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PHYSICAL REVIEW D VOLUME 5, NUMBER 3 1 FEBRUARY 1972

Tests for the Internal Quantum Numbers of Partons

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The restrictions for the structure functions of deep-inelastic electroproduction and neutrino-induced production following from assigning partons to any representation of SU(2) × SU(2) × Y are derived. Apart from the general parton-model assumptions only isospin invariance and positivity are used. Comparison with experiment allows us to exclude all models which have only partons of spin $\frac{1}{2}$, isospin $\le \frac{1}{2}$, and integral charge.

I. INTRODUCTION

One of the popular models for deep-inelastic electron- and neutrino-nucleon scattering is Feynman's parton model.[1-4] In this model one views the nucleon as built of constituents, partons, which scatter incoherently. It is then natural to ask for the quantum numbers of partons. On a more fundamental level one may hope to identify partons with the bare quanta of some underlying field theory and ask for the quantum numbers of the basic fields which build up the electromagnetic and weak currents.

The relation of Callan and Gross[5] gives a direct test for the spin of partons.

If partons have only spin $\frac{1}{2}$, the longitudinal structure functions vanish; for spin 0, the transverse ones vanish. In the present article we will derive the restrictions for the structure functions which follow from internal-quantum-number assignments to partons. The basic fields are assumed to carry a representation of SU(2) × SU(2) × Y, where Y is the hypercharge. We exploit only isospin invariance and positivity. No assumptions on the momentum distribution of partons etc. are made. The generalization to SU(3) × SU(3) would be straightforward but tedious.

All notations and definitions are taken from Ref. 6. Bjorken's scaling functions[7] will be defined by

$$\begin{aligned} F_1(x) &= \lim 2MW_1(\nu, Q^2), \\ F_2(x) &= \lim \nu W_2(\nu, Q^2), \\ F_3(x) &= \lim \nu W_3(\nu, Q^2), \\ F_L(x) &= \frac{1}{x}F_2(x) - F_1(x), \end{aligned} \tag{1.1}$$

where $x = Q^2/2M\nu$ and the limit is taken for $\nu \to \infty$, $Q^2 \to \infty$, x fixed. The Cabibbo angle of the weak current will be set equal to zero.

II. DERIVATION OF POSITIVITY DOMAIN

We shall assume the existence of SU(2)×SU(2) and hypercharge currents, formed from fundamental spin-$\frac{1}{2}$ and spin-0 fields.

$$J_\mu^{a,\pm} = \bar\psi \tfrac{1}{2}(T^a\gamma_\mu \pm T_5^a\gamma_\mu\gamma_5)\psi + i\,\phi^T \tfrac{1}{2}(t^a \pm t_5^a)\partial_\mu\phi\,, \quad a=1,2,3 \qquad (2.1)$$

$$J_\mu^Y = \bar\psi Y\gamma_\mu\psi + i\,\phi^T y \partial_\mu \phi\,, \qquad (2.2)$$

where we have chosen a Hermitian basis for the scalar fields ϕ which implies that t^a and t_5^a are skew symmetric. $\frac{1}{2}(T^a \pm T_5^a)$ and $\frac{1}{2}(t^a \pm t_5^a)$ form SU(2)×SU(2) representations:

$$[\tfrac{1}{2}(T^a \pm T_5^a), \tfrac{1}{2}(T^b \pm T_5^b)] = i\epsilon_{abc}\tfrac{1}{2}(T^c \pm T_5^c)\,, \qquad (2.3)$$

$$[\tfrac{1}{2}(t^a \pm t_5^a), \tfrac{1}{2}(t^b \pm t_5^b)] = i\epsilon_{abc}\tfrac{1}{2}(t^c \pm t_5^c)\,. \qquad (2.4)$$

The parton model expresses the structure functions in terms of parton densities in the nucleon. For electroproduction, for instance, one finds[1]

$$F_2^\gamma(x) = x\sum P_N \sum_{i=1}^N f_N^i(x) Q_i^{\,2}\,, \qquad (2.5)$$

where Q_i is the charge of ith parton. In complete analogy to nonrelativistic nuclear physics we shall rewrite Eq. (2.5) as an expectation value in the nucleon state of a one-particle operator, acting in the Fock space of partons.

