Isovector Spectral Function and K_{l3} Decays*

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(Received 23 September 1971)

An inequality relating the parameter λ_+ of K_{l3} decays to the spectral function of the isovector electromagnetic current is derived, assuming only Weinberg's first spectral-function sum rule. It is shown that a large value for λ_+ (e.g., $\lambda_+ = 0.045$) is not compatible with ρ saturation of the isovector spectral function. Since the principal tests of the sum rule have relied on the additional assumption that such resonance saturations are possible, some doubt is cast on the validity, or at least on the previous applications, of the sum rule. Implications for the Weinberg mass relations, c-number Schwinger terms, and the e^+e^--annihilation cross section are discussed.

I. INTRODUCTION

The K_{l3} decays, such as $K^+ \to \pi^0 l^+ \nu_l$, where l is an electron or muon, have received a great deal of experimental and theoretical interest recently.[1] The most interesting aspect of these decays is that the form factor $f_-(t)$, which would be zero if SU(3) were an exact symmetry, has a surprisingly large magnitude. Another interesting experimental fact is that the slope[1] λ_+ of the approximately linear form factor $f_+(t)$ is around 0.045; this is much larger than would be expected from simple K^* dominance of the $\Delta I = \frac{1}{2}$, $\Delta S = 1$ weak current. In Sec. II we derive a rigorous inequality relating λ_+ to an integral of the spectral function of this current. For the present experimental value of λ_+, we find, as expected, that the spectral function cannot be dominated by the K^* resonance. In Sec. III we tentatively accept the validity of the first Weinberg sum rule[2] in order to derive an additional inequality relating λ_+ and the spectral-function integrals of the isovector and isoscalar electromagnetic currents. The Weinberg sum rule, of course, depends on the assumption of c-number Schwinger terms and on the convergence of certain integrals. For the current experimental value of λ_+, the inequality is not compatible with the assumption of resonance saturation of the spectral functions of the electromagnetic current. This in itself does not contradict the Weinberg sum rule; however, most of the applications and tests of the sum rule have depended crucially on this extra resonance-saturation assumption. Hence, if λ_+ really is large, either the sum rule is wrong or it will have to be reinterpreted. This and other implications of our result are discussed in Sec. IV. Finally, in Sec. V we discuss the possible errors in our numerical results.

II. DERIVATION

Now let us derive the bound. The hadronic part of the matrix element for $K^+ \to \pi^0 l^+ \nu_l$ is

$$\langle \pi^0(p') | V_\mu^\dagger(0) | K^+(p)\rangle = [1/(2\pi)^3(4p_0p_0')^{1/2}][f_+(t)(p+p')_\mu + f_-(t)(p-p')_\mu], \tag{1}$$

where $V_\mu^\dagger$ is the $\Delta S = 1$, $\Delta I = \frac{1}{2}$ vector current ($V_\mu^\dagger \equiv V_{4,\mu} - iV_{5,\mu}$ in octet notation), and $t = (p-p')^2$. In the SU(3)-symmetric limit we would have $f_-(t) = 0$ and $f_+(0) = -1/\sqrt{2}$.

By analytically continuing (1) to the matrix element $\langle 0 | V_\mu^\dagger | \pi^0(p')K^+(p)\rangle$ and by applying standard reduction techniques, one can show that

$$\mathrm{Im} f_+(t) = \{[(2\pi)^3 2p_0]^{1/2}/4k\} \times \sum_n [(2\pi)^4 \delta^4(p+p'-p_n) \times \langle K^+(p) | j_\pi(0) | n\rangle \langle n | \hat{p}\cdot\vec{V}(0) | 0\rangle]. \tag{2}$$

Here, k is the center-of-mass three-momentum of the $K^+\pi^0$ system, $\sum_n$ is a sum over intermediate states n with momentum p_n, and j_π is the pion source function. The matrix element of j_π is proportional to the complex conjugate of the scattering amplitude for $K^+\pi^0 \to n$. Because of the octet nature of V_μ only intermediate states n of total isospin $\frac{1}{2}$ are included in the sum. Also, we have selected a space component of V_μ, so only states with total angular momentum $J = 1$ enter the sum (the time component of the nonconserved current would couple with states of $J = 0$).

Of course, $f_+(t)$ is analytic except for a right-hand cut starting at the $K^+\pi^0$ scattering threshold $t_0 = (m_K + \mu)^2$, where μ is the pion mass. The physical region for K_{l3} decays is from $t = m_l^2$ to $t = (m_K - \mu)^2$.

