PHYSICAL REVIEW D VOLUME 5, NUMBER 1 1 JANUARY 1972

Radiative Weak Decays of Hyperons*

Y. Ernesto Nagai
Department of Physics, Michigan State University, East Lansing, Michigan 48823
(Received 27 September 1971)

Current-algebra techniques and Low's soft-photon theorem are applied to evaluate the amplitudes of the radiative weak decays of hyperons. The calculated branching ratio and asymmetry parameter for the decay $\Sigma^+ \to p + \gamma$ are in very good agreement with recently measured values.

I. INTRODUCTION

Insufficient experimental information on two-body radiative weak decays of hyperons has prevented the selection of the best model among several proposed since the first papers on the subject appeared more than a decade ago. These models predict decay rates and branching ratios that are in rough agreement with each other and with the available data. However, the first experimental determination[1] of the asymmetry parameter for the decay $\Sigma^+ \to p + \gamma$ gave the unexpected result $a = -1.03^{+0.52}_{-0.42}$, much larger than the theoretical predictions. The soft-pion and soft-photon approach, considered by Ahmed,[2] subsequently led to a value for the asymmetry parameter consistent with the experimental value given above, but at the expense of an internal inconsistency in the calculation as pointed out by Ram Mohan.[3] When the inconsistency is removed, Ahmed's calculation also yields a negligible asymmetry parameter for $\Sigma^+ \to p + \gamma$.

In this paper we consider the soft-pion–soft-photon method and obtain excellent results by properly avoiding inconsistencies and by using the values of the parameters found in our previous analysis of nonleptonic decays of hyperons.[4] The method essentially consists in relating the amplitudes for the two- and three-body radiative weak decays, $\alpha \to \beta + \gamma$ and $\alpha \to \beta + \pi^0 + \gamma$, via the soft-pion theorem. By expanding the three-body radiative decay amplitude in powers of the photon momentum according to Low's soft-photon theorem,[5] it is possible to express the decay amplitude for $\alpha \to \beta + \gamma$ in terms of the amplitude for the nonradiative process $\alpha \to \beta + \pi^0$.

In Sec. II we present the details of the formalism. The parity-conserving (pc) and the parity-violating (pv) amplitudes, as well as the quantities accessible to experimental determination, are given in Sec. III in terms of the parameters found in nonradiative decays of hyperons.[4] An Appendix provides the final explicit expressions for the amplitudes.

II. FORMALISM

Let $R^{\mu}_{\beta\alpha}$ be the transition amplitude for the three-body radiative weak decay

$$\alpha(p) \to \beta(p') + \pi^0(q') + \gamma(k) \tag{1}$$

with normalization defined by

$$\langle \beta(p')\pi^0(q')\gamma(k)|\mathcal{H}_{WE}(0)|\alpha(p)\rangle = \frac{i}{(2\pi)^6}\frac{1}{(2k_0 2q_0')^{1/2}}\frac{1}{N_\alpha N_\beta}\epsilon_\mu R^{\mu}_{\beta\alpha}, \tag{2}$$

where $N_\alpha = (E_\alpha/M_\alpha)^{1/2}$ and E_α is the total energy of the particle α with mass M_α. The soft-pion theorem[6] applied to $R^{\mu}_{\beta\alpha}$ gives

$$\epsilon_\mu R^{N\mu}_{\beta\alpha}(q'=0) = \lim_{q'\to 0}[(m_\pi^2/c)\epsilon_\mu q'_\lambda S^{\mu\lambda}_{\beta\alpha}(q') - \epsilon_\mu R^{B\mu}_{\beta\alpha}(q')] - (m_\pi^2/c)(2\pi)^{9/2}(2k_0)^{1/2}N_\alpha N_\beta \langle \beta(p')\gamma(k)|[F_3^5, \mathcal{H}_{WE}]_-|\alpha(p)\rangle\,, \tag{3}$$

where the surface term $S^{\mu\lambda}_{\beta\alpha}$ is defined by

$$\epsilon_\mu S^{\mu\lambda}_{\beta\alpha}(q') = -i(2\pi)^{9/2}(2k_0)^{1/2}N_\alpha N_\beta \int d^4x\, e^{iq'\cdot x}\langle \beta(p')\gamma(k)|T(\mathfrak{F}_3^{5\lambda}(x)\mathcal{H}_{WE}(0))|\alpha(p)\rangle\,. \tag{4}$$

