

## Energy Independence of Inclusive Reactions\*

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The question of the energy dependence of inclusive reactions such as  $a + b \rightarrow c + \text{anything}$  is discussed. Factorization arguments are used to show that the condition that  $ab\bar{c}$  and  $ab$  be exotic for energy independence leads to the energy independence of a number of nonexotic reactions. It is suggested on the basis of simple Regge-exchange diagrams that perhaps a sufficient condition for energy independence, when  $b$  fragments into  $c$ , is that  $ab\bar{c}$ ,  $ab$ , and  $a\bar{c}$  be exotic.

The recent work of Mueller<sup>1</sup> showed that the amplitude for the single-particle distribution

$$a + b \rightarrow c + \text{anything} \quad (1)$$

was related through unitarity to the forward elastic three-body amplitudes

$$a + b + \bar{c} \rightarrow a + b + \bar{c}. \quad (2)$$

Using this result, Chan *et al.*<sup>2</sup> developed a Regge phenomenology in which they predicted on the basis of duality that when  $ab\bar{c}$  is an exotic channel the cross section for reaction (1) is energy-independent.

Ellis *et al.*<sup>3</sup> have shown that this criterion is not sufficient by demonstrating that factorization leads to the energy independence of the cross section even for certain cases where  $ab\bar{c}$  is not exotic. They suggested that the correct criteria for energy independence are that both the  $ab\bar{c}$  and  $ab$  channels be exotic. We have investigated whether indeed these two conditions are sufficient and found, again through use of factorization, that they are not.

We consider the invariant cross section corresponding to single-particle spectra which, in the notation of Ref. 2, is given by

$$\omega \frac{d\sigma}{d^3p} \equiv f(s; p_{\parallel}^b, p_{\perp}^2), \quad (3)$$

where

$$f(s; p_{\parallel}^b, p_{\perp}^2) = A(p_{\parallel}^b, p_{\perp}^2) + B(p_{\parallel}^b, p_{\perp}^2) s^{-1/2}. \quad (4)$$

We have assumed that the  $P'$ ,  $\omega$ ,  $A$ , and  $\rho$  trajectories are exchange-degenerate with a common intercept  $\alpha(0) \approx 0.5$ . The longitudinal momentum of  $c$  in the rest frame  $b$  and the transverse momentum are given by  $p_{\parallel}^b$  and  $p_{\perp}$ . We consider the two reactions with  $K^+$  and  $K^0$  incident:

$$K^{0,+} + p \rightarrow \Delta^- + \text{anything}. \quad (5)$$

For both of these reactions both  $ab$  and  $ab\bar{c}$  are exotic; hence, according to Ellis *et al.*,<sup>3</sup> the Regge

contributions cancel, viz.,

$$B^{0,+} = \gamma_{P'}^K (\gamma_{P'}^{N\bar{\Delta}} - \gamma_{\omega}^{N\bar{\Delta}}) \pm \gamma_A^K (\gamma_A^{N\bar{\Delta}} - \gamma_{\rho}^{N\bar{\Delta}}) = 0. \quad (6)$$

We have factored the  $KK$  and  $N\bar{\Delta} - N\bar{\Delta}$  Regge vertices and made use of the exchange-degenerate result that

$$\gamma_{P'}^K = \gamma_{\omega}^K \quad \text{and} \quad \gamma_A^K = \gamma_{\rho}^K. \quad (7)$$

Adding and subtracting Eqs. (6), we find that

$$\gamma_{P'}^{N\bar{\Delta}} = \gamma_{\omega}^{N\bar{\Delta}} \quad \text{and} \quad \gamma_A^{N\bar{\Delta}} = \gamma_{\rho}^{N\bar{\Delta}}. \quad (8)$$

