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Second-Class Currents: Some Model-Independent Considerations and Conclusions

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We study the problem of second-class currents, and attempt to clarify the situation, by working within the framework of a formulation that makes no appeal to any specific dynamical model of hadronic interactions or nuclear structure. While much of the paper is pedagogical, several new theorems and results are presented. The import of recent experiments is discussed, and a scheme for classification of second-class currents is proposed.

I. INTRODUCTION

Although the notion of "second-class currents" was introduced by Weinberg[1] over 13 years ago, it is only in the last year due to the remarkable experiments of Wilkinson, Alburger, and collaborators[2] that the subject has attracted widespread attention among theoretical physicists.[3] Regrettably, most of the recent discussion, if not entirely incorrect or misleading, has been within the framework of models which are, in many cases, oversimplified, to say the least. As a result, the precise significance of the experimental results[4] of Wilkinson, Alburger, *et al.* has been obscured.

The purpose of this paper is to attempt to clarify the situation, by working within the framework of a formulation that makes no appeal to any specific dynamical model of hadronic interactions or nuclear structure. We shall look upon β transitions of a nucleus as the β transitions of the nucleus as a whole, rather than in terms of β transitions undergone by "off-shell" nucleons in the nucleus; in effect, we shall treat nuclei as "elementary particles" of *arbitrary spin.*[5] It is immaterial then whether a nucleus such as C^{12} is presumed to consist of six protons and six neutrons, or 12 neutrons and six quark-antiquark pairs.

While much of this paper is frankly pedagogical, we do report some new results – new either in the sense that they did not heretofore exist in the literature, or in the sense that they have now been liberated from model-dependent derivations. We are led to the recognition that the notion of second-class currents, as introduced by Weinberg, embraces a variety of currents with different theoretical and experimental implications; a systematic study of the subject must therefore entail a subclassification. Such a subclassification is proposed in Sec. VII of this paper.

In Sec. II of this paper we introduce a perspicuous notation. Section III is devoted to the question of whether one can distinguish between first- and second-class currents by looking at the space-time properties of their matrix elements; in this section we state and prove an extension of a theorem on Hermitian currents due to Durand, De Celles, and Marr.[6] In Sec. IV we develop a general kinematical formalism[7] for dealing with semileptonic interactions and state some relevant formulas for decay rates and ft values; we also show that for transitions within an isomultiplet, and *only* for transitions within an isomultiplet, interference effects between first- and second-class currents of the same isospin (or isospins differing by 0 mod 2) vanish in the limit of zero lepton mass.[8] Section V is devoted to a more detailed study of these interference effects; it is shown that if the second-class current is conserved, the difference of ft values for mirror transitions must at least have a linear dependence on the sum of the energy releases.[9] In Sec. VI we introduce and discuss the topic of conjugate transition. Section VII deals with the subclassification mentioned earlier. In Sec. VIII we discuss the conclusions that can be drawn from present experiments and other possible tests for second-class currents.

II. NOTATION Etc.

In the following we denote first-class, charge-raising, hypercharge-conserving,[10] vector and axial-vector currents by $V_\mu^{(+)}$ and $A_\mu^{(+)}$, respectively; second-class currents with the same quantum numbers will be denoted by $\overline{V}_\mu^{(+)}$ and $\overline{A}_\mu^{(+)}$. It is understood that these are Heisenberg operators, carrying the full time dependence generated by the hadronic Hamiltonian; effects arising from the exchanges of π's, ω's, or U^{235}-$\overline{U}^{235}$ pairs are, therefore, all correctly included.[11] The definition of class implies that

$$G V_\mu^{(+)} G^{-1} = V_\mu^{(+)}, \tag{2.1}$$

$$G A_\mu^{(+)} G^{-1} = -A_\mu^{(+)}, \tag{2.2}$$

$$G \overline{V}_\mu^{(+)} G^{-1} = -\overline{V}_\mu^{(+)}, \tag{2.3}$$

$$G \overline{A}_\mu^{(+)} G^{-1} = \overline{A}_\mu^{(+)}. \tag{2.4}$$

Charge-lowering currents are *defined* via an isotopic rotation

$$J_\mu^{(-)} = \eta e^{-i\pi I_2} J_\mu^{(+)} e^{+i\pi I_2} \quad (J = V, A, \overline{V}, \text{ or } \overline{A}), \tag{2.5}$$

where the phase factor $\eta = (-1)^{I_J - 1}$, if J transforms as a $(2I_J + 1)$-component tensor operator of the isospin group.[12]

