cay $\pi \to \mu\nu e^+e^-$ also.

$$\frac{\Gamma(\pi \to \mu\nu e^+e^-)}{\Gamma(\pi \to \mu\nu)} = 0.633\times10^{-7}.$$

The contribution of the structure terms in this case is less than 1%.

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CP Violation in τ Decays

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By making use of postulated commutation relations of a *CP*-violating Hamiltonian with the vector and axial-vector charges of chiral SU(2) × SU(2) symmetry, we obtain a theoretical prediction of asymmetry in the slope parameters of $\tau^\pm$ decay in terms of the $K_L \to 2\pi^0$ parameter ϵ'. We expect $\Delta = (a^+ - a^-)/(a^+ + a^-) \sim 4\epsilon'$, where $a^\pm$ are the slope parameters in the linear matrix-element approximation of $\tau^\pm$ decay. With the present experimental limit, $|\epsilon'| \lesssim \frac{1}{5}|\epsilon|$, this gives an upper limit for Δ of 10^{-3}.

I. INTRODUCTION

Although much time has elapsed since *CP* violation was first observed experimentally,[1] there has been little success in determining the precise form of the interaction. Most of the experimental data on *CP* violation come from observing the two-pion decay modes of the neutral-*K*-meson system. By observing the charged and neutral decays of K_L and K_S, the two (complex) parameters η_{+-} and η_{00} are measured and values for ϵ and ϵ' deduced.[2] The *CP* parameters ϵ and ϵ' characterize quite different physical origins of the *CP* violation and so it is essential that their value be determined carefully. The parameter ϵ characterizes a mass shift in the neutral-*K* system which need not come from the weak interaction itself, and the parameter ϵ' is related to a matrix element of a *CP*-violating weak interaction.

Two approaches to the origin of these parameters have evolved; (1) a new interaction is postulated or (2) the existing weak-interaction current-current form is modified. One of the approaches of the first type is the superweak theory which associates the breakdown of *CP* invariance with a new $|\Delta S| = 2$ interaction which gives rise to a mass shift and also to a nonzero ϵ.[3] The superweak theory predicts that $\epsilon \neq 0$ and $\epsilon' \equiv 0$ since there is no primary interaction that violates *CP* invariance. Two theories of the second type have been proposed, one introducing phases between the vector and axial-vector currents in the Cabibbo current of the weak interaction and another model introducing neutral currents as the source of the CP violation.[4,5] Of course any modification of the weak interaction has severe limits placed on it due to the body of CP-conserving data that exist. The neutral-current theory (which is on the verge of being ruled out due to inconsistencies with CP-conserving decays) is constructed so that it has the $\Delta I=\frac{1}{2}$ rule and so $\epsilon'\equiv 0$, while Glashow's model of introducing phases in the Cabibbo current admits solutions ranging from $\epsilon'\equiv 0$ to $|\epsilon'| \sim |\epsilon|$ by adjusting the phases.[6]

In view of the various models for CP violation it is very important to find the relative sizes of ϵ and ϵ'. Unfortunately, it is extremely difficult to observe the neutral-two-pion decay and so another observation is desirable. One observation that can be made is the three-pion decay of the charged K mesons, where the CP violation will show up as a difference in the slope parameters of the transition rates. This observation has the advantage that it occurs only in a $\Delta I=\frac{3}{2}$ channel, so that if it can be related to CP violation in $K_L \to 2\pi$, it is determined by ϵ' alone. This means that the experimental information on $K_L \to 2\pi$ can be used to predict the asymmetry in $\tau^\pm$ decays. It is the purpose of this work to establish a relation between the CP parameter ϵ' and the slope asymmetry in τ^+ and τ^- decay. Section II deals with the current-algebra techniques to be used and Sec. III discusses the resulting linear approximation and defines the slope parameters. Section IV defines the weak-interaction Hamiltonians and the commutation relations needed for current-algebra techniques, and Sec. V is concerned with the computation of the slope asymmetry. Finally, Sec. VI relates this to other experiments and discusses the results.

