residues of these leading Regge poles factorize in the manner specified in Sec. III. Thus, contributions from cuts are ignored. Such assumptions are admittedly not well established even for elastic amplitudes, let alone Mueller amplitudes. These very assumptions however have been invoked in the past to obtain interrelationships between different hadronic fragmentation processes.[2] We have also assumed that forward elastic (photon-hadron and hadron-hadron) amplitudes are dominated for leading Regge poles with factorizable residues. Finally, in Sec. V it is assumed that if the well-known functions W_1 and νW_2 scale in the $\nu \gg q^2$ region, then so must the contributions of each individual Regge trajectory. This is a very natural assumption, in the absence of which a very delicate conspiracy must exist (see Ref. 5) between these contributions in order that their sum in the form of $W_1(\nu, q^2)$ and $\nu W_2(\nu, q^2)$ be a function only of ν/q^2 in a dense $\nu \gg q^2$ domain in the ν-q^2 plane.

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[1]For recent summaries of the experimental and theoretical progress in electroproduction, see E. D. Bloom *et al.*, MIT-SLAC Report No. SLAC-PUB-796, 1970 (unpublished), presented at the Fifteenth International Conference on High-Energy Physics, Kiev, U.S.S.R., 1970, and F. J. Gilman, SLAC Report No. SLAC-PUB-842, 1970 (unpublished).

[2]Literature on hadronic inclusive processes is by now quite rich. For our purposes, the references which will be of use are A. H. Mueller, Phys. Rev. D 2, 2963 (1970); Chan Hong-Mo *et al.*, Phys. Rev. Letters 26, 672 (1971).

[3]See, for instance, S. D. Drell and T.-M. Yan, Phys. Rev. Letters 24, 855 (1970).

[4]H. D. I. Abarbanel, M. L. Goldberger, and S. B. Treiman, Phys. Rev. Letters 22, 500 (1969); H. Harari, *ibid.* 22, 1078 (1969).

[5]S. Rai Choudhury and R. Rajaraman, Phys. Rev. D 2, 2728 (1970); R. Rajaraman and G. Rajasekaran, *ibid.* 3, 266 (1971).

[6]R. W. Anthony *et al.*, Phys. Rev. Letters 26, 38 (1971). The high-energy data of J. W. Elbert and A. R. Erwin are also quoted in this paper.

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Spin Dependence in Inclusive Reactions*

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We treat the spin dependences of high-energy inclusive reactions from the viewpoint of a J-plane analysis. Measurements of various spin correlations are shown to bear directly on questions such as factorization of residues at Regge poles and the strengths of J-plane cuts. We provide examples of experiments which test these ideas using polarized beams, polarized targets, or polarized products. Electroproduction and high-energy neutrino reactions are viewed as sources for polarized spin-one "beams," and the implications for the processes are drawn in detail. Finally, we consider scaling properties of these latter reactions and discuss produced-particle multiplicity in the scaling region.

I. INTRODUCTION

The qualitative features exhibited in hadronic multiparticle production reactions provide a source for discrimination among various models of the strong interactions. A useful separation of these production processes has been made by Feynman[1] who invites us to consider separately *exclusive* reactions in which all final produced particles are detected and *inclusive* processes in which only a selected subset of produced objects are observed, the coordinates of the rest being summed over. Many of the complications of production reactions are washed out in inclusive processes and, indeed, a variety of models[2] agree on the limiting behavior to be expected at very high energies for the invariant inclusive differential cross sections. In particular, the phenomena of production of large numbers of slow-moving particles in the barycentric system – so-called pionization[3] – and of a limiting distribution for the production of fragments associated with the beam or target – namely, limiting fragmentation – appear as common attributes connected with diffraction scattering.

To distinguish among the alternatives one will clearly have to look beyond the gross features of the production spectrum found by measuring only the momenta involved. In this paper we propose to examine in some detail the consequences for spin dependence in inclusive processes which follow within the multiperipheral or general Regge-pole model.[4] It transpires that there are a number of definite statements that one can make based on the properties of J-plane singularities generally agreed upon in the context of two-body physics.[5] The crucial observation in what follows is that the inclusive cross section is given by a piece of a *forward* absorptive amplitude. This means that over-all helicity is conserved, and, once one accepts the idea that Regge poles mediate the reaction, whenever any particle is properly isolated from the rest by a Regge exchange, its individual helicity will be conserved to leading order in s. (This statement requires certain technical assumptions which will be given precisely below.)

Typically one will conclude on this basis that certain spin correlations are suppressed by powers of s relative to other allowed transitions. The absence of this suppression would be telling evidence against either the properties of Regge poles one takes quite for granted; e.g., factorization, or, possibly, the use of a Regge analysis in inclusive phenomena. We propose looking for the effects presented here in a variety of experiments. The first kind requires a polarized beam and demands looking in the fragmentation region of the target. Since quite high-energy polarized photon beams are presently available at SLAC, one can probably make these tests rather soon. Another form of this experiment involves producing polarized virtual-photon or weak-current "beams" in electroproduction or neutrino reactions. This is simple since the photon or weak-current polarization vector $\bar{u}\gamma_\alpha u$ or $\bar{u}\gamma_\alpha(1\pm\gamma_5)u$ is at one's disposal by varying the lepton parameters. We discuss these processes in some detail and, while we are at it, consider implications of scaling behavior for them, and make some remarks about the multiplicity of produced hadrons in the scaling limit.

A second kind of experiment is symmetric to these and involves inclusive production on polarized targets. Unhappily, targets of significant polarization are not readily at hand, so the matters here may be more difficult to attain in practice. The skeptic may properly be concerned about our testing the Regge models with photons since there are any number of well-known technical problems[5]

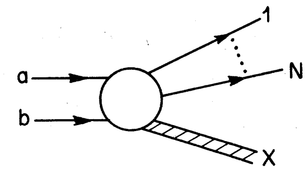

FIG. 1. An inclusive reaction.

associated with photon-induced reactions from the point of view of the J plane. The last kind of experiment we discuss, therefore, is one which requires a measurement on the polarization of a produced particle at zero transverse momentum. If this particle decays it will analyze its own polarization, but one would still need another produced object to provide an axis about which to measure the decay parameters. The necessity of producing three hadrons in the final state may thus cut down on the expected rate and make the experiment difficult.

II. INCLUSIVE-REACTION KINEMATICS

We will be concerned in this work with the differential cross section associated with the process $a+b\to 1+2+\cdots+N+X$, where a and b are incident particles carrying momenta p_a and p_b, respectively, and N particles of momenta $p_1,\ldots,p_N$ are detected, while X represents an undetected group of hadrons whose coordinates are summed over (see Fig. 1). Spin labels $s_a, s_b, s_1,\ldots,s_N$ will appear when needed.