The Schwarz inequality

$$\left|\sum_n A_n^* B_n\right|^2 \le \left(\sum_n |A_n|^2\right)\left(\sum_m |B_m|^2\right),$$

true for any complex numbers A_n and B_n, can now be applied to (2), yielding

$$|\mathrm{Im} f_+(t)|^2 \le (\pi t^{3/2}/6k)\sigma_1^{(1)}(t)\rho^{(1)}(t)/t = (2\pi^2 t^{3/2}/k^3)\sin^2\delta_1^1\,\rho^{(1)}(t)/t\,. \quad (3)$$

In (3), $\sigma_1^{(1)}(t)$ is the total $J=1$, $I=\frac{1}{2}$, $K\pi$ cross section and $\delta_1^1(t)$ is the p-wave, $I=\frac{1}{2}$ elastic phase shift. We have used the fact that the total cross section for $K^+\pi^0$ to scatter into states of $J=1$, $I=\frac{1}{2}$ is just

$$\tfrac{1}{3}\sigma_1^{(1)} = (4\pi/k^2)\sin^2\delta_1^1\,.$$

In (3) the spectral function $\rho^{(1)}(t)$ is defined by

$$(2\pi)^3\sum_n \delta^4(q-p_n)\langle 0|V_\mu^\dagger(0)|n\rangle\langle n|V_\nu(0)|0\rangle = (-g_{\mu\nu}+q_\mu q_\nu/q^2)\rho^{(1)}(q^2)+q_\mu q_\nu \rho^{(0)}(q^2). \quad (4)$$

By using the Schwarz inequality again, now in integral form, we have

$$\left|\int_{t_0}^\infty \frac{\mathrm{Im} f_+(t)\,dt}{G(t)}\right|^2 \le \left|\int_{t_0}^\infty \frac{|\mathrm{Im} f_+(t)|\,dt}{G(t)}\right|^2 \le \left(\int_{t_0}^\infty \frac{2\pi t^{3/2}\sigma_1^{(1)}(t)\,dt}{6kG(t)^2}\right)\times\tfrac{1}{2}\int_{t_0}^\infty \frac{\rho^{(1)}(t)\,dt}{t} \le \left(\int_{t_0}^\infty \frac{2\pi t^{3/2}\sigma_B(t)\,dt}{6kG(t)^2}\right)\times\tfrac{1}{2}\int_{t_0}^\infty \frac{\rho^{(1)}(t)\,dt}{t}\,, \quad (5)$$

where $G(t)$ is any positive definite function of t and $\sigma_B(t)$ is any upper bound on $\sigma_1^{(1)}(t)$. (We have used the fact that $\rho^{(1)}$ is positive definite. In Sec. III we also use the positive definiteness of $\rho^{(0)}$.)

By choosing $G(t)$ appropriately, the left-hand side can be made into a dispersion integral for experimentally known values of $f_+(t)$. For example, if we choose $G(t)=\pi t(t-t_1)$, where t_1 is in the range $0 \le t_1 \le (m_K-\mu)^2$, the left-hand side of (5) becomes

$$\left|\frac{1}{\pi}\int_{t_0}^\infty \frac{\mathrm{Im} f_+(t)\,dt}{t(t-t_1)}\right|^2 = \left|\frac{f_+(t_1)-f_+(0)}{t_1}\right|^2. \quad (6)$$

In the physical region for K_{l3} decays, $f_+(t)$ is well fitted by the formula[1]

$$f_+(t)=f_+(0)(1+\lambda_+ t/\mu^2). \quad (7)$$

Hence, the left-hand side of (5) is just $[f_+(0)\lambda_+/\mu^2]^2$. Notice that at this point we are not making any crucial use of the approximate linearity of $f_+(t)$. We are merely using (7) as a reasonably accurate parametrization of the experimental data in the region in which $f_+(t)$ is actually measured. There is a great deal of experimental uncertainty in the value of λ_+. We shall tentatively use the value[1] $\lambda_+=0.045\pm0.012$. This is to be compared with the K^*-dominance value of $\lambda_+=\mu^2/m_{K^*}{}^2=0.0225$. Unfortunately, $f_+(0)$ is not well known experimentally, but by combining results of Refs. 3 and 4 we can estimate[5] $f_+(0)\approx -0.89/\sqrt{2}$.