II. REDUCTION FORMULA

To compute the matrix elements for τ decay, we will use the soft-pion emission formula developed by Weinberg and the techniques of applying this formula to $K\to 3\pi$ decay which were developed by Abarbanel.[7,8] These techniques have been utilized elsewhere in a calculation of the charge asymmetry in K_L decay.[9] We treat all three pions as soft and find that the matrix element for the three-pion decay is[10]

$$F_\pi{}^3\langle\pi^\alpha(q_1)\pi^\beta(q_2)\pi^\gamma(q_3)|H_w(0)|K^\rho\rangle = -\tfrac{1}{6}\{\langle 0|[F^{5\alpha}(0),[F^{5\beta}(0),[F^{5\gamma}(0),H_w(0)]]]|K^\rho\rangle + \text{permutations of } (\alpha,\beta,\gamma)\}$$
$$+\tfrac{1}{2}F_\pi[D^{(1)}_{\lambda\gamma}(\alpha)H^{\beta\lambda}_\mu(q_3-q_1)_\mu + D^{(1)}_{\lambda\beta}(\alpha)H^{\gamma\lambda}_\mu(q_2-q_1)_\mu + D^{(1)}_{\lambda\gamma}(\beta)H^{\alpha\lambda}_\mu(q_3-q_2)_\mu]\,, \tag{2.1}$$

where we have used partial conservation of axial-vector current (PCAC) in the form $\partial_\mu A^\alpha_\mu = m_\pi{}^2F_\pi\varphi^\alpha_\pi$ and we have defined

$$H^{\alpha\lambda}_\mu = i\int d^4x\, e^{-i(q_2+q_3)\cdot x}\langle\pi^\alpha|T\{V^\lambda_\mu(x)H_w(0)\}|K^\rho\rangle\ . \tag{2.2}$$

In obtaining Eq. (2.1) we have assumed the validity of the once-integrated equal-time SU(2)$\times$SU(2) commutation relations given by Gell-Mann[11]:

$$[F^\alpha(t),V^\beta_\mu(y)]_{t=y_0} = D^{(1)}_{\gamma\beta}(\alpha)V^\gamma_\mu(y)\,, \tag{2.3a}$$

$$[F^\alpha(t),A^\beta_\mu(y)]_{t=y_0} = D^{(1)}_{\gamma\beta}(\alpha)A^\gamma_\mu(y)\,, \tag{2.3b}$$

$$[F^{5\alpha}(t),V^\beta_\mu(y)]_{t=y_0} = D^{(1)}_{\gamma\beta}(\alpha)A^\gamma_\mu(y)\,, \tag{2.3c}$$

$$[F^{5\alpha}(t),A^\beta_\mu(y)]_{t=y_0} = D^{(1)}_{\gamma\beta}(\alpha)V^\gamma_\mu(y)\,, \tag{2.3d}$$

where $D^{(I)}_{\gamma\beta}(\alpha)=[I(I+1)]^{1/2}(I1\beta\alpha|I\gamma)$ and the F's (F^5's) are the space integrals of the V_0's (A_0's). By dispersing the $H^{\alpha\lambda}_\mu$ defined in Eq. (2.2) in the variables $\nu=-q\cdot q_x/m_x$, $q=q_2+q_3$, we find that it can be written

$$H^{\alpha\lambda}_\mu = \sqrt{3}\begin{pmatrix}8 & 8 & 8_a\\ \lambda & K^\rho & K^n\end{pmatrix}\langle\pi^\alpha|H_w(0)|K^n\rangle\frac{(2q_K-q)_\mu}{m_\pi{}^2-m_K{}^2}\,, \tag{2.4}$$

where we have kept only the π and K pole terms and

$$\begin{pmatrix}8 & 8 & 8_a\\ \lambda & K^\rho & K^n\end{pmatrix}$$

is an SU(3) Clebsch-Gordan coefficient.[12] Substituting this result into Eq. (2.1) and retaining only the lowest-order terms in the pion momenta, we find

