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Finite Proton-Neutron Mass Difference and Scale Invariance*

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A representation of the forward Compton amplitude in which the A_2 meson breaks scale invariance is shown to be consistent with existing data for the difference between the proton and neutron structure functions $\nu W_2^p - \nu W_2^n$, while ensuring a finite proton-neutron mass difference ΔM. The conjecture that $W_L \equiv W_1 + (\nu^2/q^2)W_2 \to 0$ as $\nu \to \infty$ for fixed q^2 leads to an expression for ΔM in terms of measurable quantities.

I. INTRODUCTION

We begin with the almost obligatory remark that despite intensive study in recent years, the problem of the proton-neutron mass difference ΔM has remained unsolved. The conjecture[1] that the electromagnetic interaction, in first-order approximation, should give a good estimate for ΔM led to Cottingham's[2] formula for the self-mass of a hadron, δM, in terms of the forward amplitude for Compton scattering. Harari[3] considered the exchange of Regge poles in the crossed channel, and showed that the $\Delta I = 2$ mass differences are adequately obtained from the Born terms in the Cottingham formula, while the $\Delta I = 1$ mass differences could have an additional contribution from the subtraction term for the $T_1(\nu, q^2)$ amplitude, because its behavior is dominated by the A_2 Regge pole. Pagels[4] showed that if the structure functions $W_1(\nu, q^2)$ and $\nu W_2(\nu, q^2)$ are scale-invariant in the Bjorken limit,[5] $-q^2 \to \infty$ with $\omega = -2M\nu/q^2$ fixed, then the self-mass δM diverges unless some unlikely cancellations occur among terms in the Cottingham formula.

We take the position that while divergent self-masses are acceptable, a theory of self-masses must predict the observed finite proton-neutron mass difference. Within the framework of the Cottingham formula, this means that the differences $W_1^p - W_1^n$ and $\nu W_2^p - \nu W_2^n$ cannot have a *nontrivial* Bjorken limit if the proton-neutron mass difference is finite.

II. FORMULA FOR MASS DIFFERENCE

The formula for the self-mass of a hadron is given by

$$\delta M = \frac{i\alpha}{(2\pi)^3} \int \frac{d^4q\, T_{\mu\nu}(\vec{q}, q^0) g^{\mu\nu}}{q^2 + i\epsilon}, \qquad (1)$$

where $\epsilon^\mu \epsilon^\nu T_{\mu\nu}$ is the forward Compton amplitude for scattering of photons of four-momentum q off hadrons of four-momentum P, and $\alpha = e^2/4\pi$. $T_{\mu\nu}$ can be expanded in terms of two Lorentz-invariant functions of q^2 and $\nu = P\cdot q/M$:

$$T_{\mu\nu}(\vec{q}, q^0) = \frac{1}{M^2}\left(P_\mu - \frac{P\cdot q}{q^2} q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2} q_\nu\right) T_2(\nu, q^2) - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) T_1(\nu, q^2). \qquad (2)$$

The Cottingham formula is obtained by a Wick rotation in the variable ν, giving the result

$$\delta M = \frac{\alpha}{2\pi^2} \int_0^{-\infty} \frac{dq^2}{q^2} \int_0^{\sqrt{-q^2}} d\nu\, (-q^2 - \nu^2)^{1/2} T(i\nu, q^2), \qquad (3)$$

where

$$T(\nu, q^2) \equiv T_{\mu\nu}(\vec{q}, q^0) g^{\mu\nu}. \qquad (4)$$

Following Harari,[3] we assume a once-subtracted, fixed-q^2 dispersion relation for $T_1(\nu, q^2)$, while $T_2(\nu, q^2)$ requires no subtraction:

$$T_1(\nu, q^2) = T_1(0, q^2) + 2\nu^2 \int_0^\infty \frac{W_1(\nu', q^2)\,d\nu'}{\nu'(\nu'^2 - \nu^2)} , \quad (5)$$

$$T_2(\nu, q^2) = 2\int_0^\infty \frac{\nu' W_2(\nu', q^2)\,d\nu'}{\nu'^2 - \nu^2} , \quad (6)$$

where the structure functions for inelastic electron-hadron scattering, W_1 and W_2, are given by

$$W_i(\nu, q^2) = (1/\pi)\,\mathrm{Im}T_i(\nu, q^2) . \quad (7)$$

In order to calculate the subtraction constant $T_1(0, q^2)$ we shall assume,[6] consistent with the data, that