Now that the left-hand side of (5) is known, we can use (5) as a lower bound on the integral of $\rho^{(1)}(t)$. In order to do this we must find some appropriate upper bound σ_B on $\sigma_1^{(1)}$. The values of $\delta_1^1(t)$ are known experimentally up to about $t^{1/2}=1.2$ GeV (Ref. 6) (δ_1^1 is fitted beautifully by a K^* Breit-Wigner formula in this region, but we have used the actual data points given in Ref. 6). Above 1.2 GeV we have upper-bounded $\sin^2\delta_1^1$ by unity (probably a gross overestimation). We have estimated that the experimental errors in the low-energy values of $\sin^2\delta_1^1$ will not affect our final results by more than 10%.

All quantities in (5) are therefore known except for the spectral-function integral; for this quantity we have a very conservative lower bound. Defining

$$I(t_1)=\frac{1}{3\pi}\int_{t_0}^\infty \frac{\sigma_B(t)\,dt}{kt^{1/2}(t-t_1)^2}, \quad (8)$$

we have

$$\frac{1}{2}\int_{t_0}^\infty \frac{\rho^{(1)}(t)\,dt}{t} \ge \left(\frac{f_+(0)\lambda_+}{\mu^2}\right)^2 \frac{1}{I(t_1)}\,. \quad (9)$$

We can parametrize the left-hand side of (9) as $C\times 0.037 f_+(0)^2$ GeV^2, where $0.037 f_+(0)^2$ GeV^2 is the value the integral would have if K^* dominance were valid [$0.0375 f_+(0)^2$ GeV^2 in the narrow-width approximation]. Canceling the common $f_+(0)^2$ factor and evaluating $I(t_1)$ for $t_1=0$, we find

$$C>1.6\times10^3\lambda_+{}^2\,. \quad (10)$$

For the experimental value $\lambda_+=0.045$, this implies $C>3.3$. That is, the spectral-function integral must be more than three times as large as the resonance-saturation value. For $\lambda_+=0.033$ (the one-standard-deviation value), $C>1.8$.

As a check on the correctness of (10) we insert the K^*-dominance value of $\lambda_+=0.0225$, for which $C=1$. Our inequality yields $C>0.82$, suggesting that our inequality is not too far from an equality.

III. APPLICATIONS

Our inequality (9) is not in itself very interesting. From the largeness of λ_+ we could have guessed that K^* domination of $\rho^{(1)}$ is not valid. However, it has been shown by Glashow, Schnitzer, and

Weinberg[2] that if the relevant Schwinger terms are c numbers and if the integrals converge, then

$$\frac{1}{2}\int_{t_0}^{\infty}\left(\frac{\rho^{(1)}(t)}{t}+\rho^{(0)}(t)\right)dt=\int_{4\mu^2}^{\infty}\frac{\rho_V(t)}{t}dt$$

$$=\int_{9\mu^2}^{\infty}\frac{\rho_S(t)}{t}dt\,, \qquad (11)$$

where $\rho_V(t)$ and $\rho_S(t)$ are the spectral functions of the conserved (electromagnetic) isovector and isoscalar currents. The $\frac{1}{2}$ is due to our normalization of V_μ.

Equation (11) is a special case of what is known as Weinberg's first (fast convergent) sum rule. Combining (9) and (11), we find

$$\int_{4\mu^2}^{\infty}\frac{\rho_V(t)}{t}dt\geq\frac{1}{2}\int_{t_0}^{\infty}\frac{\rho^{(1)}(t)}{t}dt\geq\left(\frac{f_+(0)\lambda_+}{\mu^2}\right)^2\frac{1}{I(t_1)}\,. \qquad (12)$$

Let us define the number L by

$$\int_{4\mu^2}^{\infty}\frac{\rho_V(t)}{t}dt=\frac{L m_\rho{}^2}{f_\rho{}^2}\approx L\times 0.0244\ \mathrm{GeV}^2\,, \qquad (13)$$

where m_ρ is the ρ mass and $m_\rho{}^2/f_\rho$ is the photon-ρ "coupling constant."[7] If the common assumption that (13) can be saturated by a zero-width ρ resonance is true,[8] then $L=1$. The finite-width ρ-saturation hypothesis gives $L\approx 0.92$. Therefore, (12) becomes

$$L\geq\left(\frac{f_\rho}{m_\rho}\frac{f_+(0)\lambda_+}{\mu^2}\right)^2\frac{1}{I(t_1)}\,. \qquad (14)$$

Choosing $t_1=0$ yields $L>974\,|\lambda_+|^2$; for $\lambda_+=0.045$, this gives $L>1.97$. If we choose t_1 in the physical region (where λ_+ is actually measured), we obtain bounds that are almost as stringent. For $t_1=0.03$ GeV^2, for example, $L>900\,|\lambda_+|^2$; for $\lambda_+=0.045$, $L>1.82$. It must be noted that the stated error for λ_+ is ± 0.012.[1] For $\lambda_+=0.033$ the bounds on L are around 1.