Since the charge-conjugation operator commutes with $\exp(i\pi I_2)$ [our phase convention is such that $C(I_1, I_2, I_3)C^{-1} = (-I_1, I_2, -I_3)$], the definition in Eq. (2.5) implies that if $J_\mu^{(+)}$ has a definite C parity, so does $J_\mu^{(-)}$, and the two C parities are identical. The same is, of course, true of G, P, and T. We may therefore write

$$C J_\mu^{(\pm)} C^{-1} = \eta_C J_\mu^{(\pm)\dagger}, \tag{2.6}$$

$$P J_\mu^{(\pm)}(\vec{x}, t) P^{-1} = \eta_P g^{\mu\nu} J_\nu^{(\pm)}(-\vec{x}, t), \tag{2.7}$$

$$T J_\mu^{(\pm)}(\vec{x}, t) T^{-1} = \eta_T g^{\mu\nu} J_\nu^{(\pm)}(\vec{x}, -t), \tag{2.8}$$

$$G J_\mu^{(\pm)} G^{-1} = \eta_G J_\mu^{(\pm)}, \tag{2.9}$$

where

$$\eta_C \eta_P \eta_T = -1 \quad \text{for } all\ J \quad (TCP \text{ theorem}), \tag{2.10}$$

$$\eta_G \eta_P = +1, \quad J = V, A \tag{2.11}$$

$$= -1, \quad J = \overline{V}, \overline{A}. \tag{2.12}$$

Equations (2.5), (2.6), and (2.9) imply that

$$J_\mu^{(\pm)} = -\eta \eta_T \eta_G \eta_P J_\mu^{(\mp)\dagger}, \tag{2.13}$$

a relationship that will prove useful later. If one assumes a "$\Delta I = 1$ rule," one may set $\eta = +1$. (Here "$\Delta I = 1$ rule" means simply that the current carries $I_J = 1$; it should *not* be confused with the charge-symmetry condition $\eta\eta_T\eta_G\eta_P = +1$ which cannot be simultaneously respected by first- and second-class currents without violating T invariance.)

We shall refer to a current as normal, under time reversal, if $\eta_T = +1$, abnormal if $\eta_T = -1$.

Finally, the charges associated with these currents will be defined in the usual way:

$$Q_J^{(\pm)} = \int J_0^{(\pm)}(\vec{x}, t)\, d^3x. \tag{2.14}$$

III. SPACE-TIME AND ISOTOPIC PROPERTIES OF MATRIX ELEMENTS OF CURRENT OPERATORS

In this section we address ourselves to the following question: Can one distinguish between first- and second-class currents by looking at the space-time properties of their matrix elements? We show that this is possible only for matrix elements between two members of an isomultiplet. For these cases we obtain the theorem that with a "ΔI-odd rule" the matrix elements of second-class axial-vector currents and first-class vector currents are divergenceless; with a "ΔI-even rule" the matrix elements of first-class axial-vector and second-class vector currents are divergenceless. This theorem is an extension, to non-Hermitian currents, of a theorem on Hermitian currents due to Durand, DeCelles, and Marr.

We use invariantly normalized helicity states, and define a partially reduced vertex Γ_μ via

$$\langle \vec{p}_f, \lambda_f; n_f, I_f^3 | J_\mu^{(+)}(0) | \vec{p}_i, \lambda_i; n_i, I_i^3 \rangle = (-1)^{I_f - I_f^3} \begin{pmatrix} I_f & I_J & I_i \\ -I_f^3 & 1 & I_i^3 \end{pmatrix} \Gamma_\mu^{\lambda_f \lambda_i}(\vec{p}_f, n_f; \vec{p}_i, n_i). \tag{3.1}$$

Here n indicates internal quantum numbers other than I^3, the 3 component of isospin. ($I_J^3 = 1$ since in a $\beta^\mp$ transition $\Delta I^3 = \pm 1$.) Equation (3.1) is, of course, rendered possible by the Wigner-Eckart theorem, in isospin space. Using Eqs. (2.5), (2.13), (3.1), the relationship

$$e^{-i\pi I_2} |\vec{p}, \lambda; n, I^3\rangle = (-1)^{I - I^3} |\vec{p}, \lambda; n, -I^3\rangle, \tag{3.2}$$

and some elementary properties of the 3-j symbols, we find

$$\Gamma_\mu^{\lambda_f \lambda_i}(\vec{p}_f, n_f; \vec{p}_i, n_i) = (-1)^{I_f - I_i + I_J + 1} \eta_G \eta_P \eta_T \Gamma_\mu^{\lambda_i \lambda_f}(\vec{p}_i, n_i; \vec{p}_f, n_f)^*. \tag{3.3}$$

