along our contour correspond to the scaling points. Following the remark in Ref. 6, the sum rules become identities as $m_0/m \to \infty$. For $m_0 >> m$, m_0/m finite, it would be more precise to subdivide the unit semicircle into three regions instead of two: (a) The Regge region, $0 < \phi < O(m^2/m_0^2)$, (b) The Bjorken-limit region, $O(m^2/m_0^2) \lesssim \phi \lesssim O(m/m_0)$, and $O(m_0/m) \gtrsim |\omega| \gtrsim O(1)$, and (c) The Johnson-Low-Bjorken-limit region, $O(m/m_0) \lesssim \phi \lesssim \pi$, and $O(1) \gtrsim |\omega| \gtrsim 0$. The JLB for $\mathcal{T}_2(\omega, q^2)$, $|q^2| \to \infty$, $|\omega| \to 0$, is also given by $\mathfrak{F}_2(\omega)$ for $|\omega| \to 0$. In this limit $\mathcal{T}_2$ vanishes as $O(\omega)$ and is given by $\mathcal{T}_2 \cong 4\omega \int_1^\infty F_2(\omega')/\omega'^2 d\omega'$. This last remark does not change the form of the sum rules (3.13) and (3.15).

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Renormalizable Symmetry Model of Weak Interactions*

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A renormalizable model of weak interactions is presented. The leptons and the SU(3) quarks are classified in an SU(4)×SU(2) symmetry scheme in which the quantum numbers are charge, baryon number, lepton number, and weakness – a quantum number that replaces the usual second lepton number. The coupling of a sextet of spinless bosons to the leptons and hadrons gives a theory of weak interactions that has universality, the correct selection rules, renormalizability, conserved vector current, and $V-A$ in the local limit. Elastic $e\nu_\mu$ scattering is predicted. The intermediate scalar boson leads to μe and ee pairs in the conventional high-energy neutrino experiments but no $\mu\mu$ pairs are predicted. Some neutral-current effects are expected and these are consistent with present data.

I. INTRODUCTION

The development of a renormalizable theory of weak interactions has received considerable attention,[1,2] but so far no entirely satisfactory solution has been proposed. Soon after weak interactions were found to proceed by the $V-A$ current-current Hamiltonian,[3] a renormalizable theory using spin-zero bosons was proposed by Tanikawa and Watanabe.[1] Their theory was based on the fact that a $V-A$ interaction can be reexpressed in terms of scalar and pseudoscalar interactions by means of the Fierz transformation[4]

$$\overline{\psi}_a\gamma_\mu(1+\gamma_5)\psi_b\overline{\psi}_c\gamma_\mu(1+\gamma_5)\psi_d = -2\overline{\psi}_a(1-\gamma_5)\psi_{\bar{c}}\overline{\psi}_{\bar{b}}(1+\gamma_5)\psi_d, \quad (1)$$

where $\bar{b}$ and $\bar{c}$ are the antiparticles of b and c. By introducing a semiweak coupling of spin-zero bosons to the various densities of the form $\overline{\psi}_{\bar{b}}(1+\gamma_5)\psi_d$, a renormalizable theory is produced.

The two main criticisms[5] of the Tanikawa-Watanabe approach are that universality is accidental and the conserved vector current plays no role. Of these two faults the former is more serious as the conserved vector current can be introduced in a number of ways. For example, one could couple the pseudoscalar mesons to intermediate spinor particles in such a way that the lepton current in the effective second-order Lagrangian is coupled to the conserved isospin current if the right relations exist between the coupling constants and between the masses of the intermediate particles. This approach makes universality even more accidental. The conserved vector current may also be introduced by coupling the leptons and the SU(3) quarks to intermediate scalar bosons in such a way that the conserved vector current in quark form appears in the effective Lagrangian. This approach again requires certain degeneracies in coupling constants and masses and therefore makes universality accidental unless a reason can be found for the existence of these degeneracies.[6] A possible reason for such degeneracies is that the particles involved belong to irreducible representations of a symmetry group under whose transformations the weak-interaction Hamiltonian is invariant. This point of view is adopted in this paper.

In Sec. II the symmetry of weak interactions is discussed. Section III is devoted to the interesting leptonic processes while Sec. IV outlines the application of the theory to conventional semileptonic processes. The associated production of SU(3) quarks and intermediate scalar bosons is discussed in Sec. V. Sections VI and VII treat neutral-current effects and nonleptonic processes.