$$F_\pi{}^3\langle\pi^\alpha(q_1)\pi^\beta(q_2)\pi^\gamma(q_3)|H_w(0)|K^\rho\rangle = -\tfrac{1}{6}\{\langle 0|[F^{5\alpha}(0),[F^{5\beta}(0),[F^{5\gamma}(0),H_w]]]|K^\rho\rangle + \text{permutations of } (\alpha,\beta,\gamma)\}$$
$$+\frac{F_\pi\sqrt{3}}{m_\pi{}^2-m_K{}^2}\bigg[D^{(1)}_{\lambda\gamma}(\alpha)\begin{pmatrix}8 & 8 & 8_a\\ \lambda & K^\rho & K^n\end{pmatrix}\langle\pi^\beta|H_w(0)|K^n\rangle q_K\cdot(q_3-q_1)$$
$$+D^{(1)}_{\lambda\beta}(\alpha)\begin{pmatrix}8 & 8 & 8_a\\ \lambda & K^\rho & K^n\end{pmatrix}\langle\pi^\gamma|H_w(0)|K^n\rangle q_K\cdot(q_2-q_1)$$
$$+D^{(1)}_{\lambda\gamma}(\beta)\begin{pmatrix}8 & 8 & 8_a\\ \lambda & K^\rho & K^n\end{pmatrix}\langle\pi^\alpha|H_w(0)|K^n\rangle q_K\cdot(q_3-q_2)\bigg]. \tag{2.5}$$

This expression has been utilized to obtain the slopes for the CP-conserving τ decays with remarkably good agreement.[8] If we take the CP-conserving results to be an indication of the validity of Eq. (2.5), then essentially any deviations from the experimental results will come from the form of the Hamiltonian chosen.

We are only concerned with the $\tau^\pm$ decays at present and so we will focus our attention on the decays $K^\pm\to\pi^\pm\pi^\pm\pi^\mp$, i.e., we will put $\alpha=\beta$ in Eq. (2.5). Carrying this out we have

$$F_\pi^3\langle\pi^\alpha(q_1)\pi^\alpha(q_2)\pi^\gamma(q_3)|H_w(0)|K^\rho\rangle = -\tfrac{1}{3}(C^{\alpha\alpha\gamma}+C^{\alpha\gamma\alpha}+C^{\gamma\alpha\alpha})$$
$$+\frac{F_\pi\sqrt{3}\,D^{(1)}_{\lambda\gamma}(\alpha)}{m_\pi{}^2-m_K{}^2}\begin{pmatrix}8 & 8 & 8_a\\ \lambda & K^\rho & K^n\end{pmatrix}\langle\pi^\alpha|H_w(0)|K^n\rangle q_K\cdot(2q_3-q_1-q_2), \tag{2.6}$$

where we have defined the commutator terms as

$$C^{\alpha\beta\gamma}\equiv\langle 0|[F^{5\alpha}(0),[F^{5\beta}(0),[F^{5\gamma}(0),H_w(0)]]]|K^\rho\rangle. \tag{2.7}$$

Equation (2.6) will be utilized to compute the slope asymmetry in $\tau^\pm$ decay when the Hamiltonian contains both CP-even and CP-odd components.

III. STRUCTURE OF MATRIX ELEMENT

Experimentally it is found that the rate for the decay $K^\rho\to\pi^\alpha\pi^\alpha\pi^\gamma$ depends linearly on the kinetic energy of π^γ, the "odd" pion. This is the so-called linear approximation which was first suggested by Weinberg.[13] Thus, we write

$$|M(K^\pm\to\pi^\pm\pi^\pm\pi^\mp)|^2\equiv|M^\pm|^2=C^\pm(1+a^\pm Y), \tag{3.1}$$

where $Y\equiv(3T_3-Q)/Q$, $Q=T_1+T_2+T_3$, and $C^\pm$ and $a^\pm$ are constants. The a's are the slope parameters and it is the asymmetry in the a^+ and a^- that we wish to compute. Various attempts to explain this simple form have been made, with one of the most successful being that of Abarbanel[8] discussed in Sec. II. For example, Eq. (2.5) predicts that $a^+=0.26$ while experimentally it is found that $a^+=0.247\pm0.005$.[14] This very satisfying agreement lends some support to the validity of the calculational techniques we are using.

