broke, G. Carlson, D. E. Groom, J. W. Keuffel, J. L. Morrison, and J. L. Osborne, Phys. Rev. Letters 27, 160 (1971).

[6]H. E. Bergeson, G. L. Bolingbroke, D. E. Groom, J. W. Keuffel, and J. L. Osborne, in *Proceedings of the Fifteenth International Conference on High Energy Physics, Kiev, U.S.S.R., 1970* (Atomizdat, Moscow, 1971).

[7]J. R. Wayland, W. G. Cantrell, N. M. Duller, P. J. Green, and W. R. Sheldon, Nucl. Phys. B32, 527 (1971).

[8]M. G. K. Menon and P. V. Ramana Murthy, in *Progress in Cosmic Ray and Elementary Particle Physics*, edited by J. G. Wilson and S. A. Wouthuysen (North-Holland, Amsterdam, 1967), Vol. 9, p. 161; also see J. C. Barton and C. T. Stockel, Can. J. Phys. 46, S318 (1968).

[9]K. Kobayakawa, Nuovo Cimento 47B, 156 (1967).

[10]A. M. Aurela and A. W. Wolfendale, Ann. Acad. Sci. Fennicae, Ser. A 227, 1 (1967); I. C. Appleton, M. T. Hogue, and B. C. Rastin, Nucl. Phys. 26B, 365 (1971).

[11]J. Babecki and B. Furmańska, Lett. Nuovo Cimento 4, 521 (1970).

[12]J. D. Bjorken, S. Pakvasa, W. Simmons, and S. F. Tuan, Phys. Rev. 184, 1345 (1969).

[13]G. W. Carlson, J. W. Keuffel, and J. L. Morrison, in Proceedings of the Twelfth International Conference on Cosmic Rays, Hobart, 1971 (unpublished), Vol. 4, p. 1412.

PHYSICAL REVIEW D VOLUME 5, NUMBER 3 1 FEBRUARY 1972

Empirical Test for Composite Hadrons

G. Rajasekaran
Tata Institute of Fundamental Research, Bombay 5, India
(Received 14 September 1971)

We describe an empirical test to distinguish hadrons which are composites in the low-threshold channels from those which can be understood only from a more global point of view. This test is based on the existence of a pole in the K matrix in the latter case. Thus, the absence of a K pole signals a composite hadron. The test is applicable to hadrons coupled to S-wave two-body channels only. The properties of the K pole are analyzed using a field-theoretic model, and the pole and the residues are related to the wave-function renormalization constant of the hadron. We also construct coupling-constant sum rules which are connected to the test for compositeness. Applications to $\Lambda(1405)$ and other hadrons are described.

I. INTRODUCTION

One of the most striking features of hadron physics is just the fact that there are so many hadrons. Two viewpoints have been prevalent in the attempts to understand this rich hadron spectrum. We may broadly characterize these as follows:

(A) Local model. This point of view regards any observed hadron as a composite (bound or resonant) formed by forces in the adjacent hadron channels. Effects due to the very distant channels are treated, if at all, as an afterthought. Most of the old bootstrap-model calculations belong to this type.

(B) Global model. The neighboring hadron channels are essentially ignored in this point of view and the dynamics of the existence of the observed hadrons are sought elsewhere. A simple version of this type of approach is the so-called naive quark model, which considers the observed hadrons as bound states of some so-far unobserved heavy quarks. Among the more sophisticated versions of this philosophy, one may cite the models based on the infinite-component wave equation, the infinite-dimensional representations of current algebra, the currently popular zero-width dual models, etc. The common feature of all these models is to ignore the coupling of a particular hadron to its neighboring channels or to treat it as small.

The experimentally observed hadron spectrum being so complex and the interactions among the various hadrons being so diverse, it is reasonable to expect that the two models (A) and (B) may play a complementary role in understanding the hadron spectrum. In other words, it is quite possible, for instance, that the global point of view does encompass many of the hadrons, but, nevertheless, some of the hadrons may owe their existence to special local features, and may be essentially composites formed by strong attractive forces in the neighboring channels. We shall call the hadrons which can be understood on the basis of the local model as class-A hadrons, and those requiring the global model as class-B hadrons.

In a dynamical calculation which explicitly takes into account only the forces in the adjacent channels, a hadron of class A will come out as a pole in the S matrix, whereas a hadron of class B will have to be put in by hand as a Castillejo-Dalitz-Dyson[1] pole. In an equivalent field theory, the former corresponds to a composite hadron, while the latter corresponds to an elementary one. So, we have the correspondence

class A ~ composite,

class B ~ elementary.

But these adjectives "composite" and "elementary" used in their wider connotation may be misleading.[2] We use these words only because of a lack of better terminology, and they are to be understood strictly within the context of the local and global models already described.

If it is granted that the multifarious aspects of the hadronic world cannot be handled by a single model, then it is of importance to know which of the observed hadrons belong to class A and which to class B. As has been pointed out some time ago,[3,4] there exists a simple phenomenological criterion which may allow one to decide whether a particular hadron H belongs to class A or B. This criterion is that the K matrix (or the reaction matrix) for the set of the neighboring two-body S-wave channels, which must include at least one closed channel, should have a pole near the mass of the hadron H if H belongs to class B. Of course, the S matrix always has a pole corresponding to a particle. We argue that the K matrix *also* should have a pole in the neighborhood if the particle is not formed by forces in the neighboring channels.

In contrast to other methods, this K-pole test is a low-energy test and hence is easier to apply. But the test is applicable only to hadrons coupled to S-wave two-body channels.

It turns out that the K-pole test is closely related to earlier work by Ezawa, Muta, and Umezawa,[5] as well as by Weinberg.[6] In fact, one can use Weinberg's expressions for the scattering length and effective range to show the existence of the pole of the K matrix, which, in the single-channel case, is simply $(k\cot\delta)^{-1}$. Such a pole is not present for the $^3S_1$ nucleon-nucleon phase shift and hence the deuteron belongs to class A. By directly focusing attention on the pole of the complete K matrix for a multichannel process, a certain amount of conceptual simplicity can be achieved, and further, the test is now applicable to stable as well as unstable hadrons.

The purpose of the present paper is to give a more complete discussion of this problem than was attempted earlier. We first give a general argument based on the smoothness of the K-matrix elements to show the existence of the K pole in the neighborhood of the "elementary" hadron (Sec. II). This is then confirmed by explicit calculation using a field-theoretic model (Sec. III and Appendix). The position of the K pole as well as its residues are expressed in terms of the wave-function renormalization constant[7] of the hadron. In the next two sections, higher partial waves and the hierarchy of multichannel K matrices are discussed, and the meaning of the K-pole test is made more precise. We then construct another equivalent test for "elementarity" in the form of sum rules for the physical coupling constants of the hadron (Sec. VI). The tests are applied to the case of $\Lambda(1405)$ in Sec. VII which also includes a discussion of the SU(3) breaking in the coupling constants of $\Lambda(1405)$. Possibilities of applying the tests to other hadrons are indicated in Sec. VIII. Brief comments on a few other aspects of the problem are offered in Sec. IX.

II. A GENERAL ARGUMENT

Consider a two-body two-channel S-wave scattering. Given the 2×2 S matrix describing the two-channel process, one can introduce the T and K matrices as follows:

$$S = 1 + 2ik^{1/2}Tk^{1/2}, \tag{2.1}$$

$$T = K(1 - ikK)^{-1}, \tag{2.2}$$

where

$$k = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix},$$

k_1, k_2 being the momenta in the two channels in the c.m. system.

The elements of the T matrix considered as functions of the total energy W in the c.m. system have branch points at the two thresholds $m_1+\mu_1$ and $m_2+\mu_2$; in contrast, the K matrix is regular at these points. We shall write

$$K^{-1} = \begin{pmatrix} a & b \\ b & c \end{pmatrix}, \tag{2.3}$$

where a, b, and c are real for real W and we can represent them by smooth functions of W even in the neighborhood of the thresholds. From Eqs. (2.2) and (2.3) we get

$$T = \begin{pmatrix} c - ik_2 & -b \\ -b & a - ik_1 \end{pmatrix}\Delta^{-1}, \tag{2.4}$$

where

$$\Delta = ac - b^2 - k_1k_2 - (ik_1c + ik_2a).$$

We are considering the case where the mass of the hadron H lies between the two thresholds. The hadron H will then occur as a resonance in the lower channel, which we denote as the channel 1. So, let us look at T_{11}, which from Eq. (2.4) is

$$T_{11} = \left(a - \frac{b^2}{c + |k_2|} - ik_1\right)^{-1}, \tag{2.5}$$

where we have put $k_2 = +i|k_2|$. Identifying with the usual form for elastic scattering,

$$T_{11}=(k_1\cot\delta_1-ik_1)^{-1}, \tag{2.6}$$

where δ_1 is the phase shift in channel 1, we get

$$k_1\cot\delta_1=a-\frac{b^2}{c+|k_2|}. \tag{2.7}$$

At the resonance $W=W_0$, δ_1 passes through $\frac{1}{2}\pi$ and so

$$\left(a-\frac{b^2}{c+|k_2|}\right)_{W=W_0}=0.$$

Or,

$$(\det K^{-1}+a|k_2|)_{W=W_0}=0, \tag{2.8}$$

where

$$\det K^{-1}=ac-b^2.$$

In Figs. 1 and 2 we have plotted the functions $\det K^{-1}$ and $-a|k_2|$ schematically. Their intersection determines the resonance position W_0. Since a, b, and c are smooth functions of W, $\det K^{-1}$ can be approximated by a straight line over the small region of interest. On the other hand, $-a|k_2|$ has the form $\sim(m_2+\mu_2-W)^{1/2}$ near the threshold $m_2+\mu_2$. There are two distinct possibilities for $\det K^{-1}$ as illustrated in Figs. 1 and 2. In Fig. 1, $\det K^{-1}$ has a large slope and passes through zero in the neighborhood and so the K matrix has a pole at that point. In Fig. 2, $\det K^{-1}$ has a small slope and stays constant over the region of interest and so there is no pole for the K matrix.

