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Calculation and Experimental Proof of the Transverse Shift Induced by Total Internal Reflection of a Circularly Polarized Light Beam

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Wiegrefe, Fedorov, Costa de Beauregard, and Schilling have discussed the transverse energy flux existing in total reflection of an elliptically polarized light beam, the latter two proposing formulas for the transverse shift of the reflected beam. We have calculated the transverse shift by an energy-flux-conservation argument similar to Kristoffel's and to Renard's in their deduction of the longitudinal Goos-Hänchen shift, thus obtaining a formula different from those of the previous authors. We have also tested experimentally the existence of the transverse shift, in the optimal case of circular polarization and quasilimit total reflection, by using two slightly different multiplying procedures. Our measurements definitely vindicate our own formula for the transverse shift against both Costa de Beauregard's and Schilling's. The relevance of our results in connection with noncollinearity of velocity and momentum of the spinning photon inside the evanescent wave is very briefly discussed.

I. INTRODUCTION

It is now well known that the Poynting vector inside an inhomogeneous plane wave (that is, a formally plane wave with a complex propagation vector) is, in general, oblique on the phase planes. It seems that Boguslawski[1] was the first to call attention to this point in 1912 for the case of two interfering plane waves propagating in an absorbing medium. Soon after, Wiegrefe[2] considered in 1914–1916 the much more interesting case of Fresnel's evanescent wave, where the real and imaginary parts of the propagation vector are orthogonal to each other and there is no energy absorption. He calculated the Poynting vector inside the evanescent wave generated by an incident plane wave of arbitrary linear polarization, thus displaying, in general, a nonzero component of this vector normal to the incidence plane. It is surprising that he did not mention that this remarkable transverse energy flux turns out to be considerably stronger when the incident beam is elliptically polarized, and is indeed maximal (given the incidence angle) when the evanescent wave is circularly polarized. Rose and Wiegrefe[3] have attempted to prove experimentally the existence of this transverse energy flux, but, in the light of subsequent work, it is clear today that their approach was not the best.

In 1929 a remarkable phenomenon associated with the classical, longitudinal, energy flux inside Fresnel's evanescent wave was predicted theoretically on the basis of a stationary-phase argument by Picht[4]: a parallel shift of the reflected light ray inside the incidence plane, as if the photons performed some tunneling inside the evanescent wave before coming back in the medium of higher index. This longitudinal shift of the reflected pencil was indeed proved experimentally in 1947 by Goos and Hänchen.[5] Later, Acloque and Guillemet[6] and Osterberg and Smith[7] demonstrated in a very striking fashion the tunneling of the photons inside the evanescent wave.

In 1955 Fedorov[8] called attention to Wiegrefe's papers and to the fact that an elliptic polarization of the incident beam entails a much larger transverse energy flux. He produced a compact calculation of the Poynting vector inside Fresnel's evanescent wave and, in the general case, announced that a lateral shift of the reflected beam should be associated with the transverse energy flux, just as the Goos-Hänchen shift is associated with the longitudinal energy flux. However, he produced no calculation of the expected transverse shift.

In 1964–1965 Costa de Beauregard, not aware of the previous writings, approached the subject with a different motivation: noncollinearity of velocity and momentum of spinning particles,[9] which he discussed for propagating electron[10] or photon[11] waves. In the total reflection of an incident pure plane wave, translational invariance in the direction orthogonal to the incidence plane entails that the photon's momentum has a zero component in this direction, while, as previously said, in general the energy flux has a nonzero component. A Fourier analysis[12] clearly shows that this phenomenon is not an artifact inherent in an idealized situation, and that it is still present in the more realistic case of a bundle of waves.

In 1965 Schilling[13] produced a synthetic calculation of the longitudinal and lateral shifts of a totally reflected beam by using a stationary-phase argument. A peculiarity of his approach is that the field quantities inside the evanescent wave do not enter his calculations. As for the final result, he added to the Noether type[14] formula for the Goos-Hänchen shift a formula for the transverse shift which, in the light of our own theoretical and experimental work, is of the correct sign and order of magnitude, but is $1/\cos^2 i$ times too small (i denoting the incidence angle).

Our[15] calculation of the new lateral shift essentially uses the transverse energy flux inside Fresnel's evanescent wave; it consists of an energy-conservation argument similar to the one used by Kristoffel[16] and by Renard[17] in their deductions of the longitudinal Goos-Hänchen shift. Apart from being compact and simple, it yields a value for the new transverse shift that is not only different from those given by the previous authors,[11,13] but is also unambiguously supported by our measurements.[18] Ricard[19] has also produced a calculation of the longitudinal and transverse shifts of the reflected beam which, though not based on the energy-flux-conservation argument, yields exactly the same results.

In our measurements it is clear that we could not use the Goos-Hänchen[5] multiplying procedure, because then the transverse displacements would be opposite at the alternating total reflections. There are two obvious ways out of this difficulty. One is to reverse the circular polarization of the incident beam between two alternating total reflections; this is possible (as explained below) by using a prism the section of which is an isosceles triangle and the two sides of which receive a metallic reflecting coating. The other solution is to use additive rather than alternating successive reflections, as is possible inside a prism with a regular polygonal section. These two techniques have been used with equal success and complete consistency of the results.

The new transverse shift is much smaller than the Goos-Hänchen longitudinal shift: At its maximum, that is, for circular polarization and for an incidence angle very near the limiting angle of total reflection, it goes to a limit that is finite and of the order of half a wavelength; let us recall that the longitudinal shift goes in principle to infinity when the incidence angle approaches its limiting value, and that it was in fact of some ten wavelengths in Goos and Hänchen's experiments. For these reasons our experiment is *a priori* more delicate than Goos and Hänchen's and additional refinements are needed.

The first of these is that the circular polarization state of the beam must be preserved in sign and in magnitude at each of the successive total reflections. Fresnel's formulas show that this will be the case provided that each total reflection occurs extremely near the limiting case. This, in turn, requires high precision in the realization of both of our multiplying prisms.

Contrary to Goos and Hänchen, it would be difficult for us to mark a zero point outside our beam. Fortunately this difficulty can be appropriately bypassed by marking our beam by a rectilinear object and illuminating one half of it by right and the other by left circularly polarized light: This will double the effect.