We now note that the matrix element written in Eq. (2.6) is of the form

$$\langle\pi^\alpha(q_1)\pi^\alpha(q_2)\pi^\gamma(q_3)|H_w(0)|K^\rho\rangle=E^\alpha+\sigma^\alpha(s-s_3), \tag{3.2}$$

where α is + (−) for the τ^+ (τ^-) mode, $s_i\equiv(q_K-q_i)^2$, and $3s=s_1+s_2+s_3$. Squaring Eq. (3.2) and keeping terms linear in s and lower, we obtain the correspondence with Eq. (3.1) with the identifications

$$a^\alpha=-\tfrac{4}{3}m_KQ\frac{\sigma^\alpha}{E^\alpha}. \tag{3.3}$$

The parameters σ^α and E^α are given by Eq. (2.6) as

$$E^\alpha=-\frac{1}{3F_\pi{}^3}(C^{\alpha\alpha\gamma}+C^{\alpha\gamma\alpha}+C^{\gamma\alpha\alpha}) \tag{3.4a}$$

and

$$\sigma^\alpha=\frac{3\sqrt{3}\,D^{(1)}_{\lambda\gamma}(\alpha)}{2F_\pi{}^2(m_\pi{}^2-m_K{}^2)}\begin{pmatrix}8 & 8 & 8_a\\ \lambda & K^\rho & K^n\end{pmatrix}\langle\pi^\alpha|H_w(0)|K^n\rangle. \tag{3.4b}$$

If any CP violation occurs in the $\tau^\pm$ decays, one of the ways it can appear is in a difference between the slopes a^+ and a^-. Defining the parameter Δ by

$$\Delta\equiv\frac{a^+-a^-}{a^++a^-}, \tag{3.5}$$

it is a measure of the amount of CP violation present in the charged-K decays. Since we are dealing only with charged K mesons, a measure of Δ is also a direct measure of ϵ', the parameter characterizing the primary CP-violating weak interaction.[15] This is the purpose of the present work, to establish this connection and make a prediction for the magnitude of Δ. Combining Eqs. (3.3), (3.4), and (3.5) we may write

$$\Delta = \frac{1 - R_E/R_\sigma}{1 + R_E/R_\sigma}, \tag{3.6}$$

where $R_x \equiv x^+/x^-$. Equations (3.4) will thus allow us to compute the value of Δ, once we have specified the commutation relations implied in these equations.

IV. HAMILTONIANS

We will divide the weak Hamiltonian into its CP-even and CP-odd parts as

$$H_w = H_w^{(+)} + H_w^{(-)}, \tag{4.1}$$

where $(CP)H_w^{(\pm)}(CP)^{-1} = \pm H_w^{(\pm)}$ and $(CPT)H_w^{(\pm)}(CPT)^{-1} = H_w^{(\pm)}$. We know that deviations from the $\Delta I = \frac{1}{2}$ rule are small for $K^+ \to 3\pi$ decays, so we will take Suzuki's model for $H_w^{(+)}$ which says that the CP-conserving Hamiltonian transforms like $(\frac{1}{2}, 0)$ under $SU(2) \times SU(2)$.[16,17] This makes the commutation relation of $H_w^{(+)}$ with the vector and axial-vector charges the same,

$$[F^\alpha(0), H_\rho^{(+)}(\tfrac{1}{2})] = [F^{5\alpha}(0), H_\rho^{(-)}(\tfrac{1}{2})] = D_{\rho'\rho}^{(1/2)}(\alpha) H_{\rho'}^{(+)}(\tfrac{1}{2}), \tag{4.2}$$

where ρ is the third component of isospin.

