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[8]We are indebted to Professor A. Pais for this comment.

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Time Reversal and β Decay: A Generalization of the Kim-Primakoff Theorem*

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Kim and Primakoff have shown that certain T-violating currents cannot contribute to neutron β decay because of their properties under charge symmetry. Here we show that the same currents cannot contribute to strangeness-conserving β-decay transitions within any isomultiplet. We also extend the result to strangeness-violating decays by means of V spin.

By analyzing the axial-vector current in terms of time reversal (T) and charge symmetry (CS), Kim and Primakoff[1] have shown that the absence of T-violating effects in neutron β decay does not necessarily prove semileptonic weak interactions to be T-invariant. Certain T-violating terms may be present in the current, but because of their behavior under CS, they cannot contribute to transitions from one member of an isodoublet to the other. Their existence, therefore, can only be detected in other types of isospin transition.

Here we shall generalize the Kim-Primakoff theorem by showing that those parts of the axial-vector current that cannot contribute to neutron decay cannot, in fact, contribute to β-decay transitions within any isomultiplet. Definitive tests of T invariance in strangeness-conserving weak interactions must therefore involve decays from one isomultiplet to another. A similar result applies to strangeness-violating semileptonic processes.

The strangeness-conserving axial-vector current can be written as a sum of terms which are either "normal" (n) or "abnormal" (a) under T, and which are either "regular" (r) or "irregular" (i) under CS:[1]

$$A_\lambda = \sum_{\substack{x=n,a\\ y=r,i}} A_\lambda^{(x)(y)}. \tag{1}$$

Under T we have

$$TA_\lambda^{(x)(y)}T^{-1} = -a_{xy}(A_\lambda^{(x)(y)})^\dagger , \quad a_{nr}=a_{ni}=+1, \quad a_{ar}=a_{ai}=-1, \tag{2}$$

and under CS, we find

$$e^{i\pi I_2}A_\lambda^{(x)(y)}e^{-i\pi I_2} = -b_{xy}(A_\lambda^{(x)(y)})^\dagger , \quad b_{nr}=b_{ar}=+1, \quad b_{ni}=b_{ai}=-1. \tag{3}$$

Terms which are either $(n)(r)$ or $(a)(i)$ belong to the first-class category of currents, and terms which are $(n)(i)$ or $(a)(r)$ belong to the second class.[2]

When we take matrix elements of A_λ between states α and β, we obtain terms which are either pure axial vector (A), or induced pseudoscalar (P), or induced pseudotensor (T).[1] The corresponding form factors are

$$F(N,x,y;\alpha\to\beta), \quad N=A,P,T, \quad x=n,a, \quad y=r,i, \tag{4}$$

their dependence upon momentum transfer having been suppressed. As shown by Kim and Primakoff,[1] the Hermiticity of the semileptonic Hamiltonian implies that

$$F(N,x,y;\ \alpha\to\beta) = c_N F(N,x,y;\beta\to\alpha)^*, \quad c_N = \begin{cases} +1 & \text{for } N\equiv A,P \\ -1 & \text{for } N\equiv T, \end{cases} \tag{5}$$

and the time-reversal properties of $A_\lambda^{(x)(y)}$ require

$$\begin{aligned} F(N,x,y;\alpha\to\beta) &= a_{xy}c_N F(N,x,y;\beta\to\alpha) \\ &= a_{xy}F(N,x,y;\ \alpha\to\beta)^*. \end{aligned} \tag{6}$$

Charge symmetry tells us that

$$F(N,x,y;\ \alpha\to\beta) = b_{xy}F(N,x,y;\ \alpha'\to\beta'), \tag{7}$$

where α' and β' are states charge-symmetric to α and β, respectively. Combining Eqs. (6) and (7), we obtain the relation

$$F(N,x,y;\ \alpha\to\beta)=a_{xy}b_{xy}c_N F(N,x,y;\ \beta'\to\alpha')\,. \tag{8}$$