For *general* n_i, n_f, Eq. (3.3) relates two *different* vertex functions; specification of the class of the current, therefore, does not put any constraints on the form factors which occur in a given vertex function. The only cases in which a constraint is obtained are those in which n_i and n_f are two members of the *same* isotopic multiplet. For then $n_i \equiv n_f$ and

$$\Gamma_\mu^{\lambda_f\lambda_i}(\vec{p}_f, n_f; \vec{p}_i, n_i) = (-1)^{I_J+1}(\eta_G\eta_P)\Gamma_\mu^{\lambda_i\lambda_f}(\vec{p}_i, n_f; \vec{p}_f, n_i)^* . \tag{3.4}$$

In order to study the matrix element of the divergence, it is convenient to go to the brick-wall frame in which $\vec{p}_f + \vec{p}_i = 0$, and choose the z axis to lie along $\vec{p}_i$. If we denote the matrix elements of the divergence by Γ_D, we have

$$\Gamma_D^{\lambda_f\lambda_i} = i(p_f - p_i)^\mu \Gamma_\mu^{\lambda_f\lambda_i} \tag{3.5}$$

$$= i2|\vec{p}_i|\Gamma_3^{\lambda_f\lambda_i}(\vec{p}_i) \quad \text{(brick-wall frame)} . \tag{3.6}$$

Now rotational invariance tells us that

$$\Gamma_3^{\lambda_f\lambda_i}(\vec{p}_i) = \delta_{-\lambda_f,\lambda_i}\Gamma_3^{-\lambda_i,\lambda_i}(\vec{p}_i)$$

$$= \delta_{-\lambda_f,\lambda_i}(-1)^{2s+1}\Gamma_3^{-\lambda_i,\lambda_i}(\vec{p}_f) , \tag{3.7}$$

and consideration of the discrete operations $P\mathfrak{D}(0,\pi,0)$ and $T\mathfrak{D}(0,\pi,0)$ leads to

$$\Gamma_3^{-\lambda_i,\lambda_i}(\vec{p}_i) = \eta_P(-1)^{2s}\Gamma_3^{\lambda_i,-\lambda_i}(\vec{p}_i) , \tag{3.8}$$

$$\Gamma_3^{-\lambda_i,\lambda_i}(\vec{p}_i) = \eta_T\Gamma_3^{-\lambda_i,\lambda_i}(\vec{p}_i)^* . \tag{3.9}$$

Equations (3.4), (3.7), (3.8), and (3.9) together imply that

$$[1 + (-1)^{I_J+1}\eta_G]\Gamma_3^{-\lambda_i,\lambda_i}(\vec{p}_i) = 0 . \tag{3.10}$$

$\Gamma_D^{\lambda_f,\lambda_i}$ therefore vanishes in the brick-wall frame, and hence in all frames, if $1 + (-1)^{I_J+1}\eta_G \neq 0$. Consequently, with a "$\Delta I$-odd rule" first-class vector and second-class axial-vector currents, when sandwiched between the states of an isomultiplet, are effectively conserved. With "ΔI-even rules," the foregoing statement modifies in an obvious fashion. Note that the above result is true for both T-normal and T-abnormal currents.

IV. DECAY RATES AND ft VALUES

Let the leptonic current be denoted by l_μ and the total hadronic current by J_μ. The rate for the β^- reaction $N_i(Z) \to N_f(Z+1) + e^- + \bar{\nu}_e$ may be written in the form

$$\Gamma^{(-)} = \frac{G^2}{4M_i}\left(\frac{1}{2s_i+1}\right)\int \sum_{\text{spins}} |\langle f|J_\mu^{(+)}|i\rangle\langle e^-\bar{\nu}_e|l^\mu|0\rangle|^2 F^{(-)}(2\pi)^4\delta^4(p_f + p_{e^-} + p_{\bar{\nu}} - p_i)\, d^9\Omega . \tag{4.1}$$

Here p_a indicates the momentum of particle a, M_i and s_i are the mass and spin, respectively, of the parent nucleus, G is the weak-coupling constant, and F is a correction factor to take account of the Coulombic interaction between the lepton and the daughter nucleus.[13] $d^9\Omega$ is an invariant volume element in the final-state phase space.

We define leptonic and hadronic tensors, $L^{\mu\nu}$ and $H^{\mu\nu}$, via

$$L_{\mu\nu} = \sum_{\text{spins}} \langle e^-\bar{\nu}_e|l_\mu|0\rangle\langle e^-\bar{\nu}_e|l_\nu|0\rangle^* \tag{4.2}$$

$$= 4[(q_\mu q_\nu - K_\mu K_\nu) - g_{\mu\nu}(q^2 - m_e^2) - i\epsilon_{\mu\nu\lambda\rho}q^\lambda K^\rho] , \tag{4.3}$$

$$H_{\mu\nu}^{(+)} = \sum_{\text{spins}} \langle f|J_\mu^{(+)}|i\rangle^*\langle f|J_\nu^{(+)}|i\rangle \tag{4.4}$$