II. BASIC SYMMETRY SCHEMES

The parallel between leptons and possible basic SU(3) hadrons has long been recognized and exploited.[7-13] In one of the earliest schemes[7,10] to explain the eightfold way, Gell-Mann introduced four basic particles: a unitary singlet b^0 ($Y=I=0$) and a unitary triplet u^0, d^-, S^- ($Y=\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}$; $I_3=\frac{1}{2}, -\frac{1}{2}, 0$). The baryons came from $(bt\bar{t})$, $(btt\bar{t}\bar{t})$, etc. and the mesons from $(t\bar{t})$, $(tt\bar{t}\bar{t})$, etc. The parallel of (b^0, u^0, d^-, S^-) with the leptons $(\nu_e, \nu_\mu, e^-, \mu^-)$ is obvious. As the leptons fall naturally into two charge doublets (ν_e, e^-), (ν_μ, μ^-), one can carry the parallel further and arrange the hadrons in a similar way: (u^0, d^-), (b^0, S^-). One can then discuss a weak-interaction symmetry scheme whose basic constituents are the four charge doublets. However, it is more convenient and more elegant to use the SU(3) quarks,[10] $q_1q_2q_3$, as the basic hadrons. To make a parallel with the lepton structure a unitary singlet quark q_0 ($Q=\frac{2}{3}$, $Y=I=0$) is needed. Then one can arrange the quarks in two charge doublets, (q_1, q_2) and (q_0, q_3), and discuss the weak-interaction symmetry in terms of the two lepton charge doublets and the two hadron charge doublets.

Consider the two quartets

$$\psi^{\uparrow} = (\nu_e, \nu_\mu, q_1, \bar{q}_3),$$
$$\psi^{\downarrow} = (e, \mu, q_2, \bar{q}_0). \tag{2}$$

Both $\psi^{\uparrow}$ and $\psi^{\downarrow}$ belong to the same $\{4\}$ representation of an SU(4) symmetry group[14] which has the following quantum numbers:

(a) *Baryon number, B.* The usual assignment is made: $B=0$ for the leptons, $\frac{1}{3}$ for q_1 and q_2, and $-\frac{1}{3}$ for $\bar{q}_3$ and $\bar{q}_0$.

(b) *Lepton number, L.* In place of the usual two lepton numbers we define two traceless operators. The first we call lepton number and define it by $L=0$ for the quarks, $L=1$ for $e^-, \nu_e, \mu^+, \bar{\nu}_\mu$, and $L=-1$ for $e^+, \bar{\nu}_e, \mu^-, \nu_\mu$. This lepton-number assignment[15] is consistent with observed selection rules. The place of a second lepton number is taken by the third quantum number, weakness.

(c) *Weakness, W.* Define $W=1$ for $e^-, \nu_e, \mu^-, \nu_\mu$ and $W=-1$ for $q_1, q_2, \bar{q}_3, \bar{q}_0$. Weakness conservation prohibits some reactions that B and L conservation alone would allow. We consider a few examples. [Later on, we shall break the SU(4) symmetry so that $\Delta W = -2\Delta Y$ reactions are allowed. The following reactions are forbidden by this selection rule.] Important prohibited reactions are

$$\pi^- \to e^- + \nu_\mu,$$

$$\mu^- \to e^- + \nu_\mu + \nu_\mu,$$

and[16]

$$K^0 \to \pi^+ + (\text{any two leptons with } Q_{\text{tot}} = -1).$$

For baryons the selection rule prohibits

$$\Sigma^+ \to n + \mu^+ + \bar{\nu}_e$$
$$\to n + e^+ + \bar{\nu}_\mu$$

which are allowed by L and B conservation.

The quantum numbers of the SU(4) quartets are summarized in Table I.

The fundamental "quark" of the weak-interaction symmetry is an SU(4) quartet, each member of which is a charge doublet. Thus, the operative symmetry is SU(4)×SU(2). Introduce weak isospin, i, to describe the charge doublets. Then the SU(4)×SU(2) quark is $\psi = (\psi^{\uparrow}, \psi^{\downarrow})$ with $\psi^{\uparrow}$ having $i_3 = \frac{1}{2}$ and $\psi^{\downarrow}$ having $i_3 = -\frac{1}{2}$. The charge of any particle is given by

$$Q = i_3 - \tfrac{1}{4}W - \tfrac{1}{2}B - \tfrac{1}{4}N_q, \tag{3}$$

where N_q is the number of quartet particles in the representation. Thus one sees that an alternate but equivalent choice for an SU(4) quantum number would be "weak hypercharge," $y = -\frac{1}{2}W - B$.

