Nonleptonic Decays of Hyperons*

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Once-subtracted dispersion relations are written for the amplitudes of the nonleptonic decays of hyperons. The subtraction constants are given by equal-time commutators and, after the low-mass baryon intermediate states are separated, the remaining dispersion integrals over higher-mass resonances are evaluated in the framework of the Regge theory. A four-parameter best-fit solution to both the s- and p-wave amplitudes is found to be in very good agreement with experiment.

I. INTRODUCTION

Ever since the powerful techniques of current algebra have been applied to nonleptonic decays of hyperons, each new effort to calculate the s- and p-wave amplitudes for these decays has led to more numerical puzzles than would seem reasonable for such deceptively simple processes. In fact, after the initial success of Suzuki and Sugawara[1] in describing the s-wave amplitude on the basis of current algebra, partial conservation of axial-vector current (PCAC), and soft-pion extrapolation, several authors[2,3] have attempted simultaneous fits of both the s- and p-wave amplitudes. The only partial agreement of their results with experiment has led to refinements to the theory in various forms,[4-6] but without appreciable changes in the discrepancy, or with the introduction of too many parameters and *ad hoc* additions.

Here we present a new analysis of the problem by applying dispersion-relation techniques to the scattering of a spurion from an hyperon. In a particular form of amplitudes considered by Okubo,[7] the scattering process formally reduces to the decay process in the limit of vanishing four-momentum of the spurion. In this approach the Regge behavior of the scattering amplitude at high energies requires one subtraction to the dispersion relation in the energy variable, the subtraction point being chosen such that the calculable soft-pion amplitude gives the subtraction constant. Then the low-mass baryon pole contribution is separated from the dispersion integral, the remaining part of the integral coming from higher-mass resonances. Difficulties in evaluating this latter resonance contribution with a minimum of free parameters has previously led to its neglect without justification.

We propose, within the scheme of current×current interaction and octet dominance, to implement the dispersion approach by making further application of the Regge theory. Our method of evaluating the resonance contribution consists in assuming Regge behavior for the scattering amplitude at high energies and extrapolating this form of amplitude to the lower-energy region. In this way the result of the higher-mass integration is just the real part of the Regge amplitude from the t-channel exchanges, slightly modified due to the once-subtracted form of the dispersion relation. This approach has its qualitative justification in the concept of local duality which has been explored in the realm of high-energy phenomenology.[8] Simply stated, local duality says that the Regge amplitude, when extrapolated to lower energies, represents the true amplitude in an average sense.

In Sec. II we present the formalism and show explicitly how to evaluate the resonance contribution in the framework of the Regge theory and local duality. Parametrization of the s- and p-wave amplitudes in terms of unknown weak coupling constants is given in Sec. III, and in Sec. IV we discuss the results obtained. An appendix gives the final expressions used for the numerical fit.

II. FORMALISM

The energy-momentum conservation for the decay

$$\alpha(p) \to \beta(p') + \pi^a(q') \tag{1}$$

requires $p = p' + q'$. Then the invariant s- and p-wave amplitudes are constants, and we are not able to write dispersion relations directly for this reaction. This difficulty can be avoided by considering the associated scattering process of a spurion κ from the hyperon α,

$$\kappa(q) + \alpha(p) \to \beta(p') + \pi^a(q'), \tag{2}$$

and properly taking the limit $q \to 0$ in the scattering amplitudes so as to recover the amplitudes for the decay (1).

