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Inelastic Photoproduction of Spin-One W^+ Bosons in a Quark-Parton Model*

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We apply the quark-parton model to the inelastic photoproduction of single W^+ bosons, with any number of hadrons present in the final state. We compute the total cross section and the energy and angular distributions of the bosons. We find the energy distribution to be peaked near the high-energy region, and a very sharp forward peak in the angular distribution. Experimental difficulties will make it hard to treat this process as a test of the model which, however, can be used to obtain an estimate of the various cross sections.

I. INTRODUCTION

The treatment of hadrons as composite particles has received considerable attention following the SLAC experiments on inelastic electron-nucleon scattering.[1] A popular version of such an approach is the parton model, whose simplicity and physical intuitiveness have allowed a large variety of applications, including inelastic Compton and neutrino scattering.[2] The experiments proposed or performed so far probe the structure of hadrons through either the electromagnetic or weak currents, while the process considered in this paper involves an interference between these two currents. As such it is a further test of the model, although experimentally a much more difficult one.

Photoproduction of the W boson has been discussed frequently in the literature, and total and differential cross sections for the elastic process have been given by various authors.[3] It took the SLAC experiment to draw attention to the fact that one had no *a priori* reason to neglect the inelastic production process in which any number of hadrons are produced along with the particle (or particles) of interest. The calculation reported here gives an estimate of the number of W bosons produced in inelastic photon scattering, together with their angle and energy distributions. For this purpose a definite parton model was chosen following Bjorken and Paschos.[4] Variations (within the parton model) can be easily incorporated.

In Sec. II we give the elements of the model and set up some of our notation. In Sec. III we give the matrix element for the production through pointlike interactions, and perform the spin and polarization sums on the square of the matrix element. Section IV gives the results of a numerical integration of this latter quantity (multiplied by the proper probability functions) to yield the angle and energy distributions of the bosons, as well as the total cross section, for different values of the boson mass and photon energy. Conclusions and remarks will be reserved for Sec. V.

II. THE MODEL

For definiteness, we consider W^+ bosons produced by scattering photons off a proton; a calculation with W^+ replaced by W^- or the target replaced by a neutron would proceed along the same lines. We treat the proton as consisting of pointlike particles (partons) which, in an infinite-momentum frame, interact freely with the photon. It is then straightforward to calculate the matrix element for the process

$$\gamma(K)+\text{parton }(P^i)\rightarrow W^+(Q)+\text{parton }(P^j). \tag{2.1}$$

K, P^i, Q, and P^j denote the four-momenta of the photon, the initial parton, the boson, and the final parton, respectively. As usual, we obtain the cross section for the inelastic process (Fig. 1) by substituting $P^i\rightarrow x^iP$, where x^i is the fraction of the proton momentum (P) carried by the interacting parton; multiplying the square of the matrix element by $P(N)$, the probability of finding N partons in the proton, and by $f_N(x)$, the probability of finding a parton with momentum xP in such a configuration; averaging over all N-parton configurations; integrating over x from 0 to 1; and summing over all possible N.

To compute the cross section it is necessary to work in a more specific model by choosing $P(N)$ and $f_N(x)$. Data on electron and neutrino scattering do not, at the present, distinguish among the many possibilities.[5] We shall therefore adopt the simple model of Ref. 4 in which the constituents of the proton are taken to be 3 "valence" quarks of spin $\frac{1}{2}$ plus an infinite sea of quark-antiquark pairs with equal numbers of each quark type, and a constant joint distribution of the x^i's for all the partons is assumed. The latter assumption leads to

$$f_N(x)=(N-1)(1-x)^{N-2}. \tag{2.2}$$

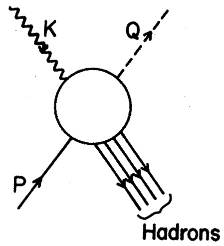

FIG. 1. The process $\gamma + p \to W^+ +$hadrons.