Suppose that in the decay $\alpha\to\beta+e^-+\bar{\nu}$ the states α and β are members of the same isomultiplet with isospin quantum numbers (I,I_z) and (I,I_z+1), respectively. The states α' and β' must also belong to this isomultiplet and their respective quantum numbers are $(I,-I_z)$ and $(I,-(I_z+1))$. Equation (8) now takes the form

$$F(N,x,y;\ I_z\to I_z+1)=a_{xy}b_{xy}c_N F(N,x,y;\ -(I_z+1)\to -I_z). \tag{9}$$

Because the z component of isospin changes by one unit in the transition $\alpha\to\beta$, the current A_λ must be at least an isovector, and it may even contain higher tensors. We write it as a sum of isotensors of rank k $(k=1,2,3,\dots)$,

$$A_\lambda=\sum_{k=1,2,\dots}A_\lambda^{(k)}, \tag{10}$$

and we denote the form factors associated with each $A_\lambda^{(k)}$ by a superscript (k). From the Wigner-Eckart theorem,[3] the ratio of the two form factors in Eq. (9) for a fixed value of k is equal to the ratio of Clebsch-Gordan coefficients

$$\langle (I,I_z),(k,1)|I,I_z+1\rangle/\langle (I,-I_z-1),(k,1)|I,-I_z\rangle.$$

Thus, we can rewrite Eq. (9) as

$$F^{(k)}(N,x,y;\ I_z\to I_z+1) =(-1)^{k+1}a_{xy}b_{xy}c_N F^{(k)}(N,x,y;\ I_z\to I_z+1). \tag{11}$$

Whenever the phase factor in Eq. (11) is negative, the corresponding matrix element vanishes. The appropriate condition, namely,

$$a_{xy}b_{xy}c_N=(-1)^k \tag{12}$$

does not depend upon the isospin of the parent and daughter states α and β, and so terms which do not contribute to neutron decay do not contribute to decays within any isomultiplet. For an isovector current $(k=1)$, which is the case of most interest, the noncontributing terms belong to the second class for axial-vector (A) and induced-pseudoscalar (P) couplings,[1] and to the first class for pseudotensor coupling.

If we want to extend this type of analysis to transitions from one isomultiplet to another, then we must make use of a symmetry higher than isospin. For example, in $\Sigma^\pm\to\Lambda$ β decay, we might assume that the axial-vector current belongs to an SU(3) octet; then, because there is no F-type coupling between Σ and Λ states,[4] it follows that

$$F(N,x,y;\ \Sigma^-\to\Lambda)=-c_N F(N,x,y;\ \Sigma^+\to\Lambda)^*. \tag{13}$$

Combining this result with the CS properties of the current, we find that

$$F(N,x,y;\ \Sigma^-\to\Lambda)=b_{xy}c_N F(N,x,y;\ \Sigma^-\to\Lambda)^*\,. \tag{14}$$

Thus, the axial-vector and induced-pseudoscalar couplings contain no second-class terms, and the pseudotensor no first-class ones; however, no T-violating currents are excluded in this case.

Equation (14) is not as strong a result as Eq. (11) because SU(3) is not an exact symmetry of strong interactions. If the effect of SU(3) breaking is to introduce non-octet components into $\Sigma^\pm$ and Λ states, then the magnitude of these components measures the extent to which Eq. (14) should hold. This magnitude is expected to be of order 10%, even though metastable hadron states are remarkably pure octets.