The differential cross section for this reaction is given in the a, b barycentric system as

$$\Delta^{1/2}(s, m_a^2, m_b^2)d\sigma(a+b\to 1+2+\cdots+N+X) = \prod_{j=1}^{N}\frac{d^3p_j}{\pi m_j^2 E_j}M(p_a, p_1,\ldots,p_N, p_b), \quad (1)$$

where the flux of the incoming beam is expressed via the relative momentum $p=\Delta^{1/2}(s, m_a^2, m_b^2)/2\sqrt{s}$ with

$$s=(p_a+p_b)^2, \quad (2)$$

and

$$\Delta(x, y, z)=(x+y-z)^2-4xy. \quad (3)$$

The dynamics is contained in

$$M(p_a, p_1,\ldots,p_N, p_b)=\frac{1}{2(16\pi^2)^N}\int d^4y_1\cdots d^4y_N\,d^4Z_2\cdots d^4Z_N\exp\Big(i\sum_{j=1}^{N}p_j\cdot y_j\Big)\vec{K}_{y_1}\cdots\vec{K}_{y_N}(2E_a2E_b)^{1/2}$$

$$\times\langle ab\,\text{in}|\,(J_1(y_1)\phi_2(y_2)\cdots\phi_N(y_N))_+(J_1(0)\phi_2(Z_2)\cdots\phi_N(Z_N))_+|\,ab\,\text{in}\rangle$$
$$\times(2E_a2E_b)^{1/2}\overleftarrow{K}_{Z_2}\cdots\overleftarrow{K}_{Z_N}\exp\left(-i\sum_{j=2}^{N}p_j\cdot Z_j\right), \tag{4}$$

where $\phi_j(y)$ is the field operator for particle j, and $J_j(y)$ is the associated source density which arises as $K_y\phi_j(y)=J_j(y)$. E_j is the energy of particle j.

The parametrization of phase space is chosen so that when p_i is labeled by[6]

$$p_i=m_i\,(\cosh\beta_i\cosh\theta_i,\ \sinh\beta_i\cos\phi_i,\ \sinh\beta_i\sin\phi_i,\ \cosh\beta_i\sinh\theta_i), \tag{5}$$

we have

$$\frac{d^3p_i}{\pi m_i^2E_i}=\frac{d\phi_i}{2\pi}d\theta_i\,d(\cosh\beta_i)^2. \tag{6}$$

This choice for naming the components of p_i decouples the longitudinal momentum $m_i\cosh\beta_i\sinh\theta_i$ in an effective manner from transverse momentum. Thus, an invariant squared subenergy $(p_i+p_j)^2$, which plays a significant role in the Regge analysis to follow, will become large when the difference $\theta_i-\theta_j$ is large, regardless of the β's which govern the transverse momentum. This is readily seen from

$$p_i\cdot p_j=M_iM_j[\cosh\beta_i\cosh\beta_j\cosh(\theta_i-\theta_j)-\sinh\beta_i\sinh\beta_j\cos(\phi_i-\phi_j)]. \tag{7}$$

In fact, the relevant values of β will be small since observed transverse momenta are strongly damped.

Now $M(p_a, p_1, \ldots, p_N, p_b)$ is related to a piece of the $N+2$ to $N+2$ *forward* absorptive amplitude as exhibited in Fig. 2. To properly discuss the multi-Regge analysis of this, one makes a multiple crossed-channel partial-wave analysis based on the SO(1, 3) little group of the null momentum transfer between pairs of particles.[6] One may conveniently envision the analysis by lining up the particles according to their longitudinal-boost angle θ as in Fig. 3 and imagining that across each link in the figure one makes an ordinary two-body SO(1, 3) partial-wave decomposition.

We only need the following results of that labor. Consider the inclusive reaction in the barycentric system of a and b and choose the incoming three-momenta along the z axis,

$$p_a=m_a(\cosh\theta_a,\ 0,\ 0,\ -\sinh\theta_a) \tag{8}$$

and

$$p_b=m_b(\cosh\theta_b,\ 0,\ 0,\ \sinh\theta_b), \tag{9}$$

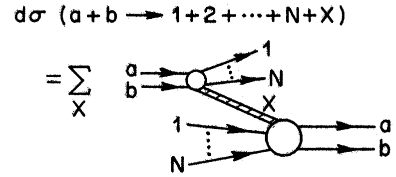

FIG. 2. The relation between M and a piece of the three-to-three forward absorptive part.

with

$$m_a\sinh\theta_a=m_b\sinh\theta_b. \tag{10}$$

When the squared subenergy $(p_1+p_a)^2$, governed by $\theta_1-(-\theta_a)$, is large, each Regge pole at α contributes a leading factor to the asymptotic expansion of M,

$$\exp(\theta_1+\theta_a)(\alpha-|M_{a1}+\Delta h_a|), \tag{11}$$

where M_{a1} is a label of the SO(1, 3) representation, into which the Regge (or Lorentz) pole is put, which corresponds to the minimum ordinary angular momentum in the representation.[7] In order to enter in (11) M_{a1} must be less than or equal to $\min(2j_a, 2j_1)$; j_a = spin of a, and j_1 = spin of 1. In (11) Δh_a is the difference in helicity between the incoming (in the sense of Fig. 3) state of a and the departing state of a. In general, a will contain a mixture of helicities and a number of helicity transitions will be present. Similarly, when the squared subenergy $(p_N+p_b)^2$ is large, each Regge pole contributes a factor

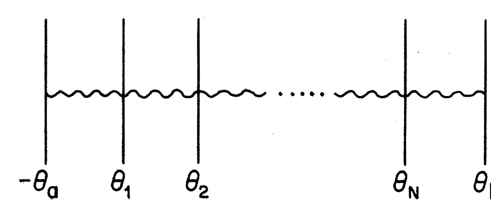

FIG. 3. A representation of the kinematics of the $N+2$ to $N+2$ forward amplitude in terms of the longitudinal-boost angles, θ_i, of each particle. The SO(1,3) partial-wave decomposition is performed in the channels indicated by the wavy lines.

$$\exp(\theta_N+\theta_b)(\alpha-|M_{Nb}+\Delta h_b|)\,, \tag{12}$$

with Δh_b the helicity flip at the b-b–Regge-pole vertex in Fig. 3. Here, of course, the relevant $M_{Nb}\leq \min(2j_N, 2j_b)$. Therefore, the leading term in the asymptotic behavior (11) requires the helicity flip Δh_a to have magnitude M_{a1}, or, symmetrically, the leading behavior in (12) requires the magnitude Δh_b to be equal to M_{Nb}. Corrections to this are reduced at least one power of the large subenergy.