$$\lim_{|\nu| \to \infty;\, q^2 \text{ fixed}} T_L(\nu, q^2) = 0, \quad (8)$$

where

$$T_L(\nu, q^2) = T_1(\nu, q^2) + (\nu^2/q^2)T_2(\nu, q^2) ,$$

and we define

$$W_L(\nu, q^2) = (1/\pi)\,\mathrm{Im}T_L(\nu, q^2) . \quad (9)$$

Therefore, we can write an unsubtracted, fixed-q^2 dispersion relation for T_L, obtaining for the longitudinal amplitude

$$T_L(\nu, q^2) = 2\int_0^\infty \frac{\nu' W_L(\nu', q^2)\,d\nu'}{\nu'^2 - \nu^2} , \quad (10)$$

and for the subtraction constant

$$T_1(0, q^2) = 2\int_0^\infty W_L(\nu', q^2)\frac{d\nu'}{\nu'} . \quad (11)$$

Here $T_1(0, q^2)$ is determined in terms of the measurable quantity $W_L(\nu, q^2)$. It should be noted that (8) is equivalent to the statement that $t_1(\nu, q^2)$ satisfies an unsubtracted, fixed-q^2 dispersion relation, where[2,3]

$$T_{\mu\nu}(\vec{q}, q^0) = (q^2 g_{\mu\nu} - q_\mu q_\nu)t_1(\nu, q^2) + [-\nu^2 g_{\mu\nu} - q^2 P_\mu P_\nu/M^2 + (\nu/M)(P_\mu q_\nu + P_\nu q_\mu)]t_2(\nu, q^2) . \quad (12)$$

By using (2) with the representations (5) and (6) and substituting (11) into (3), we have

$$\delta M = \frac{\alpha}{\pi^2}\int_0^{-\infty}\frac{dq^2}{q^2}\int_0^{\sqrt{-q^2}} d\nu\,(-q^2 - \nu^2)^{1/2}\left[-3\left(\int_0^\infty W_L(\nu', q^2)\frac{d\nu'}{\nu'} - \nu^2\int_0^\infty \frac{W_1(\nu', q^2)\,d\nu'}{\nu'(\nu'^2 + \nu^2)}\right) + \left(1 + \frac{\nu^2}{q^2}\right)\int_0^\infty \frac{\nu' W_2(\nu', q^2)\,d\nu'}{\nu'^2 + \nu^2}\right] . \quad (13)$$

We observe that the nucleon poles at $2M\nu = -q^2$, in (13), will give the usual Born contribution to the self-mass.[7]

By performing the ν integration in (13), and defining

$$\Delta W_i(\nu, q^2) = W_i^p(\nu, q^2) - W_i^n(\nu, q^2) \quad \text{and} \quad \Delta W_L(\nu, q^2) = W_L^p(\nu, q^2) - W_L^n(\nu, q^2) , \quad (14)$$

we obtain for the proton-neutron mass difference

$$\Delta M = \Delta M^{\mathrm{Born}} + \frac{\alpha}{2\pi}\int_0^\infty \frac{dq^2}{q^2}\int_{\nu_t}^\infty \nu\,d\nu\left(3\Delta W_1(\nu, -q^2)\left[1 - \left(1 + \frac{q^2}{\nu^2}\right)^{1/2} + \frac{q^2}{2\nu^2}\right] + \Delta W_2(\nu, -q^2)\left\{\left(1 + \frac{q^2}{\nu^2}\right)^{1/2} - \frac{\nu^2}{q^2}\left[1 - \left(1 + \frac{q^2}{\nu^2}\right)^{1/2}\right] - \frac{3}{2}\right\} - \frac{3}{2}\frac{q^2}{\nu^2}\Delta W_L(\nu, -q^2)\right) , \quad (15)$$

where ν_t is the inelastic threshold. By expanding the integrand in (15) in powers of q^2/ν^2, we see that the terms involving ΔW_1 and ΔW_2 are positive provided ΔW_1 and ΔW_2 are positive, while the contribution of ΔW_L to ΔM is positive or negative, depending on the sign of W_L.