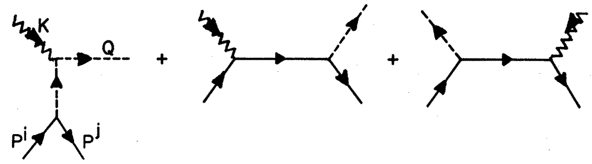

FIG. 2. Feynman diagrams for $\gamma + P^i \to W^+ + P^j$.

We shall also take (following Ref. 4)

$$P(N) = \frac{1}{(1-\ln 2)N(N-1)}. \tag{2.3}$$

In terms of the familiar quark triplet, the weak hadronic current is written as

$$J_\mu = \bar{\mathcal{P}}\gamma_\mu(1-\gamma_5)(\mathcal{N}\cos\theta_C + \lambda\sin\theta_C), \tag{2.4}$$

where $\mathcal{P}$, $\mathcal{N}$, λ are the quark field operators (with charges $\frac{2}{3}$, $-\frac{1}{3}$, $-\frac{1}{3}$) and θ_C is the Cabibbo angle.

The justification for the use of the parton model in this process will be discussed in Sec. V. Here, however, we should mention that our assumption about the distribution functions is too specific at this experimental stage to provide us with a test of the model, and our aim is only to estimate the inelastic cross sections.

III. CROSS SECTION FOR POINT PARTICLES

The matrix element corresponding to the sum of the three diagrams in Fig. 2 is given by

$$\begin{aligned} M^{ij} = iG_W e\epsilon^\mu(K)\epsilon^{*\alpha}(Q)\bar{u}(P^j)\bigg[&\frac{Q^i\gamma_\alpha(1-\gamma_5)(\not{K}\gamma_\mu + 2P^i_\mu)}{2K\cdot P^i} + \frac{(Q^i-1)(2P^i_\mu - 2Q_\mu - \gamma_\mu\not{K})\gamma_\alpha(1-\gamma_5)}{1-2Q\cdot P^i} \\ &+\gamma^\delta(1-\gamma_5)\frac{(K-2Q)_\mu g_{\alpha\delta} + K_\delta g_{\alpha\mu} - K_\alpha g_{\mu\delta} + (Q-K)_\delta(K\cdot Q g_{\alpha\mu} - K_\alpha Q_\mu)}{2K\cdot Q}\bigg]u(P^i)J^{ij}, \end{aligned} \tag{3.1}$$

where

$$G_W^{\,2} = \frac{GM_W^{\,2}}{\sqrt{2}}, \quad G \simeq \frac{10^{-5}}{M_p^{\,2}} \quad (M_p = \text{proton mass}),$$

$$J^{ij} = \langle j \,|\, I^- \,|\, i\rangle\cos\theta_C + \langle j \,|\, U^- \,|\, i\rangle \sin\theta_C .$$

Q^i denotes the charge of the ith parton in units of $|e|$, and the polarization vector of the photon (boson) has been denoted by $\epsilon^\mu(K)$ ($\epsilon^\alpha(Q)$). We have assumed a point electromagnetic interaction for the boson with no anomalous magnetic moment. Our units are in $c=\hbar=1$, and the boson mass $M_W = 1$. It can be easily checked that M^{ij} is gauge-invariant.

We take the square of the matrix element and sum or average over spins and polarizations, using

$$\epsilon^\mu(K)\epsilon^{*\nu}(K) \to -g^{\mu\nu},$$

$$\epsilon^\alpha(Q)\epsilon^{*\beta}(Q) \to -g^{\alpha\beta} + Q^\alpha Q^\beta,$$

to obtain

$$\frac{1}{4}\sum_{j,\,\text{spins, pol.}} |M^{ij}|^2 = \frac{e^2 G_W^{\,2}}{16 m_i^{\,2}} F(P^i, Q^i, K, Q)\sum_j |J^{ij}|^2, \tag{3.2}$$