In Eq. (3) we have separated the amplitude $R^{\mu}_{\beta\alpha}$ into Born ($R^{B\mu}_{\beta\alpha}$) and non-Born ($R^{N\mu}_{\beta\alpha}$) parts. The desired amplitude for the two-body radiative decay, $\alpha \to \beta + \gamma$, is proportional to the equal-time commutator (ETC) term. Since we know how to evaluate the surface Born term, the next step consists in relating $R^{\mu}_{\beta\alpha}$ to the amplitude $M_{\beta\alpha}$ of the nonradiative decay $\alpha \to \beta + \pi^0$:

$$\langle \beta(p')\pi^0(q')|\mathcal{H}_W(0)|\alpha(p)\rangle = \frac{i}{(2\pi)^{9/2}} \frac{1}{(2q_0')^{1/2}} \frac{1}{N_\alpha N_\beta} M_{\beta\alpha}. \quad (5)$$

Now, by keeping terms of the order k^{-1} and k^0 in the expansion in k of the amplitude $R^{\mu}_{\beta\alpha}$, Low's soft-photon theorem relates $R^{\mu}_{\beta\alpha}$ to $M_{\beta\alpha}$. If the Born part of $M_{\beta\alpha}$ is removed from this relationship, we are left with a connection between the non-Born parts $R^{N\mu}_{\beta\alpha}$ and $M^N_{\beta\alpha}$. In the soft-pion limit, $q' \to 0$, Low's soft-photon theorem then reads

$$\epsilon_\mu R^{N\mu}_{\beta\alpha}(q'=0) = e\bar{u}(p')\left[\left(\not\epsilon + \frac{\mu_\beta}{2M_\beta}\not k\not\epsilon\right)\frac{1}{\not p + \not k - M_\beta}\bar{M}^N_{\beta\alpha}(q'=0) + \bar{M}^N_{\beta\alpha}(q'=0)\frac{1}{\not p - \not k - M_\alpha}\left(\not\epsilon + \frac{\mu_\alpha}{2M_\alpha}\not k\not\epsilon\right)\right]u(p), \quad (6)$$

where $\bar{M}^N_{\beta\alpha}$ is the amplitude $M^N_{\beta\alpha}$ without the Dirac spinors.

Now consider the soft-pion formula[4,6] for $M_{\beta\alpha}$:

$$M^N_{\beta\alpha}(q'=0) = \lim_{q'\to 0}\left[(m_\pi{}^2/c)q'_\lambda T^\lambda_{\beta\alpha}(q') - M^B_{\beta\alpha}(q')\right] - (m_\pi{}^2/c)(2\pi)^3 N_\alpha N_\beta \langle\beta(p')|[F^5_3, \mathcal{H}_W]_-|\alpha(p)\rangle, \quad (7)$$

where the surface term $T^\lambda_{\beta\alpha}$ is defined by

$$T^\lambda_{\beta\alpha}(q') = -i(2\pi)^3 N_\alpha N_\beta \times \int d^4x\, e^{iq'\cdot x}\langle\beta(p')|T(\mathcal{F}^{5\lambda}_3(x)\mathcal{H}_W(0))|\alpha(p)\rangle. \quad (8)$$

The ETC term in (7) is just the current-algebra term of the nonradiative decay $\alpha \to \beta + \pi^0$. Note that in our analysis the current-algebra term alone does not represent the entire observed amplitude for the nonradiative decay.

From Eqs. (3), (6), and (7) we see that the on-mass-shell amplitude for $\alpha \to \beta + \gamma$ is given in terms of the off-mass-shell surface Born terms and the current-algebra term for $\alpha \to \beta + \pi^0$. These off-mass-shell terms take different values for derivative and nonderivative couplings of pseudoscalar mesons to baryons. In fact, working in the limit of exact unitary symmetry and assuming nonderivative coupling for the $\bar{B}B\pi$ vertex, Ahmed[2] obtained a relatively large pv amplitude for the decay $\Sigma^+ \to p + \gamma$ coming from these off-mass-shell surface Born terms. On the other hand, assuming unitary symmetry, CP invariance, and the usual octet dominance, Hara has shown[7] that the pv amplitudes for $\Sigma^+ \to p + \gamma$ and $\Xi^- \to \Sigma^- + \gamma$ are zero in the current $\times$ current interaction picture. This apparent contradiction has its origin in the use of the nonderivative coupling for the $\bar{B}B\pi$ vertex as pointed out by Ram Mohan.[3] Therefore, within the framework of current algebra and the current $\times$ current interaction, the derivative coupling should be used consistently throughout the calculation.