Let us now consider the classification mentioned earlier. If the hadron H belongs to class B, it does not have much to do with the channels 1 and 2. So, if we move the threshold $m_2+\mu_2$ to the left, for instance, the value of W_0 should be relatively unaffected and so $m_2+\mu_2-W_0$ should become arbitrarily small. This is in fact the case in Fig. 1. For small perturbations in the threshold in which we are interested, the most sensitive dependence on the threshold $m_2+\mu_2$ comes from $|k_2|$, and thus $\det K^{-1}$ and a can be left unchanged. So, as $m_2+\mu_2$ moves to the left, the curve $-a|k_2|$ will move along with it, whereas the line $\det K^{-1}$ stays put. The point of intersection, W_0, moves only a little because of the steepness of the line $\det K^{-1}$, and $m_2+\mu_2-W_0$ approaches zero. So a class-B hadron is consistent with Fig. 1, i.e., with the presence of a K pole.

One can go further. If the hadron H belongs to class B, then it should be possible to move the threshold $m_2+\mu_2$ to the left of W_0 so that the resonance H continues to exist even above $m_2+\mu_2$. To show that this indeed is the case for Fig. 1, one should now write down the formulas for a two-channel resonance. Let us use the eigenphase shifts δ_α, δ_β obtained through diagonalizing by means of an orthogonal transformation U:

$$(k^{1/2}Kk^{1/2})^{-1}=U\begin{pmatrix}\cot\delta_\alpha & 0\\ 0 & \cot\delta_\beta\end{pmatrix}U^{-1}. \tag{2.9}$$

At the two-channel resonance $W=W_0$ occurring above $m_2+\mu_2$, one of the eigenphase shifts passes through $\frac{1}{2}\pi$. So, we have

$$(\det K^{-1})_{W=W_0}=(k_1k_2\cot\delta_\alpha\cot\delta_\beta)_{W=W_0}=0. \tag{2.10}$$

Since in Fig. 1 $\det K^{-1}$ does pass through zero in the neighborhood, we see that the resonance H continues to exist even above the threshold $m_2+\mu_2$. This is exactly what one would expect of a class-B hadron. In fact, the position of the resonance in this case is just the K pole.

In the case of Fig. 2, one can easily see that as $m_2+\mu_2$ moves to the left, W_0 also moves to the left by about the same extent, and hence the difference $m_2+\mu_2-W_0$ remains approximately constant. One cannot decrease $m_2+\mu_2-W_0$ arbitrarily in this case and the existence of H is intimately connected with the channel 2. So, the case of absence of a nearby K pole (i.e., Fig. 2) corresponds to a class-A hadron.

III. A FIELD-THEORETIC MODEL

We construct a Lee-type model with two S-wave channels denoted by B_1P_1 and B_2P_2 having the same quantum numbers as the particular hadron H under consideration. Such a model is very convenient for the purpose of comparing the elementary H with the composite H. If the bare coupling constants g_1^0 and g_2^0 for the vertices HB_1P_1 and HB_2P_2 as well as the bare mass m_H^0 of H were finite, the model describes the case of the elementary H (class B). The case of composite H (class A) is obtained[8] by taking the limit $g_1^0\to\infty$, $g_2^0\to\infty$, $m_H^0\to\infty$, keeping $(g_1^0)^2/m_H^0$ and $(g_2^0)^2/m_H^0$ finite.

The model is described by the Hamiltonian

$$\mathcal{H}=m_H^0H^\dagger H+m_1B_1^\dagger B_1+m_2B_2^\dagger B_2+\int d^3k[\omega_1P_1^\dagger(k)P_1(k)+\omega_2P_2^\dagger(k)P_2(k)]$$
$$-(4\pi)^{1/2}g_1^0\int d^3k\frac{u(k)}{(2\pi)^{3/2}(2\omega_1)^{1/2}}[H^\dagger B_1P_1(k)+\text{H.c.}]-(4\pi)^{1/2}g_2^0\int\frac{d^3k\,u(k)}{(2\pi)^{3/2}(2\omega_2)^{1/2}}[H^\dagger B_2P_2(k)+\text{H.c.}], \tag{3.1}$$

where $\omega_i=(\vec{k}^2+\mu_i^2)^{1/2}$ and $u(k)$ is the cutoff function. The T matrix in the $B_1P_1-B_2P_2$ sector is

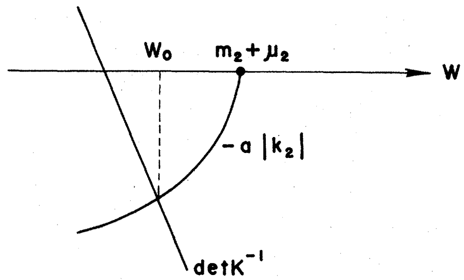

FIG. 1. Plots of $\det K^{-1}$ and $-a|k_2|$ against W with K pole.

$$T_{ij}=\frac{1}{D(W)}\lambda_i\lambda_j u(k_i)u(k_j),$$
$$D(W)=\frac{1}{(g_0)^2}(m_H^0-W)-F(W), \tag{3.2}$$

where we have put

$$g_i^0=\lambda_i g_0 \tag{3.3}$$

and we have introduced the function $F(W)$ defined by

$$F(W)=\lambda_1^{\,2}\frac{1}{\pi}\int_{m_1+\mu_1}^{\infty}dW'\frac{k_1'u^2(k_1')}{W'-W-i\epsilon}+\lambda_2^{\,2}\frac{1}{\pi}\int_{m_2+\mu_2}^{\infty}dW'\frac{k_2'u^2(k_2')}{W'-W-i\epsilon}\,. \tag{3.4}$$

Let us also define the principal-value function:

$$F_P(W)=\lambda_1^{\,2}\frac{P}{\pi}\int_{m_1+\mu_1}^{\infty}dW'\frac{k_1'u^2(k_1')}{W'-W}+\lambda_2^{\,2}\frac{P}{\pi}\int_{m_2+\mu_2}^{\infty}dW'\frac{k_2'u^2(k_2')}{W'-W}\,.$$

The denominator function of the K matrix involves $F_K(W)$ defined by

$$F_K(W)=F(W)-ik_1\lambda_1^{\,2}u^2(k_1)-ik_2\lambda_2^{\,2}u^2(k_2)\,. \tag{3.5}$$

In fact, the K matrix is given by precisely the same expression as the T matrix above with $F(W)$ replaced by $F_K(W)$. Note that $F(W)$ is an analytic function of W with branch points at the thresholds, whereas $F_P(W)$ is nonanalytic. The function $F_K(W)$ is also analytic (provided the cutoff function is analytic) but without the branch points at the thresholds.

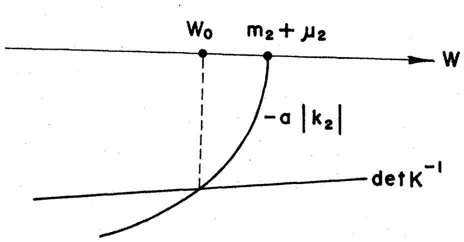

FIG. 2. Plots of $\det K^{-1}$ and $-a|k_2|$ against W without K pole.

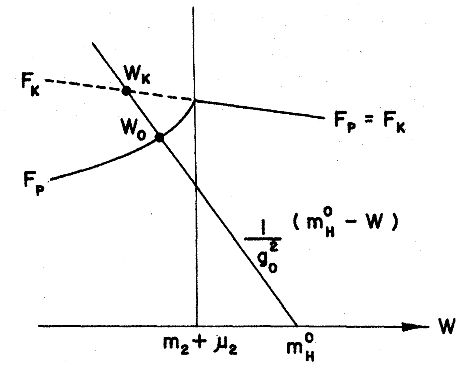

FIG. 3. Graphical determination of W_0 and W_K with elementary particle (S-wave case).

