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Scattering of a Charged Vector Meson in a Static Field at High Energies*

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We study here the scattering of a charged vector meson (without the anomalous magnetic moment) in a static field. The scattering amplitude for such a process is obtained explicitly in the case of a central field. We find that the simple-exponentiation form of the eikonal approximation does not occur.

I. INTRODUCTION

More than a decade ago, a number of authors discovered[1] that the scattering amplitude of a high-energy scalar meson (or a fermion without the anomalous magnetic moment) in a static field takes the simple-exponentiation form of the eikonal approximation. Two years ago, several authors have found that, at high energies, this simple-exponentiation form holds for the multi-photon-exchange amplitude of electron-electron scattering[2,3] or pion-pion scattering.[4] There remained the question of whether the simple-exponentiation form in fact occurs in *all* high-energy amplitudes.

Lately, it has become clear that this cannot be true. When the particles in collision have finite sizes or internal degrees of freedom, the simple-exponentiation form does not occur.

Several examples of this breakdown of simple exponentiation have been given. In field-theoretic models, for instance, the failure of exponentiation is due to the fact that a particle in collision develops a structure by creating particles.[5-7] Since field-theoretic models are difficult to handle, we have also sought easier and physically more transparent examples in potential scattering. In particular, we have studied the following two cases: (i) potential scattering with more than one channel[8]; (ii) potential scattering of a fermion with an anomalous magnetic moment.[9] In both instances, breakdown of the simple-exponentiation form occurs. Case (ii) is closely related to case (i): A fermion has two spin states which do not decouple at high energies if the fermion has an anomalous magnetic moment.

In this paper, we shall study the scattering of a charged vector meson in a static field. We shall show that the simple-exponentiation form does not occur even if the vector meson has no anomalous magnetic moment. Physically, this is because the spin states of a vector meson do not decouple at high energies even if the vector meson has no anomalous magnetic moment – a fact which can be easily checked by examining the Born term. Thus the case of the vector meson is equivalent to the potential-scattering case with more than one channel.

II. VECTOR MESON IN STATIC FIELD

We here consider the interaction of a charged vector meson[10] with a static field. We shall designate the magnetic moment of this vector meson to be $(1+\kappa)(e/2M)S$, where S and M are the spin and the mass of the vector meson, respectively. The wave function ϕ_ν of a vector meson in a static field $V(\vec{x})$ satisfies[11]

$$\partial_\mu G_{\mu\nu} - M^2\phi_\nu + ie\kappa\phi_\mu F_{\mu\nu} = 0\,, \qquad (2.1)$$

where

$$G_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu\,,$$

$$\partial_\mu = \frac{\partial}{\partial x_\mu} - ieA_\mu\,,$$

$$A_\mu = i\delta_{\mu 4}V(\vec{x})\,,$$

and

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu}\,.$$

Equation (2.1) can also be written as

$$\frac{\partial^2}{\partial x_\mu \partial x_\mu}\phi_\nu - \frac{\partial^2}{\partial x_\nu \partial x_\mu}\phi_\mu - M^2\phi_\nu = ie\left(A_\mu \frac{\partial \phi_\nu}{\partial x_\mu} + \frac{\partial}{\partial x_\mu}(A_\mu \phi_\nu) - A_\mu \frac{\partial \phi_\mu}{\partial x_\nu} - \frac{\partial}{\partial x_\mu}(A_\nu \phi_\mu)\right) + e^2(A_\mu A_\mu \phi_\nu - A_\mu A_\nu \phi_\mu) - ie\kappa\phi_\mu F_{\mu\nu}. \tag{2.2}$$

Applying ∂_ν to (2.1), we get

$$\partial_\nu \partial_\mu G_{\mu\nu} - M^2 \partial_\nu \phi_\nu + ie\kappa \partial_\nu \phi_\mu F_{\mu\nu} = 0, \tag{2.3}$$

which will be useful later. The boundary condition is

$$\lim_{z \to -\infty} \phi_\nu(x) = e^{-i(Et - pz)} a_\nu, \tag{2.4}$$

where a_ν is a constant and z is the third component of the four-vector x_μ.