Finally, since the effect we are looking for is of the order of the wavelength, it is important that it is not hidden inside a diffraction pattern. For this reason our rectilinear object was a phase object.[20]

II. CALCULATION OF THE LONGITUDINAL AND TRANSVERSE SHIFTS OF THE REFLECTED BEAM IN TOTAL REFLECTION

We will base our demonstration on the classical Fresnel-Maxwell formulas for total reflection occurring on the plane $z=0$ separating the vacuum $z<0$ from a nonabsorbing homogeneous medium $z\geqslant 0$ with real permittivity ϵ and permeability $\mu\equiv 1$, and thus of admittance $Y=\sqrt{\epsilon}$ and index of refraction $n=c\sqrt{\epsilon}$. It is noteworthy that everything significant for us occurs in the evanescent wave, that is, *in vacuo*; it is thus clear that we are dealing with properties of the electromagnetic field

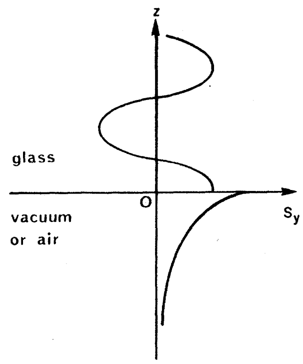

FIG. 1. Transverse component of the Poynting vector above and below the reflecting plane.

in itself, the presence of matter merely creating the appropriate boundary conditions.

The well-known Fresnel-Maxwell formulas for the incident, i, reflected, r, and transmitted (in vacuum), t, plane waves are, apart from phase factors P,

$$\begin{aligned}
&E^i = (\gamma_i E_\parallel^i, E_\perp^i, -\alpha_i E_\parallel^i),\\
&H^i = (-Y\gamma_i E_\perp^i, YE_\parallel^i, Y\alpha_i E_\perp^i),\\
&E^r = (-\gamma_r r_\parallel E_\parallel^i, r_\perp E_\perp^i, -\alpha_i r_\parallel E_\parallel^i),\\
&H^r = (Y\gamma_i r_\perp E_\perp^i, Yr_\parallel E_\parallel^i, Y\alpha_i r_\perp E_\perp^i),\\
&E^t = (\gamma_t \tau_\parallel E_\parallel^i, \tau_\perp E_\perp^i, -\alpha_t \tau_\parallel E_\parallel^i),\\
&H^t = (-\gamma_t \tau_\perp E_\perp^i, \tau_\parallel E_\parallel^i, \alpha_t \tau_\perp E_\perp^i),
\end{aligned} \tag{1}$$

with the symbols $\parallel$ and $\perp$ denoting linear polarization parallel and perpendicular to the incidence plane $y=0$; α, $\beta\equiv 0$, and γ the projecting cosines of the propagation vectors k; and the r's and τ's the reflection and transmission coefficients. Between them and the angular frequency ω one has the formulas

$$\begin{aligned}
&k_i = k_r = n\omega/c, \quad k_t = \omega/c,\\
&k_i\alpha_i = k_t\alpha_t, \quad \text{or} \quad n\alpha_i = \alpha_t .
\end{aligned} \tag{2}$$

Total reflection occurs when $\alpha_i > \alpha_l \equiv 1/n$ (that is, $\alpha_t = n\alpha_i > 1$), so that α_t is imaginary with the expression $j(\alpha_t^2 - 1)^{1/2}$. The transmitted wave is then, according to Fresnel's theory, a formal plane wave with a complex propagation vector, the real k_x^t and imaginary k_z^t parts of which are orthogonal to each other. In this case the reflection and transmission coefficients assume the values

$$\begin{aligned}
&r_\parallel = \frac{\gamma_i\alpha_l - j(\alpha_i^2-\alpha_l^2)^{1/2}}{\gamma_i\,\alpha_l^2 + j(\alpha_i^2-\alpha_l^2)^{1/2}},\\
&r_\perp = \frac{\gamma_i - j(\alpha_i^2-\alpha_l^2)^{1/2}}{\gamma_i + j(\alpha_i^2-\alpha_l^2)^{1/2}},
\end{aligned} \tag{3}$$

$$\begin{aligned}
&\tau_\parallel = \frac{2\gamma_i\alpha_i}{\gamma_i\alpha_l^2 + j(\alpha_i^2-\alpha_l^2)^{1/2}},\\
&\tau_\perp = \frac{2\gamma_i}{\gamma_i + j(\alpha_i^2-\alpha_l^2)^{1/2}},
\end{aligned} \tag{4}$$

entailing

$$r_\parallel^* r_\parallel = r_\perp^* r_\perp = 1 . \tag{5}$$

Using (1), (2), (3), (4) and the definition of the Poynting vector,

$$\vec{S} \equiv \tfrac14(\vec{E}^* \times \vec{H} + \vec{E} \times \vec{H}^*), \tag{6}$$

we find, in the evanescent wave,

$$\vec{S}^t = (\alpha_i/2\alpha_l)\exp[2k_i(\alpha_i^2-\alpha_l^2)^{1/2}z](\tau_\perp^*\tau_\perp E_\perp^{i*}E_\perp^i + \tau_\parallel^*\tau_\parallel E_\parallel^{i*}E_\parallel^i, -(j/\alpha_l)(\alpha_i^2-\alpha_l^2)^{1/2}(\tau_\perp^*\tau_\parallel E_\perp^{i*}E_\parallel^i - \text{c.c.}), 0) \tag{7}$$

and inside the medium, where the incident and reflected wave are superposed,

$$\vec{S}^{ir} = \tfrac12 Y_1\alpha_i(E_\perp^{i*}E_\perp^i(1 + r_\perp^* r_\perp + N^* r_\perp + N r_\perp^*) + E_\parallel^{i*}E_\parallel^i(1 + r_\parallel^* r_\parallel + N^* r_\parallel + N r_\parallel^*), E_\perp^{i*}E_\parallel^i(N^* r_\parallel - N r_\perp^*) + \text{c.c.}, 0), \tag{8}$$

with

$$N \equiv p^{*r}p^i = \exp[j(\vec{k}_r^* - \vec{k}_i)\cdot\vec{r}\,] . \tag{9}$$

Thus, either in the medium or in the vacuum, there is an energy flux in the x direction, which was of course expected, and no energy flux in the z direction, which is also very natural. The important point for us is that there is in general a nonzero energy flux in the y direction normal to the incidence plane; Fig. 1 shows the z dependence of the S_y component, in the quasilimiting case, for a left circularly polarized incident beam (right circular polarization would yield the symmetrical curve).

