Scattering of Pions by Any Target in Dual-Resonance Models

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(Received 16 August 1971)

Guided by the Lovelace $\pi\pi$ formula, a set of constraints is proposed for any realistic Veneziano formulas for the scattering of pions by any target. One constraint, the vanishing of integrals over the (s,u) term in the forward direction due to a mechanism of local mass cancellations, is tested for $\pi\Sigma$, πN, $\pi\rho$, and $\pi\Delta$ scattering. This leads to the following predictions: $F/(F+D)=1.4\pm0.1$ for the ratio of octet baryon–vector- (tensor-) meson couplings; $\Gamma_{A_1\rho\pi}\approx\Gamma_{\rho\pi\pi}/\sqrt{2}\approx 90$ MeV; $g_{\Delta^{++}\Delta^{+}\pi^{+}}{}^2/4\pi=35$ in agreement with SU(6).

The Lovelace[1] formula for $\pi\pi$ scattering is still a major achievement in the field of Veneziano-type[2] models. The formula, having duality and crossing symmetry naturally built in, accounts, roughly, for the known physics of the $\pi\pi$ system in the whole range of energies. For detailed fits one requires unitarized versions of the model,[3] but we shall not attempt to include such features in our discussion.

The attempts to apply Veneziano's formula to other four-particle amplitudes has not always met with complete success and certainly has never achieved the general agreement with experiment and theory of the $\pi\pi$ formula. In particular, trouble occurs when baryons are included. For instance, both a straight πN generalization of the Veneziano formula[4] and a relativistic dual quark-model formulation[5] failed to give sensible low-energy parameters.

Instead of imposing from the beginning a particular form for the amplitude, we try in this paper to extract from the Veneziano-Lovelace formula some basic properties. These properties, which are derived from the dual structure and not from the specific form of the Veneziano amplitude, are then classified and their consequences explored. We shall concentrate our attention only on scattering of pions on an arbitrary target T because pion-mass extrapolations are small and allow direct comparison with soft-limit results, but what we have to say could equally be extended to other pseudoscalar-meson-target reactions.

We work in the framework of dual-resonance models[6] and assume that planar duality holds, i.e., the scattering amplitude $A(s,t,u)$ has the quark underlying structure given by duality diagrams.[7] It can then be written as a sum of (s,t), (u,t), and (s,u) terms, each term exhibiting poles in two channels s and t, etc. According to the physical situation, these quark-model nonexotic poles are interpreted either as resonances or Regge poles. The terms $V(s,t)$ and $V(u,t)$ have the same quark structure and show no particular symmetry under $s\leftrightarrow t$ and $u\leftrightarrow t$ interchanges, respectively. The (s,u) diagram does not change when seen from s or u channels, and the corresponding $U(s,u)$ term is then taken as even under $s\leftrightarrow u$ interchange:

$$U(s,u)=U(u,s)\,. \tag{1}$$

Keeping in mind that a pure $U(s,u)$ term is t-channel exotic ($I_t=2$) and the $s\leftrightarrow u$ crossing properties of the t-channel isospin amplitudes, we write the simplest and most general t-channel isospin amplitudes for πT scattering in the form

$$A_0^t=\beta[V(s,t)+V(u,t)]+\delta U(s,u)\,, \tag{2a}$$

$$A_1^t=\alpha[V(s,t)-V(u,t)]\,, \tag{2b}$$

$$A_2^t=\gamma U(s,u)\,. \tag{2c}$$

In principle, $V(s,t)$ and $U(s,u)$ are not the same function, and of the four Veneziano Clebsch-Gordan coefficients α, β, δ, and γ two can be absorbed in the normalization of $V(s,t)$ and $U(s,u)$. Note that throughout this paper we shall never assume a specific form for $V(s,t)$ and $U(s,u)$.

We now consider the soft-pion limit $(q_1{}^2, q_2{}^2\to 0)$ in the reaction

$$\pi(q_1)+T(p_1)\to\pi(q_2)+T(p_2)$$

and define the variable $\nu=q_1\cdot(p_1+p_2)/2m_T$, m_T being the mass of the target. If in the region $\nu\simeq 0$ the s-wave part of the amplitude is vanishingly small and allows an expansion in the variable ν, then the s-wave scattering length is approximately given by

$$a=m_\pi\left.\frac{\partial A(\nu,t=0)}{\partial\nu}\right|_{\nu\to 0}\,. \tag{3}$$