In light of the above arguments we use derivative coupling for the strong $\bar{B}B\pi$ vertex, in which case the surface Born terms of (3) and (7) vanish. Then, from (3), (6), and (7) we obtain

$$(2\pi)^{9/2}(2k_0)^{1/2}N_\alpha N_\beta\langle\beta(p')\gamma(k)|\mathcal{H}_{WE}(0)|\alpha(p)\rangle = e\bar{u}(p')\left[\left(\not\epsilon + \frac{\mu_\beta}{2M_\beta}\not k\not\epsilon\right)\frac{1}{\not p' + \not k - M_\beta}(c_{\beta\alpha} - \gamma_5 v_{\beta\alpha}) + (c_{\beta\alpha} - \gamma_5 v_{\beta\alpha})\frac{1}{\not p - \not k - M_\alpha}\left(\not\epsilon + \frac{\mu_\alpha}{2M_\alpha}\not k\not\epsilon\right)\right]u(p), \quad (9)$$

where we have written the weak vertex as

$$\langle\beta(p')|\mathcal{H}_W(0)|\alpha(p)\rangle = \frac{1}{(2\pi)^3}\frac{1}{N_\alpha N_\beta}\bar{u}(p')(c_{\beta\alpha} - \gamma_5 v_{\beta\alpha})u(p). \quad (10)$$

III. pc AND pv AMPLITUDES

The most general form for the two-body radiative weak-decay amplitude[8] can be written in terms of two invariant amplitudes, pc (A') and pv (B'):

$$\langle\beta(p')\gamma(k)|\mathcal{H}_{WE}(0)|\alpha(p)\rangle = \frac{1}{(2\pi)^{9/2}}\frac{1}{(2k_0)^{1/2}}\frac{1}{N_\alpha N_\beta}e\bar{u}(p')(A'_{\beta\alpha} + \gamma_5 B'_{\beta\alpha})\not k\not\epsilon u(p). \quad (11)$$

Inserting expression (11) in the left-hand side of Eq. (9) and performing simple algebraic manipulations, we obtain

$$A'_{\beta\alpha} = \frac{1}{M_\alpha - M_\beta}\left(\frac{\mu_\beta}{2M_\beta} - \frac{\mu_\alpha}{2M_\alpha}\right)c_{\beta\alpha}$$

$$+\frac{1}{M_\alpha - M_\delta}\frac{\mu_T}{2M_T}c_{\delta\alpha} - \frac{1}{M_\gamma - M_\beta}\frac{\mu_T}{2M_T}c_{\beta\gamma},$$

$$B'_{\beta\alpha} = \frac{1}{M_\alpha + M_\beta}\left(\frac{\mu_\beta}{2M_\beta} + \frac{\mu_\alpha}{2M_\alpha}\right)v_{\beta\alpha}$$

$$+\frac{1}{M_\alpha + M_\delta}\frac{\mu_T}{2M_T}v_{\delta\alpha} + \frac{1}{M_\gamma + M_\beta}\frac{\mu_T}{2M_T}v_{\beta\gamma}, \tag{12}$$

where we have included terms arising from the electromagnetic transition $\Sigma^0 \leftrightarrow \Lambda$, with μ_T and $M_T = \frac{1}{2}(M_{\Sigma^0} + M_\Lambda)$ being the transition anomalous magnetic moment and the transition mass, respectively. Of course these transition terms are absent in the decays of charged hyperons.

If we write $c_{\beta\alpha}$ and $v_{\beta\alpha}$ as

$$c_{\beta\alpha} = 2\sqrt{2}\,(fF^6_{\beta\alpha} + dD^6_{\beta\alpha}),$$
$$v_{\beta\alpha} = 2\sqrt{2}\,iG[\alpha F^7_{\beta\alpha} + (1-\alpha)D^7_{\beta\alpha}], \tag{13}$$

then the parameters f, d, G, and α have the numerical values found in the analysis of the nonradiative decays,[4] namely,

$$(1-\alpha)/\alpha \simeq \sqrt{3}, \quad d/f = -1.09$$

and (14)

$$f = 1.513\, m_\pi, \qquad G = 11.25 m_\pi,$$

in units of $10^5 m_\pi^{-1/2}\ \mathrm{sec}^{-1/2}$.

Furthermore, the experimentally determined (total) magnetic moments of the proton and of the neutron[9] (in units of nuclear magneton) are used:

$$\mu_p(\mathrm{total}) = 2.793, \quad \mu_n(\mathrm{total}) = -1.913. \tag{15}$$

The total magnetic moments, μ_α, of the other baryons are taken to be the SU(3) values[10] multiplied by M_p/M_α. These "mass-corrected" values[11] come closer to the existing experimental values for μ_Λ and μ_{Σ^+}.