The 15 Hermitian generators of the SU(4) group may be expressed in terms of the standard traceless generators satisfying

$$[F_a{}^b, F_c{}^d] = \delta_c^b F_a{}^d - \delta_a^d F_c{}^b,$$
$$(F_a{}^b)^\dagger = F_b{}^a, \quad F_c{}^c = 0, \tag{4}$$

where the indices take the values 1, 2, 3, 4. The Hermitian generators are

$$F_i = \tfrac{1}{2}\sum_{a,b} (\lambda_i)_{ab} F_a{}^b, \tag{5}$$

with the λ_i given in Appendix A. The three diagonal λ's are chosen to be λ_3, λ_6, and λ_{15}, and the weak-interaction quantum number operators have the representation

$$L = \lambda_3, \quad B = \lambda_6, \quad W = \sqrt{2}\,\lambda_{15}. \tag{6}$$

It should be emphasized that in contradistinction to the SU(3) theory of strong-interaction symmetries, we do not look upon the higher representations of SU(4) as bound states of the SU(4) quartets. These higher representations are merely used for classification purposes; the SU(4) quartets are not building blocks and the spinless bo-

TABLE I. Quantum numbers of an SU(4) quartet.

	L	B	W
e	1	0	1
μ	−1	0	1
q_2	0	1	−1
$\bar{q}_0$	0	−1	−1

sons discussed below are not considered to be bound states of them. For these reasons the spinless bosons can be considered to have only semiweak and electromagnetic interactions.

To couple spinless bosons to the SU(4)×SU(2) quarks we must form bilinear scalars from ψ. As we wish to restrict ourselves to positive-chirality fields the scalars must be bilinear in $\bar{\psi}^C$ and ψ. (C indicates charge conjugation.) Since $\bar{\psi}^C$ is a pair of SU(4) quartets rather than antiquartets, the scalars form the sextet and decimet representations of SU(4),

$$\{4\}\times\{4\}=\{6\}+\{10\}. \tag{7}$$

The sextet consists of the antisymmetric combinations of $\bar{\psi}^C$ and ψ while the decimet consists of the symmetric combinations. Since scalar couplings have the property

$$\bar{\psi}_a^{\uparrow C}(1+\gamma_5)\psi_b^{\downarrow}=\bar{\psi}_b^{\downarrow C}(1+\gamma_5)\psi_a^{\downarrow}, \tag{8}$$

scalars can only form the weak-isosinglet sextet (6,1) and the weak-isotriplet decimet (10,3). The other possibilities, (10,1) and (6,3), vanish for scalar couplings. Thus, the scalar sextet coupling involves the introduction of six spinless bosons while the scalar decimet coupling requires 30 spinless bosons. Therefore, the sextet coupling is more attractive but only experiment[17] can determine which coupling exists.

Let X be the 4×4 antisymmetric matrix of weak-isosinglet spinless-boson fields. A convenient notation is the following:

$$X=\begin{pmatrix} 0 & V & X_e & W_e \\ -V & 0 & X_\mu & W_\mu \\ -X_e & -X_\mu & 0 & Y \\ -W_e & -W_\mu & -Y & 0 \end{pmatrix}. \tag{9}$$

The quantum numbers of this sextet of particles are given in Table II.

Now we construct an interaction that is invariant under SU(4)×SU(2) transformations. With positive-chirality fields and the sextet coupling the only (renormalizable) interaction is

$$H_0=\tfrac{1}{2}g\bar{\psi}^C X(1+\gamma_5)\psi+\text{H.c.} \tag{10}$$

Here $\bar{\psi}^C=(\bar{\psi}^{\downarrow C},-\bar{\psi}^{\uparrow C})$. More explicitly, H_0 may be written in the forms

$$\begin{aligned} H_0&=g\bar{\psi}^{\downarrow C}X(1+\gamma_5)\psi^{\uparrow}+\text{H.c.} \\ &=-g\bar{\psi}^{\uparrow C}X(1+\gamma_5)\psi^{\downarrow}+\text{H.c.} \end{aligned} \tag{11}$$