The transition amplitude for (2) can be written[9]

$$M(s,t,u)=\bar{u}(p')\{F_1(s,t,u)-\gamma_5F_2(s,t,u)+\tfrac{1}{2}\gamma\cdot(q+q')[G_1(s,t,u)+\gamma_5G_2(s,t,u)]\}u(p)\,, \tag{3}$$

where the invariant amplitudes F_i and G_i are functions of the Mandelstam variables

$$\begin{aligned} s&=(p+q)^2=(p'+q')^2\,,\\ t&=(p-p')^2=(q'-q)^2\,,\\ u&=(p-q')^2=(p'-q)^2\,,\\ s+t+u&=M_\alpha^2+M_\beta^2+m_\pi^2+q^2\,. \end{aligned} \tag{4}$$

It is convenient to take s, t, q^2 as independent variables. Also, using the Dirac equation, we can express amplitude (3) in a more suitable form as

$$M(s,t,q^2)=\bar{u}(p')\{H_1(s,t,q^2)-\gamma_5H_2(s,t,q^2)+[\gamma\cdot q,\gamma\cdot q']_-[J_1(s,t,q^2)-\gamma_5J_2(s,t,q^2)]\}u(p)\,, \tag{5}$$

where

$$H_1\equiv F_1+\frac{s-u}{2(M_\alpha+M_\beta)}G_1\,,$$

$$H_2\equiv F_2-\frac{s-u}{2M_\alpha-M_\beta)}G_2\,,$$

$$J_1\equiv\frac{G_1}{2(M_\alpha+M_\beta)},\quad\text{and}\quad J_2\equiv\frac{G_2}{2(M_\alpha-M_\beta)}.$$

In the limit $q\to 0$ we see from (4) that $s=M_\alpha^2$, $t=q'^2=m_\pi^2\simeq 0$, and $q^2=0$ so that the transition matrix for the decay (1) is given by

$$M(M_\alpha^2,0,0)=\bar{u}(p')[H_1(M_\alpha^2,0,0)-\gamma_5H_2(M_\alpha^2,0,0)]u(p)\,. \tag{6}$$

We formally identify $H_1(M_\alpha^2,0,0)$ and $H_2(M_\alpha^2,0,0)$ with the s- and p-wave decay amplitudes, respectively,

$$A=H_1(M_\alpha^2,0,0)\quad\text{and}\quad B=H_2(M_\alpha^2,0,0)\,. \tag{7}$$

Before we write down dispersion relations in s and fixed t, q^2 for the amplitude $H_i(s,t,q^2)$, we have to settle the question of subtractions. Assuming Regge behavior for $H_i(s,t,q^2)$ at high energies, the need for subtraction depends on the trajectories exchanged in the t channel. For the s-wave amplitude, $H_1(s,t,q^2)$, the exchanged trajectory in the t channel is that of K^*, whereas for the p-wave amplitude, $H_2(s,t,q^2)$, we have the trajectories of K and K_A exchanged. Although the trajectory parameters for these mesons are not firmly established at present, under the reasonable assumption of a linear trajectory with a universal slope of about 1 GeV^{-2}, we obtain the following intercepts for K^*, K, and K_A, respectively:

$$\alpha_V\simeq 0.25\,,\quad \alpha_P\simeq -0.25\,,\quad \alpha_A\simeq -0.75\,. \tag{8}$$

These numbers imply one subtraction for H_1 and none for H_2. To stay on the safer side, however, we make one subtraction to both amplitudes and write the dispersion relation

$$H_i(s,t,q^2)=H_i(s_0,t,q^2)+\frac{1}{\pi}(s-s_0)\int_{-\infty}^{+\infty}ds'\frac{\mathrm{Im}H_i(s',t,q^2)}{(s'-s_0)(s'-s-i\epsilon)}, \tag{9}$$

where the subtraction point s_0 should be chosen in such a way that we are able to compute the subtraction constant $H_i(s_0,t,q^2)$. For this purpose recall that in the soft-pion calculation we let $q'\to 0$ and find the extrapolated amplitude in terms of an equal-time commutator. In this limit $q'\to 0$ we obtain $s=p'^2=M_\beta^2$ and $t=q^2$. Therefore, if we choose $s_0=M_\beta^2$ and let $q\to 0$ in order to recover the desired decay amplitudes, the subtraction constant $H_i(M_\beta^2,0,0)$ will be given by the soft-pion amplitude.