where

$$\begin{aligned} F(P, Q^i, K, Q) = (g^{\alpha\beta} - Q^\alpha Q^\beta)\,\mathrm{Tr}\bigg\{(\not{K} + \not{P} - \not{Q} + M)\bigg[&\frac{Q^i\gamma_\alpha(1-\gamma_5)(\not{K}\gamma_\mu + 2P_\mu)}{2K\cdot P} + \frac{(Q^i-1)(2P_\mu - 2Q_\mu - \gamma_\mu\not{K})\gamma_\alpha(1-\gamma_5)}{1-2Q\cdot P} \\ &+\gamma^\delta(1-\gamma_5)\frac{(K-2Q)_\mu g_{\alpha\delta} + K_\delta g_{\alpha\mu} - K_\alpha g_{\mu\delta} + (Q-K)_\delta(K\cdot Q g_{\mu\alpha} - K_\alpha Q_\mu)}{2K\cdot Q}\bigg] \end{aligned}$$

$$\times(\not{P}+M)\left[\frac{Q^i(\gamma^\mu \not{K}+2P^\mu)\gamma_\beta(1-\gamma_5)}{2K\cdot P}+\frac{(Q^i-1)\gamma_\beta(1-\gamma_5)(2P^\mu-2Q^\mu-\not{K}\gamma^\mu)}{1-2Q\cdot P}\right.$$
$$\left.+\gamma^\sigma(1-\gamma_5)\frac{(K-2Q)^\mu g_{\beta\sigma}+K_\sigma g^\mu_\beta-K_\beta g^\mu_\sigma+(Q-K)_\sigma(K\cdot Q g^\mu_\beta-K_\beta Q^\mu)}{2K\cdot Q}\right]\Bigg\}\,. \tag{3.3}$$

We have put $P^i\cdot P^i=M^{i2}=M^{j2}$.

The trace was taken on the Brookhaven CDC 6600 using the SCHOONSHIP program developed by M. Veltman, and some of the terms were checked by hand. The result is given in the Appendix.

In terms of s, t, and ν, where (m = proton mass)

$$s=(P+K)^2\,,$$
$$t=(K-Q)^2\,,$$
$$\nu=P\cdot(K-Q)/m\,,$$

the differential cross section is (after replacing P^i by xP)

$$\frac{d\sigma}{d\nu\,dt}=\frac{-\alpha G_W{}^2}{16}\int_0^1\frac{1}{x^2(s-m^2)^2}\sum_N\langle\sum_{i,j}F(xP,Q^i,K,Q)|J^{ij}|^2\rangle_N\;P(N)f_N(x)\delta\left(\nu+\frac{t}{2mx}\right)dx$$
$$=\frac{-\alpha G_W{}^2}{16}\frac{1}{x^2(s-m^2)^2\nu}\sum_N\langle\sum_{i,j}F(xP,Q^i,K,Q)|J^{ij}|^2\rangle_N P(N)xf_N(x)\,, \tag{3.4}$$

with $x=-t/2m\nu$ and $\alpha=e^2/4\pi$. Within the model described in Sec. II,

$$\left\langle\sum_{i,j}F(xP,Q^i,K,Q)|J^{ij}|^2\right\rangle_N=\tfrac{3}{2}F(Q=\tfrac{2}{3})-\tfrac{1}{2}F(Q=\tfrac{1}{3})+\tfrac{1}{6}N[F(Q=\tfrac{2}{3})+F(Q=\tfrac{1}{3})]\,, \tag{3.5}$$

which is independent of θ_C since the model treats the $\mathfrak{N}$ and λ quarks symmetrically. The replacement of a quark by an antiquark changes only the sign of Q^i (however, only an $\bar{\mathfrak{N}}$ or $\bar{\lambda}$ can produce a charge = +1 particle). $F(Q)$ is a shorthand for $F(xP,Q^i,K,Q)$.