This interaction is universal in the sense that it gives the same coupling constant for muon decay and for β decay. However, it prohibits hypercharge-changing processes between known particles; in other words, the interaction H_0 conserves hypercharge to all orders as long as neither a q_0 nor an intermediate scalar boson (ISB) is emitted or absorbed. To introduce the $\Delta Y\neq 0$ interaction, we make the canonical transformation[18]

$$\begin{aligned} q_1&\to q_1'=q_1\cos\theta-q_0\sin\theta\,, \\ q_0&\to q_0'=q_0\cos\theta+q_1\sin\theta\,. \end{aligned} \tag{12}$$

Thus

$$\psi\to\phi=\begin{pmatrix} \nu_e, & \nu_\mu, & q_1', & \bar{q}_3 \\ e^-, & \mu^-, & q_2, & \bar{q}_0' \end{pmatrix} \tag{13}$$

and the weak-interaction Hamiltonian becomes

$$H_0\to H=\tfrac{1}{2}g\bar{\phi}^C X(1+\gamma_5)\phi\,. \tag{14}$$

The first-order vertices represented by this interaction are shown in Fig. 1. The interaction H has the following properties:

(1) It leads to the correct selection rules for all known weak processes.

(2) It leads to the $V-A$ current-current form of interaction when the mass M of the intermediate scalar boson is large compared to the momentum transfer of the process under consideration (the "local limit").

(3) The Cabibbo[19] form of universality is built in.

(4) The theory is renormalizable.

(5) The leptons couple to the conserved vector current in the local limit.

(6) Weakness is not conserved by H, but the selection rule $\Delta W=-2\Delta Y$ obtains.

(7) Weak isospin is not conserved. Conservation of weak isospin will be seen to be equivalent

TABLE II. Quantum numbers of the boson sextet.

	Q	B	L	W
V	1	0	0	−2
$W_{e,\mu}$	$\frac{2}{3}$	$+\frac{1}{3}$	−1, 1	0
$X_{e,\mu}$	$\frac{1}{3}$	$-\frac{1}{3}$	−1, 1	0
Y	0	0	0	2

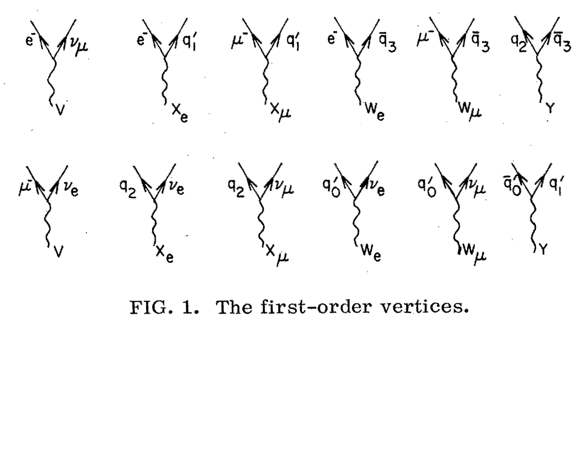

FIG. 1. The first-order vertices.

to the $\Delta I = \frac{1}{2}$ rule.

We outline these results in the following sections.

III. PURE LEPTONIC PROCESSES

The leptonic processes go via the interaction

$$H_L = gV[\overline{e}^+(1+\gamma_5)\nu_\mu - \overline{\mu}^+(1+\gamma_5)\nu_e] + \text{H.c.} \tag{15}$$

and this gives rise to the effective interaction in second order for processes that go by the exchange of a V:

$$g^2 \int d^4y[\overline{e}^+(1+\gamma_5)\nu_\mu(x) - \overline{\mu}^+(1+\gamma_5)\nu_e(x)] \times \Delta_c(x-y)[\overline{\nu}_\mu(1-\gamma_5)e^+(y) - \overline{\nu}_e(1-\gamma_5)\mu^+(y)]. \tag{16}$$