There are three reasons why we wish to separate the pole contribution from the dispersion integral in (9). First, we need the pole term to remove the well-known ambiguity[10] associated with the soft-pion extrapolation. Second, the soft-pion term (subtraction constant) plus the pole contribution reproduce the old results so that the magnitude and form of the remaining integral provides a possible explanation for the numerical puzzles associated with the problem. Third, the removal of the pole makes the idea of extrapolating the Regge amplitude down to low energies more plausible, as discussed below. Then, denoting H_i^P for the pole contribution to the dispersion integral, we rewrite (9) as

$$H_i(s)=H_i(s_0)+H_i^P(s)+\frac{1}{\pi}(s-s_0)\int_{-\infty}^{+\infty}ds'\frac{\mathrm{Im}H_i'(s')}{(s'-s_0)(s'-s-i\epsilon)}, \tag{10}$$

where H_i' stands for the remaining part of H_i after the removal of the one-baryon intermediate states. Also the arguments of the H's have been simplified for ease of notation. Note that the point $s=M_\alpha^2$, to which we wish to extrapolate in order to obtain the decay amplitude, lies at the lower end of the resonance region, and since we already extracted the large pole contribution from this region, the remaining amplitude H_i' can be assumed to be represented approximately by the extrapolated Regge amplitude according to the concept of local duality. In what follows we elaborate on this qualitative idea and evaluate the remaining integral explicitly.

With the help of an identity, the integral in (10) can be written as

$$\int_{-\infty}^{+\infty} ds' \frac{\mathrm{Im}H_i'(s')}{(s'-s_0)(s'-s-i\epsilon)} = \frac{i\pi}{s-s_0}\mathrm{Im}H_i'(s) + P\int_{-\infty}^{+\infty} ds' \frac{\mathrm{Im}H_i'(s')}{(s'-s_0)(s'-s)}. \tag{11}$$

Now the assumption of local duality implies

$$P\int_{-\infty}^{+\infty} ds' \frac{\mathrm{Im}H_i'(s')}{(s'-s_0)(s'-s)} = P\int_{-\infty}^{+\infty} ds' \frac{\mathrm{Im}H_i^R(s')}{(s'-s_0)(s'-s)}, \tag{12}$$

where the Regge amplitude[11] $H_i^R(s)$ has the form

$$H_i^R(s) \equiv \gamma_i \frac{1 \mp e^{-i\pi\alpha_i}}{\sin\pi\alpha_i} s^{\alpha_i}. \tag{13}$$

Here the minus sign refers to K^* and K_A trajectories, and the plus sign to that of K. Inserting the Regge form (13) into Eq. (12) and performing the integration, we obtain

$$\int_{-\infty}^{+\infty} ds' \frac{\mathrm{Im}H_1'(s')}{(s'-s_0)(s'-s-i\epsilon)} = \frac{i\pi}{s-s_0}\mathrm{Im}H_1'(s) + \frac{\pi\gamma_V}{s-s_0}(s^{\alpha_V} - s_0{}^{\alpha_V})\tan\tfrac{1}{2}\pi\alpha_V, \tag{14a}$$

$$\int_{-\infty}^{+\infty} ds' \frac{\mathrm{Im}H_2'(s')}{(s'-s_0)(s'-s-i\epsilon)} = \frac{i\pi}{s-s_0}\mathrm{Im}H_2'(s) + \frac{\pi\gamma_A}{s-s_0}(s^{\alpha_A} - s_0{}^{\alpha_A})\tan\tfrac{1}{2}\pi\alpha_A - \frac{\pi\gamma_P}{s-s_0}(s^{\alpha_P} - s_0{}^{\alpha_P})\cot\tfrac{1}{2}\pi\alpha_P. \tag{14b}$$