With $P(N)$ and $f_N(x)$ given by Eqs. (2.2) and (2.3) we can perform the necessary sums over N:

$$\sum_N P(N)xf_N(x)=\frac{1}{1-\ln 2}\left(\frac{x}{2(1-x)^2}\right)\left(\ln\frac{2-x}{x}-2(1-x)\right)\,, \tag{3.6}$$

$$\sum_N NP(N)xf_N(x)=\frac{1-x}{(1-\ln 2)(2-x)}\,. \tag{3.7}$$

Substituting Eqs. (3.5), (3.6), and (3.7) into Eq. (3.4) and expressing $2m\nu$ in terms of $m_f{}^2$, the invariant

TABLE I. Total cross sections in units of 10^{-35} cm^2 for $\gamma+p\to W^+ +$hadrons, with $M_W=5$ and 10 GeV/c^2. The last column lists the total cross sections (in same units) for $\gamma+p\to W^+ + n$ with the nucleons treated as point particles.

Lab energy of the photon (GeV)	Inelastic σ_T for $M_W=5$ GeV/c^2	Inelastic σ_T for $M_W=10$ GeV/c^2	Elastic σ_T for $M_W=5$ GeV/c^2
100	1.15	0.11	0.83
200	2.15	0.58	1.35
400	3.40	1.35	1.83
600	4.24	1.92	2.06
800	4.87	2.39	2.20
1000	5.37	2.77	2.29
1200	5.79	3.11	2.35
1400	6.15	3.40	2.40
1600	6.46	3.67	2.43
1800	6.74	3.91	2.45
2000	6.98	4.13	2.47

mass squared of the final hadrons, through $2m\nu = m_f^2 - m^2 - t$, we obtain

$$\frac{d\sigma}{dm_f^2 dt} = \frac{\alpha G_W^2}{16(s-m^2)^2}\frac{1}{xt}\left[\left[\tfrac{3}{2}F(Q=\tfrac{2}{3}) - \tfrac{1}{2}F(Q=\tfrac{1}{3})\right]\frac{1-x}{2(1-\ln 2)(1-x)^2}\left(\ln\frac{2-x}{x} - 2(1-x)\right) + \tfrac{1}{6}\left[F(Q=\tfrac{2}{3}) + F(Q=\tfrac{1}{3})\right]\frac{1-x}{(1-\ln 2)(2-x)}\right]. \tag{3.8}$$

IV. RESULTS

The total cross section is obtained by integrating Eq. (3.8).[6] The limits on t and m_f are

$$\left.\begin{matrix} t^{\max} \\ t^{\min} \end{matrix}\right\} = 1 - 2K_0(q_0 \pm q),$$

$$m_f^{\max} = \sqrt{s} - 1, \quad m_f^{\min} = m + m_\pi,$$

where

$$q_0 = \frac{s+1-m_f^2}{2\sqrt{s}}, \quad q = (q_0^2 - 1)^{1/2}, \quad K_0 = \frac{s-m^2}{2\sqrt{s}}, \quad m_\pi = \pi^0 \text{ mass}.$$

Numerical results (using Gaussian quadrature) with $M_W = 5$ and 10 GeV/c^2 for various incoming photon energies are given in Table I.

The total differential (in the energy and angle of the W boson in the laboratory frame) cross section is given by

$$\frac{d\sigma}{dE_W d\cos\theta} = 4mE_\gamma(E_W^2 - 1)^{1/2}\frac{d\sigma}{dm_f^2 dt}, \tag{4.1}$$

where θ is the angle which the boson makes with the direction of the photon, and E_W is its energy in the laboratory frame. The energy of the photon, E_γ, is of course given by $(s-m^2)/2m$.