(Here $\overline{e}^+$ means the adjoint of the field e^+.) In the local limit, $\Delta_c(x) \to \delta(x)/M^2$, one gets the usual $V-A$ interaction for muon decay with the aid of the Fierz transformation (1) and with

$$g^2 = \sqrt{2}\, GM^2. \tag{17}$$

In the ISB theory the selection rules for elastic weak leptonic processes are different from the $V-A$ and intermediate vector boson (IVB) selection rules. Here the processes

$$\nu_\mu + e^\pm \to \nu_\mu + e^\pm \tag{18}$$

are allowed while the sextet coupling forbids[17]

$$\nu_e + e^\pm \to \nu_e + e^\pm \tag{19}$$

in second order. Thus, the presence of $e\nu_\mu$ scattering will be a critical test of this theory while the presence or absence of $e\nu_e$ scattering will decide between the decimet and sextet couplings, respectively. The data of Reines and Gurr[17] seem to indicate the presence of $e\overline{\nu}_e$ scattering, thus favoring the deciment coupling, but the backgrounds are large. The elastic rates observed by Steiner[20] and by Cundy *et al.*[21] are intermediate[22] between those expected for $e\nu_e$ and $e\nu_\mu$. These elastic scattering experiments are the most promising tests of the ISB theory since neutrino beams from accelerators generally contain less than 1% ν_e's. Thus $e\nu_\mu$ scattering should show up easily in the next generation of neutrino experiments if it is to show up at all.

Astrophysical evidence[23] for stellar energy loss through processes like photoneutrino pair production

$$\gamma + e^- \to e^- + \nu_e + \overline{\nu}_e$$

is equally good evidence for the process

$$\gamma + e^- \to e^- + \nu_\mu + \overline{\nu}_\mu$$

so no conclusions can be made on this basis.

The intermediate boson for leptonic processes can be produced in the reactions

I. $\nu_\mu + Z \to V^- + e^+ + Z$,

II. $\overline{\nu}_\mu + Z \to V^+ + e^- + Z$,

where Z is a nucleus which exchanges a virtual photon with e and V. These reactions should be contrasted with those predicted by the IVB theory

I′. $\nu_\mu + Z \to W^+ + \mu^- + Z$,

II′. $\overline{\nu}_\mu + Z \to W^- + \mu^+ + Z$.

Searches[24-26] for the IVB have concentrated on looking for the $\mu\mu$ pairs and $e\mu$ pairs resulting from I′ and II′ when W decays. In contrast, reactions I and II lead only to μe and ee pairs when V decays. So far no $\mu\mu$ pairs satisfying the necessary criteria have been found, but Bernardini *et al.*[25] report six $e\mu$ pairs while background can account for about three pairs. On the other hand, Burns *et al.*[26] find no $e\mu$ pairs, so the experimental situation is unclear. If ISB and IVB production rates are about the same, these experiments would seem to indicate $M \gtrsim 2$ GeV. (Preliminary calculations[27] indicate that ISB rates may be considerably less than IVB rates, so M could be appreciably less than 2 GeV.)

The decay modes of the intermediate scalar boson, V, are

$$V^- \to e^- + \nu_\mu$$
$$\to \mu^- + \nu_e.$$

The branching ratio is very close to one and the lifetime is $\sqrt{2}\,\pi/GM^3$ which is about

$$\tau = 3.0\times 10^{-19}(M_p/M)^3 \text{ sec}. \tag{20}$$

IV. SEMILEPTONIC PROCESSES

The interaction (14) leads to the usual Cabibbo form of the semileptonic interaction in the local limit. While this is obvious, it may still be instructive to review the derivation briefly.

The electron semileptonic processes result from the interaction

$$g\{X_e[\overline{e}^+(1+\gamma_5)q_1' - \overline{q}_2^C(1+\gamma_5)\nu_e] + W_e[\overline{e}^+(1+\gamma_5)q_3^C - \overline{q}_0'(1+\gamma_5)\nu_e]\} + \text{H.c.} \tag{21}$$

The resulting inelastic terms in the effective second-order interaction in the local limit are

$$H' = -\sqrt{2}\, G[\overline{e}^+(1+\gamma_5)q_1'\overline{\nu}_e(1-\gamma_5)q_2^C + \overline{e}^+(1+\gamma_5)q_3^C\,\overline{\nu}_e(1-\gamma_5)q_0'] + \text{H.c.} \tag{22}$$

Upon using the Fierz transformation (1) one gets

$$H' = (G/\sqrt{2})\overline{\nu}_e\gamma_\mu(1+\gamma_5)e^- \times[\overline{q}_2\gamma_\mu(1+\gamma_5)q_1' - \overline{q}_3\gamma_\mu(1-\gamma_5)q_0'] + \text{H.c.} \tag{23}$$