$$F_2^{\gamma r}(x) = x\{\langle r|\chi^\dagger(x)Q^2\chi(x)|r\rangle + \langle r|\phi^\dagger(x)q^2\phi(x)|r\rangle\}\,, \qquad (2.6)$$

where r = proton or neutron and $\chi(x)$ and $\phi(x)$ are the annihilation operators for spin-$\frac{1}{2}$ and spin-0 partons with

$$\chi(x) = \begin{pmatrix}\psi(x)\\ \bar\psi(x)\end{pmatrix}. \qquad (2.7)$$

Q and q are the charge matrices for spin-$\frac{1}{2}$ and spin-0 partons, respectively. Similar expressions hold for neutrino-induced production. The crucial step is to introduce 2 matrices:

$$\langle r,\alpha|M_T|s,\beta\rangle = \langle r|\chi_\alpha^\dagger\chi_\beta|s\rangle\,, \qquad (2.8)$$

$$\langle r,\alpha|M_L|s,\beta\rangle = \langle r|\phi_\alpha^\dagger\phi_\beta|s\rangle\,, \qquad (2.9)$$

and to observe that M_T and M_L are positive matrices, invariant under SU(2), acting in a space with representations $D(j_1)\times D(j_2)\times D(\frac{1}{2})$, if the fields χ and ϕ carry the representation (j_1, j_2) of SU(2) ×SU(2) and r, s = proton, neutron. It is a standard problem of the addition of 3 angular momenta to find the reduced matrix elements of M_T and M_L. Positivity requires the reduced matrix to be positive. This can then be translated in restrictions on the measurable structure functions which are just linear combinations of the reduced matrix elements of M_T and M_L.

In this way one finds the following positivity domain $\mathfrak{D}$ corresponding to a representation (j_1, j_2, y) of SU(2)×SU(2)×Y for spin-$\frac{1}{2}$ partons [the corresponding antipartons belong to $(j_2, j_1, -y)$]:

$$\begin{aligned}
F_1^{\gamma p,\,\gamma n} &= \sum m_l[\tfrac{1}{3}(l+\tfrac{1}{2})^2 + \tfrac{1}{4}y^2 \pm \tfrac{1}{3}y - \tfrac{1}{3}(\vec c\cdot\vec n_l)(1\pm y)]\,,\\
(F_1+F_3)^{\nu p+\nu n} &= \sum m_l \tfrac{16}{3} j_2(j_2+1)\,,\\
(F_1-F_3)^{\nu p+\nu n} &= \sum m_l \tfrac{16}{3} j_1(j_1+1)\,, \qquad (2.10)\\
(F_1+F_3)^{\nu p-\nu n} &= \sum m_l(-\tfrac{4}{3})(1+2\vec b\cdot\vec n_l)\,,\\
(F_1-F_3)^{\nu p-\nu n} &= \sum m_l(-\tfrac{4}{3})(1+2\vec a\cdot\vec n_l)\,,
\end{aligned}$$

where $\|j_1-j_2|-\frac{1}{2}| \le l \le j_1+j_2+\frac{1}{2}$ and $\vec a$, $\vec b$, $\vec c$ are 3-vectors such that

$$\vec a + \vec b + \vec c = 0\,, \qquad (2.11)$$

$$|\vec a| = j_1+\tfrac{1}{2},\quad |\vec b| = j_2+\tfrac{1}{2},\quad |\vec c| = l+\tfrac{1}{2}\,, \qquad (2.12)$$

and $m_l \ge 0$, $|\vec n_l| \le 1$. If $\vec a$, $\vec b$, $\vec c$ are linearly dependent, $\vec n_l$ has to be replaced by a unit vector in the direction of the 3-vector $\vec a$, $\vec b$, or $\vec c$ of greatest magnitude.

The edges of the positivity domain are obtained by setting in Eq. (2.10) all except one of the m_l equal to zero and $|\vec n_l| = 1$.

For spin-0 partons one finds, if ϕ belongs to a representation $(j_1, j_2, y) + (j_2, j_1, y)$ of SU(2)×SU(2) ×Y,

$$\begin{aligned}
F_L^{\gamma p+\gamma n} &= \sum m_{kl}\, 2[\tfrac{1}{3}k(k+1) + \tfrac{1}{4}y^2]\,,\\
F_L^{\gamma p-\gamma n} &= \sum m_{kl}(-\tfrac{2}{3}y)[l(l+1) - k(k+1) - \tfrac{3}{4}]\,, \qquad (2.13)\\
F_L^{\nu p+\nu n} &= \sum m_{kl}\tfrac{8}{3}[j_1(j_1+1) + j_2(j_2+1)]\,,\\
F_L^{\nu p-\nu n} &= \sum m_{kl}\tfrac{4}{3}[l(l+1) - k(k+1) - \tfrac{3}{4}]\,,
\end{aligned}$$

where $|j_1-j_2| \le k \le j_1+j_2$, $|k-\frac{1}{2}| \le l \le k+\frac{1}{2}$, and $m_{kl} \ge 0$. The edges of the positivity domain are again obtained by allowing only one of the m_{kl} to be nonzero.