A similar but stronger bound can be obtained by taking $G(t)=\pi(t-t_1)^3/(t-m_{K^*}{}^2)$ in (5). This $G(t)$ effectively suppresses the K^* contribution to the $\sigma_1^{(1)}$ integral. The left-hand side of (5) becomes $|f'(t_1)+(t_1-m_{K^*}{}^2)f_+''(t_1)/2\,|^2$. We can then repeat all of the steps that led to (14) to derive a lower bound on L in terms of the second derivative $f_+''(t_1)$, the form of which the reader can easily write down.

Little is known experimentally about $f_+''(t_1)$ [the data fitted by (7) are relatively insensitive to it], but if we make the *ad hoc* assumption that $f_+''(t_1)/f_+(0)$ is no larger than it would be if $f_+(t)$ were dominated by a K^* pole, and that still $f_+'(t_1)=f_+(0)\lambda_+/\mu^2$, then we can show, for example, that $L>10.0$ for $\lambda_+=0.045$ and $L>2.3$ for $\lambda_+=0.033$ (all t_1 up to 0.05 GeV^2 give bounds this big). Because of the uncertainty in $f_+''(t_1)$ we shall make no further use of this second bound on L.

IV. IMPLICATIONS

We have seen in Sec. III that if λ_+ really is much larger than the K^*-pole value, and if the Weinberg sum rule is true, then $\rho_V(t)$ must have important contributions other than the ρ resonance. The principal tests of the spectral-function sum rule (and therefore of the c-number Schwinger-term hypothesis) have relied on the additional assumption that each spectral function can be saturated by a resonance (the ρ for ρ_V). In Ref. 2, for example, this extra assumption is used to predict $m_{A_1}/m_\rho=\sqrt{2}$, which agrees very well with experiment. (Actually, this particular result also requires the second and less convergent Weinberg sum rule.) We now see, however, that if $\lambda_+\geq 0.045$, then the resonance-saturation approximation is false. In this case, we would have to conclude that either (i) the Weinberg sum rules are still true, and they are satisfied separately by the resonance and nonresonance parts of the spectral functions, or (ii) the sum rules are not true, and the success of the mass relations must be accidental or due to some other origin.

It has been suggested[9,10] that the spectral-function integrals might not converge (the convergence of the ρ_V and ρ_S integrals requires that the total cross section for $e^+e^-\to$hadrons through one photon must decrease faster than $1/t^2$ for large t). In this case the sum rules cannot be valid [possibility (ii) above]. It is also conceivable that the integrals diverge but that the sum rules are still true for the resonance part of the spectral functions. We will include this (in some formal sense) as a special case of possibility (i). We must emphasize that if possibility (i) is true, it is empirical. It does not follow from the arguments in Ref. 2.

We consider our results [that large λ_+ implies (i) or (ii)] to be much more modest than the possibility that the spectral integrals diverge.[9,10] However, Pestieau and Terazawa's result[9] is based on the (experimentally unknown) asymptotic behavior of the reaction $e^+e^-\to H+$anything, where H is a hadron, and the conclusion of Bég *et al.*[10] depends on the assumption of asymptotic scale invariance. Our result, though limited, is very direct and does not require any additional assumptions.

The spectral function ρ_V is related to the e^+e^--annihilation cross section (into hadrons with $I=1$) by

$$\rho_V(t)=t^2\sigma_{I=1}(t)/16\pi^3\alpha^2 \qquad (15)$$

to lowest order in α. Again, we can say that if $\lambda_+\geq 0.045$ [and if (11) is true], then this cross section must contain something besides the ρ resonance. This is supported by recent measurements of the pion form factor,[11] measured in the reaction $e^+e^- \to \pi^+\pi^-$, which does not decrease nearly as fast as would be expected from simple ρ dominance.