When matrix elements of H' are taken between states of known particles, only the q_1 parts of q_1' and q_0' contribute since the q_0 terms only connect states that differ by a fractional hypercharge. Therefore, the effective hadron current in (23) is

$$J^h_\mu = \overline{q}_2\gamma_\mu(1+\gamma_5)q_1\cos\theta - \overline{q}_3\gamma_\mu(1-\gamma_5)q_1\sin\theta . \quad (24)$$

The vector part of the strangeness-conserving current is the conserved isospin-lowering current, $\overline{q}_2\gamma_\mu q_1$. The strangeness-nonconserving current is of the $V+A$ type; in other theories it is of the $V-A$ type. Thus a measurement of the sign of the ratio g_A/g_V will provide another test of this theory. Preliminary data are not conclusive one way or the other. For the process $\Sigma^- \to ne\nu$, the Heidelberg group gets[28]

$$g_A/g_V = 0.20 \pm 0.28 .$$

$V+A$ predicts 0.31 while $V-A$ predicts −0.31. The experimental measurements for $\Lambda \to pe\nu$ are confusing. One group[29] gets 0.65 ± 0.09 from the $e\nu$ correlation, while more recently by studying the up-down asymmetry in the decay another group[30] gets

$$0.32^{+0.17}_{-0.13} .$$

The $V \pm A$ prediction is ∓ 0.72. Further measurements of g_A/g_V are clearly needed for strangeness-changing semileptonic decays.

V. PRODUCTION OF X AND W

The scalar bosons responsible for the semileptonic processes are produced in association with the SU(3) quarks. Useful processes will be

$$\nu_\mu + Z \to Z + \overline{q}_2 + \overline{X}_\mu ,$$
$$\nu_\mu + Z \to Z + q_1 + \overline{W}_\mu ,$$
$$\nu_\mu + Z \to Z + q_0 + \overline{W}_\mu .$$

The nucleus Z is needed, of course, for energy-momentum balance, and it augments the rate by absorbing the virtual photon coherently. If q_0, q_1, and q_2 have a greater mass than X and W, they will decay very quickly into them giving the effective reactions

$$\nu_\mu + Z \to Z + X_\mu + \overline{X}_\mu + \nu_\mu$$
$$\to Z + X_e + \overline{X}_\mu + \nu_e$$

and

$$\nu_\mu + Z \to Z + W_\mu + \overline{W}_\mu + \nu_\mu$$
$$\to Z + W_e + \overline{W}_\mu + \nu_e .$$

The thresholds for these reactions will be considerably higher than the threshold for V production so one could not expect to see these reactions with presently available neutrino beams.

If q_0, q_1, and q_2 have a smaller mass than X and W, the intermediate bosons will decay into them and the effective reactions will be

$$\nu_\mu + Z \to Z + q_2 + \overline{q}_2 + \nu_\mu$$
$$\to Z + q_1 + \overline{q}_2 + \mu^-$$
$$\to Z + \overline{q}_1 + q_1 + \nu_\mu$$
$$\to Z + \overline{q}_3 + q_1 + \mu^- .$$

The reactions for q_0 production are obtained from these by replacing any or all of the q_1's by q_0's. If these reactions take place they will be useful production mechanisms for quarks as they are virtually free of strong-interaction effects.

VI. NEUTRAL-CURRENT EFFECTS

The weak-interaction Hamiltonian, H, gives rise—in second order in the local limit—to terms that resemble the neutral-current interactions of Oakes[31] and of Glashow et al.[13] For the semileptonic processes the neutral-current terms are

$$-\frac{G}{\sqrt{2}}\{[\overline{e^-}\gamma_\mu(1+\gamma_5)e^- + \overline{\mu^-}\gamma_\mu(1+\gamma_5)\mu^-][\overline{q}_1'\gamma_\mu(1+\gamma_5)q_1' - \overline{q}_3\gamma_\mu(1-\gamma_5)q_3]$$
$$+[\overline{\nu}_e\gamma_\mu(1+\gamma_5)\nu_e + \overline{\nu}_\mu\gamma_\mu(1+\gamma_5)\nu_\mu][\overline{q}_2\gamma_\mu(1+\gamma_5)q_2 - \overline{q}_0'\gamma_\mu(1-\gamma_5)q_0']\} + \text{H.c.}$$