The best justification of Eq. (3) is the success of the current-algebra calculations of scattering lengths.[8,9] Theoretically, Eq. (3) is on more secure grounds for an amplitude odd under isospin crossing because such an amplitude is constrained to vanish at $\nu=0$.[10] This is in fact the amplitude we are most interested in in this paper. We extend Eq. (3) to each planar dual amplitude, defining in this way the a_{st}, a_{ut}, and a_{su} contributions to the s-wave scattering lengths. Using the $s \leftrightarrow u$ crossing properties of the t-channel amplitudes, we obtain from Eqs. (2) the following expressions for the scattering lengths:

$$a_0^t = \beta(a_{st} + a_{ut}) + \delta a_{su} = 0\,, \tag{4a}$$

$$a_1^t = \alpha(a_{st} - a_{ut}) = 2\alpha a_{st}\,, \tag{4b}$$

$$a_2^t = \gamma a_{su} = 0\,. \tag{4c}$$

Note that if Eq. (3) is true the addition of a Pomeranchuk-like term to A would not affect the scattering lengths because it is even under $s \leftrightarrow u$ crossing. For $\pi\pi$ scattering with physical pions, the zero-pion-mass approximation of Eq. (3) is not correct.[9] In particular, because of the additional symmetry, one has $a_{su} = a_{st}$, modifying Eqs. (4a) and (4c).

If we now go to the high-energy limit in the forward direction, then

$$\operatorname{Im} A(s, t\simeq 0, u) \underset{s\to\infty}{\sim} \operatorname{Im} V(s, t\simeq 0)\,, \tag{5}$$

which implies that the resonance contributions from $\operatorname{Im}U(s,u)$ at $t\simeq 0$ do not add up at high energy to form Reggeons, but rather compensate among themselves. Such compensations are achieved in chiral schemes,[11,12] as in the $\pi\pi$ Veneziano formula,[1,13,14] by the inclusion of low-lying particles (daughters). For example, in the $\pi\pi$ case the mass-degenerate ρ and ϵ have equal and opposite contributions to $\operatorname{Im}U(s,u)$. We now extend these ideas to other processes.

As $\operatorname{Im}U(s,u)$ does not contribute at high energy in the forward direction, one can write for $U(s,u)$ superconvergent relations in the form

$$\int \nu^k \operatorname{Im}U(s,u)\,d\nu = 0 \quad (t\simeq 0)\,. \tag{6}$$

Equation (6) holds for all odd integers k. The most natural way of achieving this is by cancellations between high- and low-partial-wave contributions in each local mass region. Then Eq. (6) would be expected to hold also for even k. In our applications we restrict k to a value, $k=-2$, that provides convergence even for amplitudes which are not superconvergent, and thus safely allows saturation with a few resonances. The test for superconvergence then becomes the local cancellation of the integral.

It is important to remark that the superconvergence of $U(s,u)$ is not derived here from the presence of exotic states in the t channel, but appears as a consequence of the dual planar structure of the amplitude. When an exotic t channel is present, $I_t = 2$, Eq. (6) coincides with the superconvergent relations of Brout *et al.*,[14] but as shown below in the case of πN scattering it is also valid when there are no exotic channels.

At this stage we compare our Eqs. (1)–(6), which we think should be kept in a Veneziano formula for the πT scattering amplitude, with the Lovelace expression. Equations (1) and (2) are satisfied. Equations (3) and (4) are also satisfied up to terms in m_π^2 in the limit of linear expansion of the denominator Γ functions.[13] Equation (5) is obviously satisfied. Equation (6) is exact in the zero-width resonance approximation. Note that the Veneziano formula for πK scattering[15] also satisfies the equations that refer to $V(s,t)$ and $V(u,t)$ terms [there is no $U(s,u)$ term in πK scattering]. In the case of the $\pi\eta$ system, conditions (3) and (4) are not satisfied and the Lovelace formula then is not correct.[16]

As the next step we discuss the consequences of imposing on the πT amplitude the constraints of the additivity quark model in the version proposed in previous work[17]: quark-model additivity is additivity of $V(s,t)$ π-quark duality diagrams generating the $V(s,t)$ π-T diagram. We express the high-energy additivity rule in the following way:

$$\operatorname{Im}\langle \pi T | A | \pi T\rangle \underset{s\to\infty;\, t\approx 0}{\approx} \sum_i \operatorname{Im}\langle \pi Q_i | A | \pi Q_i\rangle = n \operatorname{Im} V_Q(s, t\simeq 0)\,, \tag{7}$$

where $\operatorname{Im}V_Q(s,t\simeq 0)$, a universal function of s, is the amplitude for the basic π–nonstrange-quark Q interaction and n is the number of interacting quarks in T. Note that the additivity rule (7) does not work for the real part of the amplitude because then $V(u,t)$ and $U(s,u)$ terms also contribute. Via duality and finite energy sum rules (FESR), the high-energy curve when extrapolated down to the low-energy region must be, on the average, equal to the low-energy contributions. In this way the additivity rule (7) for the imaginary part of the planar dual amplitude $V(s,t)$ can be extended to the whole range of energies.

