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Study of the Decays $K \to \mu\nu e^+e^-$ and $K \to e\nu\mu^+\mu^-$

S. Krishna and H. S. Mani
Department of Physics, Indian Institute of Technology, Kanpur, India
(Received 25 June 1971)

The decays $K \to \mu\nu e^+e^-$ and $K \to e\nu\mu^+\mu^-$ have been discussed in a hard-meson model. The structure-dependent effects can be seen in the $K \to \mu\nu e^+e^-$ mode only at large values of the mass of the virtual photon. However, in $K \to e\nu\mu^+\mu^-$ decay the structure-dependent terms predominate, even though the branching ratio is small.

We have studied the decays $K \to \mu\nu e^+e^-$ and $K \to e\nu\mu^+\mu^-$ in a "hard-meson" model – an extension of the Schnitzer and Weinberg hard-pion approach.[1] Study of the decay $K \to \mu\nu e^+e^-$ was suggested some time ago by Chu, Ebata, and Scott[2] as a possible test for violation of time-reversal invariance. The decays may also be of interest in studying the structure of the strangeness-changing weak vertex.

The kinematics is defined by $K^+(k) \to \nu(q_1) + \mu^+(q_2) + e^-(p_1) + e^+(p_2)$, with $r = p_1 + p_2$ and $q = q_1 + q_2$. The amplitude for $K^+ \to \mu^+\nu e^+e^-$ may be written

$$T = \frac{e^2 G \sin\theta}{\sqrt{2}} \bar{u}(p_1)\gamma^\lambda v(p_2) \bar{u}(q_1)\gamma^\sigma(1-\gamma_5)\left(\sqrt{2}\, F_K \frac{\gamma\cdot(k-q_1) - m_\mu}{(k-q_1)^2 - m_\mu^2}\gamma_\lambda k_\sigma - M_{\lambda\sigma}\right)\frac{1}{r^2 + i\epsilon} v(q_2), \tag{1}$$

where

$$M_{\lambda\sigma} = \int d^4x\, e^{ir\cdot x}\langle 0|T\{J_\lambda^{\text{em}}(x)[A_\sigma^{4-i5}(0) - V_\sigma^{4-i5}(0)]\}|K^+(k)\rangle$$
$$= A\epsilon_{\lambda\sigma\alpha\beta} r^\alpha k^\beta + B\left(\frac{r_\lambda r_\sigma}{r^2} - g_{\lambda\sigma}\right) + C\left(\frac{k\cdot r k_\sigma r_\lambda}{r^2} - k_\lambda k_\sigma\right) + D\left(\frac{k_\lambda r_\sigma}{k\cdot r} - g_{\lambda\sigma}\right) + E\left(\frac{k_\lambda q_\sigma}{k\cdot r} + g_{\lambda\sigma}\right). \tag{2}$$

A, B, C, D, and E are, in general, functions of r^2 and $k\cdot r$.

The contribution of the axial-vector current in (2) is calculated by the Schnitzer-Weinberg method. The relevant matrix element is expressed in terms of the Schnitzer-Weinberg proper vertices as

$$\int d^4x\, d^4y\, e^{ir\cdot x - ik\cdot y}\langle 0|T\{J_\lambda^a(x)A_\sigma^b(0)\partial^\mu A_\mu^c(y)\}|0\rangle$$
$$= f_{abc}\frac{F_K m_K^2 g_{V_a}^{-1} g_{K_A}^{-1}}{k^2 - m_K^2}\Delta_{\sigma\nu}^{K_A}(q)\Delta_{\lambda\eta}^{V_a}(r)\Gamma^{\eta\nu}(k,q) + f_{abc}\frac{F_K^2 m_K^2 g_{V_a}^{-1}}{(k^2 - m_K^2)(q^2 - m_K^2)} q_\sigma \Delta_{\lambda\eta}^{V_a}(r)\Gamma^\eta(k,q),$$

where V_a stands for ρ or ω. Now using $SU(3)\otimes SU(3)$ current commutation relations and conservation of electromagnetic current, the vertices may be evaluated following Schnitzer and Weinberg. The result obtained is

$$\Gamma_{\lambda\sigma}(k,q)=g_{V_a}{}^{-1}g_{K_A}{}^{-1}M_{K_A}{}^2 M_{V_a}{}^2 F_K\left[g_{\lambda\sigma}+M_{K_A}{}^{-2}(q_\lambda q_\sigma - q^2 g_{\lambda\sigma})+\frac{F_K{}^{-2}g_{K_A}{}^2}{M_{K_A}{}^2}(M_{K_A}{}^{-2}-M_{V_a}{}^{-2})(r_\lambda r_\sigma - r^2 g_{\lambda\sigma}) + \frac{\delta g_{K_A}{}^2}{F_K{}^2 M_{K_A}{}^4}(k_\lambda r_\sigma - k\cdot r g_{\lambda\sigma})\right],$$