We give explicitly the four equations contained in (2.1):

$$[(E - eV)^2 + \vec{\nabla}^2 - M^2]\phi_j + i(E - eV)\frac{\partial}{\partial x_j}\phi_0 - \frac{\partial}{\partial x_j}\vec{\nabla}\cdot\vec{\phi} + ie\kappa\phi_0 \frac{\partial}{\partial x_j}V(\vec{x}) = 0, \quad j = 1, 2, \text{ and } 3 \tag{2.5}$$

$$(\vec{\nabla}^2 - M^2)\phi_0 - i\vec{\nabla}\cdot[E - eV(\vec{x})]\vec{\phi} + ie\kappa\vec{\phi}\cdot\vec{\nabla}V(\vec{x}) = 0. \tag{2.6}$$

Putting

$$\Phi_\nu = e^{iE(t-z)}\phi_\nu, \tag{2.7}$$

we rewrite (2.5) and (2.6) as

$$\left(-2EeV + 2iE\frac{\partial}{\partial z} + \vec{\nabla}^2 - M^2 + e^2V^2\right)\vec{\Phi}_\perp + i(E - eV)\vec{\nabla}_\perp\Phi_0 - \vec{\nabla}_\perp\left[\left(iE + \frac{\partial}{\partial z}\right)\Phi_3 + \vec{\nabla}_\perp\cdot\vec{\Phi}_\perp\right] + ie\kappa\Phi_0\vec{\nabla}_\perp V(\vec{x}) = 0, \tag{2.8}$$

$$(-2EeV + E^2 + \vec{\nabla}_\perp^2 - M^2 + e^2V^2)\Phi_3 + i(E - eV)\left(iE + \frac{\partial}{\partial z}\right)\Phi_0 - \left(iE + \frac{\partial}{\partial z}\right)\vec{\nabla}_\perp\cdot\vec{\Phi}_\perp + ie\kappa\Phi_0\frac{\partial}{\partial z}V(\vec{x}) = 0, \tag{2.9}$$

and

$$\left(-E^2 + 2iE\frac{\partial}{\partial z} + \vec{\nabla}^2 - M^2\right)\Phi_0 - i\left(iE + \frac{\partial}{\partial z}\right)[E - eV(\vec{x})]\Phi_3 - i\vec{\nabla}_\perp\cdot[E - eV(\vec{x})]\vec{\Phi}_\perp + ie\kappa\vec{\Phi}\cdot\vec{\nabla}V(\vec{x}) = 0. \tag{2.10}$$

In the above equations, $\vec{\Phi}_\perp$ and $\vec{\nabla}_\perp$ are the transverse parts (containing the first and the second component) of $\vec{\Phi}$ and $\vec{\nabla}$, respectively.

For the convenience of later calculations, we shall choose the dependent variables in (2.8)–(2.10) to be Φ_0, $\Phi_- = \Phi_0 - \Phi_3$, and $\vec{\Phi}_\perp$. Then (2.8)–(2.10) become

$$\left(2i(E - eV)\mathfrak{D} - ie\frac{\partial V}{\partial z} + \mathfrak{D}^2 + \vec{\nabla}_\perp^2 - M^2 - \vec{\nabla}_\perp\vec{\nabla}_\perp\cdot\right)\vec{\Phi}_\perp + [ie\kappa(\vec{\nabla}_\perp V) - \mathfrak{D}\vec{\nabla}_\perp]\Phi_0 + [i(E - eV) + \mathfrak{D}]\vec{\nabla}_\perp\Phi_- = 0, \tag{2.11}$$