Integrating Eq. (4.1) over $\cos\theta$, we obtain the energy distribution of the W's. The limits are given by

$$\cos\theta^{\max} = 1,$$

$$\cos\theta^{\min} = \frac{2(E_\gamma + m)E_W - [s+1-(m+m_\pi)^2]}{2E_\gamma(E_W^2-1)^{1/2}}.$$

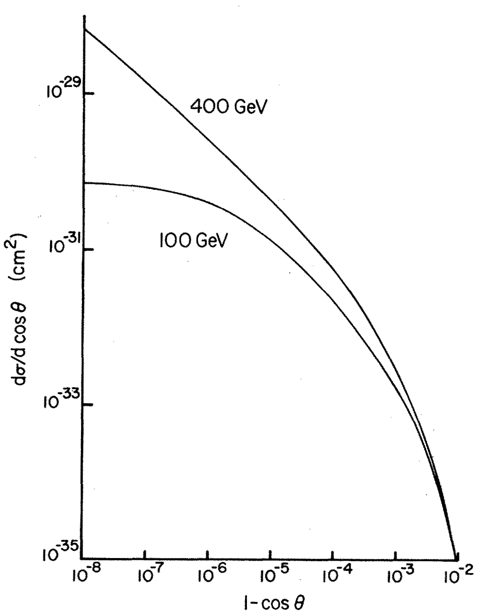

FIG. 3. The angular distribution $d\sigma/d\cos\theta$ of W^+ bosons for $M_W = 5$ GeV/c^2, $E_\gamma = 100$ and 400 GeV.

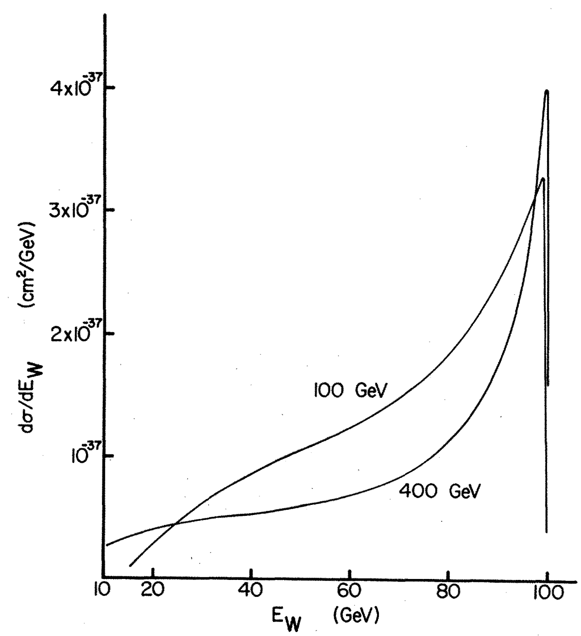

FIG. 4. The energy distribution $d\sigma/dE_W$ of W^+ bosons for $M_W = 5$ GeV/c^2, $E_\gamma = 100$ and 400 GeV (the energy scale for the latter case has been divided by 4).

To obtain the angular distribution of the boson, we integrate over its energy with the limits

$$\left.\begin{matrix} E_W^{\max} \\ \\ E_W^{\min} \end{matrix}\right\} = \frac{1}{2(s+E_\gamma{}^2\sin^2\theta)}\left((E_\gamma+m)[s+1-(m+m_\pi)^2] \pm E_\gamma\cos\theta\{[s+1-(m+m_\pi)^2]^2-4s-4E_\gamma^2\sin^2\theta\}^{1/2}\right).$$

The distributions in angle and energy are displayed in Figs. 3 and 4, respectively.

As a check on our calculation and for purposes of comparison we have computed the cross section for elastic photoproduction of W bosons, treating the nucleons as point particles with no anomalous magnetic moment (the last diagram in Fig. 2 does not contribute in this limit). The results are given in column 4 of Table I. These were compared and found to agree with a similar calculation by Fearing *et al.*, who also report on the effect of including the charge, magnetic, and weak form factors of the nucleons (see the third paper of Ref. 3).