Note that the equality (12) is less restrictive than local duality itself. By taking the value of the integrals obtained in (14) into the expression (10), we get

$$A = K_1 + \mathrm{Re}H_1^P(M_\alpha{}^2) + \gamma_V(M_\alpha{}^{2\alpha_V} - M_\beta{}^{2\alpha_V})\tan\tfrac{1}{2}\pi\alpha_V,$$

$$B = K_2 + \mathrm{Re}H_2^P(M_\alpha{}^2) + \gamma_A(M_\alpha{}^{2\alpha_A} - M_\beta{}^{2\alpha_A})\tan\tfrac{1}{2}\pi\alpha_A - \gamma_P(M_\alpha{}^{2\alpha_P} - M_\beta{}^{2\alpha_P})\cot\tfrac{1}{2}\pi\alpha_P, \tag{15}$$

where we have written K_i for the current-algebra term and have eliminated the soft-pion ambiguity between the surface and the pole terms. From the intercepts given in (8) and the small mass splitting among the octet baryons, we see that $M_\alpha{}^{2\alpha_P} - M_\beta{}^{2\alpha_P}$ is proportional to $M_\alpha{}^{2\alpha_A} - M_\beta{}^{2\alpha_A}$. Also recent studies[12] suggest the same D/F ratio for the strong $\bar{B}BK$ and $\bar{B}BK_A$ vertices. Therefore, we can replace the two resonance contributions to the p-wave amplitude B by an effective term and write

$$A = K_1 + \mathrm{Re}H_1^P(M_\alpha{}^2) + \gamma_V(M_\alpha{}^{2\alpha_V} - M_\beta{}^{2\alpha_V})\tan\tfrac{1}{2}\pi\alpha_V, \tag{16a}$$

$$B = K_2 + \mathrm{Re}H_2^P(M_\alpha{}^2) + \gamma'(M_\alpha{}^{2\alpha_A} - M_\beta{}^{2\alpha_A})\tan\tfrac{1}{2}\pi\alpha_A. \tag{16b}$$

Of course the form of the resonance terms is reminiscent of the t-channel pole models.[13]

III. PARAMETRIZATION OF THE s- AND p-WAVE AMPLITUDES

The strong $\bar{B}BP$ and the weak $\bar{B}B\kappa$ vertices are parametrized[14] as follows:

$$\langle\beta(p')|j_P^a(0)|\delta(p_n)\rangle = \frac{1}{(2\pi)^3}\frac{1}{N_\beta N_\delta}\bar{u}(p')i\gamma_5 u(p_n)K^a_{\beta\delta}, \tag{17}$$

$$\langle\beta(p')|\mathcal{H}(0)|\delta(p_n)\rangle = \frac{1}{(2\pi)^3}\frac{1}{N_\beta N_\delta}\bar{u}(p')(c_{\beta\delta} - \gamma_5 v_{\beta\delta})u(p_n), \tag{18}$$

where

$$N_\beta \equiv (E_\beta/M_\beta)^{1/2}, \quad E_\beta = (\vec{p}'^2 + M_\beta{}^2)^{1/2},$$

$$K^a_{\beta\delta} \equiv 2g_{\pi NN}[\alpha F^a_{\beta\delta} + (1-\alpha)D^a_{\beta\delta}],$$

$$c_{\beta\delta} \equiv 2(fF^6_{\beta\delta} + dD^6_{\beta\delta}),$$

$$v_{\beta\delta} \equiv 2iG[\alpha F^7_{\beta\delta} + (1-\alpha)D^7_{\beta\delta}].$$