We now consider the reaction

$$n + p \rightarrow \Delta^- + \text{anything}. \quad (9)$$

Exploiting factorization, relation (8), and the exchange degeneracy of the nucleon vertex residues, viz.,

$$\gamma_{P'}^N = \gamma_{\omega}^N \quad \text{and} \quad \gamma_A^N = \gamma_{\rho}^N, \quad (10)$$

we obtain the result that

$$\begin{aligned} B(n + p \rightarrow \Delta^- + \text{anything}) \\ = \gamma_{P'}^N (\gamma_{P'}^{N\bar{\Delta}} - \gamma_{\omega}^{N\bar{\Delta}}) - \gamma_A^N (\gamma_A^{N\bar{\Delta}} - \gamma_{\rho}^{N\bar{\Delta}}) \equiv 0. \end{aligned} \quad (11)$$

The channel  $n\bar{p}\bar{\Delta}$  which has the quantum numbers of the  $\Delta^{++}$  is not exotic. This leads us to conclude that the condition that  $ab$  and  $ab\bar{c}$  be exotic is not a sufficient condition for assuming energy independence. Consideration of the possible single-Regge-pole exchange diagrams shown in Fig. 1 suggests that the sufficient condition for the energy independence is the satisfaction of the criteria

$$ab\bar{c} \text{ exotic}, \quad (12a)$$

$$ab \text{ exotic}, \quad (12b)$$

$$a\bar{c} \text{ exotic}. \quad (12c)$$

Conditions (12a), (12b), and (12c) ensure that for the Regge exchanges represented by Figs. 1(a), 1(b), and 1(c), respectively, the energy-dependent

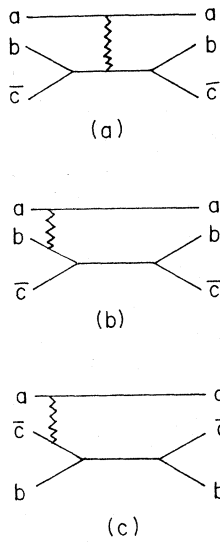


FIG. 1. Possible Regge-exchange diagrams for elastic three-body collisions.

part of the cross section will vanish. Imposing condition (12c), the reaction  $K^0 p \rightarrow \Delta^- + \text{anything}$

is no longer exotic, and hence only  $B^+$  of (6) equals zero, eliminating the result shown in (10). Of the nine reactions listed in Ref. 3 for which  $B \neq 0$  only five remain when the condition that  $a\bar{c}$  be exotic is also imposed. They are

$$K^+ N \rightarrow \left\{ \begin{array}{l} \pi^- \\ K^- \\ \bar{p} \end{array} \right\} + \text{anything} \quad (13)$$

and

$$pN \rightarrow \left\{ \begin{array}{l} K^- \\ \bar{p} \end{array} \right\} + \text{anything}. \quad (14)$$

In order to demonstrate through factorization the necessity of the additional condition suggested by Fig. 1(c) that  $a\bar{c}$  be exotic, we have had to consider inclusive reactions in which the  $\Delta^-$  is the final product. While the measurement of this inclusive reaction might prove difficult experimentally, the conclusions reached theoretically are independent of these difficulties. Experimental tests of these ideas are easily realized through comparison of reactions (13) and (14) with  $K^+ N \rightarrow \pi^+ + \text{anything}$  or  $pN \rightarrow \pi^+ (\text{or } K^+) + \text{anything}$ .

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<sup>1</sup>A. H. Mueller, Phys. Rev. D 2, 2963 (1970).

<sup>2</sup>Chan Hong-Mo, C. S. Hsue, C. Quigg, and J.-M. Wang,

Phys. Rev. Letters 26, 672 (1971).

<sup>3</sup>J. Ellis, J. Finkelstein, P. H. Frampton, and M. Jacob, Phys. Letters 35B, 227 (1971).

### Erratum

#### Hard-Meson Current Algebra and the Breakdown of Partial Conservation of Axial-Vector Current,

M. G. Miller [Phys. Rev. D 4, 757 (1971)]. The correction is as follows: Equation (81), which *now* reads

$$\Gamma(\varphi \rightarrow 3\pi)|_{\text{TH}} = (0.064 \pm 0.25) \text{ MeV},$$

should read

$$\Gamma(\varphi \rightarrow 3\pi)|_{\text{TH}} = (0.64 \pm 0.25) \text{ MeV}.$$