$$\Gamma_\lambda(k,q)=g_{V_a}{}^{-1}\left\{-M_{V_a}{}^2(k+q)_\lambda+\tfrac{1}{2}\left[1-\frac{F_K{}^{-2}g_{K_A}{}^2}{M_{K_A}{}^4}(M_{K_A}{}^2-M_{V_a}{}^2)\right][r^2(k+q)_\lambda - r_\lambda r\cdot(k+q)] + \frac{\delta g_{K_A}{}^2 M_{V_a}{}^2}{F_K{}^2 M_{K_A}{}^4}(k\cdot r q_\lambda - k_\lambda r\cdot q)\right\}.$$

Substituting these in $M_{\lambda\sigma}$ the values of B, C, D, and E are obtained. These are

$$B=\sum_{V=\rho,\omega}\frac{F_K r^2}{\sqrt{2}\,(M_V{}^2-r^2)}\left\{\frac{k\cdot r}{q^2-M_K{}^2}\left[1-\frac{g_{K_A}{}^2}{F_K{}^2 M_{K_A}{}^4}(M_{K_A}{}^2-M_V{}^2)+\frac{\delta g_{K_A}{}^2 M_V{}^2}{F_K{}^2 M_{K_A}{}^4}-(k\cdot r)^{-1}(k^2-q^2+M_V{}^2)\right] + \frac{g_{K_A}{}^2 M_V{}^2 F_K{}^{-2}}{q^2-M_{K_A}{}^2}(M_V{}^{-2}-M_{K_A}{}^{-2})\left(1+\frac{k\cdot r}{M_{K_A}{}^2}\right)-1\right\},$$

$$C=\sum_{V=\rho,\omega}\frac{F_K r^2}{\sqrt{2}\,(M_V{}^2-r^2)}\left\{\frac{1}{q^2-M_K{}^2}\left[-1+\frac{g_{K_A}{}^2}{F_K{}^2 M_{K_A}{}^4}(M_{K_A}{}^2-M_V{}^2)-\frac{\delta g_{K_A}{}^2 M_V{}^2}{F_K{}^2 M_{K_A}{}^4}+(k\cdot r)^{-1}(k^2-q^2+M_V{}^2)\right] - \frac{g_{K_A}{}^2 M_V{}^2 M_{K_A}{}^{-2} F_K{}^{-2}}{q^2-M_{K_A}{}^2}(M_V{}^{-2}-M_{K_A}{}^{-2})\right\},$$

$$D=\sum_{V=\rho,\omega}\frac{F_K k\cdot r r^2}{\sqrt{2}\,(r^2-M_V{}^2)}\left\{\frac{1}{q^2-M_K{}^2}\left[1-\frac{g_{K_A}{}^2}{F_K{}^2 M_{K_A}{}^4}(M_{K_A}{}^2-M_V{}^2)+\frac{\delta g_{K_A}{}^2 M_V{}^2}{F_K{}^2 M_{K_A}{}^4}-(k\cdot r)^{-1}(k^2-q^2+M_V{}^2)\right] + \frac{M_V{}^2 g_{K_A}{}^2}{M_{K_A}{}^2 F_K{}^2(q^2-M_{K_A}{}^2)}(M_V{}^{-2}-M_{K_A}{}^{-2}+\delta r^{-2})\right\},$$

$$E=\sqrt{2}\,F_K.$$

The vector-current piece in the left-hand side of (2) contributes only to A, the coefficient of the totally antisymmetric term. The value of A at $r^2=0$ has been obtained by Sarkar and others[3] in discussing the decay $K\to\mu\nu\gamma$. $A(r^2)$ is obtained from their value by assuming vector dominance of the electromagnetic current. We therefore obtain

$$|A|=\frac{\sqrt{2}\,g_{K^*}G_{K^{*+}K^+\gamma}}{|q^2-M_{K^*}{}^2|}\,\frac{M_\rho{}^2}{M_\rho{}^2-r^2}.$$