Here $F^a_{\beta\delta} = -if_{a\beta\delta}$ and $D^a_{\beta\delta} = d_{a\beta\delta}$. In the tadpole model[4] G takes the form

$$G = \xi g_{\pi NN}\frac{A(K_1^0 \to 0)}{m_K{}^2}. \tag{19}$$

The amplitude $A(K_1^0 \to 0)$ for the transition of the K_1^0 into the vacuum can be related to the amplitude $A(K_1^0 \to 2\pi^0)$ in the soft-pion limit: $A(K_1^0 \to 0) = 2F_\pi{}^2 A(K_1^0 \to 2\pi^0)$ with $F_\pi = 0.95 m_\pi$. The suppression factor ξ accounts for the coupling-constant shift from the SU(3) values in the $\bar{B}BK$ vertex as well as for the uncertainty involved in using the physical value of $A(K_1^0 \to 2\pi^0)$ instead of the unknown soft-pion limiting value. A dispersion-theoretical evaluation[15] of the $\bar{B}BK$ coupling constants found consistency with the SU(3) values, but bootstrap calculations[16] suggested a considerable reduction from those values. Regarding the uncertainty involved in the soft-pion extrapolation, a hard-pion study[17] of the decays $K\to 2\pi$ yielded corrections to the usual current-algebra values of about 10%. Therefore, assuming relatively small coupling-constant shift from SU(3) values, we expect ξ to be nearly unity.

Furthermore, we assume the coupling of the K^* to the baryons to be of F type only, whereas the $\bar{B}BK_A$ and $\bar{B}BK$ vertices have both F- and D-type couplings with the same mixing parameter α for both vertices. As for the weak vertex of the resonance term, we take it proportional to D^6 for both the s- and p-wave amplitudes.[13] With the definitions and assumptions stated above, the s- and p-wave amplitudes (16) can be written more explicitly as

$$A=\frac{\sqrt{2}}{F_\pi}[F^a,c]_{-\beta\alpha}+\frac{(M_\alpha-M_\beta)K^a_{\beta\delta}v_{\delta\alpha}}{(M_\delta+M_\beta)(M_\alpha+M_\delta)}-\frac{(M_\alpha-M_\beta)v_{\beta\gamma}K^a_{\gamma\alpha}}{(M_\alpha+M_\gamma)(M_\gamma+M_\beta)}+\gamma_V K^b_{\beta\alpha}d_{6ab}(M_\alpha^{2\alpha_V}-M_\beta^{2\alpha_V})\tan\tfrac{1}{2}\pi\alpha_V\,, \tag{20a}$$

$$B=\frac{\sqrt{2}}{F_\pi}[F^a,v]_{-\beta\alpha}+\frac{(M_\alpha+M_\beta)K_{\beta\delta}c_{\delta\alpha}}{(M_\delta+M_\beta)(M_\alpha-M_\delta)}-\frac{(M_\alpha+M_\beta)c_{\beta\gamma}K^a_{\gamma\alpha}}{(M_\alpha+M_\gamma)(M_\gamma-M_\beta)}+\gamma' K^b_{\beta\alpha}d_{6ab}(M_\alpha^{2\alpha_A}-M_\beta^{2\alpha_A})\tan\tfrac{1}{2}\pi\alpha_A\,, \tag{20b}$$

where we have extracted the SU(3) dependence from γ_V and γ' defined in (16), and introduced the current-algebra and pole terms given in previous analyses.[3,6] For convenience we rewrite the above expressions in terms of current-algebra (C), pole (P), and resonance (R) parts:

$$\begin{aligned}A&=\frac{1}{F_\pi}A^C+R\frac{G}{m_\pi}A^P+A^R\,,\\ B&=\frac{G}{F_\pi}B^C+\frac{R}{m_\pi}B^P+B^R\,,\end{aligned} \tag{21}$$

where $R\equiv g_{\pi NN}m_\pi/\sqrt{2}M_N$ and, for example, in the case of the Λ decay,

$$A^C(\Lambda^0_-)=\frac{1}{\sqrt{6}}(3f+d)\,,$$

$$A^P(\Lambda^0_-)=\frac{1}{\sqrt{6}}\frac{M_\Lambda-M_N}{M_N+M_N}\frac{2M_N}{M_\Lambda+M_N}+\frac{2}{\sqrt{6}}(1-\alpha)(1-2\alpha)\frac{M_\Lambda-M_N}{M_\Lambda+M_\Sigma}\frac{2M_N}{M_\Sigma+M_N}\,,$$