Other transitions between different isomultiplets occur in strangeness-violating semileptonic decay. Because their selection rules $\Delta S=\Delta Q=1$ are equivlent to[5]

$$\Delta V_z=1,\quad \Delta Y_V=0\,, \tag{15}$$

we can analyze these decays in exactly the same way as strangeness-conserving ones, except that we use the V-spin subgroup[5,6] of SU(3) instead of isospin. As the analog of Eq. (11) we obtain the result

$$\tilde{F}^{(l)}(N,x,y;\ V_z\to V_z+1) =(-1)^{l+1}a_{xy}\tilde{b}_{xy}c_N\tilde{F}^{(l)}(N,x,y;\ V_z\to V_z+1) \tag{16}$$

for the component $\tilde{A}^{(l)}(N,x,y)$ of the $\Delta S=1$ axial-vector current which behaves as an lth-rank tensor in V space. The coefficient a_{xy} is the same as in Eq. (2), and $\tilde{b}_{xy}$ comes from the V-spin analog of Eq. (3).[4]

Equation (16) is not as powerful in its consequences for hyperon decay as its isospin counterpart in Eq. (11). It does imply that components of the axial-vector current for which

$$a_{xy}\tilde{b}_{xy}c_N=(-1)^l \tag{17}$$

cannot contribute to the decays $\Sigma^-\to ne^-\bar{\nu}$ and $\Xi^0\to\Sigma^+e^-\bar{\nu}$, both of which are transitions within a V doublet.[4] But, because Σ^0 and Λ^0 are not eigenstates of V spin,[5,6] it has no simple consequences for decays like $\Xi^-\to\Sigma^0e^-\bar{\nu}$ and $\Lambda\to pe^-\bar{\nu}$.

Currents obeying Eq. (17) cannot contribute to $\Omega^-\to\Xi^{*0}e^-\bar{\nu}$ because it is a transition from one member of a V-spin quartet to another.[5,6] They can, however, contribute to $\Omega^-\to\Xi^0e^-\bar{\nu}$, which is of the type $\frac{3}{2}\to\frac{1}{2}$ in V space. Thus, we see that the best tests of T invariance in $\Delta S=1$ decays are to be found in processes which either involve a Σ^0 or Λ in the initial or final state, or which occur between one V multiplet and another.

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High-Energy Effects in the N/D Method*

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A discussion of high-energy effects in the N/D method is presented and arguments are given that difficulties of the type described by Coulter can be avoided.

In a recent article,[1] Coulter has given an account of an interesting study of high-energy effects in the N/D method. Starting with a standard resonance form, the generalized potential[2]

$$B(s)=\frac{1}{\pi}\oint_L \frac{\operatorname{Im}A(s')}{s'-s}\,ds'$$

is calculated for $s\geqslant 4$ directly from the phase shifts. Then $B(s)$ is used in an N/D calculation, and it is found that when the original phase shifts have an asymptotic behavior corresponding to a Castillejo-Dalitz-Dyson (CDD) pole, $\delta\to\pi$ as $s\to\infty$, the method completely fails to reproduce them. But when the asymptotic behavior is changed to a non-CDD form, $\delta\to 0$ as $s\to\infty$, the resonance is reproduced in a satisfactory way. The use of a narrow-width approximation or a Breit-Wigner form in the crossed channel implies the existence of CDD effects, and it is suggested that an attempt to obtain a direct-channel non-CDD resonance from one of these is inconsistent and can be meaningful only if high-energy effects are negligible. Since the amplitude was the same throughout the entire low-energy region in both of Coulter's calculations, it seems that high-energy effects are important, and that the use of this type of resonance in the crossed channel is in serious error.

Here we would like to suggest that the importance of the high-energy form of the direct-channel phase shift does not necessarily imply equal importance to the high-energy form in the crossed channel.[3] The usual input is obtained from the t-channel partial-wave series, which is convergent only for $s>-32$. Since the left-hand cut is given by an integral over t from 4 to $4-s$, the only t values that contribute directly are $4<t<36$. Thus only small changes can result from changing a high-t phase shift from a CDD form to a non-CDD form. Of course, one could use a truncated series for $s<-32$ (with a cutoff to avoid divergences in the integration), so that large t would formally contribute. However, the divergence of the full series makes this procedure highly suspect.

In any realistic calculation, only the part of the left-hand cut $-32<s<0$ should be directly calculated from the crossed-channel partial-wave expansion. One of us (J.D.) has recently discussed[4] a modified form of the Balázs method by which