III. MODEL FOR THE STRUCTURE FUNCTIONS

In order to discuss the proton-neutron mass difference, we shall consider a model for the forward Compton amplitude valid for large ν. We have

$$T_1^{p,n}(\nu, q^2) = -\pi\left[\sum_{i=P,P'} A_i\left(\frac{\sqrt{\omega}}{1+\sqrt{\omega}} + \frac{\sqrt{-\omega}}{1+\sqrt{-\omega}} - B_i\right)\beta_i^{p,n}\left(\frac{\nu}{-q^2 + m_0^2}\right)^{\alpha_i}\xi_i + A_{A_2}\left(\frac{\sqrt{\omega}}{1+\sqrt{\omega}} + \frac{\sqrt{-\omega}}{1+\sqrt{-\omega}} - B_{A_2}\right)\beta_{A_2}^{p,n}\left(\frac{1}{-q^2 + \tau_{A_2}^{\,2}}\right)^{\gamma}\nu^{\alpha_{A2}}\xi_{A2}\right] , \quad (16)$$

where

$$\xi_i = (1+e^{-i\pi\alpha_i})/\sin\pi\alpha_i \quad (i=P, P'),$$
$$\xi_{A_2} = (1+e^{-i\pi\alpha_{A2}})/\sin\pi\alpha_{A_2}. \tag{17}$$

We shall assume that for large $|\nu|$,

$$T_1^{p,n}(\nu, q^2) = -(\nu^2/q^2)T_2^{p,n}(\nu, q^2), \tag{18}$$

which leads to

$$W_L(\nu, q^2) = 0 \quad (|\nu| \text{ large}). \tag{19}$$

The structure functions W_1 and W_2 are obtained from the imaginary part of (16). We have

$$W_1^{p,n}(\nu, q^2) = \sum_{i=P,P'} f_i(\omega)\beta_i^{p,n}\left(\frac{\nu}{-q^2+m_0^2}\right)^{\alpha_i} + f_{A_2}(\omega)\beta_{A_2}^{p,n}\left(\frac{1}{-q^2+\tau_{A_2}^2}\right)^{\gamma}\nu^{\alpha_{A2}}, \tag{20}$$

where the threshold factors $f_i(\omega)$ are given by (using $\alpha_P = 1$ and $\alpha_{A_2} = \alpha_{P'} = \frac{1}{2}$)

$$f_P(\omega) = A_P\left(\frac{\sqrt{\omega}}{1+\sqrt{\omega}} + \frac{\omega}{1+\omega} - B_P\right)$$

and

$$f_j(\omega) = A_j\left(\frac{\sqrt{\omega}}{1+\sqrt{\omega}} + \frac{\sqrt{\omega}+\omega}{1+\omega} - B_j\right) \quad (j=P', A_2). \tag{21}$$

The parameters A_i and B_i $(i=P, P', A_2)$ are determined by requiring that for $\omega \to \infty$,

$$f_i(\omega) \to 1 \quad (i=P, P', A_2) \tag{22}$$

and also that

$$f_i(\omega) \to 0 \quad \text{as} \quad \omega \to 1. \tag{23}$$

This gives

$$A_P = B_P = 1,$$
$$A_{A_2} = A_{P'} = 2, \tag{24}$$
$$B_{A_2} = B_{P'} = 1.5.$$

The ratio $\beta_{A2}/(\tau_{A_2})^{2\gamma}$ is fixed at $q^2 = 0$ by requiring that the model give the correct value for $\Delta\sigma_T = \sigma(\gamma p) - \sigma(\gamma n)$ in the Regge region $\nu > 2$ GeV. From earlier work,[8] this means that $\beta_{A_2}/(\tau_{A_2})^{2\gamma} = 0.14$ $\text{GeV}^{-3/2}$. The parameters β_P, $\beta_{P'}$, and m_0 are the same as in the earlier model.[8]

We shall now enumerate some properties of the amplitude (16):

(1) For fixed q^2, (16) is a real analytic function in the cut ν plane with poles in the second sheet; hence (16) satisfies fixed-q^2 dispersion relations.

(2) The model possesses the correct s-u crossing symmetry, as it is even for $\nu \to -\nu$.

(3) For fixed $|\vec{q}|$, (16) has poles only in the second sheet, and therefore a Wick rotation may be performed on (16) to obtain the Cottingham formula (3).

(4) The representation (16) is valid for all ν and q^2 outside the resonance regions [except for possible fixed poles, for example, at $J = 0$, which may have to be included in (16)[9]]. It has the threshold property $W_i^{p,n}(\omega, q^2) \to 0$ for $\omega \to 1$, and the correct Regge behavior as $\nu \to \infty$ for fixed q^2.

(5) The P and P' terms are scale-invariant in the limit $-q^2 \to \infty$, ω fixed, and when $\gamma = \alpha_{A_2}$, then also the A_2 contribution is scale-invariant in this limit, but when $\gamma \neq \alpha_{A_2}$ the A_2 term breaks scale invariance.