Each of the usual leading trajectories P, P', ρ, A_2, etc. appear to carry M quantum number zero,[5,8] since they contribute to helicity-nonflip amplitudes in two-body processes. Therefore, we are led to conclude that when $p_1\cdot p_a$, *or equivalently* $\theta_1+\theta_a$, *becomes large, the leading contribution of each Regge pole carries* $\Delta h_a=0$. Furthermore, the contribution of helicity-flip transitions is down by $|\Delta h_a|$ in powers of $p_1\cdot p_a$. The situation is clearly similar for Δh_b when $\theta_N+\theta_b$ is large.

This same line of reasoning allows one to conclude that when $\theta_1+\theta_a$ and $\theta_2-\theta_1$ are large *and* $\beta_1=0$, that is, the transverse momentum of particle 1 vanishes, then to leading order in powers of energy, $\Delta h_1=0$. What we are doing in this case is to isolate particle 1 from a and from the conglomerate $2,\ldots,N,b$ by Regge poles carrying $M=0$, thus carrying no azimuthal information in the leading order. Then we make p_1 collinear with a, which by rotational invariance means it cannot flip its helicity.

In a shorter version of this paper,[9] where many of the main results were described, the argument given here proceeded by using factorization of Regge poles in the t channel and the absence of conspiracy at $t=0$ to deduce the helicity-nonflip property just described. The equivalence of these lines of reasoning is demonstrated in detail in Ref. 8. There is, however, an additional benefit that one may derive from the factorization argument. Namely, if we consider a transition which isolates a helicity-dependent Regge-particle vertex $\beta_{h,h'}$, each form of reasoning allows us to conclude

$$\beta_{h,h'}=\delta_{h'h}\beta_h\,. \tag{13}$$

Factorization, however, allows us to go further by lifting this vertex out of the original process in which we found it, and deduce many of its properties by connecting it with simpler systems, for example, spinless particles. Thus, if parity and time reversal are good symmetries of the original process, we learn

$$\beta_{h,h'}=\beta_{-h,-h'}=\beta_{h',h}\,, \tag{14}$$

with a convenient choice of phases.

In a real sense, as we shall discuss in a moment, it is probable that only isolated poles in the J plane are going to carry $M=0$, and the unitarity of the S matrix then implies factorization of the residues,[10] so the factorization and $M=0$ properties are likely to be achieved together.

Before we go on to draw some of the conclusions of the arguments we have given, let us pause to consider how they may break down. The primary ingredient is clearly the assumption that the leading asymptotic behavior in any relevant squared subenergy s is governed by those isolated J-plane *poles* which one is acquainted with from two-body physics. It happens that this is true in the multiperipheral model,[4,6] but generalization from that may be unwarranted. What is more likely is that J-plane singularities other than poles, especially branch points, play a significant role in governing the high-s behaviors of amplitudes. Indeed, one does not expect cuts to be factorizable. In addition, since they are generated in conventional models from the contribution of two poles in the t channel, there is no apparent reason why they should carry $M=0$ and, thus, should flip helicity with no loss of powers in s.[11] From a J-plane viewpoint, one would reasonably view the failure of the predictions we will outline as evidence for strong contributions of Regge cuts.

In contrast to the simple Regge model we have followed here, eikonal-approximation models do not give a precise statement about the spin dependence of forward scattering except for the single leading order in s. There, at least in electrodynamics, one finds that the coefficient of s^1 does not flip helicity.[12] The contributions to any lower powers of s are untouched in these calculations. Furthermore, in these models it is rather clear that the J-plane structure of the diffraction contribution is not factorizable, so one would expect any of our results depending on this property to break down. To draw the contrast here a bit clearer, let us note that the Regge argument given above means that as soon as a *finite* number of Regge poles with $M=0$, not just the Pomeranchukon, describe the behavior in a squared subenergy s, then no helicity flip will occur to order s^{α_L-1}, where α_L is the leading trajectory. In the eikonal models, absence of helicity flip is only true for the diffraction component, and the lower contributions surely flip helicity. Should the tests we will momentarily propose, in fact, defend the Regge reasoning, such an outcome would pose a serious challenge to eikonal-model builders. Should they fail to do so, the situation will, alas, be more ambiguous.

III. SOME EXPERIMENTS TO TEST THESE IDEAS

A. Polarized Beams

Let us begin for simplicity with single-particle inclusive production: $a+b\to 1+X$, and let particle

b be incident along the positive z axis carrying spin label s_b. Choose the four-vectors in the center-of-mass frame of a and b,

$$p_b = m_b(\cosh\theta_b, 0, 0, \sinh\theta_b), \tag{15}$$

$$p_a = m_a(\cosh\theta_a, 0, 0, -\sinh\theta_a), \tag{16}$$

and

$$p_1 = m_1(\cosh\beta_1 \cosh\theta_1, \sinh\beta_1, 0, \cosh\beta_1 \sinh\theta_1), \tag{17}$$

where $m_a \sinh\theta_a = m_b \sinh\theta_b$, and we have chosen the produced particle to lie in the x-z plane, as we may. The differential cross section for this process is

$$\frac{d\sigma(a+b\to 1+X)}{d\theta_1\, d(\cosh\beta_1)^2} = \frac{M(\theta_1+\theta_a, \beta_1, \theta_b-\theta_1)}{2m_a m_b \sinh(\theta_a+\theta_b)}, \tag{18}$$

noting that

$$\Delta^{1/2}(s, m_a^2, m_b^2) = 2m_a m_b \sinh(\theta_a+\theta_b). \tag{19}$$

Now let the incident energy become large, so θ_a and θ_b are large, holding β_1 and $\theta_1+\theta_a$ fixed. That is, the transverse momentum of p_1 is fixed, and we select events in which the produced particle "runs along" near the negative z axis, that is, more or less in the direction of particle a. This limit is the fragmentation limit of a. In this regime $s_b = (p_1+p_b)^2$ becomes large since

$$p_1 \cdot p_b = m_1 m_b \cosh\beta_1 \cosh(\theta_b - \theta_1) \tag{20}$$

and $p_1 \cdot p_a$ remains finite since

$$p_1 \cdot p_a = m_1 m_a \cosh\beta_1 \cosh(\theta_a + \theta_1). \tag{21}$$

Suppose that M is governed by a finite sum of Regge poles with $t=0$ intercept α_i and O(3, 1) quantum number M_i. As $\theta_b - \theta_1$ gets large,