The most that we can say about the CP-violating Hamiltonian $H_w^{(-)}$ is that it may have both $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ components,

$$H_w^{(-)} = H_\rho^{(-)}(\tfrac{1}{2}) + H_\rho^{(-)}(\tfrac{3}{2}). \tag{4.3}$$

Since the vector charges generate the algebra of $SU(2)$, we have

$$[F^\alpha(0), H_\rho^{(-)}(I)] = D_{\rho'\rho}^{(I)}(\alpha) H_{\rho'}^{(-)}(I). \tag{4.4}$$

The commutator of $H_w^{(-)}$ with the axial-vector charge will be assumed to mix the isospin components of $H_w^{(-)}$. This assumption seems to require that $H_w^{(-)}$ transform at least reducibly under $SU(2) \times SU(2)$. So we take

$$[F^{5\alpha}(0), H_\rho^{(-)}(I)] = \sum_{I', \rho'} a_{II'} (I 1 \rho \alpha \,|\, I' \rho') H_{\rho'}^{(-)}(I'), \tag{4.5}$$

where the constants $a_{II'}$ are to be determined by a particular choice for $H_w^{(-)}$. These commutation relations allow us to compute the slope asymmetry, Δ.

V. COMPUTATION OF Δ

In order to compute the quantity Δ defined by Eq. (3.6), we must compute four quantities σ^+, σ^-, E^+, and E^-, where these are defined in Eqs. (3.4a) and (3.4b). We will first turn our attention to the computation of E^α.

The quantity E^α is made up of three commutator terms, each of which can be written

$$C^{\alpha\beta\gamma} = C_{1/2}^{\alpha\beta\gamma,+} + C_{1/2}^{\alpha\beta\gamma,-} + C_{3/2}^{\alpha\beta\gamma,-} \tag{5.1}$$

by virtue of the properties of the weak Hamiltonian. We have defined

$$C_I^{\alpha\beta\gamma,\pm} \equiv \langle 0 | [F^{5\alpha}(0), [F^{5\beta}(0), [F^{5\gamma}(0), H_\sigma^{(\pm)}(I)]]] | K^\rho \rangle. \tag{5.2}$$

The commutation relations (4.2) and (4.5) completely specify the form of the C's and we have

$$C_{1/2}^{\alpha\beta\gamma,+} = (\tfrac{1}{2}\sqrt{3})^3 (\tfrac{1}{2} 1 \sigma\gamma \,|\, \tfrac{1}{2}\sigma')(\tfrac{1}{2} 1 \sigma'\beta \,|\, \tfrac{1}{2}\sigma'')(\tfrac{1}{2} 1 \sigma''\alpha \,|\, \tfrac{1}{2}\sigma''')(\tfrac{1}{2}\tfrac{1}{2}\rho\sigma''' \,|\, 00) g^{(+)} \tag{5.3a}$$

and

$$C_I^{\alpha\beta\gamma,-} = \sum_{I', I''} a_{II'} a_{I'I''} a_{I''1/2} (I 1 \sigma\gamma \,|\, I'\sigma')(I' 1 \sigma'\beta \,|\, I''\sigma'')(I'' 1 \sigma''\alpha \,|\, \tfrac{1}{2}\sigma''')(\tfrac{1}{2}\tfrac{1}{2}\rho\sigma''' \,|\, 00) g^{(-)}, \tag{5.3b}$$

where we have defined $g^{(\pm)} \equiv \langle 0 \| H_w^{(\pm)} \| K \rangle$. These relations can be used to compute the ratio of the E's with the result

$$R_E = \frac{E^+}{E^-} = \frac{1 + B^+\gamma}{1 + B^-\gamma}, \tag{5.4}$$

where we have $\gamma \equiv g^{(-)}/g^{(+)}$, which is of order ϵ', and

$$B^\pm \equiv \frac{8}{3\sqrt{6}} \left[a_{11}(-\sqrt{2}\, a_{11}a_{11} + 2a_{13}a_{31}) \pm a_{13}\left(2a_{31}a_{11} + \frac{2}{\sqrt{5}} a_{33}a_{31}\right) + a_{31}\left(\mp \frac{1}{\sqrt{2}} a_{11}a_{11} + a_{13}a_{31}\right) + a_{33}\left(\mp \frac{1}{\sqrt{10}} a_{31}a_{11} - \frac{1}{5\sqrt{2}} a_{33}a_{31}\right)\right]. \tag{5.5}$$

We have written the coefficients encountered in using the commutation relation (4.5) as $a_{2I,2I'}$ and these will be determined subsequently.