The A_2 contribution is the only one that occurs in the proton-neutron mass difference. If ΔW_1 and $\nu(\Delta W_2)$ approach nontrivial scale-invariant limits, then the ΔW_1 and $\nu(\Delta W_2)$ terms in ΔM will diverge logarithmically. Explicitly, expanding the integrand in (15) in powers of $1/(q^2\omega^2)$, we have

$$\Delta M = \Delta M^{\text{Born}} + \Delta M^{\text{in}} + \Delta M^{\text{sub}}, \tag{25}$$

where

$$\Delta M^{\text{in}} = \frac{3\alpha}{4\pi}\int_0^\infty \frac{dq^2}{q^2}\int_{\omega_t}^\infty d\omega\left(\frac{M^2\Delta W_1(\omega, -q^2)}{\omega^3} + \frac{M\nu\Delta W_2(\omega, -q^2)}{2\omega^2} + (\text{higher powers})\right) \tag{26}$$

and

$$\Delta M^{\text{sub}} = -\frac{3\alpha}{4\pi}\int_0^\infty dq^2\int_{\omega_t}^\infty \frac{d\omega}{\omega}\Delta W_L(\omega, -q^2). \tag{27}$$

Because scale invariance of ΔW_1 and $\nu(\Delta W_2)$ implies only a logarithmic divergence of the mass difference, the violation of scale invariance necessary to render the mass difference finite may be very difficult to observe. In order to see this suppose we take $\alpha_{A_2} = \frac{1}{2}$ and $\gamma = 1$ in (16). Then β_{A_2} is fixed by a fit to the difference $\nu(\Delta W_2)$, which gives $\beta_{A_2} = 0.3$ $\text{GeV}^{1/2}$.

With these parameters, we can predict $\nu(\Delta W_2)$ versus ω for various fixed values of $-q^2$; the results are shown in Fig. 1. Also shown for comparison is the prediction with $\gamma = \alpha_{A_2} = \frac{1}{2}$ and $\tau_{A_2} = m_0 = 0.567$ GeV corresponding to a scale-invariant A_2 contribution as $-q^2 \to \infty$. We see that for

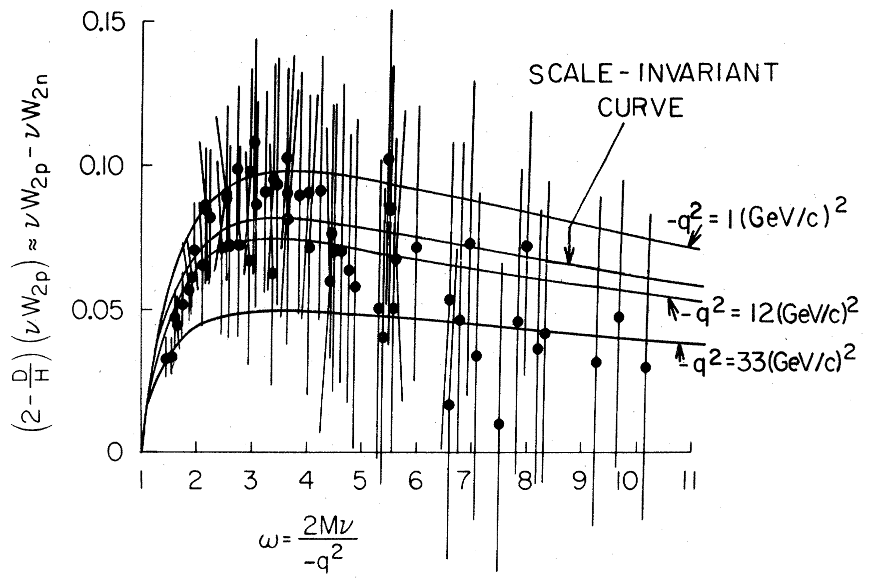

FIG. 1. The difference between the proton and neutron structure functions $\nu W_2^p - \nu W_2^n$ plotted versus ω for fixed values of $-q^2$. Also shown is our scale-invariant prediction. Glauber corrections are ignored and the data points are from Ref. 10.

$-q^2 \leq 12\ \text{GeV}^2$, which is the range of $-q^2$ in the data, the scale-invariance-breaking results are consistent with the preliminary data,[10] the largest discrepancy occurring for large values of $-q^2$. Of course, νW_2^p and νW_2^n are both scale-invariant in the Bjorken limit for this model.