We take g_{K^*} from the Das-Mathur-Okubo sum rule[4] $g_{K^*}{}^2/M_{K^*}{}^2=g_\rho{}^2/M_\rho{}^2=2F_\pi{}^2$ and $G_{K^{*+}K^+\gamma}$ from SU(3) symmetry, the quark model, and the experimental $\omega\to\pi^0\gamma$ rate.[5] Now $M_{\lambda\sigma}$ with the above values of A, B, C, D, and E is substituted[6] in (1) and the matrix element is squared. The phase-space integrations involved are

$$\Gamma(K\to\mu\nu e^+e^-)=\frac{1}{(2\pi)^8}\int\frac{m_e{}^2 m_\mu}{2m_K p_{10}p_{20}q_{20}}\,d^3q_1 d^3q_2\, d^3p_1 d^3p_2\sum_{\text{spins}}|T|^2\delta^4(k-p_1-p_2-q_1-q_2)$$

$$=\frac{1}{(2\pi)^8}\frac{m_e{}^2 m_\mu}{2m_K}\int d\mu^2\delta(r^2-\mu^2)\theta(r_0)d^4r\int\frac{d^3q_1 d^3q_2}{q_{20}}\delta^4(k-r-q_1-q_2)\int\frac{d^3p_1 d^3p_2}{p_{10}p_{20}}\sum_{\text{spins}}|T|^2\delta(r-p_1-p_2). \qquad (3)$$

We show that the integrations over p_1 and p_2 may be easily performed using invariance arguments. T from (1) may be written $T=l_\lambda T^\lambda$, where $l_\lambda=\bar{u}(p_1)\gamma_\lambda v(p_2)$; then

$$\sum_{\text{spins}}|T|^2=\sum_{\text{spins}} l_\lambda l_{\lambda'}^\dagger T^\lambda (T^{\lambda'})^\dagger = L_{\lambda\lambda'}\sum_{\text{spins}} T^\lambda(T^{\lambda'})^\dagger, \qquad (4)$$

where

$$L_{\lambda\lambda'}=\sum_{\text{spins}} l_\lambda l_{\lambda'}^\dagger=\frac{1}{m_e^2}[p_{1\lambda}p_{2\lambda'}+p_{2\lambda}p_{1\lambda'}-(p_1\cdot p_2+m_e^2)g_{\lambda\lambda'}].$$

Now all the dependence on p_1 and p_2 in $\sum_{\text{spins}}|T|^2$ is contained in $L_{\lambda\lambda'}$. So the integration over p_1 and p_2 in (3) is over $L_{\lambda\lambda'}$ only. This may be written

$$\int\frac{d^3p_1}{p_{10}}\frac{d^3p_2}{p_{20}}L_{\lambda\lambda'}\delta(r-p_1-p_2)=Ig_{\lambda\lambda'}+Jr_\lambda r_{\lambda'};\tag{5}$$

from this, I and J may be easily evaluated. The value of I is

$$I=-\frac{2\pi}{3m_e^2}\mu^2\left(1+\frac{2m_e^2}{\mu^2}\right)\left(1-\frac{4m_e^2}{\mu^2}\right)^{1/2}.\tag{6}$$

Conservation of electromagnetic current implies $r^\lambda T_\lambda=0$. So only I need be evaluated. Substituting (4), (5), and (6) in (3)

$$\Gamma(K\to\mu\nu e^+e^-)=\frac{1}{(2\pi)^8}\frac{m_e^2m_\mu}{2m_K}\left[-\frac{2\pi}{3m_e^2}\mu^2\left(1+\frac{2m_e^2}{\mu^2}\right)\left(1-\frac{4m_e^2}{\mu^2}\right)^{1/2}\right]$$
$$\times\int d\mu^2\delta(r^2-\mu^2)\theta(r_0)d^4r\int\frac{d^3q_1d^3q_2}{q_{20}}\delta(k-r-q_1-q_2)\sum_{\text{spins}}T_\lambda(T^\lambda)^\dagger.$$

Of the remaining integrations, five may be performed trivially using the δ functions. Three angular integrations are performed easily after choosing suitable axes. Of the three nontrivial integrations left, one was performed analytically and the two final ones were evaluated numerically in a computer. For the process $K\to\mu\nu e^+e^-$ with the sum of electron and positron energies greater than 20 MeV, we obtain a branching ratio of

$$\frac{\Gamma(K\to\mu\nu e^+e^-)}{\Gamma(K\to\mu\nu)}=0.159\times10^{-5}$$

of which the contribution of the structure-dependent terms (those involving A, B, C, D, and their squares) is 0.29×10^{-6} or 18%. The variation of the differential decay rate as a function of μ^2 is shown in Fig. 1. We notice that the structure-dependent contribution becomes prominent at higher values of μ^2. But the over-all contribution to the branching ratio is small because of the decreasing phase space.[7]

The decay $K\to e\nu\mu^+\mu^-$ is of more interest in this context. Here the over-all branching ratio is smaller than the $K\to\mu\nu e^+e^-$ case. But the number is almost completely due to the structure-dependent terms, the contribution of the terms involving F_K only being negligible.

$$\frac{\Gamma(K\to e\nu\mu^+\mu^-)}{\Gamma(K\to\mu\nu)}=0.245\times10^{-7}.$$

With only bremsstrahlung terms the branching ratio $\simeq10^{-13}$. The large structure dependence is due to two reasons. First, the mass of the single lepton can be factored out of the matrix element involving only F_K terms. In the case of the $K\to e\nu\mu^+\mu^-$ decays these are therefore suppresse[c] Second, the minimum permissible value of μ^2 is $(2m_\mu)^2$ as against $(2m_e)^2$ in the first decay. At this value of μ^2, the structure-dependent terms are al ready much larger than the terms involving F_K only.