Note that the T and K matrices in this model are factorizable and their inverse matrices do not exist. This defect is remedied in a more realistic model studied in the Appendix. But the conclusions arrived at in the present section on the basis of the simpler model stand essentially unchanged.

As before, let us arrange so that the physical H occurs as a resonance in channel 1 at $W=W_0$ between the two thresholds. The phase shift δ_1 can be calculated from T_{11} in Eq. (3.2):

$$k_1\cot\delta_1=\left(\frac{m_H^0-W}{(g_0)^2}-F_P(W)\right)\frac{1}{\lambda_1^{\,2}u^2(k_1)}$$
$$\text{for } m_1+\mu_1<W<m_2+\mu_2\,. \tag{3.6}$$

Since δ_1 passes through $\frac{1}{2}\pi$ at $W=W_0$, we have

$$\frac{1}{(g_0)^2}(m_H^0-W_0)-F_P(W_0)=0\,. \tag{3.7}$$

Our aim is to look for a pole of K (if any) corresponding to the H particle. If the K pole occurs at $W=W_K$, then we should have

$$\frac{1}{(g_0)^2}(m_H^0-W_K)-F_K(W_K)=0\,. \tag{3.8}$$

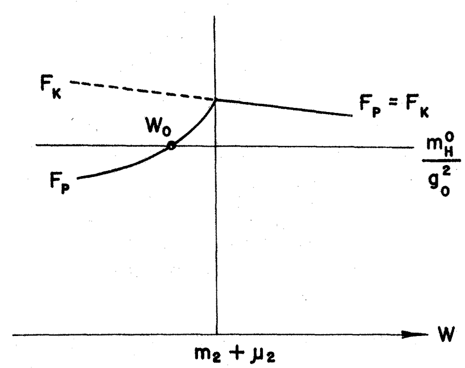

FIG. 4. Graphical determination of W_0 and W_K with composite particle (S-wave case).

We have to solve Eqs. (3.7) and (3.8) for W_0 and W_K. A graphical solution helps to visualize the situation. Note that $F_K(W)$ is a smooth function of W. On the other hand, $F_P(W)$ has a kink (discontinuous derivative) at the thresholds. One can see that

$$F_P(W)=F_K(W) \quad \text{for} \quad W>m_2+\mu_2,$$
$$F_P(W)=F_K(W)-\lambda_2{}^2|k_2|u^2(k_2) \qquad (3.9)$$
$$\text{for} \quad m_1+\mu_1<W<m_2+\mu_2\,.$$

We will put $u\approx 1$ in the low-energy region of interest. The functions $F_P(W)$ and $F_K(W)$ along with the straight line $(m_H^0-W)/(g_0)^2$ are plotted schematically in Fig. 3. The intersection of $F_P(W)$ with $(m_H^0-W)/(g_0)^2$ gives W_0, while the intersection of $F_K(W)$ with $(m_H^0-W)/(g_0)^2$ gives W_K. Thus the physical resonance at W_0 is followed by a K pole at W_K to the left of W_0. So far we have an elementary H present in our model.

As already mentioned, the composite-H case is obtained by taking the limit $(g_0)^2\to\infty$, $m_H^0\to\infty$, keeping $m_H^0/(g_0)^2$ constant. The only change is that the straight line now has zero slope, and this is shown in Fig. 4. We still want the physical H at W_0 and so the straight line $m_H^0/(g_0)^2$ has to intersect $F_P(W)$ at W_0. But it does not intersect $F_K(W)$, and hence there is no K pole in the neighborhood.

The above argument for the K pole is clearly based on the closeness in value of F_K and F_P. Although F_K has to coincide with F_P at the threshold $m_2+\mu_2$, it is possible to imagine F_K in Fig. 3 to have a rapid variation for W below $m_2+\mu_2$ in such a way as not to cut the straight line $(m_H^0-W)/(g_0)^2$. However, this is implausible, since the dependence of F_K and K^{-1} on energy is generally[9] governed by the range of the interaction, and for short-range interactions rapid variation with respect to energy is not expected. In any case, if this happens, and a pole in the K matrix is avoided, K^{-1} will then have a surprisingly large variation with respect to energy, which can be checked empirically. Summarizing, we may say that the K-pole test is applicable if K^{-1} has a small dependence on energy and W_0 is close to $m_2+\mu_2$.

Actually, in the limiting case of W_0 occurring very close to the threshold, we can show that $F_K(W)$ has a dominant constant contribution. To show it, let us write $F(W)$ in the once-subtracted form

$$F(W)=F(W_0)+(W-W_0)\left(\frac{\lambda_1{}^2}{\pi}\int_{m_1+\mu_1}^{\infty}\frac{dW'k_1'u^2(k_1')}{(W'-W_0)(W'-W)}+\frac{\lambda_2{}^2}{\pi}\int_{m_2+\mu_2}^{\infty}\frac{dW'k_2'u^2(k_2')}{(W'-W_0)(W'-W)}\right). \qquad (3.10)$$

Consider the second integral,

$$\int_{m_2+\mu_2}^{\infty}\frac{k_2'dW'u^2(k_2')}{(W'-W_0)(W'-W)}=\int_0^{\infty}\frac{k'dE'u^2(k')}{(E'+|E_0|)(E'-E)},$$

where we have put $W'=m_2+\mu_2+E'$ and similarly for W and W_0. Since $E'\approx k'^2/2\mu_2$ in the nonrelativistic limit, for small E', the integrand is $E'^{1/2}/[(E'+|E_0|)(E'-E)]$ and the integral diverges at the lower limit for $|E_0|\to 0$ and $E\to 0$. So, for small $|E_0|$ and E, the integral is dominated by the small-E' contribution, and hence it can be estimated[10] by using the nonrelativistic formula $E'=k'^2/2\mu_2$. Note that in that case, we do not need the cutoff function for convergence at the upper end. Also, since the first integral is much smaller than the second for small $|E_0|$ and small E, an exact evaluation of the former is not crucial for our argument. So, we may conveniently evaluate that also using the nonrelativistic kinematics and ignoring the cutoff. With these, the integrals in Eq. (3.10) can be obtained in closed form and we find

$$F(W)=F(W_0)+i\lambda_1{}^2(k_1^0-k_1)+i\lambda_2{}^2(k_2^0-k_2),$$

where k_1^0 and k_2^0 are the momenta evaluated at W_0. Comparing with Eq. (3.5), we see that

$$F_K(W)=F_K(W_0)\,. \qquad (3.11)$$

We have shown that in the limit of W_0 occurring very close to the threshold $m_2+\mu_2$, F_K can be treated as a constant, and in this case we can get a compact formula for the K-pole W_K. For, Eqs. (3.7) and (3.8) become

$$\frac{1}{(g_0)^2}(m_H^0-W_0)-F_K+\lambda_2{}^2(2\mu_2|E_0|)^{1/2}=0,$$
$$\frac{1}{(g_0)^2}(m_H^0-W_K)-F_K=0, \qquad (3.12)$$

where $E_0=W_0-(m_2+\mu_2)$, and similarly we define E_K. Eliminating F_K from Eqs. (3.12), we get

$$E_K=E_0-(g_2^0)^2(2\mu_2|E_0|)^{1/2}\,. \qquad (3.13)$$

This formula involves the bare coupling constant g_2^0. It can be expressed in terms of the renormalization constant. To do this, we note that in the narrow-width approximation, Eq. (3.6) leads to the following expression for the width of the resonance:

$$\tfrac{1}{2}\Gamma=k_1(g_1^0)^2\left(1+(g_0)^2\frac{dF_P}{dW}\right)^{-1}, \qquad (3.14)$$

where the right-hand side is evaluated at W_0. This

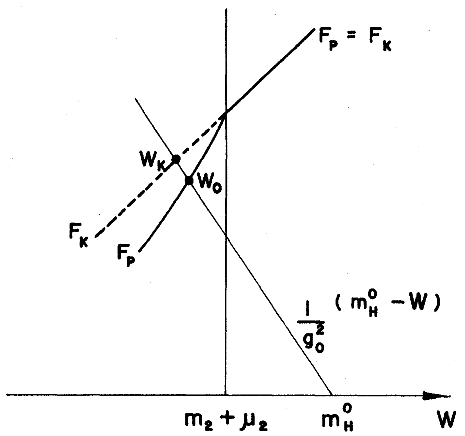

FIG. 5. Graphical determination of W_0 and W_K with elementary particle.