Now we turn to the computation of σ^α defined in Eq. (3.4b). Again, by virtue of the Hamiltonian structure, we can write

$$\sigma^\alpha \sim \chi_{1/2}^{\alpha,+} + \chi_{1/2}^{\alpha,-} + \chi_{3/2}^{\alpha,-}, \tag{5.6}$$

where we have defined $\chi_I^{\alpha,\pm} = \langle \pi^\alpha | H_\sigma^{(\pm)}(I) | K^n \rangle$. We can use soft-pion techniques and write

$$\chi_I^{\alpha,\pm} = -\frac{1}{F_\pi}\langle 0 | [F^{5\alpha}(0), H_\sigma^{(\pm)}(I)] | K^n \rangle, \tag{5.7}$$

where the commutator was determined previously. Utilizing the commutation relations and Eq. (3.4b), we find for the ratio of the σ's

$$R_\sigma = \frac{\sigma^+}{\sigma^-} = \frac{1 + A^+\gamma}{1 + A^-\gamma}, \tag{5.8}$$

where we have defined

$$A^\pm \equiv \frac{1}{\sqrt{3}}(\pm 2a_{11} + a_{31}). \tag{5.9}$$

Combining Eqs. (5.4) and (5.8) with Eq. (3.6), we find for Δ

$$\Delta = \tfrac{1}{2}(A^+ - A^- + B^- - B^+)\gamma, \tag{5.10}$$

where we have only retained terms to lowest order in γ (order ϵ'). Substituting the value for $A^\pm$ and $B^\pm$, we obtain

$$\Delta = \frac{8}{3\sqrt{6}}\left(\frac{3\sqrt{2}}{4}a_{11} - 4a_{11}a_{31}a_{13} - \frac{2}{\sqrt{5}}a_{13}a_{33}a_{31} + \frac{1}{\sqrt{2}}a_{31}a_{11}{}^2 + \frac{1}{\sqrt{10}}a_{33}a_{31}a_{11}\right)\gamma. \tag{5.11}$$

Now we notice that the basic structure of this expression is that Δ is a product of (model-dependent) numbers of order unity and a number of order ϵ'. It is this feature that allows a measurement of Δ to place restrictions directly on the CP parameter ϵ'.

In order to relate Δ directly to ϵ' we will make use of techniques developed in a previous paper, to which the reader is referred for details.[9] The parameter ϵ' defined by

$$\epsilon' = \frac{1}{\sqrt{2}}\frac{\operatorname{Im} A_2}{A_0}e^{i(\pi/2+\delta_2-\delta_0)} \tag{5.12}$$

can be related to $\gamma = g^{(-)}/g^{(+)}$ by using the commutation relations and the soft-pion techniques sketched in Sec. II. Neglecting the phases (since we only want an order-of-magnitude estimate), we have

$$|\epsilon'| = \frac{2}{\sqrt{3}}a_{31}\left(a_{11} + \frac{1}{\sqrt{5}}a_{33}\right)|\gamma|. \tag{5.13}$$

Combining this with Eq. (5.11), we obtain the result

$$|\Delta'| = \frac{\sqrt{2}}{6}\left[a_{31}\left(a_{11} + \frac{1}{\sqrt{5}}a_{33}\right)\right]^{-1} \times \left(\frac{3\sqrt{2}}{4}a_{11} - 4a_{11}a_{31}a_{13} - \frac{2}{\sqrt{5}}a_{13}a_{33}a_{31} + \frac{1}{\sqrt{2}}a_{31}a_{11}{}^2 + \frac{1}{\sqrt{10}}a_{33}a_{31}a_{11}\right)|\epsilon'|. \tag{5.14}$$

In order to obtain a numerical value for Δ, we must specify the values of the coefficients and make an estimate for the magnitude of ϵ'.