Note that only strangeness-conserving currents appear. The neutrino neutral current couples to a current that can be broken up into isoscalar, isospinor, and isovector parts. The isovector part is

$$\tfrac{1}{2}[\overline{q}_1\gamma_\mu(1+\gamma_5)q_1 - \overline{q}_2\gamma_\mu(1+\gamma_5)q_2] + \tfrac{1}{2}\sin^2\theta[\overline{q}_1\gamma_\mu(1-\gamma_5)q_1 - \overline{q}_2\gamma_\mu(1-\gamma_5)q_2] .$$

We can ignore the isospinor parts as they do not contribute to matrix elements between known particle states. If the reaction

$$\nu_\mu + p \to \nu_\mu + n + \pi^+$$

is dominated by the $N^*(1236)$, then only the isovector current contributes and we can predict

$$R = \frac{\sigma(\nu_\mu p \to \nu_\mu n \pi^+)}{\sigma(\nu_\mu p \to \mu^- p \pi^+)} = \tfrac{1}{9} , \quad (25)$$

if we ignore $\sin^2\theta$ terms. This is the same result that Glashow *et al.*[13] obtained. The present experimental limit[21] is $R \leqslant 0.08 \pm 0.04$, which is not inconsistent with our results.

The neutral-current terms also give rise to a substantial cross section for $\nu p \to \nu p$, but nothing can be calculated because the relative size and sign of the isoscalar contributions are unknown.

Consider processes like $K_L^0 \to \mu^+\mu^-$. When the masses of q_0 and q_1 are equal, such strangeness-changing neutral-current processes are strictly forbidden[32] to all orders because prior to the canonical transformation, (12), strangeness was conserved. Even if the masses M_0 and M_1 are quite different, the rate for such decays will be small. Consider the fourth-order diagrams of Fig. 2. Each diagram is separately logarithmically divergent but the sum of the two is finite; the least convergent parts give an amplitude for $K_0 \to \mu^+\mu^-$ which is[33]

$$\tilde{T} = -\cos\theta \sin\theta\, G f_K \frac{iM^2 l}{4\sqrt{2}\,\pi^2} \frac{1}{(2\pi)^{9/2}} \left(\frac{m_\mu^{\,2}}{2k_0 k_0' P_0}\right)^{1/2} \times 2 m_\mu \bar{u}(k)\gamma_5 v(k'), \quad (26)$$

where

$$l \equiv \frac{M_1^{\,2}}{M_1^{\,2}-M^2} \ln\frac{M_1^{\,2}}{M^2} - \frac{M_0^{\,2}}{M_0^{\,2}-M^2}\ln\frac{M_0^{\,2}}{M^2}\,. \quad (27)$$

More convergent terms are smaller by a factor of the order

$$(m_K/M)^2(M_1^{\,2}-M_0^{\,2})l^{-1}\,.$$

Similar calculations[34,35] with an IVB yield the result with a cutoff Λ^2 replacing $4M^2 l$. The present upper limit[36] on the branching ratio for $K_L^0 \to \mu^+\mu^-$ thus restricts the masses by

$$M l^{1/2} \lesssim 7\,\mathrm{GeV}\,. \quad (28)$$

This restriction is, of course, quite acceptable.

VII. NONLEPTONIC PROCESSES

The nonleptonic processes are described by the "3-4" part of the interaction

$$H_{\mathrm{nl}} = gY[\bar{q}_3(1+\gamma_5)q_2 - \bar{q}_0'(1+\gamma_5)q_1'] + \mathrm{H.c.} \quad (29)$$

In the local limit in second order the effective interaction is

$$H'' = -\sqrt{2}\,G'\bar{q}_3(1+\gamma_5)q_2 \times[\bar{q}_1(1-\gamma_5)q_1 - \bar{q}_0(1-\gamma_5)q_0] + \mathrm{H.c.}, \quad (30)$$

where $G' = G\cos\theta\sin\theta$ and the terms odd in q_0 have been omitted for the usual reason. The usual[37] phenomenological analysis of nonleptonic reactions is applicable to this interaction. Note that while the first factor in (30) is an isospinor, the second factor is not a pure isoscalar, so the $\Delta I = \frac{1}{2}$ rule does not follow. Presumably, one should infer that the matrix elements of the $\Delta I = \frac{3}{2}$ parts of H'' are small as a result of the nature of the quark binding in the known particles.