Now we recall Kristoffel's[16] and Renard's[17] method for calculating the longitudinal Goos-Hänchen[5] shift. According to formulas (7), where a common real exponential factor is present, there is a net energy flux inside the evanescent wave through the semiplanes $x = cte$, $z<0$. Assuming (Fig. 2) that, per unit length in the y direction, this flux $\int_{-\infty}^0 S_x^t\,dz$ is equal to the incident flux coming inside a prism of oblique thickness L_x, and of course also to the outgoing flux through a prism of the same thickness L_x, one obtains

$$L_x = \frac{1}{S_z^r}\int_{-\infty}^0 S_x^t\,dz , \tag{10}$$

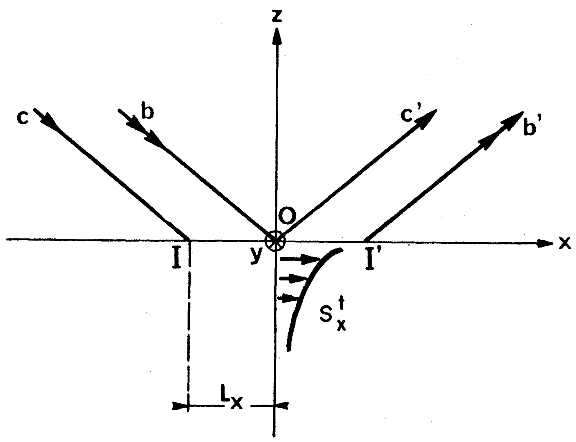

FIG. 2. Kristoffel's and Renard's reasoning for deducing the formula of the Goos-Hänchen longitudinal shift through conservation of the energy flux.

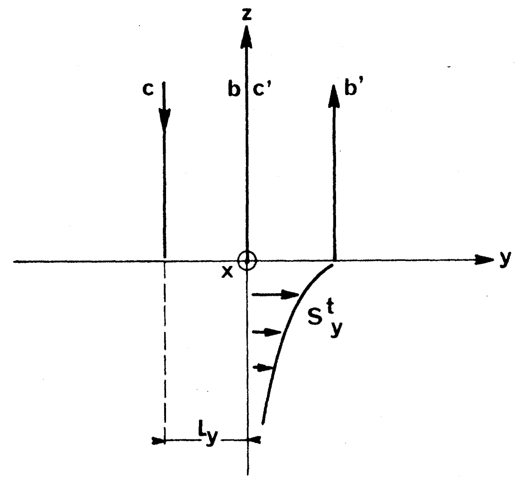

FIG. 3. Our reasoning for deducing the formula of the new transverse shift through conservation of energy flux

whence

$$L_x = \frac{\alpha_i(\tau_\perp^*\tau_\perp E_\perp^{i*}E_\perp^i + \tau_\parallel^*\tau_\parallel E_\parallel^{i*}E_\parallel^i)}{2k_i(\alpha_i^2-\alpha_l^2)^{1/2}(1-\alpha_i^2)^{1/2}(E_\parallel^{i*}E_\parallel^i + E_\perp^{i*}E_\perp^i)}\,. \tag{11}$$

This expression, first obtained by Kristoffel, is in excellent agreement with the measured value of the Goos-Hänchen longitudinal shift.

It is clear that we can apply an entirely similar procedure for the transverse energy flux, thus predicting (Fig. 3) a lateral shift of the reflected beam with the expression

$$L_y = \frac{1}{S_z^r}\int_{-\infty}^{0} S_y^t\,dz \tag{12}$$

and explicit value

$$L_y = \frac{-j\alpha_i(\tau_\perp^*\tau_\parallel E_\perp^{i*}E_\parallel^i - \tau_\parallel^*\tau_\perp E_\parallel^{i*}E_\perp^i)}{2k_t(1-\alpha_i^2)^{1/2}(E_\perp^{i*}E_\perp^i + E_\parallel^{i*}E_\parallel^i)}\,. \tag{13}$$

Additional remarks pertaining to the latter result are (Fig. 1) the following:

(1) The integral $\int_0^Z S_y^{ir}dz$ is a sinusoidal function of Z with zero mean value, so that $\int_0^{+\infty} S_y^{ir}dz$ is zero in the sense of Fourier integrals.

(2) As the discontinuity of S_y at $z=0$ is a simple step function, $\int_{-\epsilon}^{+\epsilon} S_y\,dz \to 0$ when $\epsilon \to 0$. Thus $\int_{-\infty}^{0} S_y dz = \int_{-\infty}^{+\infty} S_y\,dz$.

Among other cases, the transverse shift L_y is zero when $E_\parallel^i = 0$ or $E_\perp^i = 0$, which was evident from a symmetry argument. Incidentally, L_y is nonzero in the case of oblique linear polarization of the incident beam[2] (almost conserved in the quasilimiting case); however, the transverse shift remains very small in this case.[21]

For given α_i, $|\tau_\parallel E_\parallel^i|$, and $|\tau_\perp E_\perp^i|$, $|L_y|$ is maximum for $\tau_\perp E_\perp^i = \pm j\tau_\parallel E_\parallel^i$, that is, left or right circular polarization inside the evanescent wave. In this case, formulas (2) and (7) yield $\pm 2\omega S_y^t = \partial_z S_x^t$, in accord with Costa de Beauregard's[11] general formula $\pm 2\omega S_y = \partial_z S_x - \partial_x S_z$. Thus, as far as the magnitude of the effect is concerned, circular polarization of the *incident* beam is not the optimal case. It is, however, the most convenient case, and for that reason the one we have used in our experimental studies. In the general case, the formula

$$S_y^t = \tfrac{1}{2}\,\omega\,\frac{j(\tau_\parallel^* E_\parallel^{i*}\tau_\perp E_\perp^i - \tau_\perp^* E_\perp^{i*}\tau_\parallel E_\parallel^i)}{\tau_\perp^*\tau_\perp E_\perp^{i*}E_\perp^i + \tau_\parallel^*\tau_\parallel E_\parallel^{i*}E_\parallel^i}\,\partial_z S_x^t \tag{14}$$

follows from (2) and (7).

Using the general formulas (11) and (13) for the longitudinal L_x and transverse L_y shifts of the reflected beam in the particular case of circular polarization $E_\perp^i = \pm jE_\parallel^i$, we find

$$L_x^{(c)} = \frac{\lambda_i}{8\pi}\,\frac{\alpha_i(\tau_\parallel^*\tau_\parallel + \tau_\perp^*\tau_\perp)}{(\alpha_i^2-\alpha_l^2)^{1/2}(1-\alpha_i^2)^{1/2}} = \tfrac{1}{2}(L_x^{\parallel} + L_x^{\perp}) \tag{15}$$

and

$$L_y^{(c)} = \mp\frac{\lambda_i}{8\pi}\,\frac{\alpha_i(\tau_\perp^*\tau_\parallel + \tau_\parallel^*\tau_\perp)}{\alpha_i(1-\alpha_i^2)^{1/2}}\,, \tag{16}$$

where $\tau_\parallel$ and $\tau_\perp$ denote the transmission coefficients of the two linear polarization states, α_i the incidence angle's sine and $\alpha_1 \equiv 1/n$ its limiting value, $\lambda_i = \lambda/n$ the wavelength inside the medium; the sign in (16) is negative for left and positive for right circular polarization.