V. CONCLUSIONS AND REMARKS

As an attempt to justify the use of the model and, more specifically, the impulse approximation, we note that the dominant contribution comes from the last diagram of Fig. 2, where the intermediate particle can be very close to its mass shell. The situation is similar to inelastic photoproduction of massive muon pairs, a process recently studied by Jaffe,[7] who shows that the leading process is the annihilation of a parton from the incident photon on an antiparton in the hadron to produce the massive pairs (similar to what Drell and Yan[8] find in $pp\to\mu^+\mu^-+\cdots$). This approach can be directly applied to the photoproduction of W's, as time ordering of the last two diagrams in Fig. 2 will show. We assume, however, a simple structure for the photon, viz., quark-antiquark pair, much in the spirit which led us to choose the distribution functions of Bjorken and Paschos.[4]

Figures 3 and 4 show that the W's are produced mainly in the forward direction and carry most of the available energy. From our experience with e-p scattering, we expect the numbers presented here to correspond to the upper limits of the quantities involved, but the general shape at the angular and energy distributions should remain the same. The above cross sections may serve as a guide for the experimentalists.

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APPENDIX

The computer output for $F(P,Q^i,K,Q)$ defined in Eq. (3.3) is ($a=s-m^2$)

$$\begin{aligned} F(P,Q^i,K,Q) = {} & \frac{8Q^{i2}}{a^2}[a^2(2-2m^2-t)-2a(-t+m^2+1)+4m^2(1-m^2)] \\ & +\frac{16Q^i(1-Q^i)}{a(a+t-1)}[-a^2(t+2m^2)-a(t^2+2tm^2-t-2m^2)-t^2m^2+3tm^2-2t+4m^2(1-m^2)] \\ & +\frac{8Q^i}{a(1-t)}[-2a^2-a(t^2+3tm^2-t-5m^2-2)+2(tm^2-2t+m^2)] \\ & +\frac{8(1-Q^i)^2}{(a+t-1)^2}[a^2(2-t-2m^2)-2a(t^2+2tm^2-2t-3m^2+1)-t^3-2t^2m^2+2t^2+6tm^2-t-4m^4] \\ & +\frac{8(1-Q^i)}{(1-t)(a+t-1)}[2a^2-a(t^2+3tm^2-5t+2-5m^2)-t^3-3t^2m^2+4t^2+6tm^2+t-7m^2] \\ & +\frac{2}{(1-t)^2}[2a^2(2-t)-2a(t^2-3t+2)-2t^3m^2-t^3-4t^2m^2+10t^2+6tm^2+7t-16m^2]. \end{aligned}$$

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[1] E. D. Bloom *et al.*, Phys. Rev. Letters 23, 930 (1969); M. Breidenbach *et al.*, *ibid.* 23, 935 (1969); W. Albrecht *et al.*, Nucl. Phys. B13, 1 (1969); B. D. Dieterle *et al.*, Phys. Rev. Letters 23, 1187 (1969); M. L. Perl *et al.*,

ibid. 23, 1191 (1969).
[2]J. D. Bjorken and E. A. Paschos, Phys. Rev. D 1, 3151 (1970); D. N. Goswami and D. P. Majumdar, *ibid.* 4, 2090 (1971).
[3]H. R. Reiss and M. H. Cha, Phys. Rev. Letters 14, 399 (1965); W. Williamson, Jr. and R. T. Deck, Phys. Rev. 174, 1986 (1968); H. W. Fearing, M. Pratap, and J. Smith, this issue, Phys. Rev. D 5, 177 (1972).
[4]J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).
[5]See, e.g., D. J. Gross and C. H. Llewellyn Smith, Nucl. Phys. B14, 337 (1969), or C. W. Gardiner and D. P. Majumdar, Phys. Rev. D 4, 259 (1971). The e-p data indicate that an infinite number of partons is required and favor spin-$\frac{1}{2}$ partons.
[6]The model is expected to be valid for large s and $-t$. Usually one assumes that it holds for all values of t and integrates to get σ_T. See, e.g., M. Gourdin, Nucl. Phys. B29, 601 (1971).
[7]R. L. Jaffe, Phys. Rev. D 4, 1507 (1971).
[8]S. D. Drell and T.- M. Yan, Phys. Rev. Letters 25, 316 (1970).