We shall now apply these ideas to specific reactions and see how far they are satisfied in practice. We need to select an amplitude in which s-channel resonances come only from $V(s,t)$, i.e., the A_1^t amplitude [Eq. (2b)]. As a "good" FESR ($k=-2$), we take the Adler-Weisberger relation[18] interpreted as a FESR for A_1^t/ν^2,[19]

$$a_1^t=\frac{A_1^t(\nu,0)}{\nu}\bigg|_{\nu\to 0}=(\text{Born term})+\frac{2}{\pi}\int_{\nu_{\text{th}}}^{\infty}\frac{\text{Im}A_1^t(\nu',0)}{\nu'^2}d\nu'\,. \tag{8}$$

The sum rule is convergent, which simplifies the comparison of different scattering processes because no cutoffs, which may be channel-dependent, need be introduced.

To saturate the right-hand side of (8), we generalize the procedure developed in Ref. 17: In the resonance region we take only the first p-wave resonances and treat the contributions above as high-energy contributions, i.e., satisfying (7). Additivity for $\text{Im}V(s,t)$ implies, via (8), that the p-wave resonances also satisfy (7). We thus derive a set of SU(6) relations for coupling constants by considering the p-wave contributions to (8) of different πT reactions (πQ, $\pi\pi$, πK, πN, $\pi\Sigma$, $\pi\Xi$) in the zero-width approximation:

$$\left(\frac{g_{Q\pi}}{M_Q}\right)^2=\left(\frac{g_{\rho\pi\pi}}{m_\rho}\right)^2=\frac{8}{3}\left(\frac{g_{K^*K\pi}}{m_{K^*}}\right)^2 \tag{9a}$$

$$=\left(\frac{g_{N\pi}}{m_N}\right)^2-\frac{16}{3}\left(\frac{g_{\Delta N\pi}}{m_N+m_\Delta}\right)^2=\frac{1}{4}\left(\frac{g_{\Sigma\pi}}{m_\Sigma}\right)^2+\frac{1}{4}\left(\frac{g_{\Sigma\Lambda\pi}}{m_\Lambda+m_\Sigma}\right)^2+\frac{2}{3}\left(\frac{g_{\Sigma^*\Sigma\pi}}{m_{\Sigma^*}+m_\Sigma}\right)^2=\left(\frac{g_{\Xi\pi}}{m_\Xi}\right)^2+\frac{8}{3}\left(\frac{g_{\Xi^*\Xi\pi}}{m_{\Xi^*}+m_\Xi}\right)^2\,. \tag{9b}$$

To obtain relations (9b) the kinematic factors appearing in the relativistic widths were approximated by putting $(m_\Delta+m_N)/m_N\simeq 2$, etc., as is usual in SU(6) calculations. Relations (9a) and (9b) are SU(6) relations, as in the relativistic quark model[20]: the PPV (pseudoscalar, pseudoscalar, vector meson) coupling constant is proportional to m_V and the BBP (baryon, baryon, pseudoscalar meson) coupling is pseudovector in its nature with the s-channel SU(3) mixing parameter $f^s\equiv F/(F+D)=\frac{2}{5}$. We could write more SU(6) relations using target particles with higher spin (vector mesons, for instance), but then (7) should be interpreted in a spin-average sense. Also, if one substitutes a kaon for the pion more SU(6) relations are obtained. Note that we are not imposing the saturation of the Adler-Weisberger relation with p-wave resonances, but simply comparing their contributions in different processes.

As both the high-energy and low-energy contributions in the right-hand side of (8) satisfy (7), obviously the left-hand side also has to satisfy (7), i.e.,

$$a_1^t=na\,, \tag{10}$$

where a is a universal constant, the scattering length for πQ scattering. Equation (10) with Eqs. (4a) and (4c) reproduces Weisberg's universal scattering lengths for scattering of soft pions on any target.[9]

The equations within each of the sets (9a) and (9b) are experimentally fairly well satisfied, but the agreement is not so good when one equates meson ($VP\pi$) to baryon ($BB\pi$) coupling constants. The additivity relations provided by (7) are also not always well satisfied. They impose the condition of having pure F coupling in the t channel, which is too strong. However, our aim is not to check SU(6) but to stress that the vehicle for such an over-all SU(6)-consistent picture is the idea of duality. As emphasized several times by Rosner,[21] duality is less restrictive than SU(6) or quark-model additivity, and it is probably more fundamental.