Once the amplitudes are known we can calculate the decay rate[8] and the asymmetry parameter[12]:

$$\Gamma = \frac{e^2}{8\pi}\left(\frac{M_\alpha^2 - M_\beta^2}{M_\alpha}\right)^3 (|A'|^2 + |B'|^2),$$
$$a = \frac{2\,\mathrm{Re}(A'^*B')}{|A'|^2 + |B'|^2}. \tag{16}$$

The numerical results are displayed in Table I. The only experimental information available at present[1] refers to the decay $\Sigma^+ \to p + \gamma$:

$$\frac{\Gamma(\Sigma^+ \to p + \gamma)}{\Gamma(\Sigma^+ \to p + \pi^0)} = (2.6 \pm 0.3)\times 10^{-3},$$
$$a = -1.03^{+0.52}_{-0.42}. \tag{17}$$

The theoretical branching ratio and asymmetry parameter for $\Sigma^+ \to p + \gamma$ are 2.82×10^{-3} and -0.78, respectively. The agreement is excellent, although further reduction of the experimental error is highly desired. Note that the relatively large violation of unitary symmetry found in nonradiative decays gives rise to the relatively large magnitude of B' which is the symmetry-breaking amplitude for the radiative decay. Therefore, the measured asymmetry parameter of about -1.0 strongly supports the breaking of unitary symmetry in both types of decays, a result quite different from that of the pole models.[13]

ACKNOWLEDGMENTS

I am grateful to Professor Peter Signell for support and encouragement, and to Professor Wayne Repko for discussion and assistance with the manuscript.

APPENDIX

For each specific decay the general expression (12) yields the following amplitudes.

TABLE I. Theoretical radiative weak-decay amplitudes of hyperons, decay rates, and asymmetry parameters.

Decay	Amplitude ($10^5 m_\pi^{-1/2}$ sec$^{-1/2}$ GeV^{-1}) A'	B'	Decay rate (10^7 sec^{-1}) Γ	Asymmetry parameter a
$\Lambda \to n + \gamma$	−0.147	0.927	0.795	−0.309
$\Sigma^+ \to p + \gamma$	0.887	−0.429	2.328	−0.783
$\Sigma^0 \to n + \gamma$	−1.538	−0.304	6.009	+0.380
$\Xi^0 \to \Lambda + \gamma$	0.483	−0.512	0.653	−0.998
$\Xi^0 \to \Sigma^0 + \gamma$	1.574	0.226	0.846	+0.281
$\Xi^- \to \Sigma^- + \gamma$	0.0015	−0.062	0.0013	−0.049

$\Lambda \to n + \gamma$:

$$A' = -\frac{1}{M_\Lambda - M_n}\left(\frac{\mu_n}{2M_n} - \frac{\mu_\Lambda}{2M_\Lambda}\right)\frac{3f+d}{\sqrt{3}} - \frac{1}{M_{\Sigma^0} - M_n}\frac{\mu_T}{2M_T}(f-d),$$
$$B' = -\frac{1}{M_\Lambda + M_n}\left(\frac{\mu_n}{2M_n} + \frac{\mu_\Lambda}{2M_\Lambda}\right)\frac{(1+2\alpha)}{\sqrt{3}}G + \frac{1}{M_{\Sigma^0} + M_n}\frac{\mu_T}{2M_T}(2\alpha - 1)G. \tag{A1}$$

$\Sigma^+ \to p + \gamma$:

$$A' = \frac{1}{M_{\Sigma^+} - M_p}\left(\frac{\mu_p}{2M_p} - \frac{\mu_{\Sigma^+}}{2M_{\Sigma^+}}\right)\sqrt{2}\,(f-d),$$
$$B' = \frac{1}{M_{\Sigma^+} + M_p}\left(\frac{\mu_p}{2M_p} + \frac{\mu_{\Sigma^+}}{2M_{\Sigma^+}}\right)\sqrt{2}\,(2\alpha - 1)G. \tag{A2}$$

$\Sigma^0 \to n + \gamma$:

$$A' = \frac{1}{M_{\Sigma^0} - M_n}\left(\frac{\mu_n}{2M_n} - \frac{\mu_{\Sigma^0}}{2M_{\Sigma^0}}\right)(f-d) + \frac{1}{M_\Lambda - M_n}\frac{\mu_T}{2M_T}\frac{3f+d}{\sqrt{3}},$$
$$B' = \frac{1}{M_{\Sigma^0} + M_n}\left(\frac{\mu_n}{2M_n} + \frac{\mu_{\Sigma^0}}{2M_{\Sigma^0}}\right)(2\alpha - 1)G - \frac{1}{M_\Lambda + M_n}\frac{\mu_T}{2M_T}\frac{2\alpha+1}{\sqrt{3}}G. \tag{A3}$$