Using the well-known formulas

$$\tau_\parallel^*\tau_\parallel + \tau_\perp^*\tau_\perp = \frac{4\alpha_i^2(1-\alpha_i^2)(1+\alpha_l^2)}{\alpha_l^4(1-\alpha_i^2)+\alpha_i^2-\alpha_l^2}\,, \tag{17}$$

$$\tau_\perp^*\tau_\parallel + \tau_\parallel^*\tau_\perp = \frac{8\alpha_l\alpha_i^2(1-\alpha_i^2)}{\alpha_i^2-\alpha_l^2+\alpha_l^4(1-\alpha_i^2)}\,, \tag{18}$$

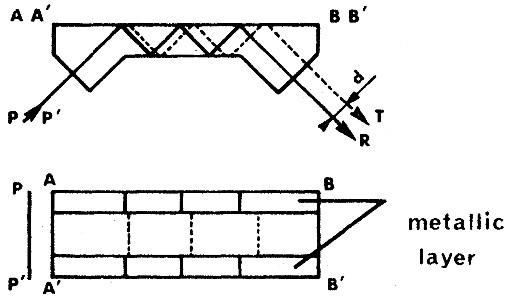

FIG. 4. Principle of the Goos-Hänchen experiment.

we rewrite (15) and (16) as

$$L_x^{(c)}=\frac{\lambda_i}{2\pi}\,\frac{\alpha_i^3(1+\alpha_l^2)(1-\alpha_i^2)^{1/2}}{(\alpha_i^2-\alpha_l^2)^{1/2}[\alpha_l^4(1-\alpha_i^2)+\alpha_i^2-\alpha_l^2]} \tag{19}$$

and

$$L_y^{(c)}=\mp\frac{\lambda^i}{\pi}\,\frac{\alpha_i^3(1-\alpha_i^2)^{1/2}}{\alpha_l^4(1-\alpha_i^2)+\alpha_i^2-\alpha_l^2}\,, \tag{20}$$

whence

$$L_y^{(c)}=\frac{(k_i^2\alpha_i^2-k_t^2)^{1/2}}{k_t T}\,L_x^{(c)}\,, \tag{21}$$

with

$$T\equiv\frac{\tau_\perp^*\tau_\perp+\tau_\parallel^*\tau_\parallel}{\tau_\perp^*\tau_\parallel+\tau_\parallel^*\tau_\perp}=\frac{k_i^2+k_t^2}{2k_ik_t}\,. \tag{22}$$

$k_i=n\omega/c$ and $k_t=\omega/c$ denote the lengths of the propagation vectors inside the medium and *in vacuo*.

It thus turns out that the transverse shift we are investigating is considerably smaller than the longitudinal Goos-Hänchen shift. For $\lambda=6328$ Å, $n=1.8$, and an incidence angle 5 min above the limiting angle, which are the conditions in one of our experiments, we find

$$L_x^{(c)}=8.36\lambda=5.3\ \mu\,,\quad L_y^{(c)}=\mp0.46\lambda=\mp0.3\ \mu\,.$$

Our detecting procedure will then have to be more sensitive than Goos and Hänchen's. Fortunately, the helicity dependence of $L_y^{(c)}$ will allow an obvious differential procedure for doubling the value of the measured displacement: We will illuminate half of the field with left and the other half with right circularly polarized light. As for multiplying the effect by using N consecutive reflections, we cannot use alternating reflections on a parallel face plate as Goos and Hänchen did (Fig. 4), because then the sign of the transverse shift would reverse each time, and no multiplication would occur. Thus we must either modify the Goos-Hänchen technique in order to reverse the helicity sign between two successive alternating reflections, or use a method where the successive reflection angles will have the same sign. We have used both of these techniques with equal success and complete consistency of the experimental results.

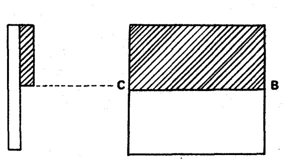

FIG. 5. The "Wolter object."

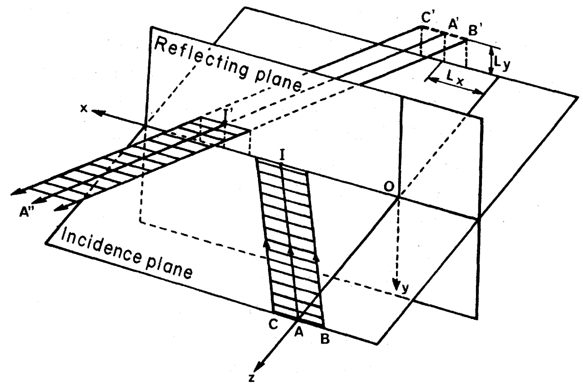

FIG. 6. Longitudinal L_x and lateral L_y shifts of a rectilinear object CAB by total reflection of a circularly polarized light beam.

III. EXPERIMENTAL METHOD

In order to test the transverse shift we are expecting, we must first mark the beam. To this

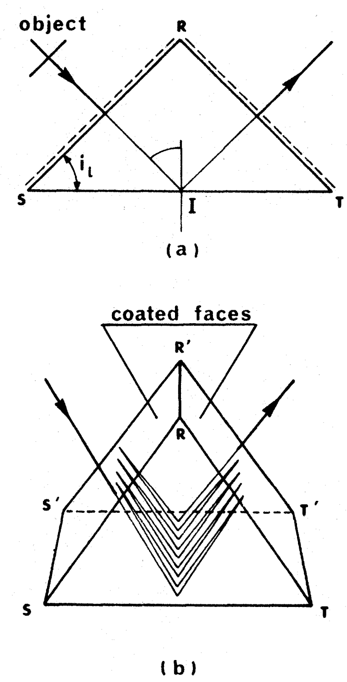

FIG. 7. The multiplying prism F: (a) cross-section view; (b) perspective view.

end we use a rectilinear object which, in order to reduce the diffraction pattern, we take to be a Wolter plate; that is, a parallel face plate half of which is covered by a layer of thickness such that a π phase shift is introduced (Fig. 5). Figure 6 then shows how the image $C'A'B'$ of the rectilinear object CAB will be shifted, in a total reflection, by the longitudinal amount L_x and the lateral amount L_y.

As previously said, we have used two different multiplying prisms. The first one, F, has an index $n = 1.52$ and its section is an isosceles triangle [Fig. 7(a)] with angles such that a beam undergoing quasilimit total reflection on the basis ST traverses orthogonally the two sides SR and RT. These sides are coated with a semitransparent metallic layer reflecting part of the beam inside the prism, as in a Pérot-Fabry interferometer.