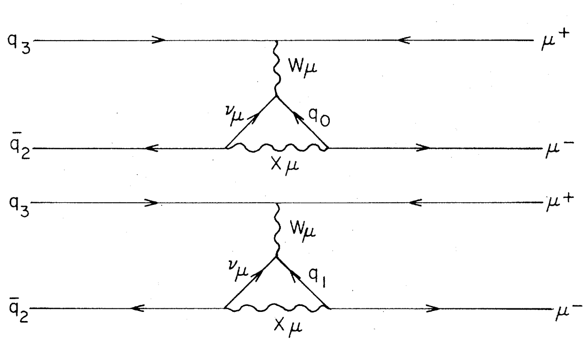

FIG. 2. Fourth-order diagrams responsible for $K_L^0 \to \mu^+\mu^-$.

VIII. DISCUSSION

The leptons and the SU(3) quarks have been classified into the basic representation of an SU(4) ×SU(2) symmetry group. A coupling of the sextet of scalar densities to a sextet of spinless bosons results in a renormalizable model of weak interactions which has many desirable features: universality, conserved vector current, correct selection rules, and $V-A$ in the local limit.

The theory is on the verge of being tested in high-energy neutrino experiments as $e\nu_\mu$ scattering is expected in second-order semiweak interactions, whereas it would only occur in fourth-order semiweak interactions in the IVB or $V-A$ theories.

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TABLE III. The nonzero antisymmetric structure constants of SU(4).

i	j	k	f_{ijk}	i	j	k	f_{ijk}	i	j	k	f_{ijk}
1	2	3	-1	3	9	14	$-\frac{1}{2}$	5	10	12	$\frac{1}{2}$
1	7	14	$-\frac{1}{2}$	3	10	11	$-\frac{1}{2}$	5	11	13	$-\frac{1}{2}$
1	8	9	$-\frac{1}{2}$	3	12	13	$\frac{1}{2}$	6	7	8	$-\frac{1}{2}$
1	10	13	$\frac{1}{2}$	4	5	6	-1	6	9	14	$\frac{1}{2}$
1	11	12	$-\frac{1}{2}$	4	7	14	$\frac{1}{2}$	6	10	11	$-\frac{1}{2}$
2	7	9	$\frac{1}{2}$	4	8	9	$-\frac{1}{2}$	6	12	13	$-\frac{1}{2}$
2	8	14	$-\frac{1}{2}$	4	10	13	$-\frac{1}{2}$	7	10	15	$-(\frac{1}{2})^{1/2}$
2	10	12	$\frac{1}{2}$	4	11	12	$-\frac{1}{2}$	8	11	15	$-(\frac{1}{2})^{1/2}$
2	11	13	$\frac{1}{2}$	5	7	9	$\frac{1}{2}$	9	12	15	$-(\frac{1}{2})^{1/2}$
3	7	8	$-\frac{1}{2}$	5	8	14	$\frac{1}{2}$	13	14	15	$-(\frac{1}{2})^{1/2}$

O'Donnell for valuable discussions.

APPENDIX A: THE SU(4) MATRICES

The 4×4 matrices, λ_i, that provide a realization of the SU(4) generators, F_i, can be written in the following convenient form. With $i=1,2,3$, the σ_i are the 2×2 Pauli matrices, and 0 and 1 are the 2×2 null and unit matrices, respectively.

$$\lambda_i=\begin{pmatrix}\sigma_i & 0\\ 0 & 0\end{pmatrix},\quad \lambda_{3+i}=\begin{pmatrix}0 & 0\\ 0 & \sigma_i\end{pmatrix},\quad \lambda_{6+i}=\frac{1}{\sqrt{2}}\begin{pmatrix}0 & \sigma_i\\ \sigma_i & 0\end{pmatrix},$$

$$\lambda_{9+i}=\frac{1}{\sqrt{2}}\begin{pmatrix}0 & i\sigma_i\\ -i\sigma_i & 0\end{pmatrix},\quad \lambda_{13}=\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},$$

$$\lambda_{14}=\frac{1}{\sqrt{2}}\begin{pmatrix}0 & i\\ -i & 0\end{pmatrix},\quad \lambda_{15}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}.$$

These matrices satisfy the commutation rules

$$[\lambda_i,\lambda_j]=2if_{ijk}\lambda_k\quad (i,j,k=1,2,\ldots,15)$$

and the nonzero antisymmetric structure constants, f_{ijk}, are given in Table III.

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