$$-[i(E - eV) + \mathfrak{D}]\vec{\nabla}_\perp\cdot\vec{\Phi}_\perp + \left(i(E - eV)\mathfrak{D} + \vec{\nabla}_\perp^2 - M^2 + ie\kappa\frac{\partial}{\partial z}V\right)\Phi_0 - [(E - eV)^2 + \vec{\nabla}_\perp^2 - M^2]\Phi_- = 0, \tag{2.12}$$

and

$$[-i(E - eV)\vec{\nabla}_\perp + ie(1 + \kappa)(\vec{\nabla}_\perp V)]\cdot\vec{\Phi}_\perp$$
$$+\left(i(E - eV)\mathfrak{D} + \mathfrak{D}^2 + \vec{\nabla}_\perp^2 - M^2 + ie\kappa\frac{\partial V}{\partial z}\right)\Phi_0 - \left((E - eV)^2 - i(E - eV)\mathfrak{D} + ie(1 + \kappa)\frac{\partial V}{\partial z}\right)\Phi_- = 0, \tag{2.13}$$

where

$$\mathfrak{D} = \frac{\partial}{\partial z} + ieV. \tag{2.14}$$

Finally, we shall rewrite (2.3) into a more convenient form. We first observe that

$$\partial_\nu\partial_\mu G_{\mu\nu} = -\partial_\nu\partial_\mu G_{\nu\mu} = -\partial_\mu\partial_\nu G_{\mu\nu} = -\partial_\nu\partial_\mu G_{\mu\nu} + ie\left(\frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu}\right) G_{\mu\nu}$$

$$= -\partial_\nu\partial_\mu G_{\mu\nu} + 2ie\frac{\partial A_\nu}{\partial x_\mu} G_{\mu\nu} ,$$

thus

$$\partial_\nu\partial_\mu G_{\mu\nu} = ie\frac{\partial A_\nu}{\partial x_\mu} G_{\mu\nu} . \tag{2.15}$$

From (2.3) and (2.15), we get

$$-ie(1+\kappa)\frac{\partial V}{\partial x_j}\left(\frac{\partial \phi_0}{\partial x_j} - i(E-eV)\phi_j\right) - M^2[-i(E-eV)\phi_0 + \vec{\nabla}\cdot\vec{\phi}] - ie\kappa\phi_0\nabla^2 V = 0 . \tag{2.16}$$

Substituting (2.7) into (2.16), we get

$$[e(1+\kappa)(E-eV)(\vec{\nabla}_\perp V) + M^2\vec{\nabla}_\perp]\cdot\vec{\Phi}_\perp + \left(ie(1+\kappa)\frac{\partial V}{\partial z}\mathfrak{D} + ie(1+\kappa)(\vec{\nabla}_\perp V)\cdot\vec{\nabla}_\perp + M^2\mathfrak{D} + ie\kappa(\nabla^2 V)\right)\Phi_0 - \left(e(1+\kappa)\frac{\partial V}{\partial z}(E-eV) + M^2 i(E-eV-i\mathfrak{D})\right)\Phi_- = 0 . \tag{2.17}$$

Although Eq. (2.17) is already contained in Eqs. (2.11)–(2.13), it is useful in later calculations.

In the following sections, we shall solve (2.11)–(2.13) to obtain the scattering amplitude in the high-energy limit.

III. THE WAVE FUNCTION

In this section we shall solve (2.11)–(2.13) in the limit $E\to\infty$, for the special case $\kappa=0$.