$$M(\theta_1+\theta_a, \beta_1, \theta_b-\theta_1) \underset{\substack{\beta_1 \text{ fixed}; \theta_1+\theta_a \text{ fixed} \\ \theta_b-\theta_1\to\infty}}{\sim} \sum_{i,h_b,h_b'} R^i_{h_b',h_b}(\theta_1+\theta_a, \beta_1) \exp(\theta_b-\theta_1)(\alpha_i - |M_i + \Delta h_b|) \tag{22}$$

for a configuration in which the helicity at the b–b–Regge-pole vertex flips by $\Delta h_b = h_b' - h_b$. The R^i are related to the residues at the Regge poles. As noted, the incoming states labeled by s_b are, in general, a mixture of definite-helicity states so the differential cross section governed by M will involve a number of helicity transitions. We have already argued that the relevant M_i are zero, so to leading order in $s \sim m_a m_b e^{\theta_a+\theta_b}$ one has

$$\frac{d\sigma(a+b\to 1+X)}{d\theta_1\, d(\cosh\beta_1)^2} \underset{\substack{\beta_1 \text{ fixed};\ \theta_1+\theta_a \text{ fixed} \\ \theta_b-\theta_1\to\infty}}{\sim} \sum_{i,h_b} f^i_{h_b,h_b}(\beta_1, \theta_1+\theta_a) s^{\alpha_i-1} + \sum_{i,h_b} g^i_{\Delta h_b=1}(\beta_1, \theta_1+\theta_a) s^{\alpha_i-2} + \cdots, \tag{23}$$

where some inconsequential constants have gone to make up the f^i and g^i from the R^i. Further, because one has only a single particle detected,

$$f^i_{h_b,h_b} = f^i_{-h_b,-h_b} \tag{24}$$

if parity is conserved.

An immediate interesting application of this comes if particle b is taken to be a real photon. Then the initial state is a mixture of $h_b = \pm 1$ which we may take as

$$|s_b\rangle = (A|+\rangle + B|-\rangle)/(|A|^2+|B|^2)^{1/2}. \tag{25}$$

To order s^{α_L-1}, where α_L is the largest of the α_i, (23) tells us that the differential cross section depends only on $|A|$ or $|B|$, while the further piece of information (24) tells us that $d\sigma(\gamma+a\to 1+X)$ does not depend on A or B at all.

Now this same conclusion may be drawn whenever $\theta_b - \theta_1$ is large enough so that the behavior of M in that variable is governed by a finite sum of $M=0$ Reggeons. Clearly, the other subenergies could get large or the transverse momentum may do as it likes, and the conclusion still holds. Thus, we find: In the photon-induced reaction $\gamma(p_b)+A(p_a)\to 1(p_1)+X$, let the squared energy $s_b = (p_b+p_1)^2$ grow so that $M(\theta_1+\theta_a, \beta_1, \theta_b-\theta_1)$ is governed by a finite sum of Regge singularities α_i with $M=0$. Then we expect that to order $(s_b)^{\alpha_L-1}$, where α_L is the largest of the α_i, $d\sigma/d\theta_1\, d(\cosh\beta_1)^2$ is independent of the state of polarization of the photon.

This differential cross section can contain only two varieties of dependence on the photon polarization. One term is, in general, independent of the polarization; the other is the coefficient of the triple product $\vec{P}_\gamma \cdot (\vec{p}_1 \times \vec{p}_b)$, where the pseudovector $\vec{P}_\gamma$ is the polarization vector of the incident photon beam. On the basis of order-of-magnitude estimates alone, the size of the spin-correlation term is at worst $\approx$ (transverse momentum of p_1)$/m_p$ relative to the other term. Therefore, if one parametrizes $d\sigma$ as

$$\frac{d\sigma(\gamma+a\to 1+X)}{d\theta_1\, d(\cosh\beta_1)^2} = F(\theta_1+\theta_a, \beta_1, \theta_b-\theta_1) + G(\theta_1+\theta_a, \beta_1, \theta_b-\theta_1)\,[\vec{P}_\gamma \cdot (\hat{p}_1 \times \hat{p}_b)], \tag{26}$$

with $\hat{p}_i$ a unit vector along $\vec{p}_i$, then we expect G/F to go to zero as $1/s_b$ as $s_b \to \infty$.

A similar conclusion follows for polarized spin-$\frac{1}{2}$ beams, since again only two states of helicity are allowed, and they are related by parity. For higher spins, polarization *independence* does not transpire, while, of course, helicity-nonflip dominance does.

Thus far, we have been able to proceed on the basis of the asymptotic dominance of $M=0$ Reggeons plus parity conservation. Helicity-nonflip dominance in single-particle inclusive production will be a test of these matters. When we come to producing more than one particle, parity alone does not lead to (24) because of the orientation angles of the produced particles relative to one another. In this case, however, we may deduce (24) by adding the assumption of *factorization* of the residues at the $M=0$ Regge poles. Factorization allows us to study the properties of the reduced residues by coupling them with the same $M=0$ Reggeon to simpler processes. Tacking our residue for the b–b–Regge-pole vertex onto a spinless target immediately gives (24) when parity is conserved.

Under the condition, then, that the residues at Regge poles factorize, we may carry over all of our results to the case of multiple-production inclusive reactions: Consider $a+b \to 1+2+\cdots+N+X$ and order the particles as in Fig. 3 in the a, b barycentric system. When $p_N \cdot p_b$ becomes large, the leading terms of $d\sigma$ have zero helicity flip at the b–b–Regge-pole vertex. If b is a photon or a spin-$\frac{1}{2}$ beam, then all spin correlations involving the polarization of the beam are suppressed by order $(p_1 \cdot p_b)^{-1}$ relative to other terms in $d\sigma$.

B. Target Polarized

The conclusions here are identical to those for polarized beams with appropriate renaming of particles a and b. Since highly polarized targets are difficult to come by, we will not dwell on this matter.

C. Polarized Products

It is unfortunate that the best polarized beams are photonic since the application of Regge phenomenology to photon-induced processes is fraught with well-advertised pitfalls. It would indeed be nice, therefore, to be able to use highly polarized hadron targets so that only hadrons are involved. Fortunately, it is possible to find a test of our arguments involving polarized produced particles.