VI. DISCUSSION

In order to evaluate Eq. (5.14) for Δ we must take a model for the CP-violating Hamiltonian so we can obtain values for the a's and we must get an estimate for ϵ'. If we take a model such as Glashow's for the Hamiltonian, then the coefficients are determined.[4] This was done in a previous paper, where the coefficients are listed.[9] If we take the piece of the Hamiltonian which transforms as $(\frac{1}{2}, 1)$ under $SU(2)\times SU(2)$ in order to get an estimate for the a's, then we have

$$a_{11} = \frac{5\sqrt{3}}{6}, \quad a_{13} = \frac{2\sqrt{3}}{3}, \quad a_{31} = -\frac{2\sqrt{6}}{3}, \quad \text{and } a_{33} = \frac{\sqrt{15}}{6}.$$

Substituting these values into Eq. (5.14), we find

$$|\Delta| \cong 4|\epsilon'|, \tag{6.1}$$

which is the result alluded to in the Introduction, i.e., that a measurement of Δ provides a direct limit on the size of ϵ'.

In order to obtain a value for ϵ' for substitution into Eq. (6.1) we must turn to the experimental information. The $K^0 \to 2\pi$ experiments do not give a clear choice for the magnitude of ϵ' other than to say that it appears to be much smaller than ϵ. We shall take the present upper limit to be $|\epsilon'| < \frac{1}{5}|\epsilon|$.[18] This gives the upper limit of Δ of 10^{-3}, which is consistent with the present experimental value of $|\Delta|_{\text{exp}} = (7.0 \pm 5.3)\times 10^{-3}$.[14]

Since none of the experimental data are as yet sufficiently sensitive to detect $\epsilon' \sim 10^{-4}$, we conclude that at present all experimental data are consistent either with a small direct CP violation in $\Delta I = \frac{3}{2}$ or with the superweak model. Experiments now in progress on $K_L \to 2\pi$ will hopefully clarify the situation by providing a better upper limit on $|\epsilon'|$.

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Tests for the Internal Quantum Numbers of Partons

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The restrictions for the structure functions of deep-inelastic electroproduction and neutrino-induced production following from assigning partons to any representation of $SU(2) \times SU(2) \times Y$ are derived. Apart from the general parton-model assumptions only isospin invariance and positivity are used. Comparison with experiment allows us to exclude all models which have only partons of spin $\frac{1}{2}$, isospin $\leq \frac{1}{2}$, and integral charge.

I. INTRODUCTION

One of the popular models for deep-inelastic electron- and neutrino-nucleon scattering is Feynman's parton model.[1-4] In this model one views the nucleon as built of constituents, partons, which scatter incoherently. It is then natural to ask for the quantum numbers of partons. On a more fundamental level one may hope to identify partons with the bare quanta of some underlying field theory and ask for the quantum numbers of the basic fields which build up the electromagnetic and weak currents.

The relation of Callan and Gross[5] gives a direct test for the spin of partons.

If partons have only spin $\frac{1}{2}$, the longitudinal structure functions vanish; for spin 0, the transverse ones vanish. In the present article we will derive the restrictions for the structure functions which follow from internal-quantum-number assignments to partons. The basic fields are assumed to carry a representation of $SU(2) \times SU(2) \times Y$, where Y is the hypercharge. We exploit only isospin invariance and positivity. No assumptions on the momentum distribution of partons etc. are made. The generalization to $SU(3) \times SU(3)$ would be straightforward but tedious.

All notations and definitions are taken from Ref. 6. Bjorken's scaling functions[7] will be defined by

$$\begin{aligned}
F_1(x) &= \lim 2MW_1(\nu, Q^2), \\
F_2(x) &= \lim \nu W_2(\nu, Q^2), \\
F_3(x) &= \lim \nu W_3(\nu, Q^2), \\
F_L(x) &= \frac{1}{x}F_2(x) - F_1(x),
\end{aligned} \tag{1.1}$$

where $x = Q^2/2M\nu$ and the limit is taken for $\nu \to \infty$, $Q^2 \to \infty$, x fixed. The Cabibbo angle of the weak