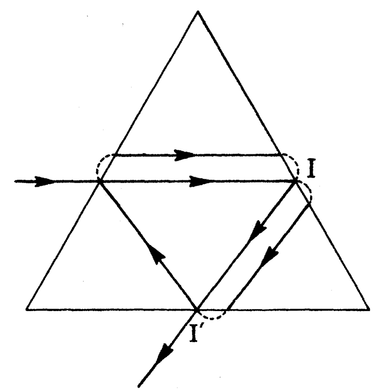

FIG. 8. The multiplying prism P (cross section).

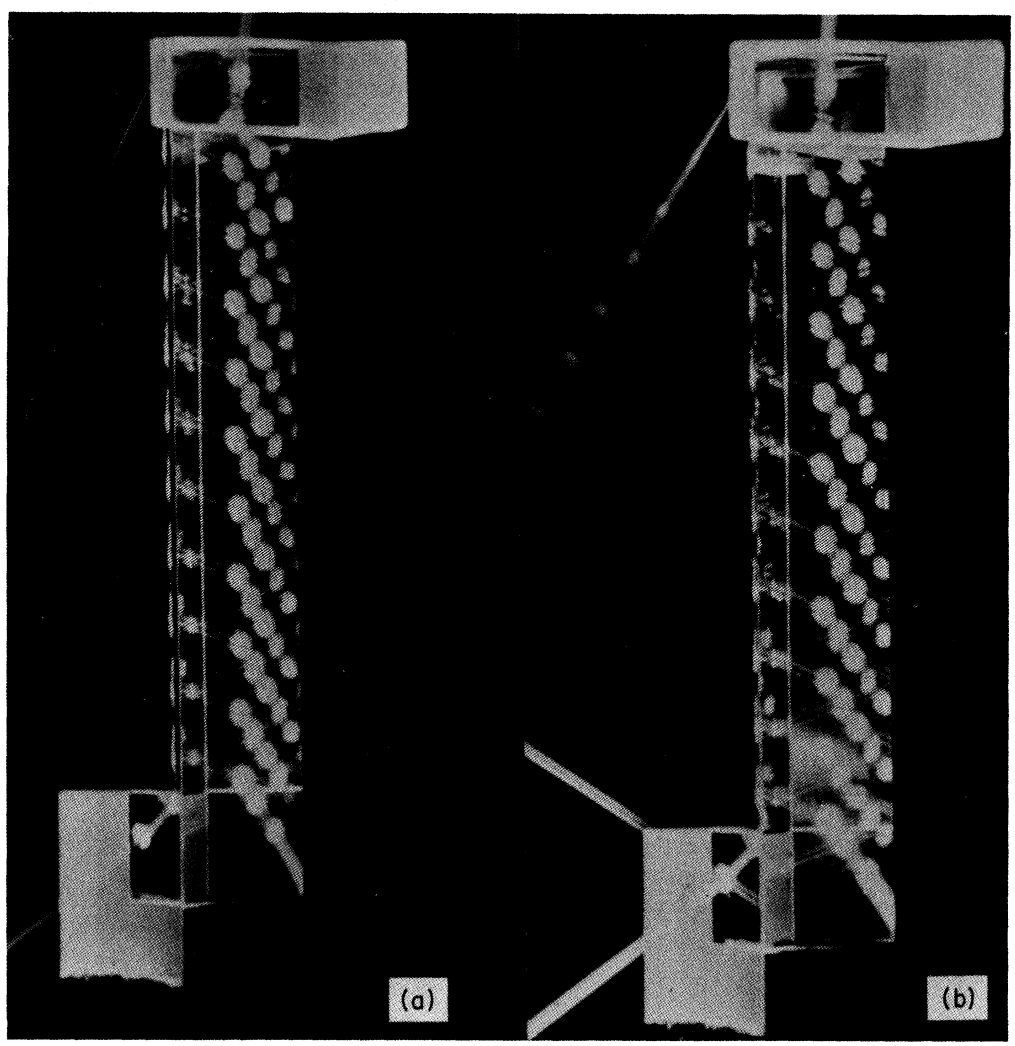

FIG. 9. The multiplying prism P, perspective view: (a) apparatus in working condition; (b) slightly deranged apparatus showing light escaping.

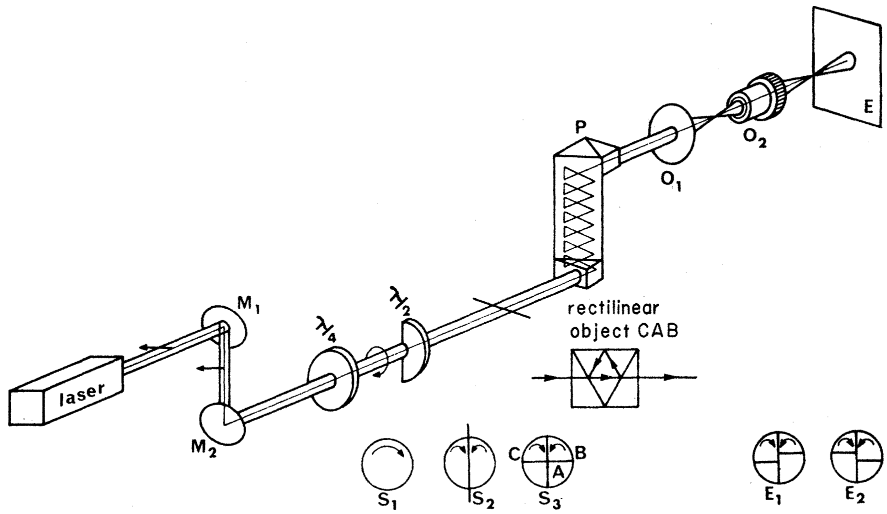

FIG. 10. Over-all experimental arrangement.

We know that circular polarization, including the helicity sign, is preserved by the quasi-limit total reflection on ST, whereas the helicity is obviously reversed by each normal reflection on the sides SR and RT, so that the successive transverse shifts will add. In order to separate the successive emerging beams we have given a slight obliquity to the incidence planes with respect to the cross-section plane of the prism [Fig. 7(b)]; this allowed us to verify that the circular polarization of the incident beam was indeed preserved in each of the 20 outgoing beams. The advantage of this procedure is its intrinsic symmetry (or quasi-symmetry), which facilitates both measurements and discussions of the results. Its drawback is that it absorbs much energy.

Our second multiplying prism, P, has an index $n=1.8$ and its section is an equilateral triangle. The light beam inside follows a helical polygon (Figs. 8 and 9), the slope of which is controllable in order to adjust the reflection angles very near, and slightly above, the limiting value of total reflection. These angles have additive projections on the section planes of the prism. In this case we use 28 total reflections. Figure 9(b) shows a slightly deranged experimental apparatus producing outgoing beams, while Fig. 9(a) shows the prism in working conditions.