$$= \sum_{a=1}^{6} H_a^{(+)}C_{\mu\nu}^a(p_f, p_i) . \tag{4.5}$$

Here

$$q^\mu = p_e^\mu + p_{\nu_e}^\mu = p_i^\mu - p_f^\mu ,$$
$$K^\mu = p_e^\mu - p_{\nu_e}^\mu , \quad P^\mu = p_f^\mu + p_i^\mu , \quad M = M_f + M_i \tag{4.6}$$

and the tensor covariants $C_{\mu\nu}$ are defined as follows:

$$C_{\mu\nu}^1 = -M^2 g_{\mu\nu} + P_\mu P_\nu , \quad C_{\mu\nu}^2 = P_\mu P_\nu , \quad C_{\mu\nu}^3 = q_\mu q_\nu ,$$
$$C_{\mu\nu}^4 = i\epsilon_{\mu\nu\lambda\rho}q^\lambda P^\rho , \quad C_{\mu\nu}^5 = P_\mu q_\nu + P_\nu q_\mu , \tag{4.7}$$
$$C_{\mu\nu}^6 = i(P_\mu q_\nu - P_\nu q_\mu) .$$

The representation, Eq. (4.5), of the hadron tensor follows from general covariance considerations.[7] The H_a are scalar functions of q^2, $q \cdot P$, and $P^2 + q^2$; for our purpose it is sufficient to display only the dependence on q^2 and $W \equiv M_f - M_i$, the energy release. We shall therefore write $H_a \equiv H_a(q^2, W)$.

Evidently $H_{\mu\nu}^{(+)} = H_{\nu\mu}^{(+)*}$; hence,

$$H_a^{(+)} = H_a^{(+)*} . \tag{4.8}$$

Time-reversal invariance implies the constraints

$$H_a^{(+)} = H_a^{(+)*} , \quad a \neq 6 \tag{4.9}$$

$$H_6^{(+)} = -H_6^{(+)*} , \tag{4.10}$$

so $H_6 = 0$ if T is conserved.

In general, the terms in $H_{\mu\nu}^{(+)}$ which stem from interference between the first- and second-class pieces can contribute to each of the invariants $H_a^{(+)}$; the a here is a space-time label and, as is evident from the discussion in Sec. III, cannot be correlated with the concept of class. In the special case in which the initial and final nuclear states are members of the same isomultiplet, a correlation does exist however; in this case Eq. (3.9) indicates that

$$H^{(+)}(p_f, p_i)_{\text{int}} = \mp H^{(+)}(p_i, p_f)_{\text{int}}^* , \tag{4.11}$$

where the minus sign holds if first- and second-class currents obey the same ΔI rule (odd or even); the plus sign holds if they obey opposite ΔI rules. In the former case, the interference term gets localized in $H_5^{(+)}$, and the interference effect vanishes in the limit of zero lepton mass.

The total decay rate may now be written in the form

$$\Gamma^{(-)}=\sum_{a=1}^{6}G_a^{(+)}(W)f_a^{(-)}, \tag{4.12}$$

where

$$f_a^{(-)}=\frac{G^2}{4M_i(2s_i+1)}\int\frac{H_a^{(+)}(q^2,W)}{G_a^{(+)}(W)}C_a^{\mu\nu}L_{\mu\nu}F^{(-)}(2\pi)^4 \times\delta^4(p_e+p_{\nu_e}+p_f-p_i)\,d^9\Omega, \tag{4.13}$$

$$G_a^{(+)}(W)=H_a^{(+)}(0,W). \tag{4.14}$$

Note that in the allowed approximation $G_a(W)/H_a(q^2,W)=1$.

We may therefore define an "f_1t value" via

$$\frac{1}{(f_1t)^-}=G_1^{(+)}(W)+\sum_{a=2}^{6}G_a^{(+)}(W)\frac{f_a^{(-)}}{f_1^{(-)}}. \tag{4.15}$$

Apart from a known, and inessential, normalization factor, this f_1t value reduces to the usual ft value[13] for a Gamow-Teller transition in the allowed approximation. (For a Fermi transition, the corresponding quantity is f_2t.)

For a positron emitter, Eq. (4.15) is replaced by

$$\frac{1}{(f_1t)^+}=G_1^{(-)}(W)+\sum_{a=2}^{6}G_a^{(-)}(W)\frac{f_a^{(+)}}{f_1^{(+)}}. \tag{4.16}$$

If the positron emitter is the mirror nucleus to the electron emitter, effects stemming from second-class currents may be measured through the parameter δ defined via

$$\delta=\frac{(ft)^+-(ft)^-}{(ft)^-} \tag{4.17}$$

$$\simeq\frac{(f_1t)^+-(f_1t)^-}{(f_1t)^-}. \tag{4.18}$$

The near equality in Eq. (4.18) would be replaced by an exact equality in the absence of forbidden corrections.