This is the point of view we take from now on when we consider the $U(s,u)$ integrals of Eq. (6). We have another specific reason for doing so: The vanishing of these integrals cannot be achieved in the framework of SU(6) quark-model $L=0$ states, as has been known for some time.[11] We are led back to the necessity of low-lying particles to saturate (6), i.e., particles below the main trajectories initiated by the SU(6) $L=0$ states. In first approximation we shall include in (6) all the observed p- and s-wave resonances in the first resonance region, in analogy with the ρ, ϵ case,[11] and, because of the convergence argument referred to above, use $k=-2$ as in the Adler-Weisberger relation.

In general, from Eqs. (2), the $U(s,u)$ term can be isolated by the combination

$$U(s,u)\propto A_0^t-(\beta/\alpha)A_1^t\,. \tag{11}$$

Equations (2), combined with the condition of no exotic states in the s channel, allow the following classification of the target particles according to their quark content:

Group 1 – only one nonstrange quark in $T(K,\Xi,\ldots)$,

$I_T=\frac{1}{2}.\ \ \delta=\gamma=0,\ \ \beta/\alpha=1\,.$

Group 2 – two nonstrange quarks in $T(\pi,\Sigma,\Lambda,\ldots)$,

$$I_T=0:\ \gamma=0,\ \ \alpha=0\,;$$
$$I_T=1:\ \delta/\gamma=-\tfrac{1}{2},\ \ \beta/\alpha=\tfrac{3}{2}\,. \tag{12}$$

Group 3 – three nonstrange quarks in $T(N, \Delta, \ldots)$,

$$I_T=\tfrac{1}{2}:\ \gamma=0,\quad \text{no constraint on } \beta/\alpha\,;$$
$$I_T=\tfrac{3}{2}:\ \delta/\gamma=-1,\quad \beta/\alpha=3\,. \tag{13}$$

In group 1 there is no $U(s,u)$ integral to satisfy, and from this point of view no low-lying particles were required in these processes. This of course is a negative result and does not preclude the existence of these particles, but it is perhaps not an unrelated coincidence that few, if any, low-lying particles coupled to πK and $\Xi\pi$ have been unmistakably detected.[22] The Ξ resonances,[22] though not definitely classified, seem to fit quite well in the two main trajectories.[23]

In group 2, apart from $\pi\pi$ itself (which leads to the results of Gilman and Harari[11]) we can investigate Eq. (6) for $\pi\Sigma$ scattering. The first p- and s-wave resonances are[22,24,25] $\Lambda(1115)$, $\Lambda(1405)$, $\Sigma(1189)$, and $\Sigma(1385)$. Using Eqs. (11) and (12) we compute the left-hand side of (6) for $k=-2$ in the zero-pion-mass limit to be

$$-(24+14)+(32+10)=-38+42=4\pm8\ \text{GeV}^{-2}\,, \tag{14}$$

which is compatible with zero on the right-hand side. Our proposed mechanism of local mass cancellations then seems to work.

In group 3 the testable case is πN scattering, but here β/α is undetermined. Looking back to Eqs. (2), taking $t\simeq 0$ and the high-energy limit, one sees that β/α is related to $f^t\equiv F/(F+D)$ by[26]

$$\beta/\alpha=4f^t-1\,. \tag{15}$$

The first p- and s-wave resonances[24,22] are now the $N(938)$, $N(1460)$, $N(1525)$ and $\Delta(1236)$, and Eq. (6) gives

$$(1-\beta/\alpha)(104+9.2+1.1)+(2+\beta/\alpha)(60.8)=0 \tag{16}$$

and, from (15),

$$f^t=1.4\pm0.1\,. \tag{17}$$

This value of f^t, larger than the SU(6) quark-model value $f^t=1$, is in reasonable agreement with the experimental determination and other theoretical predictions of $f^t\approx 1.5$ (see Ref. 27 and further references there). If our arguments about the vanishing of the $U(s,u)$ integral by cancellations in narrow mass strips are right, Eq. (17) determines a high-energy parameter from only a few low-energy resonances. Because of the rapid convergence of (6), additional high-energy contributions to (16) or to (14) would not change the results appreciably.