Consider production of two particles where the particle labeled 1 carries spin indices s_1, and other spins are averaged over. Let the longitudinal-boost angles $\theta_1+\theta_a$ and $\theta_2-\theta_1$ become large, thus isolating particle 1 from the rest of the reaction by $M=0$ Regge poles. When the transverse momentum of 1 is zero; that is, $\beta_1=0$, it is clear from Eq. (49) of Ref. 6 that helicity does not flip to order (subenergies)$^{\alpha_L-1}$. Intuitively, one sees this since $M=0$ objects carry no noncollinear information to leading order in s, so when particles 1 and a are collinear, the helicity of 1 cannot flip.

Since only helicity-nonflip transitions enter the differential cross section in this region of phase space, one may state the conclusion we have reached in the following concise manner: Consider rotating by ϕ only the spin vector of particle 1, keeping all other vectors fixed, then each helicity transition $h_1 \to h_1'$ picks up a phase $\exp i\phi(h_1-h_1')$. Since only $h_1=h_1'$ is allowed to leading order, the differential cross section will be independent of ϕ to that order.

If particle 1 subsequently decays into two objects of momenta $\vec{q}_1$ and $\vec{q}_2$ (think of producing ρ's and $\rho \to 2\pi$), then the argument just given means that $d\sigma(a+b \to 1+2+X)$ is independent of

$$\cos\phi = \frac{(\vec{p}_2 \times \vec{p}_b)\cdot(\vec{\Delta}\times\vec{p}_b)}{|\vec{p}_2\times\vec{p}_b|\,|\vec{\Delta}\times\vec{p}_b|} \tag{27}$$

to leading order in s, where $\vec{\Delta}=\vec{q}_1-\vec{q}_2$. When particle 1 has spin J, one expects, in general, that $d\sigma$ contains ϕ dependences

$$d\sigma = \sum_{M=0}^{2J}(a_M \cos M\phi) + \sum_{M=1}^{2J}(b_M \sin M\phi) \tag{28}$$

with coefficients a_M and b_M which depend on the other variables. Our analysis suggests that a_M/a_0 or b_M/a_0 for $M\neq 0$ are expected to vanish for large energies as s^{-M}.

IV. INCLUSIVE ELECTROPRODUCTION AND NEUTRINO REACTIONS

As a place where one may further test the implications of helicity-nonflip dominance, we turn to electroproduction reactions of the variety $e+p \to e'+1+2+\cdots+N+X$ or neutrino reactions $\nu_l+p \to l+1+2+\cdots+N+X$. We view the vector coupling to the leptons as a source for a polarized, off-shell, spin-one beam incident on the proton; see

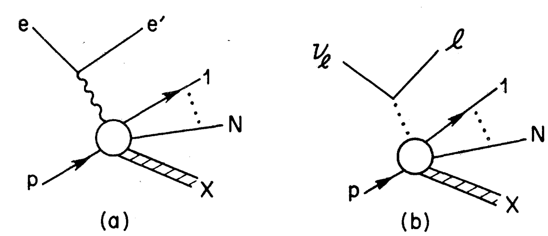

FIG. 4. (a) An inclusive electron-hadron reaction. (b) An inclusive neutrino-hadron reaction.

Fig. 4. The differential cross section for unpolarized incident electrons or muons and unpolarized target protons is still valuable for us because the hadronic matrix element of the electromagnetic or weak currents is contracted with "polarization vectors" $\bar{u}\gamma_\alpha u$ or $\bar{u}\gamma_\alpha(1-\gamma_5)u$, whose decomposition into states of definite vector-current helicity may be varied by varying the incident lepton beam energy and final scattering angle.

For electroproduction with incident lepton momentum l and outgoing lepton momentum l', we write for the production of N detected hadrons from a proton of momentum p the laboratory differential cross section

$$\frac{d\sigma\,(e+p\to e'+1+2+\cdots+N+X)}{-dE'\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\,\frac{E'}{E}\,\frac{L^{\alpha\beta}}{2E}\,W_{\alpha\beta}(q,1,2,\ldots,N,p)\prod_{j=1}^{N}\frac{d^3p_j}{\pi m_j^2 E_j}\,, \tag{29}$$

with the choice of lepton momenta

$$l=(E,E\sin\psi,0,E\cos\psi), \tag{30}$$

$$l'=(E',E'\sin(\Theta+\psi),0,E'\cos(\Theta+\psi)), \tag{31}$$

where we have neglected M_l^2 and Θ is the lab scattering angle. The angle ψ is an auxiliary quantity which is given by

$$\sin\psi=E'\sin\Theta/(E^2+E'^2-2EE'\cos\Theta)^{1/2}. \tag{32}$$

The squared four-momentum transfer

$$Q^2=-q^2=-(l-l')^2 \tag{33}$$

has been introduced and, as usual, is

$$Q^2=4EE'\sin^2(\tfrac{1}{2}\Theta). \tag{34}$$

The lepton tensor $L_{\alpha\beta}$ comes from summing over lepton spins and is explicitly

$$L_{\alpha\beta}=l_\alpha l'_\beta+l'_\alpha l_\beta-g_{\alpha\beta}\,l\cdot l'. \tag{35}$$

The interesting dynamics resides in the structure functions $W_{\alpha\beta}$ which are given as

$$W_{\alpha\beta}(q,p_1,\ldots,p_N,p)=\frac{\prod_{j=1}^{N}m_j^2}{(16\pi^2)^N}\sum_X\int\frac{d^4y}{2\pi}\,e^{-iq\cdot y}\langle p|J_\alpha(y)|p_1,\ldots,p_N,X\,\text{out}\rangle\langle X,p_1,\ldots,p_N\,\text{out}|J_\alpha(0)|p\rangle\,, \tag{36}$$

with J_α the electromagnetic current.

For neutrino- or antineutrino-induced processes the structure is essentially the same. The Fermi coupling constant replaces α/Q^2 and the lepton tensor picks up an extra antisymmetric term from V-A interference in the lepton current.

Let us write the two differential cross sections as

$$\frac{d\sigma\,(l+p\to l'+1+2+\cdots+N+X)}{d\nu\,dQ^2}=N_l\frac{E'}{E}\prod_{j=1}^{N}\frac{d^3p_j}{\pi m_j^2E_j}\,d^4k\,\delta^4\Big(k-\sum_{j=1}^{N}p_j\Big)W, \tag{37}$$

where N_l is a factor which tells if electrons or muons or neutrinos were incident,

$$N_e=N_\mu=4\pi\alpha^2/Q^4, \tag{38}$$

and

$$N_\nu=G_F{}^2/2\pi. \tag{39}$$

We have also introduced $\nu=p\cdot q/M=E'-E$ as a new and familiar variable as well as $k=\sum_{j=1}^{N}p_j$, the sum of the detected hadron momenta. The factor W will be constructed out of the lepton tensor and the structure function (36).