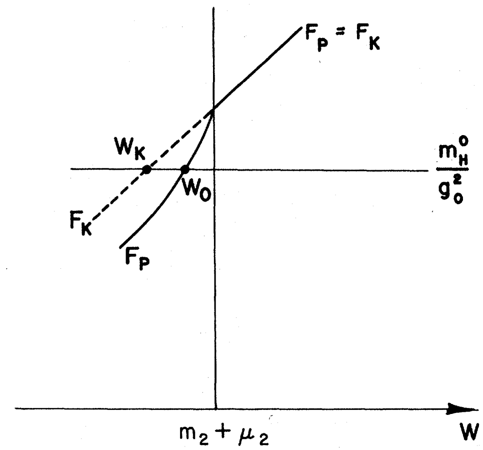

FIG. 6. Graphical determination of W_0 and W_K with composite particle.

suggests the following definitions of the physical coupling constants[11] g_i and the wave-function renormalization constant Z for H:

$$(g_i)^2 = (g_i^0)^2 Z, \quad Z = \left(1 + (g_0)^2 \frac{dF_P}{dW}\right)_{W_0}^{-1}. \tag{3.15}$$

In the limiting case, when F_K is treated as constant,

$$\left(\frac{dF_P}{dW}\right)_{W_0} \approx \lambda_2{}^2 \mu_2 |E_0|^{-1/2}$$

and so

$$Z = [1 + (g_2^0)^2 \mu_2 |E_0|^{-1/2}]^{-1}. \tag{3.16}$$

Note that in the composite-particle limit, $g_2^0 \to \infty$ and hence $Z \to 0$. Eliminating $(g_2^0)^2$ in Eqs. (3.13) and (3.16) in favor of Z, we get a remarkably simple formula for the position of the K pole:

$$E_K = \frac{1}{Z}(2 - Z)E_0. \tag{3.17}$$

In the composite-particle limit, $Z \to 0$ and the K pole runs away. But this formula for E_K can be used only for small E_0 and E_K.

In the limit of constant F_K, we can easily verify that the K matrix itself has a very simple form:

$$K_{ij} \approx \frac{g_i^0 g_j^0}{W_K - W} = \frac{g_i g_j}{Z} \frac{1}{(W_K - W)}. \tag{3.18}$$

So, in this model, the residues of the K matrix are just the bare coupling constants. It is clear that in the composite-particle limit, the residues as well as the pole of the K matrix tend to ∞, so that the K matrix becomes essentially constant.

It is important to note that the residues of this K pole have the usual sign characteristic of genuine particle poles. Hence $\det K^{-1}$ *should decrease through zero at the K pole*.[12]

Finally, we may remark that Eq. (3.18) suggests the reason why the K matrix is so useful in testing for the "elementarity" of the particle. In contrast to the S matrix (whose residues are the physical coupling constants), the K matrix reveals the "bare structure" of the particle involved.[13]

IV. HIGHER PARTIAL WAVES

So far we have considered only S-wave scattering. For higher partial waves, k' in the integral for $F(W)$ is replaced by k'^{2l+1}. This has two consequences. First, the integral cannot be dominated by the low-energy contribution, and the constant-F_K approximation fails. Second, the difference between F_K and F_P, which is now proportional to $|k|^{2l+1}$, becomes less significant for small k. The typical behavior of F_P and F_K is illustrated in Figs. 5 and 6. We see that for $l \geq 1$, H is followed by a K pole whether H is elementary or composite.

Physically, the above effect is closely related[4] to the existence of the centrifugal barrier for $l \geq 1$. Because of the barrier, it is possible for the particle H to exist with a mass even above $m_2 + \mu_2$ and the curve for $F_P = F_K$ above $m_2 + \mu_2$ has to have a positive slope, as shown in Figs. 5 and 6, in order to enable this possibility. Since F_K is a smooth function, it continues to have such a slope even below the threshold and F_P, being not very different from F_K for $l \geq 1$, also has a positive slope. This leads to the shape of the curves shown in Figs. 5 and 6. So, a centrifugal barrier always leads to a K pole, and one cannot test for elementarity in its presence.[14]

This argument suggests that even for $l = 0$, if there is a repulsive barrier surrounding the attractive potential, H will be followed by a K pole in both the elementary- and composite-particle cases.

Summarizing, we may say that the presence of a K pole is not an evidence for "elementarity" unless at the same time surrounding repulsive barriers (centrifugal or otherwise) are shown to be absent. On the other hand, the *absence of a K pole* is certainly an evidence for compositeness.

V. A PROCEDURE FOR DISCOVERING THE CONSTITUENTS

On the basis of the foregoing considerations, we may give a systematic procedure for discovering the constituents of a hadron H in an idealized world where H is coupled to a series of two-body S-wave channels of increasing thresholds.

For this purpose, let us define the hierarchy of K matrices. Consider the T matrix of dimension $m\times m$ connecting the first m channels starting with the lowest threshold. Calling it $T^{(m)}$ we can define $K^{(m)}$ of dimension $m \times m$ through Eq. (2.2), which can be rewritten as

$$(K^{(m)})^{-1} = (T^{(m)})^{-1} + ik^{(m)}, \tag{5.1}$$

where $k^{(m)}$ is the diagonal momentum matrix of dimension $m \times m$. Next include the $(m+1)$th channel. From $T^{(m+1)}$ of dimension $(m+1)\times(m+1)$ connecting the $m+1$ channels, define $K^{(m+1)}$ by

$$(K^{(m+1)})^{-1} = (T^{(m+1)})^{-1} + ik^{(m+1)}. \tag{5.2}$$

Similarly, we can define the K matrices corresponding to any number of channels.

Now, the procedure is as follows. Let W_0 be the mass or the resonance energy of the hadron H. If a certain number of channels, say n, be open at this energy W_0, it follows immediately that H is not a composite of the particles comprising these n open channels. For, an S-wave resonance cannot arise from the attractive interactions in the open channels. We assume that there are no repulsive barriers, for the sake of simplicity.

The matrix $K^{(n)}$ for these n open channels will be found to have a pole corresponding to an eigenphase shift increasing through $\frac{1}{2}\pi$ [see Eq. (2.10)]. But we should look at $K^{(n+1)}$, where $n+1$ is a closed channel. If $K^{(n+1)}$ also has a pole, then H is not a composite in the $(n+1)$th channel also. Repeat the procedure including more and more closed channels and stop at the pth channel if $K^{(p)}$ does not have a pole. The hadron H is then a composite of the particles comprising the pth channel.

The above ideal situation has been described only to bring out the ideas involved in a clear fashion. In the real world every hadron will be coupled to many-body channels and two-body channels with $l \geq 1$, occurring at higher thresholds. So, one will be able to complete the above procedure only in the fortunate cases when $K^{(p)}$ for small p does not have a pole. Also, there could be repulsive interactions in which case even though H may be a composite in the nth channel, $K^{(n)}$ may have a pole. Thus, again, only *the absence of a K pole* is of significance.

It is also useful to record the relationship between $K^{(m)}$ and the $m \times m$ submatrix of $K^{(m+1)}$. Let us write

$$K^{(m+1)} = \begin{pmatrix} \alpha & \beta \\ \tilde{\beta} & \gamma \end{pmatrix}, \tag{5.3}$$

where α is the $m \times m$ submatrix of $K^{(m+1)}$, β is a $(m+1)\times 1$ column vector, $\tilde{\beta}$ its transpose, and γ is a single element. Then,

$$K^{(m)} = \alpha + \frac{ik_{m+1}\beta\tilde{\beta}}{1 - ik_{m+1}\gamma}. \tag{5.4}$$

With respect to the $(m+1)$-channel system, $K^{(m)}$ is called the "reduced K matrix."[15]

The above relation (5.4) provides yet another way[3] of showing the existence of the pole in a lower-dimensional K matrix when the hadron is composed of particles in a very distant channel (quark channel for instance). For this, one can identify the $(m+1)$th channel as an "effective" quark channel. The strong attractive forces in the quark channel will lead to a zero of $1 - ik_{m+1}\gamma$, thus implying a pole of $K^{(m)}$; at the same time, $K^{(m+1)}$ will not have a pole.

VI. SUM RULES FOR COUPLING CONSTANTS

We shall now construct sum rules for the physical coupling constants which also can be used to test for "elementarity." For this purpose we may define the physical coupling constants as the residues of the pole of the T matrix. The T matrix necessarily has a pole (in the unphysical Riemann sheet) corresponding to the resonance. Near this complex pole W_T, we may write

$$T_{ij} \approx \frac{\gamma_i\gamma_j}{W_T - W} + \text{background term}, \tag{6.1}$$

so that

$$\gamma_i\gamma_j = \lim_{W\to W_T}(W_T - W)T_{ij}. \tag{6.2}$$

Note that the coupling constants γ_i defined here are also complex.

Let us first consider the case of a constant two-channel K matrix and write

$$K^{-1} = \begin{pmatrix} a & b \\ b & c \end{pmatrix},$$

where a, b, c are constants. Forming the T matrix out of this and extracting the residues at the pole W_T, we get

$$\gamma_1{}^2=\left[i\,\frac{dk_1}{dW}+i\,\frac{dk_2}{dW}\left(\frac{a-ik_1}{c-ik_2}\right)\right]^{-1},$$

$$\gamma_2{}^2=\left[i\,\frac{dk_1}{dW}\left(\frac{c-ik_2}{a-ik_1}\right)+i\,\frac{dk_2}{dW}\right]^{-1},$$

where all the quantities are to be evaluated at W_T. Eliminating $(a-ik_1)/(c-ik_2)$ between these two equations, we get the following sum rule for the coupling constants:

$$\left(i\gamma_1{}^2\,\frac{dk_1}{dW}+i\gamma_2{}^2\,\frac{dk_2}{dW}\right)_{W_T}=1\,. \tag{6.3}$$

What we have obtained is a multichannel generalization of Landau's formula[16] for the coupling constant.