V. STRUCTURE OF INTERFERENCE TERMS IN THE HADRON TENSOR; ENERGY DEPENDENCE OF δ

We are interested in isolating the contributions to f_1t which arise from interference between the first- and second-class pieces of the hadron current. Since this interference term, if it exists at all, is certainly small, we shall deem it legitimate to neglect radiative corrections (such as those depicted in Fig. 1) and consider only V-$\bar{V}$ and A-$\bar{A}$ interference.

Since all β transitions of interest in the present context are between different isotopic multiplets, the dominant transition – the only one in the allowed approximation – is that induced by the axial-vector current. We restrict ourselves, therefore, to A-$\bar{A}$ interference.

Let us define a tensor $h_{\mu\nu}^{(+)}$ via

$$h_{\mu\nu}^{(+)}=\sum_{\text{spins}}\langle f|\bar{A}_\mu^{(+)}|i\rangle^*\langle f|A_\nu^{(+)}|i\rangle \tag{5.1}$$

$$=\sum_{a=1}^{6}h_a^{(+)}C_{\mu\nu}^a \quad (h_4^{(+)}\equiv 0). \tag{5.2}$$

The interference terms contained in $H_{\mu\nu}^{(+)}$ may be written as

$$\delta H_{\mu\nu}^{(+)}=h_{\mu\nu}^{(+)}+h_{\nu\mu}^{(+)*}, \tag{5.3}$$

so that

$$\delta H_a^{(+)}=2\,\mathrm{Re}h_a^{(+)}. \tag{5.4}$$

If T is conserved, $\delta H_6^{(+)}=0$; $h_6^{(+)}$ is therefore purely imaginary.

In evaluating the contribution of δH to the ft value, we shall retain terms up to first order in the energy release and neglect the Coulombic correction, as well as terms which go to zero in the limit $m_e\to 0$. With these approximations one finds

$$\left(\frac{1}{(f_1t)^-}\right)_{\text{int}}=\delta H_1^{(+)}+\tfrac{1}{3}\delta H_2^{(+)}, \tag{5.5}$$

so that

$$\left(\frac{(f_1t)^+-(f_1t)^-}{(f_1t)^+(f_1t)^-}\right)_{\text{int}}=[\delta H_1^{(+)}(W^-)-\delta H_1^{(-)}(W^+)] +\tfrac{1}{3}[\delta H_2^{(+)}(W^-)-\delta H_2^{(-)}(W^+)]. \tag{5.6}$$

Since $\delta H^{(\pm)}$ arise from first-class–second-class interference,

$$\delta H_a^{(+)}(W)=-\delta H_a^{(-)}(W). \tag{5.7}$$

Equation (5.6) may therefore be written in the

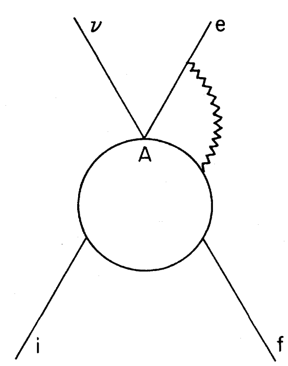

FIG. 1. Example of radiative correction which can give rise to V-A interference in the ft value.

form

$$\left(\frac{(f_1t)^+-(f_1t)^-}{(f_1t)^-}\right)_{\rm int}=a_0+a_1(W^++W^-)+O(W^2)\,, \quad (5.8)$$

where a_0 and a_1 are energy-independent constants. Nothing can be said about these constants without further specification of the nature of the second-class current. Let us assume, for example, that the $\overline{A}_\mu$ is a conserved, or effectively conserved, current. In this case one has the constraints

$$-M^2h_1+h_3q^2+h_5(q\cdot P)+ih_6(q\cdot P)=0\,, \quad (5.9)$$

$$h_1(q\cdot P)+h_2(q\cdot P)+h_5q^2-ih_6q^2=0\,. \quad (5.10)$$

Since the h_a are nonsingular, it follows that, in the limit in which nuclear recoil is neglected, both h_1 and h_2 have at least a linear dependence on the energy release. Hence,

$$a_0=0 \quad \text{if} \quad \langle\partial^\mu \overline{A}_\mu^{(+)}\rangle=0\,. \quad (5.11)$$

Finally, we note that the quantity δ defined in Eq. (4.17) can be nonzero either by virtue of second-class currents or by virtue of electromagnetic effects other than those included in the definition of the ft value. In lowest order these are (i) retardation and recoil effects in the exchange of a photon between the electron and hadronic matter, (ii) electromagnetic induction of a second-class piece in $A_\mu^{(+)}$, viz.,

$$i\int T\{A_\mu^{(+)}\mathcal{H}_{\rm em}^{\Delta I=1}(x)\}d^4x\,,$$

(iii) isospin mixing brought about by the $\Delta I=1$ and $\Delta I=2$ pieces of the electromagnetic interaction. None of these effects can be computed very reliably; the only firm statement one can make is that no $Z^2\alpha$ effects are possible.[14] We must content ourselves, therefore, with an expression for δ of the form

$$\delta=(a_0+b_0\alpha+c_0Z\alpha)+(a_1+b_1\alpha+c_1Z\alpha)(W^++W^-)\,. \quad (5.12)$$