What we shall do is to examine the helicity amplitudes for the current + proton $\to 1+2+\cdots+N+X$ inclusive cross section. This will tell us which of the various structure functions survive in the limit as $q\cdot p_1$ becomes large, for most of them will be suppressed by the mechanism we have been discussing in this paper.

In the laboratory system let us choose as our momenta

$$q=(q_0,0,0,q_3), \tag{40}$$

$$p=(M,0,0,0), \tag{41}$$

and the symmetric quantity k,

$$k=(k_0,k_T\cos\phi,k_T\sin\phi,k_L). \tag{42}$$

The structure function $W_{\alpha\beta}$ will be built out of p, q, k, and the pseudovector

$$N_\alpha = \epsilon_{\alpha\beta\sigma\nu} p^\beta q^\sigma k^\nu \tag{43}$$

$$= (0, |\vec{n}| \sin\phi, -|\vec{n}| \cos\phi, 0), \tag{44}$$

where $|\vec{n}| = Mq_3 k_T$ in the lab frame.

When the reaction is induced by electrons or muons, there are six helicity amplitudes or structure functions in general. If only one particle is detected, the number is four. For neutrino-initiated processes we will have nine transitions. We choose our structure functions as follows (neglecting terms proportional to q_μ or q_ν which will not contribute when contracted with $L_{\alpha\beta}$).

$$W_{\alpha\beta} = -g_{\alpha\beta}W_1 + \frac{p_\alpha p_\beta}{m^2} W_2 + \frac{N_\alpha N_\beta}{-N^2} W_3 + \left[\frac{(p\cdot q)(p_\alpha k_\beta + p_\beta k_\alpha)}{2m^4} - \frac{(k\cdot q)p_\alpha p_\beta}{m^4}\right] W_4 + \frac{N_\alpha p_\beta + N_\beta p_\alpha}{2m\sqrt{-N^2}} W_5$$
$$+ \frac{N_\alpha k_\beta + N_\beta k_\alpha}{2m\sqrt{-N^2}} W_6 - \frac{i\epsilon_{\alpha\beta\sigma\nu} p^\sigma q^\nu}{2m^2} W_7 + \frac{i(N_\alpha p_\beta - N_\beta p_\alpha)}{2m\sqrt{-N^2}} W_8 + \frac{i(N_\alpha k_\beta - N_\beta k_\alpha)}{2m\sqrt{-N^2}} W_9 \,. \tag{45}$$

When dealing with electroproduction of a single hadron, we need only $W_1, \ldots, W_4$. If two hadrons or more are electroproduced, then W_5 and W_6 also enter. Note they are pseudoscalars and may be taken proportional to $N\cdot p_j$, $j = 1, \ldots, N$.

To discover which of the structure functions will survive the high-energy limit, we proceed by writing out the helicity amplitudes in terms of them. This is not a very instructive exercise but the results are useful:

$$W_{\pm\pm} = W_1 + \tfrac{1}{2}W_3 \mp \tfrac{1}{2}q_3 W_7 \mp \tfrac{1}{2}k_T W_9, \tag{46}$$

$$W_{\pm\mp} = e^{\mp 2i\phi}(-\tfrac{1}{2}W_3 \pm \tfrac{1}{2}ik_T W_6), \tag{47}$$

$$W_{00} = -W_1 + \frac{q_3^{\,2}}{Q^2} W_2 + \frac{W_4}{M^2}\left(\frac{p\cdot q}{m}\frac{q_3}{\sqrt{Q^2}}(k\cdot\epsilon^0) - (k\cdot q)\frac{q_3^{\,2}}{Q^2}\right), \tag{48}$$

where

$$k\cdot\epsilon^0 = (k_0 q_3 - k_L q_0)/\sqrt{Q^2}\,, \tag{49}$$

$$W_{0\pm} = \frac{e^{\pm i\phi}}{2\sqrt{2}}\left(-\frac{p\cdot q}{m^3}\frac{k_T q_3}{\sqrt{Q^2}} W_4 \pm \frac{q_3}{\sqrt{Q^2}}(iW_5 + W_8) \pm \frac{k\cdot\epsilon^0}{m}(iW_6 + W_9)\right), \tag{50}$$

and

$$W_{\pm 0} = \frac{e^{\mp i\phi}}{2\sqrt{2}}\left(-\frac{p\cdot q}{m^3}\frac{k_T q_3}{\sqrt{Q^2}} W_4 \mp \frac{q_3}{\sqrt{Q^2}}(iW_5 - W_8) \mp \frac{k\cdot\epsilon^0}{m}(iW_6 - W_9)\right). \tag{51}$$

From these formulas we may deduce the asymptotic behavior of the structure functions W_i in the region where the squared energy $(p_1+q)^2$ becomes large, while the other momenta and Q^2 are held fixed; that is, just the limit we have discussed above the photon becomes isolated from the remainder of the process by an $M=0$ Reggeon. In this limit the large quantity $q\cdot p_1$ is proportional to ν. In general, all helicity amplitudes $W_{h',h}$ behave as ν^α for the contribution of an allowed Regge pole at α. Because of our rules above, however, we expect $W_{h',h} \to \nu^{\alpha - |h'-h|}$. From this we find that for large ν (in parentheses we indicate the Regge behavior which would occur if factorization were not valid)

$$W_1 \sim \nu^\alpha\ (\nu^\alpha), \tag{52}$$

$$W_7 \sim \nu^{\alpha-1}\ (\nu^{\alpha-1}), \tag{53}$$

$$W_2 \sim \nu^{\alpha-2}\ (\nu^{\alpha-2}),$$
$$W_3, W_6, W_9, \sim \nu^{\alpha-2}\ (\nu^\alpha), \tag{54}$$
$$W_5, W_8 \sim \nu^{\alpha-2}\ (\nu^{\alpha-1}),$$

and finally,

$$W_4 \sim \nu^{\alpha-3}\ (\nu^{\alpha-2}). \tag{55}$$

The contributions to the helicity-nonflip amplitudes in the large-ν limit are

$$W_{\pm\pm} \sim W_1 \pm \nu W_7/2M + O(\nu^{\alpha-2}) \tag{56}$$

and

$$W_{00} \sim -W_1 + (\nu^2/Q^2)W_2 + O(\nu^{\alpha-2}). \tag{57}$$

Thus, in the inclusive differential cross section we expect to see dominant in the limit we take only those terms involving W_1, W_2, and W_7 which are the familiar structure functions from inelastic electroproduction and high-energy neutrino reactions.