$$A^R(\Lambda^0_-)=\gamma_V\left(\frac{3}{2}\right)^{1/2}(M_\Lambda^{2\alpha_V}-M_N^{2\alpha_V})\tan\tfrac{1}{2}\pi\alpha_V\,,$$

$$B^C(\Lambda^0_-)=\frac{1}{\sqrt{6}}(1+2\alpha)\,,$$

$$B^P(\Lambda^0_-)=\frac{1}{\sqrt{6}}(3f+d)\frac{M_\Lambda+M_N}{M_N+M_N}\frac{2M_N}{M_\Lambda-M_N}-\frac{2}{\sqrt{6}}(1-\alpha)(f-d)\frac{M_\Lambda+M_N}{M_\Lambda+M_\Sigma}\frac{2M_N}{M_\Sigma-M_N}\,,$$

$$B^R(\Lambda^0_-)=\gamma'\frac{1}{\sqrt{6}}(1+2\alpha)(M_\Lambda^{2\alpha_A}-M_N^{2\alpha_A})\tan\tfrac{1}{2}\pi\alpha_A\,.$$

The parameters γ_V and γ' have been redefined to absorb over-all factors. The other relevant decay amplitudes are given in the Appendix.

Masses in the resonance terms are specified in GeV, and the values of the fixed constants are

$$\frac{1-\alpha}{\alpha}\simeq\sqrt{3}\,,\quad \frac{g_{\pi NN}^{\;2}}{4\pi}=14.6\,,$$

$$A(K^0_1\to 2\pi^0)=2.58\times10^{-4}\ \text{MeV}\,.$$

With $\xi=0.7$ we obtain $G=11.25m_\pi$ in units of $10^5 m_\pi^{-1/2}$ sec$^{-1/2}$. This leaves four adjustable parameters: f, d, γ_V, and γ'. The best fit to the experimental amplitudes gave the following values for these parameters: $f=1.513m_\pi$, $\gamma_V=20.52$, $\gamma'=-3.25$ in units of $10^5 m_\pi^{-1/2}$ sec$^{-1/2}$, and $d/f=-1.09$. The corresponding theoretical amplitudes are displayed in Table I with the contributions of each part given explicitly. Also shown are the experimental values.[18]

IV. DISCUSSION

Table I shows a very good over-all fit, being sufficiently accurate to discriminate between the two experimental values for the decay Σ^+_0. The only appreciable deviation occurs in the amplitude $A(\Sigma^+_+)$ in which none of the four adjustable parameters appears. Then, in the framework of current $\times$current interaction and octet dominance, the observed value $A(\Sigma^+_+)\simeq 0$ requires the symmetry-breaking terms to be negligible. However, deviations from octet dominance and from unitary symmetry have been found to occur in many other decays, indicating that the interpretation of the observed value $A(\Sigma^+_+)\simeq 0$ as an evidence for the simultaneous validity of octet dominance and unitary symmetry to a high degree of accuracy does not seem plausible. Instead, we argue that these two types of deviations tend to cancel each other in the amplitude $A(\Sigma^+_+)$ so as to reproduce the observed value. On the other hand, the vanishing p-wave amplitude, $B(\Sigma^-_-)\simeq 0$, has all three parts combined

TABLE I. Best-fit solution to both the s- and p-wave amplitudes and experimental values in units of $10^5 m_\pi^{-1/2}\ \mathrm{sec}^{-1/2}$.