Using (11), we can also place a lower bound on the integral of $\rho_S(t)/t$. For $\lambda_+ = 0.045$ and $t_1 = 0$, the bound is 1.48 times greater than the estimated ω-plus-ϕ saturation value (the experimental widths for ω, $\phi \to e^+e^-$ have been used[12]). The isoscalar spectral function is related to the cross section for $e^+e^- \to I=0$ hadrons by a relation like (15), except for an additional factor of 3 on the right.

From (11) we can combine the $I=0$ and 1 cross sections to give

$$\int_{4\mu^2}^{\infty} \frac{\rho_V(t)}{t}\, dt = \frac{3}{64\pi^3\alpha^2} \int_{4\mu^2}^{\infty} t\sigma_{e^+e^-}(t)\, dt, \qquad (16)$$

where $\sigma_{e^+e^-}$ is the total ($I=0$ plus $I=1$) annihilation cross section. Using (12), this quantity must be larger than $23.8|\lambda_+|^2$ GeV^2. For $\lambda_+ = 0.045$ this is 0.0482 GeV^2. This is to be compared with the estimated $\rho + \omega + \phi$ value of 0.0264 GeV^2.

We might mention that if $\sigma_{e^+e^-}$ is ever known experimentally to a high enough t, then our result can be used as an upper bound on λ_+

Let us now return to the Weinberg sum rule. It has been shown[13] that if the first sum rule and the resonance-saturation approximation are true, then

$$\tfrac{1}{3} m_\rho \Gamma(\rho \to e^+e^-) = m_\omega \Gamma(\omega \to e^+e^-) + m_\phi \Gamma(\phi \to e^+e^-). \qquad (17)$$

This agrees roughly, but not terribly well, with experiment.[14] Again we claim that the resonance approximation probably fails badly, so again we are led to possibilities (i) and (ii) above. Because of the partial successes of the saturated first Weinberg sum rule we would like to speculate that, for some unknown reason, it holds approximately [perhaps only in the SU(3)-symmetric limit] for the resonance part of the spectral functions. The nonresonance part may either diverge or satisfy the sum rule separately. Weinberg's second sum rule may also hold in this resonance-domination sense, but only in the limit of SU(3). This is because the analog of (17) for the second sum rule is

$$\tfrac{1}{3} m_\rho{}^3 \Gamma(\rho \to e^+e^-) = m_\omega{}^3 \Gamma(\omega \to e^+e^-) + m_\phi{}^3 \Gamma(\phi \to e^+e^-), \qquad (18)$$

which is badly broken.[14]

V. ERRORS

We would like to discuss briefly the numerical errors in our results. Our estimate of $f_+(0)$ is probably accurate to about 10%; a 10% error will affect (14) by 20%. We have bounded $\sin^2\delta_1^1$ by unity above 1.2 GeV. Variations of the K^* position and width in the low-energy region have been estimated. They should not affect (14) by over 10%. The saturation value given in (13) is uncertain, owing to uncertainties in f_ρ. The value that we have quoted corresponds to $f_\rho{}^2/4\pi = 1.9$. Most other values that have been obtained for f_ρ are larger,[7] corresponding to even smaller values for the spectral integral in the zero-width saturation hypothesis. Also, the spectral integral is about 8% smaller when a finite width is given to the resonance. Hence, the coefficient of L in (13) is the largest value compatible with ρ saturation. Finally, the most uncertain quantity is λ_+ itself. All of our conclusions depend strongly on λ_+ being almost as large as 0.045. Until λ_+ is more firmly established, all of our conclusions must be regarded as tentative.

ACKNOWLEDGMENTS

I am extremely grateful to Mahiko Suzuki for advice and encouragement throughout the course of this work, and to Namik Pak for reading the manuscript.

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*Work supported in part by the U. S. Atomic Energy Commission.

†Participating guest, Lawrence Berkeley Laboratory. National Science Foundation Graduate Fellow.

[1] L. M. Chounet and M. K. Gaillard, Phys. Letters 32B, 505 (1970); M. K. Gaillard and L. M. Chounet, CERN Report No. CERN 70-14, 1970 (unpublished); L. M. Chounet, in Proceedings of the Daresbury Study Weekend No. 2, 1971 (unpublished). Some experiments give much larger values for λ_+; for example, C. Y. Chien *et al.* [Phys. Letters 33B, 629 (1970)] find $\lambda_+ = 0.08 \pm 0.01$.

