,2}-m^2}\right)^{\alpha}_{\{\delta'}\left(\frac{\not{q}_2+m}{q_2^{\,2}-m^2}\right)^{\beta}_{\eta'}\left(\frac{\not{q}_3+m}{q_3^{\,2}-m^2}\right)^{\gamma}_{\epsilon'\}}$$

$$=3\overline{G}^{\{\delta'\eta'\epsilon'\}}_{\delta\eta\epsilon}\{m^2(m\delta^{\alpha}_{\{\delta'}\delta^{\beta}_{\eta'}\delta^{\gamma}_{\epsilon'\}}+\not{p}_{\{\delta'}^{\alpha}\delta^{\beta}_{\eta'}\delta^{\gamma}_{\epsilon'\}})I_1+[m\delta^{\alpha}_{\{\delta'}(\gamma_\mu)^{\beta}_{\eta'}(\gamma_\mu)^{\gamma}_{\epsilon'\}}+\not{p}^{\alpha}_{\{\delta'}(\gamma_\nu)^{\beta}_{\eta'}(\gamma_\nu)^{\gamma}_{\epsilon'\}}]I_2\}(2\pi)^{-12}\ ,$$

where $\{\ \}$ means symmetrization of the indices inside.

$$I_1=\int\frac{d^4q_1d^4q_2d^4q_3\delta^4(q_1+q_2+q_3-p)}{(q_1^{\,2}-m^2+i\epsilon)(q_2^{\,2}-m^2+i\epsilon)(q_3^{\,2}-m^2+i\epsilon)}\ ,$$

$$I_2=\int\frac{d^4q_1d^4q_2d^4q_3\delta^4(q_1+q_2+q_3-p)q_1\cdot q_2}{(q_1^{\,2}-m^2+i\epsilon)(q_2^{\,2}-m^2+i\epsilon)(q_3^{\,2}-m^2+i\epsilon)}\ .$$

The sum of the bubble chains is defined by

$$\mathcal{A}(p)^{\alpha\beta\gamma}_{\delta\eta\epsilon}=\overline{G}^{\alpha\beta\gamma}_{\delta\eta\epsilon}+[\mathcal{B}(p)^{\alpha\beta\gamma}_{\{\delta'\eta'\epsilon'\}}+\mathcal{B}(p)^{\alpha\beta\gamma}_{\{\alpha'\beta'\gamma'\}}\mathcal{B}(p)^{\{\alpha'\beta'\gamma'\}}_{\{\delta'\eta'\epsilon'\}}+\cdots]\overline{G}^{\{\delta'\eta'\epsilon'\}}_{\delta\eta\epsilon}$$

which gives

$$[\delta^{\alpha}_{\{\alpha'}\delta^{\beta}_{\beta'}\delta^{\gamma}_{\gamma'\}}-A(p)^{\alpha\beta\gamma}_{\{\alpha'\beta'\gamma'\}}][\mathcal{A}(p)^{\{\alpha'\beta'\gamma'\}}_{\delta\eta\epsilon}-\overline{G}^{\{\alpha'\beta'\gamma'\}}_{\delta\eta\epsilon}]=\mathcal{B}(p)^{\alpha\beta\gamma}_{\{\alpha'\beta'\gamma'\}}\overline{G}^{\{\alpha'\beta'\gamma'\}}_{\delta\eta\epsilon}\ .$$

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