VI. CONJUGATE TRANSITIONS

To every hadronic transition $N_i\to N_f$, whose amplitude may be measured in reactions such as $N_i\to N_f+e^-+\overline{\nu}_e$ or $\nu_e+N_i\to e^-+N_f$, corresponds a conjugate transition $N_f\to N_i$ whose amplitude may be measured in reactions such as $e^-+N_f\to\nu_e+N_i$ or $\overline{\nu}_e+N_f\to e^++N_i$. In this section we address ourselves to the following question: If the nuclear matrix element changes as one goes from a transition to the conjugate transition, can such change be detected by measurement of total rates?[11] Our answer is yes, in principle; in practice, however, one has to measure terms that are rather small because of their proportionality to the electron mass. This difficulty can be ameliorated by considering the reactions[8] $\nu_\mu+N_i\to\mu^-+N_f$ and $\overline{\nu}_\mu+N_f\to\mu^++N_i$.

These considerations are relevant to a discussion of second-class currents only in those special cases in which a conjugate transition may be related to a mirror transition. For didactic reasons, it is worthwhile, however, to discuss conjugate transitions in complete generality.

The hadron tensors for the transitions $N_i(p_1)\to N_f(p_1')$ and $N_f(p_2)\to N_i(p_2')$ may be written in the form

$$H_{\mu\nu}^{fi}(p_1',p_1)=\sum_a H_a^{fi}(p_1'^2,p_1^2,q_1^2)C_{\mu\nu}^a(p_1',p_1)\,, \quad (6.1)$$

$$H_{\mu\nu}^{if}(p_2',p_2)=\sum_a H_a^{if}(p_2'^2,p_2^2,q_2^2)C_{\mu\nu}^a(p_2',p_2)\,, \quad (6.2)$$

where the notation is that of Sec. IV except that we choose to indicate the dependence of the H_a on the Lorentz scalars in the problem in a slightly different fashion. Since the conjugate transition is induced by $J_\mu^{(+)\dagger}$, the definition of $H_{\mu\nu}$ implies the crossing relation

$$H_{\mu\nu}^{fi}(p_1',p_1)=H_{\nu\mu}^{if}(p_1,p_1')\,. \quad (6.3)$$

Hence,

$$H_a^{fi}(p_1'^2,p_1^2,q^2)=H_a^{if}(p_2'^2,p_2^2,q^2)\,, \quad a\neq 5 \quad (6.4)$$

$$H_5^{fi}(p_1'^2,p_1^2,q^2)=-H_5^{if}(p_2'^2,p_2^2,q^2) \quad (6.5)$$

at

$$q_1^2=q_2^2=q^2\,.$$

Consequently the result: Any difference in the nuclear matrix element between conjugate transitions can manifest itself only through the invariant H_5; it will therefore be undetectable in the limit of zero lepton mass.

In the special case in which the conjugate transition also happens to be the mirror transition, N_i and N_f must be members of an isodoublet; that the difference in nuclear matrix elements stemming from second-class currents will be visible only through H_5, in this case, has already been noted (Sec. IV).

VII. PROPOSAL FOR CLASSIFICATION OF SECOND-CLASS CURRENTS

The preceding considerations lead us to propose that second-class currents be classified as follows:

Type I. Vector current normal under time reversal. Any test for such a current is really a test for some facet of conservation of vector current (CVC).[15] To the extent that one accepts CVC, as an attractive theoretical postulate that has been vindicated by experiment, one may assume that such currents do not exist in nature.

Type II. Nonconserved axial-vector current normal under time reversal. In the presence of such currents the Cartesian components of the total axial charge,[16] viz., $Q_A^i + \overline{Q}_A^i$ ($i=1, 2, 3$), are not Hermitian operators and therefore cannot generate unitary transformations; the elegant picture of weak charges satisfying a chiral algebra and acting as generators of approximate hadron symmetries[17] therefore gets disrupted. Such currents, while theoretically very unappealing, are difficult to rule out on the basis of present experiments; the major problem is how to distinguish them from induced electromagnetic effects.