The important point for our consideration, however, is that the above sum rule as well as Landau's formula are not correct if the particle is elementary (class B). To see this, let us go back to the model studied in Sec. III and calculate the residues at the pole of the T matrix in that model. We find

$$\gamma_i\gamma_j=g_i^0g_j^0Z'\,, \tag{6.4}$$

where

$$Z'=\left(1+(g_0)^2\frac{dF}{dW}\right)_{W_T}^{-1}. \tag{6.5}$$

The renormalization constant Z' defined here is of course different from Z defined earlier. In fact Z' is complex.[17] However, what we need for our purpose is only the fact that both Z and Z' are nonzero in the elementary-particle case, while both these parameters vanish in the composite-particle limit. Using in Eq. (6.5) the constant-F_K approximation already discussed, we get

$$\left(i\gamma_1{}^2\,\frac{dk_1}{dW}+i\gamma_2{}^2\,\frac{dk_2}{dW}\right)_{W_T}=1-Z'\,. \tag{6.6}$$

Thus, in the presence of the elementary particle, the coupling constants are reduced by a factor $1-Z'$.

We have used complex coupling constants and complex Z' only because of the conceptual simplicity of considering the complete T matrix since it treats the closed and open channels alike. One can write a sum rule also in terms of the real coupling constants introduced in Eq. (3.15). In fact it is already contained in Eq. (3.16). Putting $(g_2^0)^2=(g_2)^2Z^{-1}$ in this equation, we get

$$\left(i(g_2)^2\,\frac{dk_2}{dW}\right)_{W_0}=1-Z\,. \tag{6.7}$$

This shows that if we use the real coupling constants, the sum rule involves only the closed channels, in contrast to Eq. (6.6).

As one can verify, for an arbitrary number of channels we have the following results:

$$\left(i\sum_i\gamma_i{}^2\,\frac{dk_i}{dW}\right)_{W_T}=1-Z'$$

(summation over all channels) (6.8)

$$\left(i\sum_{i,\,\mathrm{closed}}(g_i)^2\,\frac{dk_i}{dW}\right)_{W_0}=1-Z$$

(summation over closed channels). (6.9)

Because of the assumptions used in their derivation, these sum rules are valid only if all the thresholds are near the resonant energy. However, the sum rules contain a built-in mechanism by which only those channels whose thresholds are nearby contribute effectively. For, $(dk_i/dW)_{W_0}$ is infinite when the ith threshold coincides with the resonant energy W_0 and so is large only for nearby channels. Thus, in Eqs. (6.8) and (6.9), one needs to include only the nearby channels.

Using empirically determined values of the coupling constants in these sum rules, one can test for nonzero values of Z or Z'. It should be made clear that this is not an independent test for elementarity. It has precisely the same content as the K-pole test; however, it provides an alternate language for discussing the problem of elementarity.

VII. APPLICATION TO Λ(1405)

Apart from the deuteron already studied,[5,6] the hadron Λ(1405) provides an ideal application[3,4] since it is connected to the S-wave channels $\Sigma\pi$ and $N\bar{K}$, and since the experimental study of this system has been going on continuously for almost a decade. The relative positions of the $\Sigma\pi$ and $N\bar{K}$ thresholds with respect to Λ(1405) are shown in Fig. 7. To test for the compositeness of Λ(1405) one should ask whether the two-channel K matrix connecting $\Sigma\pi$ and $N\bar{K}$ has a pole in the neighborhood of Λ(1405) to its left.

Almost all[18] of the phenomenological analyses of this system are consistent with either a constant two-channel K matrix or a mildly varying one. None of them show any evidence for a pole in the K matrix. As an example, $\det K^{-1}$ from Kim's analysis[19] is shown in Fig. 7. It does not pass through zero to the left of 1405 MeV; even the sign of the slope indicates against such a possibility. One may wonder whether $\det K^{-1}$ in Fig. 7 increases through zero at a large W. Even if this happens, the residues at this K pole are opposite to that in Eq. (3.18) and this is *not* the K pole we are looking for. This will be similar to the situation occurring in the case of the deuteron, where $k\cot\delta$ $(=K^{-1})$ increases through zero at about 8 MeV for

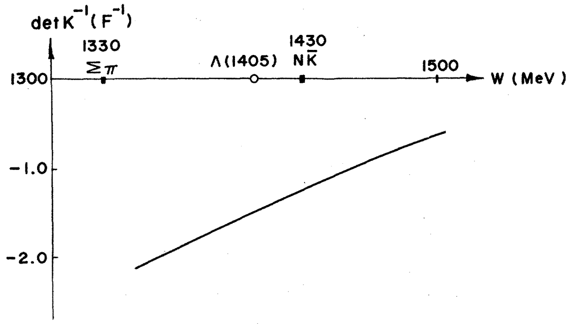

FIG. 7. Plot of Kim's $\det K^{-1}$ for $N\bar{K}, \Sigma\pi$ system (Ref. 19). Also shown along the W axis are the $\Sigma\pi$, $N\bar{K}$ thresholds, and the $\Lambda(1405)$.

$E = W - 2m_N$. The phase shift (or an eigenphase shift) *decreases slowly* through $\frac{1}{2}\pi$ at such a point [see remark (a) in Sec. IX].

More recent analyses[20,21] show that the data are consistent even with a zero slope for $\det K^{-1}$. Further, Martin, Martin, and Ross[22] attempted to fit the data directly by assuming a pole in the K matrix and found that such a fit is *not* favored by the data.

So, it may be concluded that $\Lambda(1405)$ is a composite of N and $\bar{K}$ (class A) and it cannot have much to do with global models. Actually the absence of a K pole near $\Lambda(1405)$ was indicated as early as 1960, when the first analysis was made by Dalitz and Tuan,[23] and $\Lambda(1405)$ was called a "virtual-bound-state resonance." However, the significance of this fact for global models like the quark model does not seem to have been recognized until recently.

The above conclusion is in agreement with the fact that a potential-model calculation[24] based on vector-meson exchange between baryons and mesons also indicates that $\Lambda(1405)$ is a composite.

In applying the K pole test, we have implicitly assumed that $\Lambda(1405)$ is close enough to the $N\bar{K}$ threshold. Roughly, the criterion for this closeness may be taken as $(2\mu|E_0|)^{1/2} \ll r^{-1}$, where $|E_0|$ is the "binding energy" of $\Lambda(1405)$ with respect to $N\bar{K}$, μ is the reduced mass for $N\bar{K}$, and r is the typical range of interaction. For $\Lambda(1405)$, $(2\mu|E_0|)^{1/2} = 130$ MeV and with vector-meson exchange, $r^{-1} \approx 800$ MeV. The small slope of the empirically determined[20,21] $\det K^{-1}$ indicates an even larger value for r^{-1}. So the K-pole test is clearly applicable.

We may also apply the sum rule for coupling constants. Using the empirical knowledge of the T matrix, Kim and von Hippel[25] have evaluated the coupling constants of $\Lambda(1405)$ to $\Sigma\pi$ and $N\bar{K}$:

$$g_{\Sigma\pi}^2 = 0.141 \pm 0.021, \tag{7.1}$$

$$g_{N\bar{K}}^2 = 0.64 \pm 0.08. \tag{7.2}$$

Since only the real coupling constants have been determined, the relevant sum rule to be used is

$$g_{N\bar{K}}^2\left(i\,\frac{dk_{\bar{K}}}{dW}\right)_{W_0} = 1 - Z. \tag{7.3}$$

At $W_0 = 1405$ MeV, we have

$$-i\left(\frac{dW}{dk_{\bar{K}}}\right)_{W_0} = 0.59. \tag{7.4}$$

Putting Eqs. (7.2) and (7.4) into (7.3), we have

$$1 - Z = 1.08 \pm 0.12, \tag{7.5}$$

which indicates $Z \approx 0$ for $\Lambda(1405)$ in conformity with the conclusion already reached.