$\Xi^0 \to \Lambda + \gamma$:

$$A' = \frac{1}{M_{\Xi^0} - M_\Lambda}\left(\frac{\mu_\Lambda}{2M_\Lambda} - \frac{\mu_{\Xi^0}}{2M_{\Xi^0}}\right)\frac{3f-d}{\sqrt{3}} - \frac{1}{M_{\Xi^0} - M_{\Sigma^0}}\frac{\mu_T}{2M_T}(f+d),$$
$$B' = \frac{1}{M_{\Xi^0} + M_\Lambda}\left(\frac{\mu_\Lambda}{2M_\Lambda} + \frac{\mu_{\Xi^0}}{2M_{\Xi^0}}\right)\frac{4\alpha - 1}{\sqrt{3}}G - \frac{1}{M_{\Xi^0} + M_{\Sigma^0}}\frac{\mu_T}{2M_T}G. \tag{A4}$$

$\Xi^0 \to \Sigma^0 + \gamma$:

$$A' = -\frac{1}{M_{\Xi^0} - M_{\Sigma^0}}\left(\frac{\mu_{\Sigma^0}}{2M_{\Sigma^0}} - \frac{\mu_{\Xi^0}}{2M_{\Xi^0}}\right)(f+d) + \frac{1}{M_{\Xi^0} - M_\Lambda}\frac{\mu_T}{2M_T}\frac{3f-d}{\sqrt{3}},$$
$$B' = -\frac{1}{M_{\Xi^0} + M_{\Sigma^0}}\left(\frac{\mu_{\Sigma^0}}{2M_{\Sigma^0}} + \frac{\mu_{\Xi^0}}{2M_{\Xi^0}}\right)G + \frac{1}{M_{\Xi^0} + M_\Lambda}\frac{\mu_T}{2M_T}\frac{4\alpha - 1}{\sqrt{3}}G. \tag{A5}$$

$\Xi^- \to \Sigma^- + \gamma$:

$$A' = -\frac{1}{M_{\Xi^-} - M_{\Sigma^-}}\left(\frac{\mu_{\Sigma^-}}{2M_{\Sigma^-}} - \frac{\mu_{\Xi^-}}{2M_{\Xi^-}}\right)\sqrt{2}\,(f+d),$$
$$B' = -\frac{1}{M_{\Xi^-} + M_{\Sigma^-}}\left(\frac{\mu_{\Sigma^-}}{2M_{\Sigma^-}} + \frac{\mu_{\Xi^-}}{2M_{\Xi^-}}\right)\sqrt{2}\,G. \tag{A6}$$

---

*Work supported in part by the U. S. Atomic Energy Commission.

[1]L. K. Gershwin, M. Alston-Garnjost, R. O. Bangerter, A. Barbaro-Galtieri, T. S. Mast, F. T. Solmitz, and R. D. Tripp, Phys. Rev. 188, 2077 (1969).

[2]M. A. Ahmed, Nuovo Cimento 58A, 728 (1968).

[3]L. R. Ram Mohan, Phys. Rev. D 3, 785 (1971).

[4]Y. E. Nagai, preceding paper, Phys. Rev. D 5, 60 (1972).

[5]F. E. Low, Phys. Rev. 110, 974 (1958); S. L. Adler and Y. Dothan, *ibid.* 151, 1267 (1966).

[6]L. S. Brown and C. M. Sommerfield, Phys. Rev. Letters 16, 75 (1966).

[7]Y. Hara, Phys. Rev. Letters 12, 378 (1964).

[8]R. E. Behrends, Phys. Rev. 111, 1691 (1958). The conventions for the γ matrices are those of S. Gasiorowicz, *Elementary Particle Physics* (Wiley, New York, 1966).

[9]A. H. Rosenfeld *et al.*, Rev. Mod. Phys. Suppl. 43, S1 (1971).

[10]S. Coleman and S. L. Glashow, Phys. Rev. Letters 6, 423 (1961).

[11]M. A. B. Bég and A. Pais, Phys. Rev. 137, B1514 (1965).

[12]Same as defined in Ref. 1.

[13]For example, R. Graham and S. Pakvasa, Phys. Rev. 140, B1144 (1965); L. R. Ram Mohan, *ibid.* 179, 1561 (1969).