When $\kappa=0$, (2.11)–(2.13) take the form

$$\left(2i(E-eV)\mathfrak{D} - ie\frac{\partial V}{\partial z} + \mathfrak{D}^2 + \vec{\nabla}_\perp^2 - M^2 - \vec{\nabla}_\perp\vec{\nabla}_\perp\cdot\right)\vec{\Phi}_\perp - \mathfrak{D}\vec{\nabla}_\perp\Phi_0 + [i(E-eV)+\mathfrak{D}]\vec{\nabla}_\perp\Phi_- = 0 , \tag{3.1}$$

$$-[i(E-eV)+\mathfrak{D}]\vec{\nabla}_\perp\cdot\vec{\Phi}_\perp + [i(E-eV)\mathfrak{D} + \vec{\nabla}_\perp^2 - M^2]\Phi_0 - [(E-eV)^2 + \vec{\nabla}_\perp^2 - M^2]\Phi_- = 0 , \tag{3.2}$$

and

$$[-i(E-eV)\vec{\nabla}_\perp + ie(\vec{\nabla}_\perp V)]\cdot\vec{\Phi}_\perp + [i(E-eV)\mathfrak{D} + \mathfrak{D}^2 + \vec{\nabla}_\perp^2 - M^2]\Phi_0 - \left((E-eV)^2 - i(E-eV)\mathfrak{D} + ie\frac{\partial V}{\partial z}\right)\Phi_- = 0 . \tag{3.3}$$

In the limit $E\to\infty$, (2.4) and (2.7) give

$$\lim_{z\to-\infty}\Phi_\nu(x) \sim a_\nu . \tag{3.4}$$

By Lorentz covariance, a_0 and a_3 are $O(E)$, while $\vec{a}_\perp$ is $O(1)$.

Also, Eq. (2.17) becomes, at $\kappa=0$,

$$[e(E-eV)(\vec{\nabla}_\perp V) + M^2\vec{\nabla}_\perp]\cdot\vec{\Phi}_\perp + \left(ie\frac{\partial V}{\partial z}\mathfrak{D} + ie(\vec{\nabla}_\perp V)\cdot\vec{\nabla}_\perp + M^2\mathfrak{D}\right)\Phi_0 - \left[(E-eV)\left(e\frac{\partial V}{\partial z} + iM^2\right) + M^2\mathfrak{D}\right]\Phi_- = 0. \tag{3.5}$$

A. The Wave Function in the Leading Order

We shall solve (3.1)–(3.4) in the limit $E\to\infty$. We make the asymptotic expansion

$$\Phi_0 = (E/M)\Phi_0^{(0)} + \Phi_0^{(1)} + ME^{-1}\Phi_0^{(2)} + \cdots ,$$
$$\Phi_- = (E/M)\Phi_-^{(0)} + \Phi_-^{(1)} + ME^{-1}\Phi_-^{(2)} + \cdots , \tag{3.6}$$
$$\vec{\Phi}_\perp = (E/M)\vec{\Phi}_\perp^{(0)} + \vec{\Phi}_\perp^{(1)} + ME^{-1}\vec{\Phi}_\perp^{(2)} + \cdots ,$$

and substitute it into (3.1)–(3.3). The leading term in (3.2) (proportional to E^2) immediately gives

$$\Phi_-^{(0)} = 0. \tag{3.7}$$

Similarly, the leading term in (3.1) gives

$$\mathfrak{D}\vec{\Phi}_\perp^{(0)}=0,$$

which, together with (3.4) and $\vec{a}_\perp=O(1)$, implies that

$$\vec{\Phi}_\perp^{(0)}=0. \tag{3.8}$$

Next we equate to zero the terms that are $O(E)$ in (3.1) and the terms that are $O(E^2)$ in (3.2):

$$2i\mathfrak{D}\vec{\Phi}_\perp^{(1)}-M^{-1}\mathfrak{D}\vec{\nabla}_\perp\Phi_0^{(0)}+i\vec{\nabla}_\perp\Phi_-^{(1)}=0 \tag{3.9}$$

and

$$i\mathfrak{D}\Phi_0^{(0)}-M\Phi_-^{(1)}=0. \tag{3.10}$$

Because equating the terms which are $O(E^2)$ in (3.3) to zero also gives Eq. (3.10), we still need one more equation for determining $\Phi_0^{(0)}$, $\Phi_-^{(1)}$, and $\vec{\Phi}_\perp^{(1)}$. We can go to the higher-order terms in (3.2) and (3.3), but it is much simpler to collect the leading terms in (3.5):