Figure 10 displays the whole experimental arrangement. A laser beam, with horizontal linear polarization, traverses a quarter-wave plate and then a half-wave plate covering half of the beam; the resulting beam, which illuminates the rectilinear object CAB, is thus polarized circularly with opposite helicities on its two halves. This will produce the differential doubling effect we have alluded to. After illuminating the rectilinear object CAB, the beam enters either of our two multiplying prisms, and the image of CAB is finally observed. The high luminosity of the prism P allows this image to be projected on a screen, as shown in Fig. 10.

According to the previous explanation, it is expected that the final image, as processed by the prism, assumes the form E_1 or E_2 (Fig. 10) according to the possible associations of the circular polarization states with the left and right halves of the beam. It is of course easy to switch from one association to the other by a 90° rotation of the quarter-wave plate in its plane, while a 180° rotation restores the initial state of affairs.

IV. EXPERIMENTAL DATA

The experimental results are shown on the two photographs in Fig. 11, where the circular polarization states of the two halves of the beam have been indicated. It is seen that the expected effect does exist, with the right sign. We have verified that a 180° rotation of the quarter-wave plate restores in fact the original configuration, thus excluding artifacts due to prism effects (incidentally, it is hard to imagine which prism effect could cause the observed configurations, as the quarter-wave plate is placed *before* the linear object).

The measured magnitude of the effect has also come out quite right, either with prism F or P. As our most precise measurements were performed with the P prism, we will presently discuss this case.

Due to the helical light path in the P prism, the expected transverse shift per reflection is $L_y' = L_y \times \cos\theta$, where θ denotes the angle between the incidence plane and the cross-section plane of the prism, to which the linear object CAB is parallel.

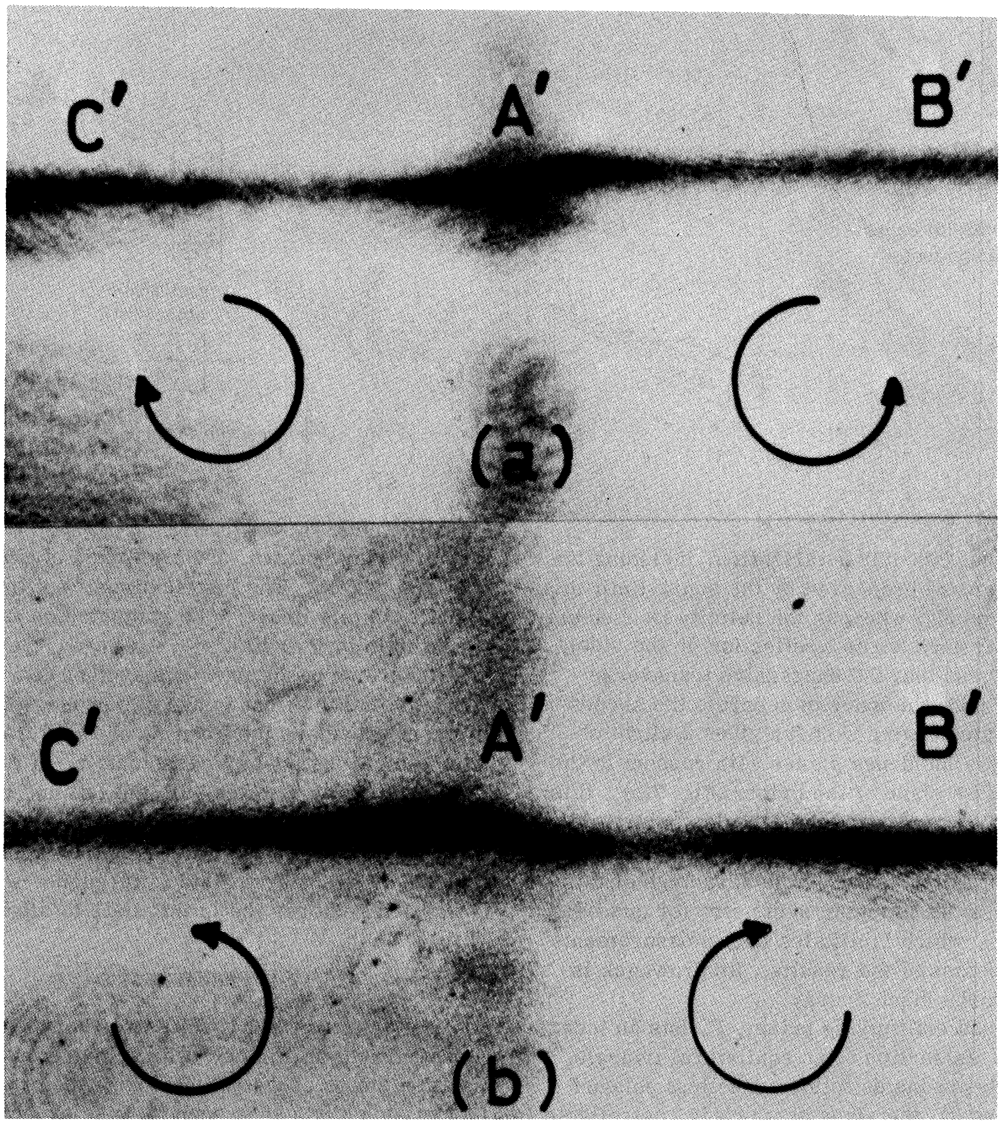

FIG. 11. Image of a rectilinear object displaying the transverse shift of a circularly polarized light beam. The circular polarization states of the two halves of the beam are indicated.

A straightforward calculation yields, in the limiting case of total reflection where $\alpha = 1/n$, $\cos\theta = [(n^2-1)/3]^{1/2}$; that is, with $n \simeq 1.8$, $\cos\theta \simeq 0.865$. For confirmation we have also measured the angle θ' between the light beam and the cross-section plane, and found $\theta' \simeq 16°$; as the relation $\cos\theta = \frac{1}{2}n \times \cos\theta'$ holds, we obtain in this way $\cos\theta \simeq 0.87$.

Now, as we have said, the wavelength of the laser radiation was $\lambda = 0.6328\ \mu$, whence, according to formula (20), $L_y^{(c)} = 0.288\ \mu$ and $L_y^{(c)\prime} = 0.288 \times 0.865\ \mu$. Taking into account the 28 successive reflections and the measured magnification 300 of our optical system, we finally obtain

$$L_{y\,\mathrm{theor}} = 4.2\ \mathrm{mm}.$$

The measurements have yielded

$$L_{y\,\mathrm{meas}} = 4.1 \pm 0.3\ \mathrm{mm}.$$

We thus conclude that the predicted effect indeed exists with the right sign and the right magnitude.

V. ADDITIONAL VERIFICATIONS AND REMARKS

Interposing a circular-light analyzer on the outgoing beam, we have verified, with both prisms F and P, that the initial circular polarization is conserved (as previously said, with the prism F this verification was made for each individual reflection).