At this point we digress a little from our main enquiry for a brief discussion of the $\Lambda(1405)$ coupling constants from the point of view of SU(3) symmetry and its breaking. Exact SU(3) symmetry with the singlet assignment of $\Lambda(1405)$ leads to the following value for the ratio $R = g_{N\bar{K}}^2/g_{\Sigma\pi}^2$:

$$R = \tfrac{2}{3}\ [\mathrm{SU}(3)]. \tag{7.6}$$

This is in violent disagreement with the empirical ratio

$$R \approx 4.5 \pm 0.8\ \text{(empirical)} \tag{7.7}$$

obtained from Eqs. (7.1) and (7.2). However, this empirical ratio is in good agreement with theoretical calculations based on potential models[24,26] as well as current algebra[26,27]:

$$R \approx 4.4\ \text{(potential models)}, \tag{7.8}$$

$$R \approx 3.2\ \text{(current algebra)}. \tag{7.9}$$

One may give a simple explanation for this large deviation of R from $\frac{2}{3}$ on the basis of the sum rule in Eq. (7.3). The point is that, if $Z = 0$, Eq. (7.3) is in effect a dynamical constraint on $g_{N\bar{K}}^2$, whereas there is no such constraint on $g_{\Sigma\pi}^2$. With such a complete dissymmetry between the two channels, the symmetry of the coupling constants is lost.[28] The existence of this constraint on $g_{N\bar{K}}^2$ also helps to explain why such diverse methods as empirical determination, potential models, and current algebra [Eqs. (7.7)–(7.9)] lead to about the same result.

If the compositeness of $\Lambda(1405)$ is granted, i.e., $Z = 0$, the relation (7.3) can in fact be used to determine $g_{N\bar{K}}^2$ merely from the value of the mass 1405 MeV and the result is

$$g_{N\bar{K}}^2 = 0.59, \tag{7.10}$$

which is almost the same as that obtained by Kim and von Hippel: 0.64 ± 0.08. Further, the value of $g_{\Sigma\pi}^2$ can be determined from the width of $\Lambda(1405)$ in the usual way. Thus, it is interesting that both

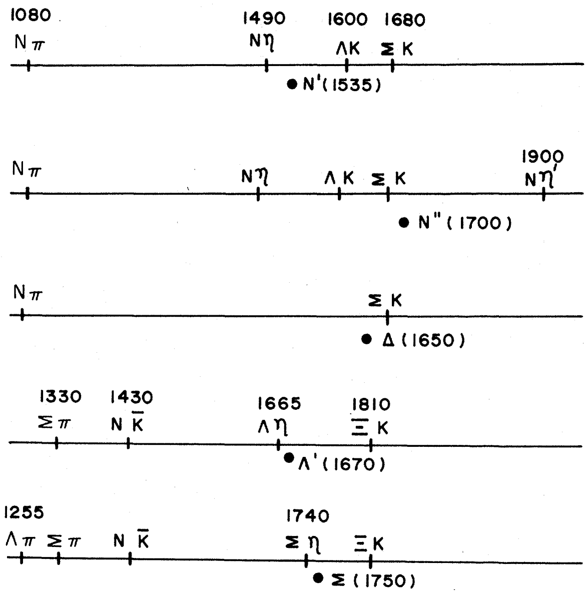

FIG. 8. Baryons coupled to S-wave channels. The threshold energies are in MeV. The hadrons under test are shown as circular dots off the energy axis to indicate that they are complex poles in the T matrix.

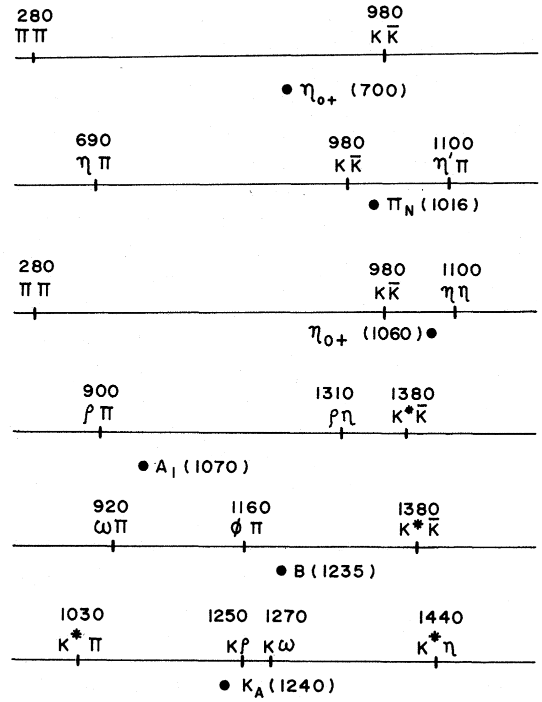

FIG. 9. Mesons coupled to S-wave channels. The threshold energies are in MeV. The hadrons under test are shown as circular dots off the energy axis to indicate that they are complex poles in the T matrix.

$g_{N\bar{K}}^2$ and $g_{\Sigma\pi}^2$ can be obtained from the resonance position and the width and a detailed empirical knowledge of the T matrix is not needed.

Finally we have to strike a note of caution. It has been particularly stressed by Dalitz[18] that our knowledge of the K matrix below the $N\bar{K}$ threshold is based on extrapolation of the experimental information available above this threshold and it may be desirable to obtain more direct knowledge of the K matrix below this threshold. This can be achieved in principle in the "virtual-target" method based on a study of the reaction $\bar{K}d \to \Sigma\pi N$. Such an analysis has been performed by Cline, Laumann, and Mapp[29] and their result is in contradiction with the conclusion that $\Lambda(1405)$ belongs to class A. Specifically, they find the coupling $N\bar{K} \to \Lambda(1405) \to \Sigma\pi$ to be much smaller than what the earlier analyses give. Using Eq. (7.3), this implies $Z \approx 1$. This is an important result since it calls for a drastic modification of most of our earlier knowledge of the K matrix for the $N\bar{K}$-$\Sigma\pi$ system. But further study of the "virtual-target" method is needed before one can reject the earlier conclusion based on so many years of experimental and phenomenological work.

VIII. POSSIBLE APPLICATIONS TO OTHER HADRONS

The possible applications are restricted by the demand that the hadrons be coupled to S-wave two-body channels. The various hadrons[30] to which the test is applicable are indicated in Figs. 8 and 9 in their proper setting with respect to the channels to which they couple. A brief discussion of these follows.

N'(1535). It is known[31] that the two-channel matrix $K^{(2)}$ connecting the open channels $N\pi$ and $N\eta$ has a pole. As already remarked in Sec. V, this is as expected on general grounds. To test for elementarity of $N'(1535)$, one should look at the K matrices with one or more closed channels. This involves fitting the data on associated production: $N\pi \to \Lambda K$ and $N\pi \to \Sigma K$ with $K^{(3)}$ and $K^{(4)}$.

N''(1700). One needs to consider $K^{(5)}$ including the $N\eta'$ channel, where η' is the pseudoscalar meson $\eta'(958)$. This may be hard.

Δ(1650). Here the problem is simpler since one can consider $K^{(2)}$ connecting $N\pi$ and ΣK, but this ignores a dominant decay mode $N\pi\pi$ (73%). Perhaps one can consider a complex $K^{(2)}$ for $N\pi$ and ΣK which takes account of $N\pi\pi$ in an indirect fashion just like the complex phase shifts.[32]

Λ'(1670) and Σ(1750). We should look at $K^{(4)}$ connecting $\Sigma\pi$, $N\bar{K}$, $\Lambda\eta$, and ΞK, and $K^{(5)}$ connecting $\Lambda\pi$, $\Sigma\pi$, $N\bar{K}$, $\Lambda\eta$, and ΞK for $\Lambda'(1670)$ and $\Sigma(1750)$, respectively. This needs fitting with $N\bar{K} \to \Xi K$.

The experimental and phenomenological analyses of these multichannel systems are yet to be done.

So, for the present, we may proceed in the reverse direction. We may use our limited theoretical knowledge of the long-range forces in the baryon-meson channels to conjecture which of the above hadrons may be composites in these channels and thus belong to class A. One may thus guess which of the K matrices will not have poles, and this information may help the phenomenological analyses of these systems. A model based on vector-meson exchange is one such possibility and this leads[33] to $N'(1535)$ and $\Lambda'(1670)$ as composites, while $\Delta(1650)$ is not a composite, the force being repulsive. On this basis we may "predict" that $K^{(4)}$ connecting $N\pi$, $N\eta$, ΛK, and ΣK as well as $K^{(4)}$ connecting $\Sigma\pi$, $N\bar{K}$, $\Lambda\eta$, and ΞK will *not* have poles corresponding to $N'(1535)$ and $\Lambda'(1670)$, whereas $K^{(2)}$ connecting $N\pi$ and ΣK will have a pole corresponding to Δ (1650).

In all the above cases, one can also try to test for nonzero values of Z and Z' by using the sum rules for coupling constants Eqs. (6.8) and (6.9) if the coupling constants are known in some way.[34] In this connection, it is interesting to note that $\Lambda'(1670)$ has a surprisingly large branching ratio (35%) for decay into $\Lambda\eta$ in spite of the tiny phase space available. ($\Lambda\eta$ threshold = 1665 MeV.) This implies a large coupling constant for $\Lambda\eta$. Retaining the $\Lambda\eta$ channel alone in Eq. (6.8) because of its extreme proximity as compared to others, one can see that large $\gamma_{\Lambda\eta}{}^2$ will lead to $Z'\approx 0$ for $\Lambda'(1670)$. This is in agreement with the expectation in the last paragraph.