The contribution of ΔM^{sub} in (25) is negative provided $W_L(\nu, -q^2)$ is positive over the range of integration. We stress that the correct sign for ΔM can only arise from ΔM^{sub} in (27), because ΔM^{Born} is known to be positive, while ΔM^{in} also appears to be positive when compared to the SLAC-MIT data. Thus, if ΔW_L is positive for a sufficiently large range of ω and $-q^2$, then it is possible that ΔM is negative. The assumption that ΔW_1 and $\nu(\Delta W_2)$ have trivial scale-invariance limits means that the divergence of ΔM^{sub} must be less than quadratic. In fact, if $\Delta W_L(\omega, q^2)$ vanishes sufficiently rapidly in both the ω and $-q^2$ directions, then the integrals in (27) will converge and ΔM will be finite.

IV. CONCLUSION

Our main assumptions have been: (1) $T_L(\nu, q^2)$ satisfies an unsubtracted, fixed-q^2 dispersion relation; (2) The differences ΔW_1 and $\nu(\Delta W_2)$ break scale invariance in the limit $-q^2 \to \infty$, ω fixed.

A model which satisfies both assumptions has been shown to give reasonable fits to the SLAC-MIT data for $\nu(\Delta W_2)$ in the region in the ω-q^2 plane experimentally investigated.

We could adopt, at this stage, the point of view that a nontrivial Bjorken scale-invariance limit for $\nu(\Delta W_2)$ is most likely valid in nature, since it is (to some people)[6] most appealing if scale invariance is not broken by any of the dominant contributions to W_1 or νW_2. However, from the point of view of the mass-difference problem, assuming that the Cottingham approach is valid, this would lead to a divergent proton-neutron mass difference, which is unacceptable. We have demonstrated that a model of $\nu(\Delta W_2)$ possessing a trivial scale-invariance limit (subject to certain assumptions about the high-energy behavior of W_L) is consistent with the present data. It is our opinion that the latter picture is more pleasing, for there are more forceful reasons for believing in a finite proton-neutron mass difference than in the existence of a nontrivial scale-invariance limit for $\nu(\Delta W_2)$.

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Form Factors for Dominant Inelastic $e^\pm$-$e^\pm$ Collisions at High Energies*

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We study and present forms for the cross section of e-e two-photon processes, $e+e \to e+e+$hadrons, analogous to the W_1, W_2 form factors in e-p inelastic collisions. In general, the cross sections are a function of ten form factors. Two do not contribute if time-reversal invariance holds. At very high energies, with no polarization measurements or with real photons, the cross sections reduce to functions of fewer form factors.

I. INTRODUCTION

Brodsky, Kinoshita, and Terazawa,[1] in their important and interesting paper, point out that in electron collisions, two-photon-exchange processes

$$e+e \to e+e+\text{hadrons} \tag{1}$$

increase logarithmically and are dominant at high energy in spite of a factor of α^4. They find that two-photon reactions are the most frequent events above 1 GeV. Balakin, Budnev, and Ginsburg,[2] Arteaga-Romero, Jaccarini, Kessler, and Parisi,[3] and Serbo[4] also consider the importance of these processes and competing reactions with an eye toward studying photon-photon interactions. Needless to say, these results give us a new insight into high-energy electron processes, and are important because they redirect our over-all view of electron-electron colliding-beam experiments.

In hopes of facilitating future experimental measurements, the double-photon cross sections were examined and a parametrization analogous to the W_1 and W_2 form factors in e-p scattering[5-7] was derived.[8] This is presented in Sec. II. In Secs. III–V, cross sections and their high-energy limits are derived for e-e, γ-e, and γ-γ collisions in terms of a number of form factors which describe the two-photon creation of hadrons. It is not within the scope of this article to examine the regions in which this production mechanism dominates over other diagrams, for instance the $C=-1$ reactions. This is being carried out in other work. Nor is the complete singularity structure[8] examined, since the aim is to present a simple derivation of the form factors analogous to the W_1 and W_2 functions of direct use to experimental measurements.

II. FORM FACTORS

Drell and Walecka,[5] von Gehlen,[6] and Gourdin[7] showed that in e-p scattering with one-photon production of hadrons (Fig. 1.), the tensor $T_{\mu\nu}$, representing the square of the photon-photon-hadron vertex summed over all polarizations, could be represented by two form factors, each a function of two variables:

$$T_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2, q\cdot p) + \frac{W_2(q^2, q\cdot p)}{m_p{}^2}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right). \tag{2}$$

One assumes that the target is unpolarized, that final polarization is not measured, and that all experimentally accessible hadron states are summed over. Current conservation,

$$q_\mu T_{\mu\nu} = q_\nu T_{\mu\nu} = 0\,, \tag{3}$$

limits us to the two functions W_1 and W_2. Since the current is a polar vector, there are no terms of the form $\epsilon_{\mu\nu\rho\sigma}p_\rho q_\sigma$.

Similarly, for the two-photon-exchange reaction