We have evaluated the branching ratio for the de

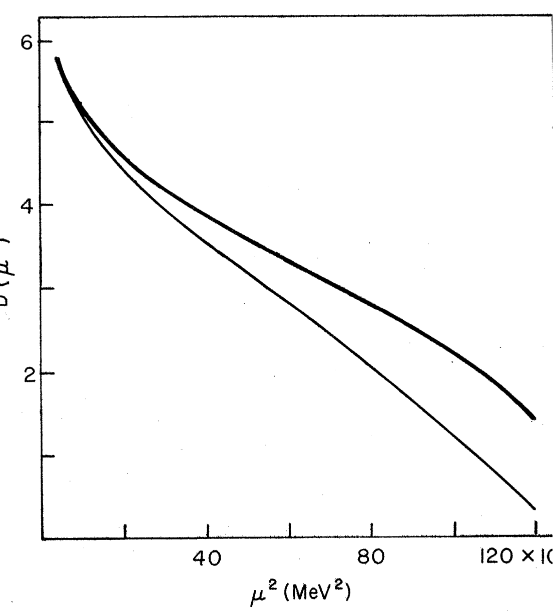

FIG. 1. Logarithm of the differential decay rate $D(\mu^2)=\log_{10}(d\Gamma/d\mu^2)$ (in an arbitrary scale) versus (mass)^2 of the virtual photon for the decay $K^+\to\mu^+\nu e^+e^-$. The lower curve is with F_K terms only. The upper curve is with the complete amplitude (F_K terms + structure term

cay $\pi\to\mu\nu e^+e^-$ also.

$$\frac{\Gamma(\pi\to\mu\nu e^+e^-)}{\Gamma(\pi\to\mu\nu)}=0.633\times10^{-7}.$$

The contribution of the structure terms in this case is less than 1%.

We would like to thank Dr. J. S. Vaishya for useful discussions.

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CP Violation in τ Decays

Edward R. McCliment
Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa 52240

and

William D. Teeters*
Department of Physical Sciences, Chicago State University, Chicago, Illinois 60621

(Received 8 November 1971)

By making use of postulated commutation relations of a CP-violating Hamiltonian with the vector and axial-vector charges of chiral SU(2) × SU(2) symmetry, we obtain a theoretical prediction of asymmetry in the slope parameters of $\tau^\pm$ decay in terms of the $K_L\to2\pi^0$ parameter ϵ'. We expect $\Delta=(a^+-a^-)/(a^++a^-)\sim4\epsilon'$, where $a^\pm$ are the slope parameters in the linear matrix-element approximation of $\tau^\pm$ decay. With the present experimental limit, $|\epsilon'|\lesssim\frac{1}{5}|\epsilon|$, this gives an upper limit for Δ of 10^{-3}.

I. INTRODUCTION

Although much time has elapsed since CP violation was first observed experimentally,[1] there has been little success in determining the precise form of the interaction. Most of the experimental data on CP violation come from observing the two-pion decay modes of the neutral-K-meson system. By observing the charged and neutral decays of K_L and K_S, the two (complex) parameters η_{+-} and η_{00} are measured and values for ϵ and ϵ' deduced.[2] The CP parameters ϵ and ϵ' characterize quite different physical origins of the CP violation and so it is essential that their value be determined carefully. The parameter ϵ characterizes a mass shift in the neutral-K system which need not come from the weak interaction itself, and the parameter ϵ' is related to a matrix element of a CP-violating weak interaction.

Two approaches to the origin of these parameters have evolved; (1) a new interaction is postulated or (2) the existing weak-interaction current-current form is modified. One of the approaches of the first type is the superweak theory which associates the breakdown of CP invariance with a new $|\Delta S|=2$ interaction which gives rise to a mass shift and also to a nonzero ϵ.[3] The superweak theory predicts that $\epsilon\neq0$ and $\epsilon'\equiv0$ since there is no primary interaction that violates CP invariance. Two theories of the second type have been proposed, one introducing phases between the vector and axial-vector currents in the Cabibbo current of the weak interaction and another model introducing neutral currents as the source