For the integrals of the structure functions, $\int_0^1 dx\,F(x)$, there are further restrictions due to the sum rules expressing total charge, hypercharge, and baryonic number of the nucleons:

$$\int_0^1 dx\{\sum m_l(-1+\vec c\cdot\vec n_l) + \sum m_{kl}[l(l+1) - k(k+1) - \tfrac{3}{4}]\} = -\tfrac{3}{2}\,, \qquad (2.14)$$

$$\int_0^1 dx(\sum m_l y + \sum m_{kl} y) = 1\,, \tag{2.15}$$

$$\int_0^1 dx(\sum m_l b) = 1\,, \tag{2.16}$$

where b is the baryon number of the partons. Equation (2.14) is of course Adler's sum rule[8]; Eqs. (2.15) and (2.16) reduce to the sum rule of Gross and Llewellyn Smith in the quark-parton model.[2]

For the first moments, $\int_0^1 dx\, xF(x)$, there is one further restriction expressing that the total longitudinal moment of partons has to add up to the longitudinal momentum of the proton. This implies

$$\int_0^1 dx\, x(\sum m_l + \sum m_{kl}) \leq 1\,. \tag{2.17}$$

III. CONCLUSION AND COMPARISON WITH EXPERIMENTS

Equations (2.10) and (2.13) express the consequences for the structure functions of deep-inelastic electroproduction and neutrino-induced production which follow from assigning a set of internal quantum numbers to partons and constitute therefore a test of these assignments. This extends our earlier findings in the quark-parton model[9] to the general case of an arbitrary SU(2)×SU(2)×Y representation. The comparison with experiment shows that some models can already be ruled out by presently existing data.

(a) *Ratio* $F_1^{\gamma n}/F_1^{\gamma p}$. Quark-parton model [SU(2) symmetry]:

$$\tfrac{1}{4} \leq F_1^{\gamma n}/F_1^{\gamma p} \leq 4\,. \tag{3.1}$$

Models with integral charges:

$$0 \leq F_1^{\gamma n}/F_1^{\gamma p} \leq \infty\,. \tag{3.2}$$

Only an isodoublet of partons with charges (0, +1) can give $F_1^{\gamma n}/F_1^{\gamma p} = 0$. Experimentally, the neutron-to-proton ratio drops from one at $x \simeq 0$ to something very near $\frac{1}{4}$ at $x \simeq 1$, if not below $\frac{1}{4}$.[10] If it drops below $\frac{1}{4}$ this would exclude the quark-parton model (assuming the longitudinal contribution to be small).

(b) *Neutrino-induced production versus electroproduction.* The total neutrino and antineutrino cross sections can be expressed as first moments of structure functions:

$$\tfrac{1}{2}\sigma^{\nu p+\nu n} = \frac{G^2ME}{\pi} Z\,, \tag{3.3}$$

$$\tfrac{1}{2}\sigma^{\bar\nu p+\bar\nu n} = \frac{G^2ME}{\pi} \bar{Z}\,, \tag{3.4}$$

$$Z = \int_0^1 dx\, x(\tfrac{1}{3}F_1^{\nu p+\nu n} - \tfrac{1}{6}F_3^{\nu p+\nu n} + \tfrac{1}{4}F_L^{\nu p+\nu n})\,, \tag{3.5}$$

$$\bar{Z} = \int_0^1 dx\, x(\tfrac{1}{3}F_1^{\bar\nu p+\bar\nu n} + \tfrac{1}{6}F_3^{\bar\nu p+\bar\nu n} + \tfrac{1}{4}F_L^{\bar\nu p+\bar\nu n})\,. \tag{3.6}$$

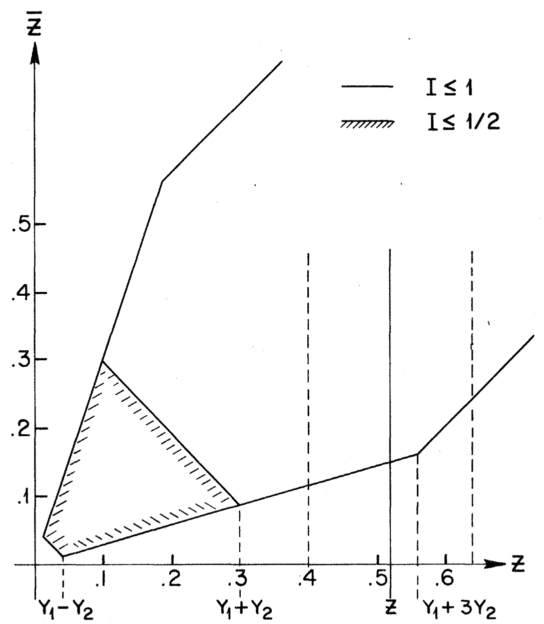

FIG. 1. Positivity domain for neutrino versus antineutrino cross sections $\frac{1}{2}\sigma^{\nu p+\nu n} = (G^2ME/\pi)Z$, $\frac{1}{2}\sigma^{\bar\nu p+\bar\nu n} = (G^2ME/\pi)\bar{Z}$ taking $Y_1 = \int dx F_2^{\gamma p}(x) = 0.17$, $Y_2 = \int dx F_2^{\gamma n}(x) = 0.13$, and assuming only integrally charged partons of spin $\frac{1}{2}$. Full line: parton isospin ≤ 1. Shaded: parton isospin $\leq \frac{1}{2}$. Experimental value for $Z = 0.52 \pm 0.12$.