Finally we write the differential cross section

$$\frac{d\sigma\,(\text{lepton}+p\to\text{lepton}'+1+\cdots+N+X)}{dQ^2\,d\nu}$$
$$= \frac{E'}{E} N_l \prod_{j=1}^{N} \frac{d^3p_j}{\pi m_j^2 E_j} d^4k\, \delta^4\Big(k - \sum_{j=1}^{N} p_j\Big)\Big[2\sin^2(\tfrac{1}{2}\Theta)W_1 + \cos^2(\tfrac{1}{2}\Theta)W_2 + \Big(\sin^2(\tfrac{1}{2}\Theta) + \frac{\sin^2\phi\, EE'\sin^2\Theta}{\nu^2+Q^2}\Big)W_3$$
$$+ \frac{W_4}{m^2}\frac{\sin\Theta}{(\nu^2+Q^2)^{1/2}}[k_L EE'\sin\Theta - \tfrac{1}{2}k_T\cos\phi(E^2-E'^2)] - \frac{(E+E')\sin\Theta\sin\phi}{2(\nu^2+Q^2)^{1/2}}W_5$$
$$- \frac{\sin\phi\, W_6\sin\Theta}{m(\nu^2+Q^2)^{1/2}}\Big(\frac{k_0(E+E')}{2} - \frac{k_T\cos\phi\sin\Theta EE'}{(\nu^2+Q^2)^{1/2}} - \frac{k_L(E^2-E'^2)}{2(\nu^2+Q^2)^{1/2}}\Big)$$
$$\mp\sin^2(\tfrac{1}{2}\Theta)(E+E')\frac{W_7}{m} \mp \tfrac{1}{2}W_8\sin\Theta\cos\phi \pm \frac{W_9}{2m}\Big(\frac{k_T}{(\nu^2+Q^2)^{1/2}}(E+E')(1-\cos\Theta)$$
$$+ \frac{k_L}{(\nu^2+Q^2)^{1/2}}(E-E')\cos\phi\sin\Theta - k_0\cos\phi\sin\Theta\Big)\Big]. \tag{58}$$

This, while not terribly transparent, has the distinct virtue that those coefficients independent of ϕ, which is just the angle we spoke of in connection with Eq. (27), multiply W_1, W_2, and W_7 alone.

V. SCALING IN INCLUSIVE LEPTON-HADRON SCATTERING

Recent experiments[13] on the reaction $e+p\to e'+X$ in the deep-inelastic region have shown a remarkable dependence on the four-momentum transfer to the hadrons. The results of the experiments are summarized by saying that the total photoabsorption cross sections for large virtual photon mass, $-Q^2 \gtrsim 2\,\text{BeV}^2$, and large energy transfer ν behave as

$$\sigma_{\text{tot}}^{\gamma(q^2)} \sim (1/Q^2)F(-m\nu/Q^2). \tag{59}$$

Alternatively we can conclude that the absorptive parts of the helicity amplitudes for forward virtual Compton scattering scale for large Q^2 and fixed $\omega = 2m\nu/Q^2$. Of the structure functions introduced in the previous sections, only W_1 and W_2 are measured in the SLAC-MIT experiment, and these depend on Q^2 and ω alone. We have that

$$W_{++}(Q^2,\omega) = W_1(Q^2,\omega) \underset{Q^2\to\infty}{\longrightarrow} W_{++}(\omega), \tag{60}$$

$$W_{00}(Q^2,\omega) = \frac{(\nu^2-Q^2)^{1/2}}{Q^2}W_2(Q^2,\omega) - W_1(Q^2,\omega) \underset{Q^2\to\infty}{\longrightarrow} W_{00}(\omega). \tag{61}$$

It is natural to inquire as to the Q^2 dependence of the more general inclusive lepton-nucleon reactions $e+p\to e'+1+\cdots+N+X$, and, in particular, to ask whether the corresponding structure functions exhibit similar scaling behavior for large Q^2. Scaling in fact might be anticipated due to the close relation between the integrated cross sections for these reactions and the total photoabsorbtion cross section. The connection, however, involves the multiplicity of produced hadrons which is, so far, a rather arbitrary function of Q^2. Therefore, we shall resort once again to a Regge-pole model for virtual-photon–hadron forward scattering, and use factorization to establish scaling, deriving as a by-product a Q^2 dependence for the multiplicity.

First let us recall that Regge theory seems to be applicable to deep-inelastic scattering[14,15] and the ω dependence of the amplitudes W_{++} and W_{00} can be fitted by a modest number of Regge poles,[16] so that

$$W_{++}(\omega) \underset{\omega\gtrsim 4}{\sim} \sum_i \beta_{++}^i \omega^{\alpha_i(0)}. \tag{62}$$

For fixed Q^2, large ν W_{++} behaves as

$$W_{++}(Q^2,\nu) \underset{\nu/Q^2\gg 1}{\sim} \sum_i \beta_{++}^i(Q^2)(\nu/\sqrt{Q^2})^{\alpha_i(0)}, \tag{63}$$

so that for (62) and (63) to be consistent, the Regge residue coupling a Regge pole at $J=\alpha_i(0)$ to two photons of virtual mass $-Q^2$ must behave, for large Q^2, as

$$\beta^i_{++}(Q^2) \underset{Q^2\to\infty}{\approx} (1/\sqrt{Q^2})^{\alpha_i(0)}\beta^i_{++}. \tag{64}$$

We now turn our attention to the inclusive cross section where, say, a single hadron of momentum P_1 is detected (the restriction to one detected hadron is nonessential). As before, this is determined in terms of a piece of the absorptive forward amplitude for

$$\gamma(Q^2)+1+\bar{p}\to\gamma(Q^2)+1+\bar{p}. \tag{65}$$

In the region where $\nu_1/\sqrt{Q^2}=q\cdot P_1/M\sqrt{Q^2}$ is relatively large (i.e., $\gtrsim 4$), we can describe this amplitude by the exchange of Regge poles between the photon and the hadrons. The essential point is that due to the factorization of the Regge residues, the dependence on the virtual photon mass is known from our previous discussion. In fact, in this region a given helicity amplitude, say $W_{++}(Q^2,\nu,P_{1T},P_{1L})$, behaves as

$$W_{++}(Q^2,\nu,P_{1T},P_{1L})\sim\sum_i\beta^i_{++}(Q^2)(\nu/\sqrt{Q^2})^{\alpha_i(0)} \times R_i(P_{1T},P_{1L}/\sqrt{\nu}), \tag{66}$$