This preservation of the rotational symmetry of the beam excludes artifacts that could come, with prism P, from the longitudinal Goos-Hänchen effect *via* the nonzero angle ϕ between two consecutive incidence planes.

An easy calculation yields $\phi \simeq 0.214\pi$, so that after 28 reflections the incidence plane is rotated by 6π, thus excluding all possibility of testing the Goos-Hänchen displacement with this apparatus. This we have nevertheless verified, by placing the rectilinear object *CAB* perpendicular to the cross-section plane of the prism, and illuminating each of its halves with orthogonal states of linear polarization. As expected, we observed that under these conditions the final image of our object remained perfectly straight, thus excluding any contribution from the Goos-Hänchen effect. (Incidentally, it is easily seen that any multiple of four reflections, as is 28, will exactly compensate the Goos-Hänchen effect in its horizontal projection, which is not the one we are using.)

VI. CONCLUSIONS

One important conclusion of our work is to select, among the different formulas that have been proposed for the transverse shift, the one that is best supported by the experimental measurements. It is certainly astonishing that, among the numerous authors who have been concerned with the transverse energy flux, only three have proposed, before our experimentation, a formula for the transverse shift, and that all three formulas are different. Let us first recall them, for the case of circular polarization inside the evanescent wave and the limiting value i_l of the incidence angle[22]:

Costa de Beauregard (Ref. 11), 1964: $L_y^c = \mp \dfrac{\lambda^i}{4\pi \tan i_l}$

Schilling (Ref. 13), 1965: $L_y^c = \mp \dfrac{\lambda^i}{\pi \tan i_l}$,

Imbert (Ref. 15), 1968; Ricard (Ref. 19), 1970: $L_y^c = \mp \dfrac{\lambda^i}{\pi \sin i_l \cos i_l}$.

Thus our value is $1/\cos^2 i_l$ times larger than Schilling's and $4/\cos^2 i_l$ times larger than Costa de Beauregard's. In the case of the prism F, $n \simeq 1.52$, $\sin i_l \simeq 0.658$, $\cos^2 i_l \simeq 0.570$, and $\frac{1}{4}\cos^2 i_l \simeq 0.142$. In the case of the prism P, $n \simeq 1.8$, $\sin i_l \simeq 0.555$, $\cos^2 i_l \simeq 0.690$, and $\frac{1}{4}\cos^2 i_l \simeq 0.172$. *Quite apart from the fact that our reasoning is much more direct than both Costa de Beauregard's and Schilling's, our experimental measurements are definitely consistent with our formula and incompatible with both Costa de Beauregard's and Schilling's.*

Another point of interest is the answer appropriate to the question, "Does our experimental result entail or not that the velocity and momentum of the spinning photon are noncollinear inside the vacuum of Fresnel's evanescent wave?" We feel that we have the right to express our feeling regarding this point, especially since the minority[9] supporting the somewhat heretical view that they are noncollinear has steadily increased through the years and has recently received the strong support of Hestenes's[23] papers and Corben's[24] paper and authoritative book.

When we rotate our quarter-wave plate by $\pm 90°$ in its plane, we observe a lateral shift of the reflected beam, that is, a shift from right to left (or vice versa) of the transverse energy flux inside the evanescent wave. However, the expectation values of the photon momentum states, i.e., of the Fourier components, are exactly the same in the left or right circular polarization states of the cylindrical incident beam. This is true also for the inhomogeneous plane waves that stand in a one-to-one correspondence with the incident plane waves, each of which has its complex propagation vector inside the *same* incidence plane as the corresponding incident plane wave. *Therefore the rotation of our quarter-wave plate cannot change the probability distribution of the (complex) momentum inside the evanescent wave, whereas it obviously changes the energy flux.* Loosely speaking, it does not change the momentum distribution, but it does change the velocity distribution. A Fourier-type analysis of the evanescent wave faithfully vindicates the foregoing views.[12]

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Frequency Dependence of the Speed of Light in Space

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To characterize the possible dispersion of the velocity of light in space (vacuum) a Cauchy-type formula, $n^2=1+A/\nu^2+B\nu^2$, is used. It is shown that relativity only allows a nonzero A term, independent of the nature of the waves or a quantization thereof. Recent experimental data provide upper bounds for A and B, limiting thereby the dispersion in the microwave, infrared, visible, and ultraviolet regions of the spectrum to less than one part in 10^{20}.

Recent observations of radio-wave,[1] visible,[2] and x-ray[3] emissions from pulsars have been interpreted to provide experimental bounds on the dispersion of light in interstellar space.[2,4,5] The dispersion or lack thereof has been discussed using the expression[2,6,7]

$$p=\frac{c}{\Delta c}\frac{\lambda_2}{\lambda_1} \tag{1}$$

to relate measurements in different regions of the spectrum giving different limits of dispersion Δc in the velocity. It was pointed out by Brown[7] that p clearly cannot be a good constant to characterize the variation of velocity with energy, because it would be infinite for $\lambda_1=\lambda_2$. Even though that difficulty could be avoided by the introduction of $(\lambda_2-\lambda_1)/\lambda_1$ in place of λ_2/λ_1 in the definition of p, there remains the more serious objection against the concept of p, in our opinion, that it suggests a linear dependence of c on λ.

It appears preferable to the present authors to represent the dispersion (if there is any) via a modified Cauchy expression of the form[8]

$$n^2=1+\frac{A}{\nu^2}+B\nu^2 \tag{2}$$

or

$$n^2=1+A'\lambda^2+\frac{B'}{\lambda^2}, \tag{3}$$

where n is defined by $c_{\text{phase}}=c_0/n$ and c_0 is the velocity of light in the absence of dispersion. The corresponding group velocities, to be used in the analysis of the experimental data, are readily calculated from the above expressions.