Let us now consider systems with zero baryonic number (Fig. 9). The K matrices required to test these are harder to get, since the experimental information has to be obtained in a very indirect manner. For instance, one may have to study the process $\pi N \to N K\bar{K}$ and extrapolate to the one-pion-exchange pole in order to extract empirical information on $\pi\pi \to K\bar{K}$. In the cases of higher-spin mesons included in Fig. 9 there are further complications due to the fact that apart from the two-body S-wave channels which alone are indicated in the figure, there are three-body channels as well as two-body D-wave channels with low thresholds. Perhaps these can be handled by using complex K matrices as mentioned earlier.

Application of the coupling-constant sum rules may prove to be easier. The large branching ratio for $\eta_{0+}(1060) \to K\bar{K}$ and the fact that $\pi_N(1016) \to K\bar{K}$ is seen in spite of the small phase space may indicate $Z'\approx 0$ for these.

IX. FINAL REMARKS

a. Levinson's theorem. We have already mentioned (Sec. VII) the slow decrease of the phase shift through $\frac{1}{2}\pi$ at $E\approx 8$ MeV in the case of the deuteron. This may be related to Levinson's theorem which states

$$\delta(E=0)-\delta(E=\infty)=\pi(n_B-n_E), \qquad (9.1)$$

where n_B is the total number of bound states, and n_E is the number of elementary particles in the channel. We know that $\delta(E=0)=\pi$ and the decrease of δ through $\frac{1}{2}\pi$ at about 8 MeV suggests $\delta(E=\infty)=0$. Using this in Eq. (9.1), one can argue for the nonelementarity of the deuteron. A similar argument can be made in the case of $\Lambda(1405)$ because of the possible increase of $\det K^{-1}$ through zero (Sec. VII). Thus, the K-pole test is consistent with Levinson's theorem. However, application of Levinson's theorem requires knowledge of the phase shift at infinite energies and in contrast the K-pole test is based on low-energy data alone.

b. Poles in unphysical sheets. The class-A hadron is also associated with a distinctly different behavior with respect to the poles of the T matrices on the various unphysical Riemann sheets. The T matrices on the different sheets can be obtained by simply changing the signs of one or more elements of the momentum matrix k in the equation

$$T^{-1}=K^{-1}-ik.$$

As a consequence, if there is a pole in K, it is generally followed by poles of T in all the unphysical sheets.[35] But in the case of the class-A hadron, which is a composite in the highest threshold, there is no pole in K although there are poles in the lower-dimensional K matrices defined in Sec. V. Correspondingly, T poles exist in the Riemann sheets reached from the unphysical region by analytic continuation below this threshold, whereas there is no T pole in the sheets obtained by continuation above this threshold. In the case of $\Lambda(1405)$ formed as a composite of $N\bar{K}$, the T matrix in the sheet reached by continuation below the $N\bar{K}$ threshold has a pole, whereas there is no pole in the sheet obtained by continuation above this threshold. This is clearly illustrated in the potential-model calculation of Logan and Wyld.[36]

c. Tests for $l\geqslant 1$. Can we find other methods to distinguish class-A from class-B hadrons in the case of $l\geqslant 1$? We have to look for finer effects in hadron spectroscopy. One such effect is the extraordinary accuracy with which certain mass formulas are satisfied experimentally. For instance, the SU(3) equal-spacing law for the decimet comprising $\Delta(1236)$, $\Sigma(1385)$, $\Xi(1530)$, and $\Omega^-(1672)$ is satisfied to an accuracy of about 3%, whereas if these are composites in the P-wave baryon-meson channels, dynamical calculations predict about 20% deviations (unless fortuitous cancellations are invoked). The hypothesis that these decimet baryons belong to class B and are only weakly coupled to the baryon-meson system leads to a natural understanding of this phenomenon.[37]

d. Regge trajectories. Our classification of the hadrons into classes A and B has the following implications for the Regge trajectories. Since the class-A hadrons are generated by the interactions in the low-threshold channels, we expect them to belong to trajectories of the potential-model type, namely trajectories which turn and fall quickly with respect to energy. Existence of indefinitely rising almost straight-line trajectories has been strongly indicated by recent developments in theory and experiment. A dynamical explanation[38] of such straight-line trajectories would require a global model, the heavy-quark model being one simple example, and so we may associate the class-B hadrons with the rising trajectories. Thus we have

class A ~ potential-model trajectory,

class B ~ indefinitely rising trajectory.

It will be interesting to check whether $\Lambda(1405)$ falls on a trajectory of the potential-model type.

ACKNOWLEDGMENT

I am deeply grateful to Professor R. H. Dalitz for the valuable correspondence I have had with him concerning this problem some years ago as well as recently.

APPENDIX

In the model considered in Sec. III, B_i and P_i could interact only through the intermediate state H. A more realistic model can be obtained by introducing another interaction between B_i and P_i. We shall study the model described by the Hamiltonian[39]

$$\mathcal{H} = m_H^0 H^\dagger H + \sum_i m_i B_i^\dagger B_i + \int d^3k \sum_i \omega_i P_i^\dagger(k) P_i(k) - (4\pi)^{1/2} \sum_i g_i^0 \int \frac{d^3k\, u(k)}{(2\pi)^{3/2}(2\omega_i)^{1/2}} [H^\dagger B_i P_i(k) + \text{H.c.}]$$
$$+ \int \frac{d^3k d^3k' u(k) u(k')}{(2\pi)^3 [2\omega_i(k) 2\omega_j(k')]^{1/2}} \sum_{i,j} h_{ij} B_i^\dagger P_i^\dagger(k) B_j P_j(k'), \tag{A1}$$

where h_{ij} are constants and we consider an arbitrary number of $B_i P_i$ channels.

This model also is exactly soluble and the T matrix for BP scattering is given by

$$\begin{aligned} T_{ij} &= T_{ij}^{(1)} + T_{ij}^{(2)}, \\ T_{ij}^{(1)} &= -u(k_i)u(k_j)[(1+hF)^{-1}h]_{ij}, \\ T_{ij}^{(2)} &= u(k_i)u(k_j)\Gamma_i\Gamma_j D^{-1}, \\ \Gamma_i &= [(1+hF)^{-1}g^0]_i, \\ D^{-1} &= m_H^0 - W - \tilde{g}^0 F(1+hF)^{-1}g^0, \\ F_i &= \frac{1}{\pi}\int_{m_i+\mu_i}^\infty dW' \frac{k_i' u^2(k_i')}{W'-W-i\epsilon}. \end{aligned} \tag{A2}$$

In these equations, matrix notation has been used: h is the square matrix with elements h_{ij}, F is a diagonal matrix with elements F_i, and g^0 is a column matrix with elements g_i^0. The two parts $T^{(1)}$ and $T^{(2)}$ describe the scattering amplitude in the absence of H and the contribution of H, respectively. Γ_i is the complete HB_iP_i vertex function and D^{-1} is the complete H propagator.

The K matrix is obtained by simply replacing F_i by F_i^K, where

$$F_i^K = F_i - ik_i u^2(k_i). \tag{A3}$$

Let us also define Γ_i^K and D^K by replacing *all* F_j occurring in Γ_i and D by F_j^K. We also need

$$F_i^P = \frac{P}{\pi}\int_{m_i+\mu_i}^\infty dW' \frac{k_i' u^2(k_i)}{W'-W} \tag{A4}$$

and we correspondingly define Γ_i^P and D^P.

For clarity we may again concentrate attention on the two-channel situation with the resonance position W_0 occurring between $m_1+\mu_1$ and $m_2+\mu_2$. We have, for the phase shift δ_1,

$$m_1+\mu_1 < W < m_2+\mu_2,$$

$$(k_1 \cot\delta_1)^{-1} = u^2(k_1)\left(\frac{(\Gamma_1^P)^2}{D^P} - \frac{h_{11}+\det hF_2^P}{\det(1+hF^P)}\right). \tag{A5}$$

The resonance position W_0 is determined by[40]

$$D^P(W_0) = 0, \tag{A6}$$

whereas the K pole W_K, if it exists, is given by

$$D^K(W_K) = 0. \tag{A7}$$

The relations between F_i^K and F_i^P are as follows:

$$\begin{aligned} &F_1^P = F_1^K,\ F_2^P = F_2^K \quad \text{for} \quad W > m_2+\mu_2, \\ &F_1^P = F_1^K,\ F_2^P = F_2^K - |k_2| u^2(k_2) \\ &\qquad\qquad \text{for} \quad m_1+\mu_1 < W < m_2+\mu_2. \end{aligned} \tag{A8}$$

In the limit of W_0 occurring very close to $m_2+\mu_2$, we may again regard F_1^K and F_2^K as constants so that from Eqs. (A6) and (A7), we get, after some algebra,

$$E_K - E_0 = -[\Gamma_2^P(W_0)]^2(2\mu_2|E_0|)^{1/2}, \tag{A9}$$

which is the relation analogous to Eq. (3.13) in the text. On the right-hand side of (A9) we have retained only the lowest-order term in $|E_0|^{1/2}$.