	Current algebra	Baryon pole	Resonance	Total (theory)	Experimental values
$A(\Lambda^0_-)$	1.241	1.113	−0.909	1.444	1.545 ± 0.024
$A(\Sigma^-_-)$	3.330	−0.437	−1.047	1.846	1.859 ± 0.017
$A(\Sigma^+_0)$	2.355	0.151	−0.740	1.766	1.568 ± 0.142
					(1.155 ± 0.187)
$A(\Sigma^+_+)$	0	−0.942	0	−0.942	0.016 ± 0.034
$A(\Xi^-_-)$	2.660	0.489	−0.954	2.195	2.020 ± 0.029
$B(\Lambda^0_-)$	8.374	4.090	−1.390	11.074	10.644 ± 0.475
$B(\Sigma^-_-)$	−3.173	2.357	0.697	−0.120	−0.549 ± 0.386
$B(\Sigma^+_0)$	−2.244	−11.203	0.493	−12.954	−11.573 ± 1.880
					(−15.713 ± 1.420)
$B(\Sigma^+_+)$	0	18.200	0	18.200	19.078 ± 0.347
$B(\Xi^-_-)$	2.244	−9.780	−0.278	−7.814	−6.831 ± 0.574

to reproduce the observed value. Thus, having several adjustable parameters, the addition of the 27 representation can be made without spoiling the present good fit for this amplitude. Similar arguments hold for the other amplitudes. In the approximation of octet dominance, therefore, we do not see the nonvanishing result for $A(\Sigma^+_+)$ in Table I as unreasonable, although the validity of our arguments remains to be seen.

Another important result displayed in Table I concerns the relative magnitude of the resonance contribution to the s-wave amplitude. Note that a suitable choice of the d/f ratio ($\simeq -0.3$) makes the current-algebra term for the s wave proportional to the observed amplitude. This has previously led to the belief that the current-algebra term was the dominant one, so that the assumption of negligible higher-mass baryon contribution to the s wave was considered to be on a good footing. Table I does not support this assumption. In contrast, the assumption works well for the p wave, explaining the rough fit of the old p-wave pole-model results.

Finally, the magnitude of the current-algebra contribution to the p-wave amplitude indicates a relatively large violation of unitary symmetry, a result quite different from that of any previous work.

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APPENDIX

For each specific decay the general expressions (20) yield the following amplitudes.

$\Sigma^- \to n + \pi^-$:

$$A^C = f - d,$$

$$A^P = -\tfrac{1}{3}(1-\alpha)(1+2\alpha)\frac{M_\Sigma - M_N}{M_\Sigma + M_\Lambda}\frac{2M_N}{M_\Lambda + M_N} + \alpha(1-2\alpha)\frac{M_\Sigma - M_N}{M_\Sigma + M_\Sigma}\frac{2M_N}{M_\Sigma + M_N},$$

$$A^R = \gamma_V(M_\Sigma^{2\alpha_V} - M_N^{2\alpha_V})\tan\tfrac{1}{2}\pi\alpha_V\,.$$

$$B^C = (2\alpha - 1),$$

$$B^P = \alpha(f-d)\frac{M_\Sigma + M_N}{M_\Sigma + M_\Sigma}\frac{2M_N}{M_\Sigma - M_N} - \tfrac{1}{3}(1-\alpha)(3f+d)\frac{M_\Sigma + M_N}{M_\Lambda + M_\Sigma}\frac{2M_N}{M_\Lambda - M_N},$$

$$B^R = \gamma'(2\alpha - 1)(M_\Sigma^{2\alpha_A} - M_N^{2\alpha_A})\tan\tfrac{1}{2}\pi\alpha_A\,.$$