$$\nu/\sqrt{Q^2}\gg 1;\quad P_{1T},P_{1L}/\sqrt{\nu}\ \text{fixed}.$$

We have used the fact that, for fixed $P_{1L}/\sqrt{\nu}$, $\nu=q\cdot P/M$ and $\nu_1=q\cdot P_1/M$ are proportional. We can then consider the behavior of W_{++} for large Q^2 (i.e., $\gtrsim 1$ BeV^2, as indicated at SLAC[13]) using (65), and derive that

$$W_{++}(Q^2,\nu,P_{1T},P_{1L}) \approx\sum_i\beta^i_{++}\,\omega^{\alpha_i(0)}R_i(P_{1T},P_{1L}/\sqrt{\nu})=W_{++}(\omega,P_{1T},P_{1L}/\sqrt{\nu}), \tag{67}$$

with ω fixed $\gtrsim 4$, $Q^2\to\infty$, and $P_{1T},P_{1L}/\sqrt{\nu}$ fixed. In conclusion, using the observed scaling and Regge behavior of the total photoabsorption cross sections, we have derived that the cross sections for inclusive electroproduction should scale in the same way in the fragmentation region of the target.

If we now combine the results of this section with those of the previous section, we see a remarkable simplification in the large-ω scaling region. As before, for electroproduction, only W_{++} and W_{00} will survive to leading order in ω. In addition, the indication is from SLAC that the W_{00} is much less important than W_{++} for large Q^2 ($\sigma_L/\sigma_T \gtrsim 0.2$). With all of this in mind, we then have for the structure function $W_{\alpha\beta}(q,P,P_1,\ldots,P_N)$, for large but fixed $\omega=2q\cdot P/Q^2$, large Q^2, and fixed P_{iT} that

$$W_{\alpha\beta}\approx\left(\frac{P_\alpha P_\beta Q^2}{m^2\nu^2}-g_{\alpha\beta}\right)W_{++}\left(\omega,P_{iT},\frac{P_{iL}}{\sqrt{\nu}}\right). \tag{68}$$

We are now in a position to determine the Q^2 dependence of the multiplicity of the hadrons produced in deep-inelastic electroproduction. This can be calculated from the differential cross section for the inclusive reaction $\gamma(Q^2)+P\to P_1+X$, where

$$\frac{d\sigma_+(\gamma(Q^2)+P\to P_1+X)}{d\theta_1\,d(\cosh\beta_1)^2}\underset{\nu\to\infty}{\sim}\frac{W_{++}}{\nu}. \tag{69}$$

Indeed, the mean multiplicity of hadrons of type 1 will be for large ν

$$n(Q^2,\nu)=\frac{1}{\sigma^{\gamma(Q^2)}_{\text{tot}}(\nu)}\int d\theta_1\,d(\cosh\beta_1)^2\frac{d\sigma_{++}}{d\theta_1\,d(\cosh\beta_1)^2}. \tag{70}$$

The usual estimate of hadron multiplicities[4] proceeds by examining the contribution of the pionization region to (70), i.e., the region where both $q\cdot P_1$ and $P_1\cdot P$ are large or where $+\theta+\Delta<\theta_1<\theta-\Delta$ for some fixed $\Delta\approx 2.5$. For fixed Q^2 one has that $W_{++}\sim\nu$ in this region and $\sigma_{\text{tot}}(\nu)\to$ const so that

$$n(Q^2,\nu)\underset{\nu\to\infty;\,Q^2\text{fixed}}{\sim}\theta_a+\theta_b-2\Delta\approx\text{const}(\ln\nu).$$

In a similar fashion we can estimate the contribution of the pionization region to the multiplicity when Q^2, $q\cdot P_1/q^2$, and $P_1\cdot P$ are large.

In this region

$$W_{++}\approx\text{const}\,\omega f(P_{iT})$$

with $\omega\gg 1$, $P_1\cdot P\gg 1$, $Q^2\to\infty$, and $\sigma^{\gamma(q^2)}_{\text{tot}}\to\text{const}/Q^2$. Since

$$\theta_a+\theta_b\approx\ln(2\nu/\sqrt{Q^2}),$$
$$P_1\cdot P=mm_1\cosh\beta_1\cosh(\theta_b-\theta_1), \tag{71}$$
$$q\cdot P_1/Q^2=(m_1/\sqrt{Q^2})\cosh\beta_1\sinh(\theta_a+\theta_1),$$

we must have in the pionization region that

$$-\theta_a+\ln(\sqrt{Q^2}/m_1)+\text{const}\le\theta_1\le\theta_b-\text{const}, \tag{72}$$

and thus

$$n(Q^2,\nu)=\int_{\text{pionization}}d\theta_1\underset{\substack{\omega\gg 1\\ Q^2\to\infty}}{\approx}\text{const}\ln\omega. \tag{73}$$

We have thus derived that the multiplicity of produced hadrons in the pionization region of deep-inelastic electroproduction *scales* and exhibits logarithmic behavior in the scaling variable ω. We cannot immediately conclude that the same is true of the total multiplicity. This is because the region of θ_1 which lies outside the pionization region increases in size as $Q^2\to\infty$ as $\ln Q^2$. In fact, when $-\theta_a-\text{const}\le\theta_1\le-\theta_a+\ln(\sqrt{Q^2}/m_1)+\text{const}$, we are in the fragmentation region of the photon, and our previous discussion of the Q^2 behavior of the inclusive cross section is inapplicable. For fixed Q^2 this region is finite and thus gives rise to a constant multiplicity. However, the multiplicity from this region could conceivably be an increasing function of Q^2. We are unable to rule this out; however, we regard it to be unlikely that this indeed happens.

VI. DISCUSSION

We have seen above that the J-plane structure, which has been uncovered in the study of two-body processes when assumed to govern the asymptotic behavior of inclusive processes, has quite striking implications for the allowed spin-dependent structure of those inclusive reactions. Briefly recounted, we have argued that whenever any particle in an inclusive process is "isolated" from the others by Regge exchange and is collinear with the incoming beam, then to leading order in the subenergy governing the Regge behavior, the helicity of that particle will not flip if the J-plane singularity is factorizable, or equivalently here since all momentum transfers are null, it carries O(3, 1) quantum number $M=0$. The primary import of our observations is to provide a rather straightforward testing ground both for the applicability of J-plane concepts to inclusive processes and, perhaps more singificantly, for the detailed examination of many of the phenomenological ideas which have grown up around Regge behavior in two-body reactions. The failure of our predictions for various kinds of spin independence must be regarded as strong evidence for non-negligible cuts in the J plane. Should their presence be indicated, the studies we are proposing can be turned around to provide a sensitive measuring ground for the properties of these cuts.

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