The physical coupling constants g_i, the wave-function renormalization constant Z for H, and the HB_iP_i vertex renormalization constants X_i can be defined by the following set of relations:

$$\begin{aligned}
(g_1)^2 &= \lim_{W\to W_0}(W_0 - W)(k_1\cot\delta_1)^{-1} \\
&= \left[(\Gamma_1^P)^2\left(\frac{dD^P}{dW}\right)^{-1}\right]_{W_0} \\
&= (g_1^0)^2 Z X_1^{-2}, \\
(g_2)^2 &= (g_2^0)^2 Z X_2^{-2}, \\
Z^{-1} &= \left(\frac{dD^P}{dW}\right)_{W_0} \\
&= \left(1 + (\Gamma_1^P)^2\frac{dF_1^P}{dW} + (\Gamma_2^P)^2\frac{dF_2^P}{dW}\right)_{W_0}, \\
X_i^{-1} &= (g_i^0)^{-1}\Gamma_i^P(W_0).
\end{aligned} \tag{A10}$$

These relations reduce to Eq. (3.15) if $X_i = 1$. The vertex-renormalization effect is one of the new features of the present model.

In the limit of $|E_0| \to 0$, we have

$$Z \approx \{1 + [\Gamma_2^P(W_0)]^2\mu_2|E_0|^{-1/2}\}^{-1}, \tag{A11}$$

so that $\Gamma_2^P(W_0)$ in Eq. (A9) can be eliminated, leading to the result

$$E_K = \frac{1}{Z}(2 - Z)E_0, \tag{A12}$$

which is the same as before.

In the same limit, the K matrix has the structure

$$K_{ij} \approx \frac{g_i g_j}{Z}\frac{1}{(W_K - W)} + b_{ij}, \tag{A13}$$

where the background term b_{ij} is

$$b_{ij} = -[(1 + hF^K)^{-1}h]_{ij}. \tag{A14}$$

So we have the desired background term in this model. Further, the residues of the K matrix are only "partially bare." They are "bare" with respect to the wave-function renormalization, but "covered" with respect to the vertex renormalization.

The sum rules for coupling constants derived in Sec. IV remain valid. Putting $[\Gamma_2^P(W_0)]^2 = (g_2)^2 Z^{-1}$ in Eq. (A11), we get

$$\left(i(g_2)^2\frac{dk_2}{dW}\right)_{W_0} = 1 - Z. \tag{A15}$$

Finally we may consider the complex coupling constants γ_i defined through the residues at the pole of the T matrix:

$$\begin{aligned}
\gamma_i\gamma_j &= \lim_{W\to W_T}(W_T - W)T_{ij} \\
&= \left[\Gamma_i\Gamma_j\left(\frac{dD}{dW}\right)^{-1}\right]_{W_T} \\
&= \frac{g_i^0 g_j^0}{X_i' X_j'}Z', \\
(Z')^{-1} &= \left(\frac{dD}{dW}\right)_{W_T} \\
&= \left(1 + (\Gamma_1)^2\frac{dF_1}{dW} + (\Gamma_2)^2\frac{dF_2}{dW}\right)_{W_T}, \\
(X_i')^{-1} &= (g_i^0)^{-1}\Gamma_i(W_T),
\end{aligned} \tag{A16}$$

where Z' and X_i' are the complex renormalization constants. Using constant F^K and putting $[\Gamma_i(W_T)]^2 = \gamma_i^2(Z')^{-1}$ in the above expression for Z', we recover the sum rule

$$\left(i\gamma_1^2\frac{dk_1}{dW} + i\gamma_2^2\frac{dk_2}{dW}\right)_{W_T} = 1 - Z'. \tag{A17}$$

---

[1]L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. 101, 453 (1956).

[2]For instance, in a field theory of interacting quarks all hadrons of class B will also be "composites." But they will be composities of quarks, whereas class-A hadrons can be considered as composites of class-B hadrons. In this sense one may say that class-B hadrons are "more elementary" than the class-A hadrons.

[3]G. Rajasekaran, in *Proceedings of the Tenth Symposium on Cosmic Rays, Elementary Particles and Astrophysics, Aligarh, 1967* (Department of Atomic Energy, Bombay), p. 521.

[4]G. Rajasekaran, *Symposia on Theoretical Physics and Mathematics*, edited by A. Ramakrishnan (Plenum, N. Y., 1969), Vol. 9, p. 43.

[5]H. Ezawa, T. Muta, and H. Umezawa, Progr. Theoret. Phys. (Kyoto) 29, 877 (1963).

[6]S. Weinberg, Phys. Rev. 137, B672 (1965).

[7]The same reservation as in our employment of the word "elementary" applies to the use of the wave-function renormalization constant also.

[8]M. J. Vaughn, R. Aaron, and R. D. Amado, Phys. Rev. 124, 1258 (1961).

[9]M. Ross and G. Shaw, Ann. Phys. (N.Y.) 9, 391 (1960); 13, 147 (1961).

[10]This argument is essentially the same as that used in Ref. 6.

[11]$(g_1)^2$, as defined by us, is also equal to the residue of the "reduced K matrix" $(k_1\cot\delta_1)^{-1}$ at its pole W_0.

[12]Of course, to calculate K^{-1} a background term should be added in Eq. (3.18). See Eq. (A13) in the Appendix.

[13]This "baring of the structure" is *only* with respect

to wave-function renormalization. See the Appendix.

[14]For instance, in the case of $\Sigma(1385)$, the three-channel K matrix containing $\Lambda\pi$, $\Sigma\pi$, and $N\bar{K}$ (closed) does have a pole (see Ref. 19) and this is no evidence for elementarity or otherwise of $\Sigma(1385)$ because of the P wave involved.

[15]R. H. Dalitz, Rev. Mod. Phys. 33, 471 (1961).

[16]L. D. Landau, Zh. Eksperim. i Teor. Fiz. 39, 1856 (1960) [Soviet Phys. JETP 12, 1294 (1961)]. I thank Professor V. Singh for originally stimulating my interest in Landau's formula.

[17]Note that Z' is dominantly real and positive to the extent that the decay width of H is small.

[18]For a recent review see R. H. Dalitz, invited talk at the Conference on Hyperon Resonances, Duke University, Durham, N. C., 1970 (unpublished).

[19]J. K. Kim, Phys. Rev. Letters 19, 1074 (1967).

[20]B. R. Martin and M. Sakitt, Phys. Rev. 183, 1345, 1352 (1969).

[21]A. D. Martin and G. G. Ross, Nucl. Phys. B16, 479 (1970).

[22]A. D. Martin, B. R. Martin, and G. G. Ross, Phys. Letters 35B, 62 (1971).

[23]R. H. Dalitz and S. F. Tuan, Ann. Phys. (N.Y.) 10, 307 (1960).

[24]R. H. Dalitz, T. C. Wong, and G. Rajasekaran, Phys. Rev. 153, 1617 (1967).

[25]J. K. Kim and F. von Hippel, Phys. Rev. 184, 1961 (1969). Apart from a factor 4π, our definition of the coupling constants differs from that of Kim and von Hippel in another respect: Our coupling constants include the statistical factor for different charge states, 3 for $\Sigma\pi$ and 2 for $N\bar{K}$.

[26]C. Weil, Phys. Rev. 161, 1682 (1967).

[27]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[28]This is true even for the coupling constants defined through the residues of the T matrix. Although the T matrix treats the closed and open channels alike, this similarity exists only at the formal level and cannot extend to the numerical values. In Eq. (6.6), dk_1/dW is dominantly real, while dk_2/dW is dominantly imaginary at the pole W_T.

[29]D. Cline, R. Laumann, and J. Mapp, Phys. Rev. Letters 26, 1195 (1971).

[30]Particle Data Group, Rev. Mod. Phys. 43, S1 (1971).

[31]F. Uchiyama-Campbell and R. K. Logan, Phys. Rev. 149, 1220 (1966).

[32]Complex K matrix has been used for $N\pi \to N\eta$ by Hendry and Moorhouse [A. W. Hendry and R. G. Moorhouse, Phys. Letters 18, 171 (1965)].

[33]H. W. Wyld, Jr., Phys. Rev. 155, 1649 (1967).

[34]Note that SU(3) cannot be used to determine unknown coupling constants from known ones since violent breaking of SU(3) is expected as in the case of $\Lambda(1405)$.

[35]See Sec. 2.4. in G. Rajasekaran, Nuovo Cimento 37, 1004 (1965).

[36]R. K. Logan and H. W. Wyld, Jr., Phys. Rev. 158, 1467 (1967).

[37]G. Rajasekaran, Phys. Rev. 159, 1488 (1967).

[38]See for instance the concluding chapter in P. D. B. Collins, Phys. Reports 1C, No. 4 (1971).

[39]This is a generalization of the model considered by Passi [J. N. Passi, Phys. Rev. D 2, 310 (1970)].

[40]Since we are studying the possible association of the resonance with the "elementary" particle H, we are not interested in the zero of $\det(1+hF^P)$.