Type III. Conserved axial-vector currents normal under time reversal. In order to avoid difficulties with parity doubling and/or the real existence of massless Goldstone bosons, one must assume that the associated charge $\overline{Q}_A^i \equiv 0$. Such chargeless currents are fully compatible with contemporary theoretical ideas. They can always be represented in the "PCTC (partial conservation of tensor current) form,"

$$\overline{A}_\mu^{(+)}(x) = \epsilon_{\mu\nu\lambda\rho}\partial^\nu M^{\lambda\rho}(x)\,, \tag{7.1}$$

where $M^{\lambda\rho}$ is a tensor with nonsingular[18] matrix elements. The representation in Eq. (7.1) (proved in the Appendix) indicates that the matrix elements of this type-III currents have a characteristic energy dependence that may enable one to distinguish between type-II and type-III currents. This is, of course, precisely what we found in Sec. V.

Type IV. Currents abnormal under time reversal. The absence of any T violation in the $\Delta Y=0$ sector may, perhaps, be regarded as evidence against the existence of such currents.[19] In any case, such currents cannot contribute to δ.

Type V. ΔI-even currents. This category may overlap with any of the previous ones. Clearly such currents can play no role in experiments involving β transitions between $I=1$ and $I=0$ nuclei.

Type VI. Currents which distinguish between electron and muon number. This category may also overlap with any of the previous ones. We list it for the simple reason that one outlandish possibility may well be associated with another. Neutrino experiments at the National Accelerator Laboratory (comparison of $\nu_\mu + n \to \mu^- + p$ and $\overline{\nu}_\mu + p \to \mu^+ + n$) can easily determine the coupling of the second-class currents (if any) to the muon number.

VIII. CONCLUDING REMARKS

In summary, the present experimental and theoretical situation with respect to second-class currents in nuclear β decay appears to be the following:

(i) There is a definite observed difference between the ft values for mirror transitions with the electronic ft value being systematically lower than the positronic ft value by 10–15%.

(ii) This difference does not show any dependence on the energy release, nor does it show any systematic Z dependence.

(iii) One infers from this that there are no type-III second-class currents; if the difference is to be attributed to second-class currents, these currents must be of type II. We recall that type-II currents do not jibe with contemporary theoretical ideas on weak charges as generators of approximate strong-interaction symmetries.

(iv) Without a model there is no way to rule out the possibility that this effect, in the nuclei in question, is electromagnetic and that there are no second-class currents. To make this distinction, experiments are needed on elementary particles, where radiative corrections are both small and relatively easier to handle. In this respect it may be of interest to note that the ft values[20] for the mirror decays $\Sigma^+ \to \Lambda^0 + e^+ + \nu_e$ and $\Sigma^- \to \Lambda^0 + e^- + \overline{\nu}_e$ also appear to show a small discrepancy, the "wallet-card" values for the relevant parameters indicating that $\delta(\Sigma) \cong -0.04$. Better determination of lifetimes etc. are needed, however, before one can attach much significance to this value for $\delta(\Sigma)$.

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APPENDIX

The representation

$$\overline{A}_\mu = \epsilon_{\mu\nu\lambda\rho}\partial^\nu M^{\lambda\rho} \quad (M^{\lambda\rho} \text{ nonsingular}) \tag{A1}$$

for currents of type III can be established as follows:

The 4-dimensional Helmholtz theorem tells us that $\overline{A}_\mu$ can always be written in the form

$$\overline{A}_\mu(x) = \partial_\mu \phi(x) + \epsilon_{\mu\nu\lambda\rho}\partial^\nu M^{\lambda\rho}(x)\,, \tag{A2}$$

where $\phi(x)$ is a pseudoscalar and $M^{\lambda\rho}$ is some tensor (of unspecified singularity structure). Since $\overline{A}_\mu$ is conserved,

$$\Box\phi(x) = 0\,. \tag{A3}$$

If we take the matrix element of Eq. (A3) between the vacuum state and any arbitrary state $\langle n|$, we find

$$P_n^2\langle n|\phi(0)|0\rangle = 0\,. \tag{A4}$$

If there are no massless particles, this implies that

$$\langle n|\phi(0)|0\rangle = 0 \text{ for all } n. \tag{A5}$$

By the Federbush-Johnson theorem, therefore,

$$\phi(x) = 0. \tag{A6}$$

Next we must establish that $M^{\lambda\rho}$ is nonsingular.[18] This follows from the constraint

$$\int \overline{A}_0 d^3x = 0, \tag{A7}$$

as may be seen most conveniently by going to momentum space.

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[1]S. Weinberg, Phys. Rev. 112, 1375 (1958).

[2]D. H. Wilkinson and D. E. Alburger, Phys. Rev. Letters 24, 1134 (1970). See D. H. Wilkinson, in Proceedings of the Royal Society of Edinburgh (to be published) for a review of the experimental data and a full set of references.