$$e(\vec{\nabla}_\perp V)\cdot\vec{\Phi}_\perp^{(1)}+\left(ie\frac{\partial V}{\partial z}\mathfrak{D}+ie(\vec{\nabla}_\perp V)\cdot\vec{\nabla}_\perp+M^2\mathfrak{D}\right)\Phi_0^{(0)}M^{-1}-\left(e\frac{\partial V}{\partial z}+iM^2\right)\Phi_-^{(1)}=0\,. \tag{3.11}$$

Making use of (3.10), we reduce (3.11) into

$$e(\vec{\nabla}_\perp V)\cdot\vec{\Phi}_\perp^{(1)}+[ieM^{-1}(\vec{\nabla}_\perp V)\cdot\vec{\nabla}_\perp+2M\mathfrak{D}]\Phi_0^{(0)}=0\,. \tag{3.12}$$

Eqs. (3.9), (3.10), and (3.12) constitute a complete set of equations which we can solve to obtain the leading terms of Φ_0, Φ_-, and $\vec{\Phi}_\perp$.

Let us set

$$\vec{H}_1=[\vec{\Phi}_\perp^{(1)}+iM^{-1}\vec{\nabla}_\perp\Phi_0^{(0)}]\exp\left(ie\int_{-\infty}^{z}V(\vec{x}_\perp,z')dz'\right),$$
$$F_0=\Phi_0^{(0)}\exp\left(ie\int_{-\infty}^{z}V(\vec{x}_\perp,z')dz'\right), \tag{3.13}$$
$$G_1=\Phi_-^{(1)}\exp\left(ie\int_{-\infty}^{z}V(\vec{x}_\perp,z')dz'\right),$$

and substitute (3.13) into (3.9), (3.10), and (3.12). We get

$$\frac{\partial\vec{H}_1}{\partial z}-\tfrac{1}{2}eM^{-1}(\vec{\nabla}_\perp V)F_0=0\,, \tag{3.14}$$

$$\frac{\partial F_0}{\partial z}+\tfrac{1}{2}eM^{-1}(\vec{\nabla}_\perp V)\cdot\vec{H}_1=0\,, \tag{3.15}$$

and

$$G_1=iM^{-1}\frac{\partial}{\partial z}F_0. \tag{3.16}$$

Note that (3.14)–(3.16) are no longer partial differential equations. Equations (3.14) and (3.15) can be written in the matrix form

$$\frac{\partial}{\partial z}\begin{bmatrix}H_x\\H_y\\F_0\end{bmatrix}=\tfrac{1}{2}eM^{-1}A\begin{bmatrix}H_x\\H_y\\F_0\end{bmatrix}, \tag{3.17}$$

where H_x and H_y are the components of $\vec{H}_1$ and

$$A=\begin{bmatrix}0&0&\partial V/\partial x\\0&0&\partial V/\partial y\\-\partial V/\partial x&-\partial V/\partial y&0\end{bmatrix}. \tag{3.18}$$

If the potential $V(\vec{x})$ is a central field, i.e., $V(\vec{x})=V(r)$, where $r=|\vec{x}|$, then

$$A=r^{-1}V'(r)B\,, \tag{3.19}$$

where

$$V'(r)=\frac{d}{dr}V(r)$$

and

$$B=\begin{bmatrix}0 & 0 & x\\ 0 & 0 & y\\ -x & -y & 0\end{bmatrix}. \tag{3.20}$$

The important point to observe is that B is independent of z. Since, in the process of solving (3.17), we can treat B as a constant, it is possible to obtain the solution of (3.17) in closed form.