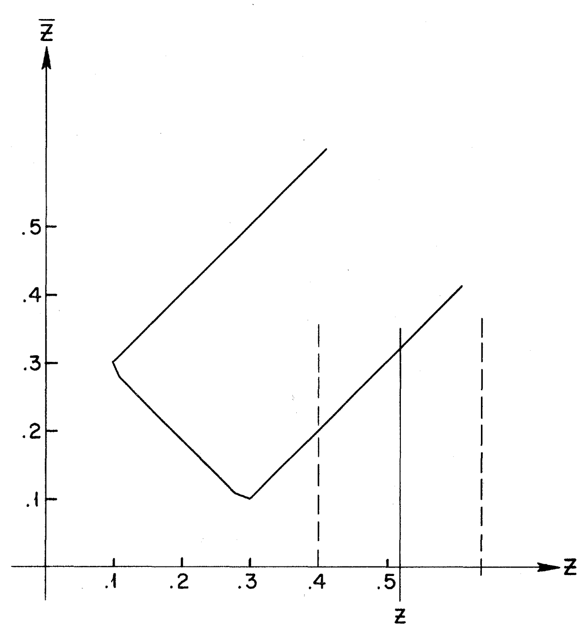

FIG. 2. Same as Fig. 1 for a model with integrally charged partons of spin $\frac{1}{2}$, isospin $\frac{1}{2}$, and spin 0, isospin ≤ 1 (for instance, the usual σ model), assuming a longitudinal contribution $\int dx\, xF_L^{\gamma p}(x) \leq 0.04$.

From the CERN neutrino experiment one knows[11]

$$Z = 0.52 \pm 0.12 . \qquad (3.7)$$

This can be compared with the first moments of electroproduction structure functions measured at SLAC[12]:

$$Y_{1,2} = \int_0^1 dx\, x(F_1^{\gamma p,\,\gamma n} + F_L^{\gamma p,\,\gamma n}) ,$$
$$L_{1,2} = \int_0^1 dx\, x F_L^{\gamma p,\,\gamma n} ; \qquad (3.8)$$

experimentally:

$$Y_1 \simeq 0.17, \quad Y_2 \simeq 0.13, \quad L_1 \lesssim 0.04 . \qquad (3.9)$$

The quark-parton model is compatible with Eqs. (3.7) and (3.9).[9] Integrally charged models are compared with experiment in Figs. 1 and 2. If one assumes that only integrally charged partons of spin $\frac{1}{2}$ contribute, one needs partons of isospin greater than $\frac{1}{2}$. This excludes, for instance, the 3-triplet model of Ref. 13 and the 4-quark model of Ref. 14. If one allows spin-0 partons, one finds that the small value of the longitudinal structure function for electroproduction does not imply the same for neutrino-induced production. There can be a σ parton which is neutral but contributes via the axial-vector current to neutrino reactions. It would, however, be odd that there should be more σ than π partons in the nucleon.

Note added in proof. In models with additional internal quantum numbers like "charm," the electromagnetic and weak currents will in general have pieces which violate charm. Since no "charmed" particles have so far been observed, experiment measures only the charm-conserving pieces of the currents. Our relations therefore test only the $SU(2)\times SU(2)\times Y$ algebra associated with the charm-conserving pieces of the currents, and charge in the preceding article always means charm-conserving piece of the charge, etc. Our relations cannot distinguish between two models if they contain the same representations of the charm-conserving $SU(2)\times SU(2)\times Y$ algebra. This was demonstrated explicitly by Lipkin[15] for the usual quark model and the Han-Nambu model.[16] Our conclusions for the 3-triplet model of Ref. 13, which is *different* from the Han-Nambu model, are unchanged.

ACKNOWLEDGMENTS

The author would like to thank Professor C. G. Callan, Professor Ph. Meyer, and Professor J. Wess for useful and stimulating discussions. The author is grateful to Dr. Carl Kaysen for the hospitality extended to him at the Institute for Advanced Study.

*Research sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, United States Air Force, under AFOSR Grant 70-1866A.

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