$\Sigma^+ \to p + \pi^0$:

$$A^C = \frac{1}{\sqrt{2}}(f-d),$$

$$A^P = \frac{1}{\sqrt{2}}(1-2\alpha)\frac{M_\Sigma - M_N}{M_N + M_N}\frac{2M_N}{M_\Sigma + M_N} - \frac{2}{\sqrt{2}}\alpha(1-2\alpha)\frac{M_\Sigma - M_N}{M_\Sigma + M_\Sigma}\frac{2M_N}{M_\Sigma + M_N},$$

$$A^R = \gamma_V \frac{1}{\sqrt{2}} (M_\Sigma^{2\alpha_V} - M_N^{2\alpha_V}) \tan\tfrac{1}{2}\pi\alpha_V ,$$

$$B^C = \frac{1}{\sqrt{2}}(2\alpha - 1),$$

$$B^P = -\frac{1}{\sqrt{2}}(f-d)\frac{M_\Sigma + M_N}{M_N + M_N}\frac{2M_N}{M_\Sigma - M_N} + \frac{2}{\sqrt{2}}\alpha(f-d)\frac{M_\Sigma + M_N}{M_\Sigma + M_\Sigma}\frac{2M_N}{M_\Sigma - M_N},$$

$$B^R = \gamma' \frac{1}{\sqrt{2}}(2\alpha - 1)(M_\Sigma^{2\alpha_A} - M_N^{2\alpha_A})\tan\tfrac{1}{2}\pi\alpha_A .$$

$\Sigma^+ \to n + \pi^+$:

$$A^C = 0,$$

$$A^P = -(1-2\alpha)\frac{M_\Sigma - M_N}{M_N + M_N}\frac{2M_N}{M_\Sigma + M_N} - \tfrac{1}{3}(1-\alpha)(1+2\alpha)\frac{M_\Sigma - M_N}{M_\Sigma + M_\Lambda}\frac{2M_N}{M_\Lambda + M_N} + \alpha(1-2\alpha)\frac{M_\Sigma - M_N}{M_\Sigma + M_\Sigma}\frac{2M_N}{M_\Sigma + M_N},$$

$$A^R = 0,$$

$$B^C = 0,$$

$$B^P = (f-d)\frac{M_\Sigma + M_N}{M_N + M_N}\frac{2M_N}{M_\Sigma - M_N} - \tfrac{1}{3}(1-\alpha)(3f+d)\frac{M_\Sigma + M_N}{M_\Sigma + M_\Lambda}\frac{2M_N}{M_\Lambda - M_N} - \alpha(f-d)\frac{M_\Sigma + M_N}{M_\Sigma + M_\Sigma}\frac{2M_N}{M_\Sigma - M_N},$$

$$B^R = 0 .$$

$\Xi^- \to \Lambda + \pi^-$:

$$A^C = \frac{1}{\sqrt{6}}(3f - d),$$

$$A^P = \frac{2}{\sqrt{6}}(1-\alpha)\frac{M_\Xi - M_\Lambda}{M_\Sigma + M_\Lambda}\frac{2M_N}{M_\Xi + M_\Sigma} - \frac{1}{\sqrt{6}}(1-2\alpha)(4\alpha-1)\frac{M_\Xi - M_\Lambda}{M_\Xi + M_\Xi}\frac{2M_N}{M_\Xi + M_\Lambda},$$

$$A^R = \gamma_V \sqrt{3/2}(M_\Xi^{2\alpha_V} - M_\Lambda^{2\alpha_V})\tan\tfrac{1}{2}\pi\alpha_V ,$$

$$B^C = \frac{1}{\sqrt{6}}(4\alpha - 1),$$

$$B^P = \frac{2}{\sqrt{6}}(1-\alpha)(f+d)\frac{M_\Xi + M_\Lambda}{M_\Sigma + M_\Lambda}\frac{2M_N}{M_\Xi - M_\Sigma} - \frac{1}{\sqrt{6}}(1-2\alpha)(3f-d)\frac{M_\Xi + M_\Lambda}{M_\Xi + M_\Xi}\frac{2M_N}{M_\Xi - M_\Lambda},$$

$$B^R = \gamma' \frac{1}{\sqrt{6}}(4\alpha - 1)(M_\Xi^{2\alpha_A} - M_\Lambda^{2\alpha_A})\tan\tfrac{1}{2}\pi\alpha_A .$$

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