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Finite Proton-Neutron Mass Difference and Scale Invariance*

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A representation of the forward Compton amplitude in which the A_2 meson breaks scale invariance is shown to be consistent with existing data for the difference between the proton and neutron structure functions $\nu W_2^p - \nu W_2^n$, while ensuring a finite proton-neutron mass difference ΔM. The conjecture that $W_L \equiv W_1 + (\nu^2/q^2)W_2 \to 0$ as $\nu \to \infty$ for fixed q^2 leads to an expression for ΔM in terms of measurable quantities.

I. INTRODUCTION

We begin with the almost obligatory remark that despite intensive study in recent years, the problem of the proton-neutron mass difference ΔM has remained unsolved. The conjecture[1] that the electromagnetic interaction, in first-order approximation, should give a good estimate for ΔM led to Cottingham's[2] formula for the self-mass of a hadron, δM, in terms of the forward amplitude for Compton scattering. Harari[3] considered the exchange of Regge poles in the crossed channel, and showed that the $\Delta I = 2$ mass differences are adequately obtained from the Born terms in the Cottingham formula, while the $\Delta I = 1$ mass differences could have an additional contribution from the subtraction term for the $T_1(\nu, q^2)$ amplitude, because its behavior is dominated by the A_2 Regge pole. Pagels[4] showed that if the structure functions $W_1(\nu, q^2)$ and $\nu W_2(\nu, q^2)$ are scale-invariant in the Bjorken limit,[5] $-q^2 \to \infty$ with $\omega = -2M\nu/q^2$ fixed, then the self-mass δM diverges unless some unlikely cancellations occur among terms in the Cottingham formula.

We take the position that while divergent self-masses are acceptable, a theory of self-masses must predict the observed finite proton-neutron mass difference. Within the framework of the Cottingham formula, this means that the differences $W_1^p - W_1^n$ and $\nu W_2^p - \nu W_2^n$ cannot have a *nontrivial* Bjorken limit if the proton-neutron mass difference is finite.

II. FORMULA FOR MASS DIFFERENCE

The formula for the self-mass of a hadron is given by

$$\delta M = \frac{i\alpha}{(2\pi)^3}\int \frac{d^4q\, T_{\mu\nu}(\vec{q}, q^0)g^{\mu\nu}}{q^2 + i\epsilon}\,, \tag{1}$$

where $\epsilon^\mu \epsilon^\nu T_{\mu\nu}$ is the forward Compton amplitude for scattering of photons of four-momentum q off hadrons of four-momentum P, and $\alpha = e^2/4\pi$. $T_{\mu\nu}$ can be expanded in terms of two Lorentz-invariant functions of q^2 and $\nu = P\cdot q/M$:

$$T_{\mu\nu}(\vec{q}, q^0) = \frac{1}{M^2}\left(P_\mu - \frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}q_\nu\right)T_2(\nu, q^2) - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)T_1(\nu, q^2)\,. \tag{2}$$

The Cottingham formula is obtained by a Wick rotation in the variable ν, giving the result

$$\delta M = \frac{\alpha}{2\pi^2}\int_0^{-\infty}\frac{dq^2}{q^2}\int_0^{\sqrt{-q^2}} d\nu\,(-q^2 - \nu^2)^{1/2}T(i\nu, q^2), \tag{3}$$

where

$$T(\nu, q^2) \equiv T_{\mu\nu}(\vec{q}, q^0)g^{\mu\nu}\,. \tag{4}$$

Following Harari,[3] we assume a once-subtracted,