Returning to the s-wave scattering lengths, deviations from the high-energy quark-model additivity, as indicated by $f^t\neq 1$, are expected to cause violations in the universal scattering lengths. Taking the πN a_1^t scattering length as the standard quantity a, the scattering lengths for $N\pi$, $\Sigma\pi$, and $\Xi\pi$ would be

$$N\pi:\ a_1^t=a\,,$$
$$\Sigma\pi:\ a_1^t=2f^ta\approx 3a\,, \tag{18}$$
$$\Xi\pi:\ a_1^t=(2f^t-1)a\approx 2a\,. \tag{19}$$

For $\Sigma\pi$, $\Xi\pi$ they are larger than predicted by universality. Neither (18) nor (19) can be unambiguously tested. It should be kept in mind that any result for scattering lengths relies on the validity of (3).

Other possible tests of Eq. (6) are more speculative. However, we shall consider $\pi\rho$ and $\pi\Delta(1236)$ scattering using spin-averaged amplitudes.

For $\pi\rho$, inserting the π, ω, and A_1 poles, Eq. (6) gives

$$4\frac{g_{\rho\pi\pi}{}^2}{m_\rho{}^2}-2g_{\omega\rho\pi}{}^2+\frac{(m_{A_1}{}^2-m_\rho{}^2)^2}{4m_{A_1}{}^4}(2g_1{}^2+g_0{}^2)=0\,, \tag{20}$$

where g_1 and g_0 are, respectively, the transverse and longitudinal couplings in the $A_1\to\rho\pi$ decay, with $\Gamma_{A_1\rho\pi}=(1/12\pi)(2g_1{}^2+g_0{}^2)q^5/m_{A_1}{}^2$. To keep consistency with our previous arguments and the local saturation of (6) with s- and p-waves only, the $A_1\to\rho\pi$ decay should occur in a purely orbital s state. This corresponds to $|g_0/g_1|\approx 1$. (For information on the experimental situation and theoretical analysis of A_1 data, see Ref. 28. To make an estimate of $\Gamma_{A_1\rho\pi}$ we allow ourselves some freedom in playing simultaneously with SU(6) and chiral symmetry. From SU(6) we borrow the relation[29] $4g_{\rho\pi\pi}{}^2/m_\rho{}^2=g_{\omega\rho\pi}{}^2$, and from chiral symmetry[11,12] (or experiment), $m_{A_1}{}^2\simeq 2m_\rho{}^2$. Neglecting terms in $m_\pi{}^2$, Eq. (20) then gives

$$\Gamma_{A_1\rho\pi}=(1/\sqrt{2})\Gamma_{\rho\pi\pi} \tag{21a}$$
$$\simeq 90\ \text{MeV}\,. \tag{21b}$$

The width predicted in (21b) is quite acceptable (experimental value[22]: $\lesssim 95$ MeV).

For $\pi\Delta$ scattering (6), saturated with the same contributions as in the πN case [the $N(1525)$ is here negligible], allows a prediction for the $\Delta\Delta\pi$ coupling constant. We use data from Sutherland's work[30] and his definition of the $\Delta\Delta\pi$ coupling: $g_{\Delta^{++}\Delta^+\pi^+}\overline{\psi}{}_\mu^{++}\gamma_5\psi_\mu^+\phi_\pi^+$. The result is

$$\frac{g_{\Delta^{++}\Delta^+\pi^+}{}^2}{4\pi}=35^{+2}_{-5}\,, \tag{22}$$

in good agreement with the SU(6) value, ~32 (Refs. 28 and 30). In Ref. 30, from the Adler-Weisberger relation, a larger value is obtained, but this, we think, is related to the general difficulty in saturating the Adler-Weisberger relation with a restricted number of resonances.[31,11,19]

We summarize the main points in our work. A generalization of the Veneziano-Lovelace formula to other πT processes should preserve fully the power of duality. This implies having simultaneously satisfactory low- and high-energy behaviors and the correct dual connection between them. Equations (1)–(6), we believe, form a sort of prescription to follow in attempts to obtain realistic Veneziano formulas. The tests of Eq. (6) in $\pi\Sigma$ and πN scattering and in $\pi\rho$ and $\pi\Delta$ scattering using local mass cancellations are encouraging.

I wish to thank Professor L. Castillejo for encouragement, suggestions, and a critical reading of the manuscript. I should also like to acknowledge a research fellowship from the Calouste Gulbenkian Foundation.

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*Work supported by a Calouste Gulbenkian Foundation Research Fellowship.

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