[3]Some of the more recent papers are S. Okubo, Phys. Rev. Letters 25, 1593 (1970); H. J. Lipkin, Phys. Letters 34B, 202 (1971); J. Delorme and M. Rho, *ibid.* 34B, 238 (1971); L. Wolfenstein and E. Henley, *ibid.* 36B, 28 (1971); H. J. Lipkin, Phys. Rev. Letters 27, 432 (1971); B. Holstein, Phys. Rev. C 3, 764 (1971).

[4]The Wilkinson-Alburger result is expressed as follows: If the effective axial-tensor Hamiltonian is written, for single nucleons, in the form

$$H_{\text{eff}} = \frac{G}{\sqrt{2}} B \overline{\psi}_p i\gamma_5 \sigma_{\mu\nu} \frac{\partial}{\partial x_\nu} \psi_n \overline{\psi}_e \gamma^\mu (1 - i\gamma_5)\psi_\nu,$$

then *comparison of an independent-particle-model calculation with experiment* yields $B < 7\times10^{-4}$ MeV^{-1} $=1.3/2M$, M being the nucleon mass. To get an orientation into orders of magnitude, we recall that the weak magnetic coupling constant is given by $(\mu_p - \mu_n)/2M \cong 3.7/2M$.

[5]A less general analysis, which is however in the same spirit, is given in Sec. III of a very recent paper by C. W. Kim and T. Fulton [Phys. Rev. D 4, 390 (1971)]. We are grateful to Professor Fulton for drawing our attention to this paper.

[6]L. Durand, P. C. DeCelles, and R. Marr, Phys. Rev. 126, 1882 (1962).

[7]This analysis is suggested by discussions of invariants in massive Compton scattering. See M. A. B. Bég, SINBI Lectures, Copenhagen (1967). A. Pais [Ann. Phys. (N.Y.) 63, 361 (1971)] has used a similar analysis for neutrino-nucleon scattering.

[8]Cf. A. Pais, Ref. 7.

[9]J. Delorme and M. Rho [Ref. 3, and Saclay report, 1971 (unpublished)] consider the consequence of a conserved second-class current *in a model-dependent context*.

[10]We restrict our discussion to the $\Delta Y = 0$ case, since the concept of class cannot be introduced for $\Delta Y \neq 0$ currents without going to the SU(3) limit.

[11]Cf. H. J. Lipkin, Ref. 3.

[12]In general, J will transform as a sum of tensor operators of different dimensionality. Without much loss of generality, we may focus our attention on an irreducible piece of J. In what follows we will use the phase conventions of M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) 7, 404 (1959).

[13]See, e.g., E. Konopinski, *Theory of Beta Radioactivity* (Oxford Univ. Press, Oxford, England, 1966).

[14]Cf. M. A. B. Bég, J. Bernstein, and A. Sirlin, Phys. Rev. Letters 23, 270 (1969).

[15]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[16]We are tacitly assuming a "$\Delta I = 1$ rule." A simple example of such a current is afforded by the following construction in terms of ω and π fields: $\overline{A}_\mu^{(+)} = i\omega_\mu \pi^{(+)}$.

[17]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[18]By "nonsingular" we mean that the matrix element is finite in the limit in which all four components of the momentum transfer approach zero at the same rate.

[19]This statement should be weighed in the light of the following observation. If one looks for T violation in β decay by measuring the correlation $f_T(p_f, p_i) \times \vec{S}_{\text{hadron}} \cdot (\vec{p}_e \times \vec{p}_\nu)$ [see, e.g., F. P. Calaprice *et al.*, Phys. Rev. Letters 18, 918 (1971)], *a small value for f_T, the correlation coefficient, need not imply small T violation.* The point is that the correlation may simply be dynamically suppressed. Thus, if the T-abnormal piece of the axial-vector current also happens to be effectively conserved, one can readily show that f_T vanishes in the limit $p_f \to p_i$. The results of Sec. III imply, then, that for transitions within an isomultiplet a ΔI-odd, T-abnormal second-class current makes no contribution to f_T in the allowed approximation. [This is a generalization of a result due to C. W. Kim and H. Primakoff, Phys. Rev. 180, 1502 (1969).] In the absence of reliable measurements of f_T in *off-multiplet transitions,* one cannot really make a firm statement about the existence or nonexistence of type-IV currents. [*Note added in proof.* Results equivalent to those stated in this footnote have been derived independently by S. P. Rosen, this issue, Phys. Rev. D 5, 760 (1972).]

[20]The relevant formula is

$$\frac{(ft)_+}{(ft)_-} = \frac{(M_{\Sigma^+} - M_\Lambda)^5 \Gamma(\Sigma^- \to \Lambda + e^- + \overline{\nu}_e)}{(M_{\Sigma^-} - M_\Lambda)^5 \Gamma(\Sigma^+ \to \Lambda + e^+ + \nu_e)}.$$