In anticipation of their use in regions of the spectrum remote from resonances, only the leading terms in ν^2 and $(1/\nu)^2$, or λ^2 and $(1/\lambda)^2$, are retained in Eqs. (2) and (3). The absence of odd powers of ν or λ is assured by the presumed symmetry with respect to reversal of the direction of time.[9]

Expressions (2) and (3) describe the frequency dependence of the speed of light in any dispersive

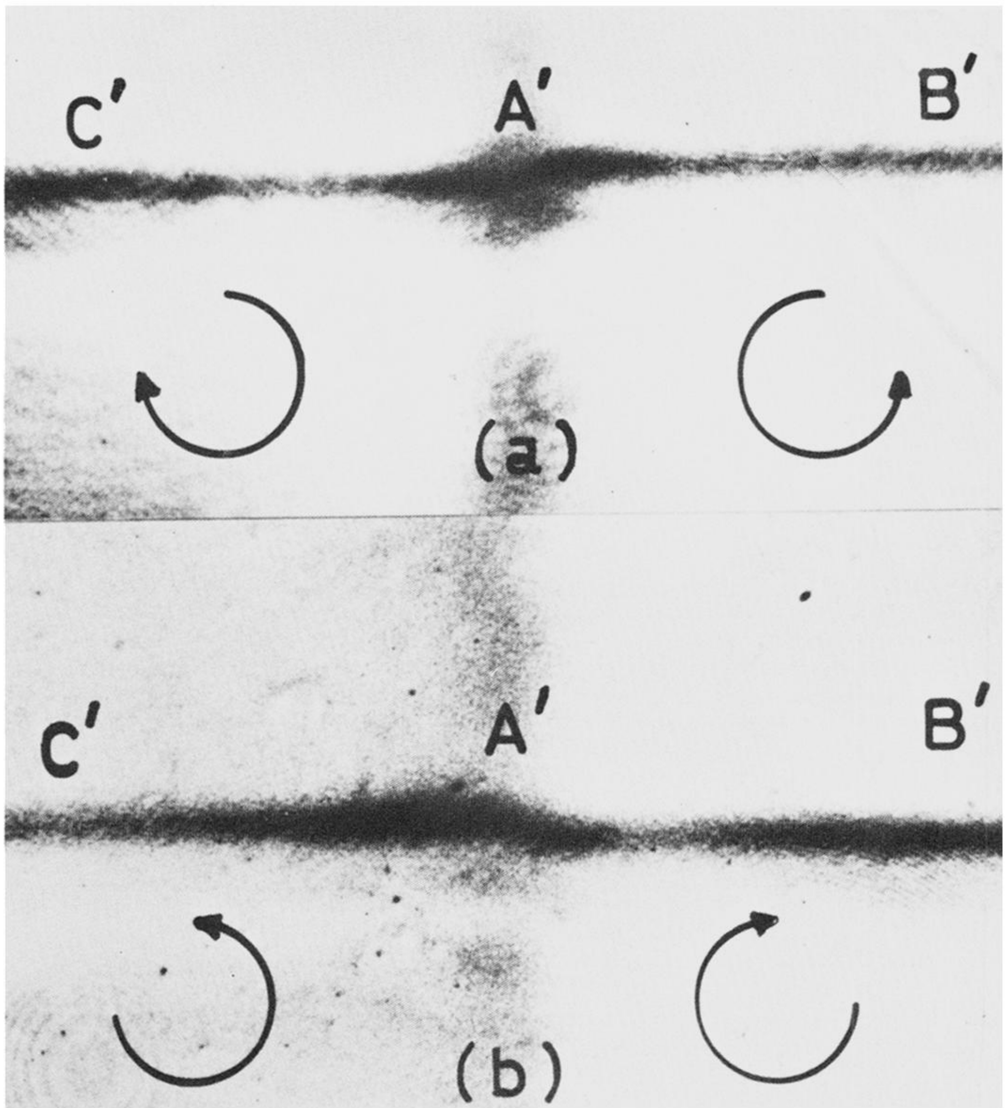

FIG. 11. Image of a rectilinear object displaying the transverse shift of a circularly polarized light beam. The circular polarization states of the two halves of the beam are indicated.

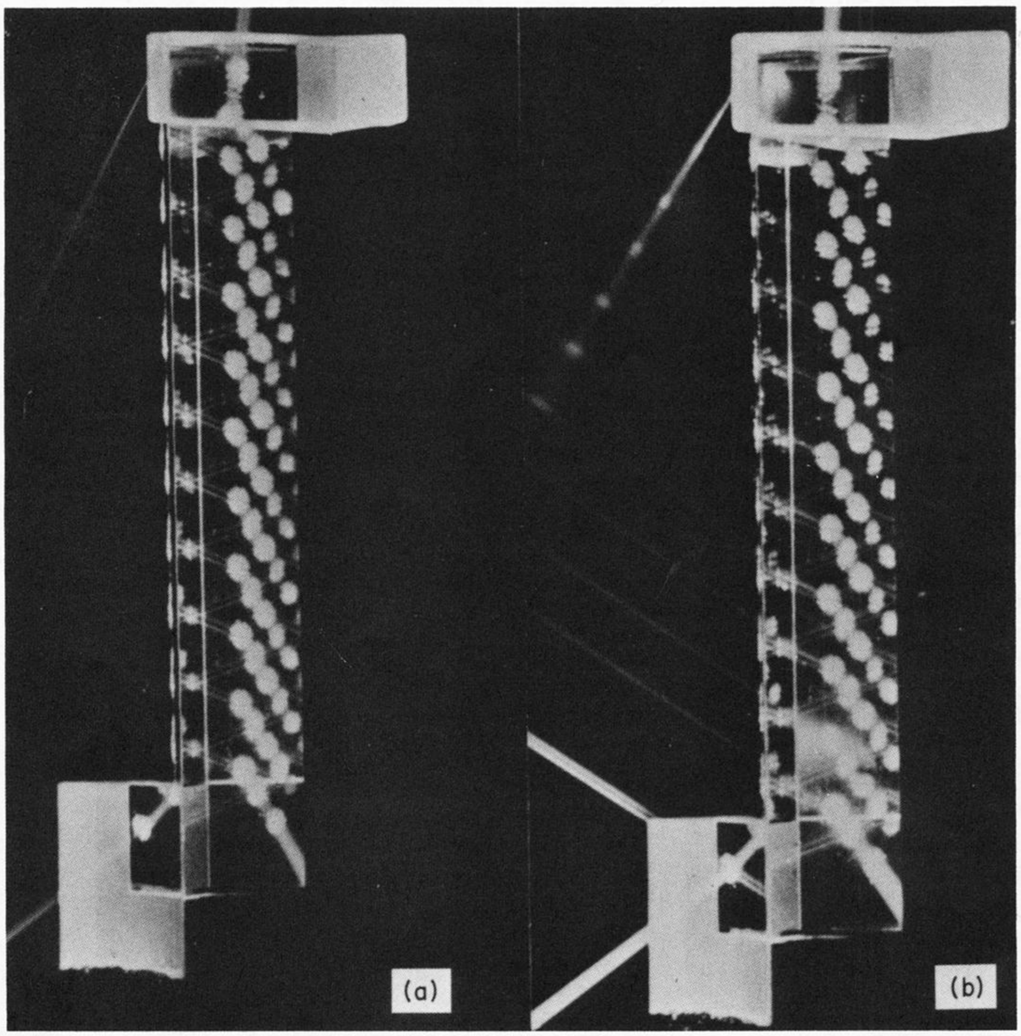

FIG. 9. The multiplying prism P, perspective view: (a) apparatus in working condition; (b) slightly deranged apparatus showing light escaping.