[2] S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters 19, 139 (1967); S. Weinberg, *ibid.* 18, 507 (1967).

[3] V. S. Mathur and S. Okubo, Phys. Rev. D 1, 3468 (1970).

[4] N. Brene, M. Roos, and A. Sirlin, Nucl. Phys. B6, 225 (1968).

[5] The bound to be derived in this section is actually independent of $f_+(0)$. A 10% error in $f_+(0)$ will change the bounds in Sec. III by about 20%.

[6] R. Mercer *et al.*, Johns Hopkins University world data tape analysis, Nucl. Phys. B32, 381 (1971).

[7] See, for example, J. J. Sakurai, in *International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969*, edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970),

p. 91.

[8]A similar technique has been used to relate various integrals of the isovector spectral function to the pion electromagnetic charge radius and to values of the pion form factor for large spacelike t. These results also tend to contradict the vector-meson saturation hypothesis. See, P. Langacker and M. Suzuki, Phys. Rev. D 4, 2160 (1971).

[9]J. Pestieau and H. Terazawa, Phys. Rev. Letters 24, 1149 (1970).

[10]M. A. B. Bég *et al.*, Phys. Rev. Letters 25, 1231 (1970).

[11]See, for example, C. Bernardini, in Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Cornell, 1971 (unpublished).

[12]M. Gourdin and E. de Rafael, Nucl. Phys. B10, 667 (1969).

[13]T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 19, 470 (1967).

[14]M. Gourdin, in Eleventh Scottish Universities' Summer School in Physics, 1970 (unpublished).

PHYSICAL REVIEW D VOLUME 5, NUMBER 3 1 FEBRUARY 1972

Second-Class Currents: Some Model-Independent Considerations and Conclusions

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(Received 20 October 1971)

We study the problem of second-class currents, and attempt to clarify the situation, by working within the framework of a formulation that makes no appeal to any specific dynamical model of hadronic interactions or nuclear structure. While much of the paper is pedagogical, several new theorems and results are presented. The import of recent experiments is discussed, and a scheme for classification of second-class currents is proposed.

I. INTRODUCTION

Although the notion of "second-class currents" was introduced by Weinberg[1] over 13 years ago, it is only in the last year due to the remarkable experiments of Wilkinson, Alburger, and collaborators[2] that the subject has attracted widespread attention among theoretical physicists.[3] Regrettably, most of the recent discussion, if not entirely incorrect or misleading, has been within the framework of models which are, in many cases, oversimplified, to say the least. As a result, the precise significance of the experimental results[4] of Wilkinson, Alburger, *et al.* has been obscured.

The purpose of this paper is to attempt to clarify the situation, by working within the framework of a formulation that makes no appeal to any specific dynamical model of hadronic interactions or nuclear structure. We shall look upon β transitions of a nucleus as the β transitions of the nucleus as a whole, rather than in terms of β transitions undergone by "off-shell" nucleons in the nucleus; in effect, we shall treat nuclei as "elementary particles" of *arbitrary spin.*[5] It is immaterial then whether a nucleus such as C^{12} is presumed to consist of six protons and six neutrons, or 12 neutrons and six quark-antiquark pairs.

While much of this paper is frankly pedagogical, we do report some new results – new either in the sense that they did not heretofore exist in the literature, or in the sense that they have now been liberated from model-dependent derivations. We are led to the recognition that the notion of second-class currents, as introduced by Weinberg, embraces a variety of currents with different theoretical and experimental implications; a systematic study of the subject must therefore entail a subclassification. Such a subclassification is proposed in Sec. VII of this paper.

In Sec. II of this paper we introduce a perspicuous notation. Section III is devoted to the question of whether one can distinguish between first- and second-class currents by looking at the spacetime properties of their matrix elements; in this section we state and prove an extension of a theorem on Hermitian currents due to Durand, De Celles, and Marr.[6] In Sec. IV we develop a general kinematical formalism[7] for dealing with semileptonic interactions and state some relevant formulas for decay rates and ft values; we also show that for transitions within an isomultiplet, and *only* for transitions within an isomultiplet, interference effects between first- and second-class currents of the same isospin (or isospins differing by 0 mod 2) vanish in the limit of zero lepton mass.[8] Section V is devoted to a more detailed study of these interference effects; it is shown that if the second-class current is conserved, the difference of ft values for mirror transitions must at least have a linear dependence on the sum of the energy