We write

$$B=|\vec{x}_\perp|^{-2}\begin{bmatrix}y & x & x\\ -x & y & y\\ 0 & i|\vec{x}_\perp| & -i|\vec{x}_\perp|\end{bmatrix}\begin{bmatrix}0 & 0 & 0\\ 0 & i|\vec{x}_\perp| & 0\\ 0 & 0 & -i|\vec{x}_\perp|\end{bmatrix}\begin{bmatrix}y & -x & 0\\ \frac{1}{2}x & \frac{1}{2}y & -\frac{1}{2}i|\vec{x}_\perp|\\ \frac{1}{2}x & \frac{1}{2}y & \frac{1}{2}i|\vec{x}_\perp|\end{bmatrix}, \tag{3.21}$$

where

$|\vec{x}_\perp|^2=x^2+y^2$. From (3.17)–(3.21), we then obtain

$$\begin{bmatrix}H_x\\ H_y\\ F_0\end{bmatrix}=|\vec{x}_\perp|^{-2}\begin{bmatrix}y & x & x\\ -x & y & y\\ 0 & i|\vec{x}_\perp| & -i|\vec{x}_\perp|\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & e^{iW} & 0\\ 0 & 0 & e^{-iW}\end{bmatrix}\begin{bmatrix}y & -x & 0\\ \frac{1}{2}x & \frac{1}{2}y & -\frac{1}{2}i|\vec{x}_\perp|\\ \frac{1}{2}x & \frac{1}{2}y & \frac{1}{2}i|\vec{x}_\perp|\end{bmatrix}\begin{bmatrix}a_1\\ a_2\\ ME^{-1}a_0\end{bmatrix}$$

$$=\begin{bmatrix}(y^2+x^2\cos W)/|\vec{x}_\perp|^2 & -xy(1-\cos W)/|\vec{x}_\perp|^2 & x\sin W|\vec{x}_\perp|\\ -xy(1-\cos W)/|\vec{x}_\perp|^2 & (x^2+y^2\cos W)/|\vec{x}_\perp|^2 & y\sin W|\vec{x}_\perp|\\ -x\sin W/|\vec{x}_\perp| & -y\sin W/|\vec{x}_\perp| & \cos W\end{bmatrix}\begin{bmatrix}a_1\\ a_2\\ ME^{-1}a_0\end{bmatrix}, \tag{3.22}$$

where

$$W=\tfrac{1}{2}eM^{-1}|\vec{x}_\perp|\int_{-\infty}^{z}V'((z'^2+|\vec{x}_\perp|^2)^{1/2})(z'^2+|\vec{x}_\perp|^2)^{1/2}\,dz', \tag{3.23}$$

and a_1, a_2, and a_0 are related to the incident wave vector and are defined in (2.4) or (3.4).

From (3.22) and (3.13), we get

$$\Phi_0^{(0)}=[-(xa_1+ya_2)|\vec{x}_\perp|^{-1}\sin W+ME^{-1}a_0\cos W]\exp\left(-ie\int_{-\infty}^{z}V(\vec{x}_\perp,z')dz'\right), \tag{3.24}$$

$$\Phi_-^{(1)}=-\tfrac{1}{2}ieM^{-2}V'(|\vec{x}|)|\vec{x}|^{-1}[(xa_1+ya_2)\cos W+ME^{-1}|\vec{x}_\perp|a_0\sin W]\exp\left(-ie\int_{-\infty}^{z}V(\vec{x}_\perp,z')dz'\right), \tag{3.25}$$

$$\Phi_x^{(1)}=[a_1(y^2+x^2\cos W)|\vec{x}_\perp|^{-2}-a_2xy(1-\cos W)|\vec{x}_\perp|^{-2}+ME^{-1}a_0x|\vec{x}_\perp|^{-1}\sin W]\exp\left(-ie\int_{-\infty}^{z}V(\vec{x}_\perp,z')dz'\right)-iM^{-1}\frac{\partial}{\partial x}\Phi_0^{(0)}, \tag{3.26}$$

and

$$\Phi_y^{(1)}=[-a_1xy(1-\cos W)|\vec{x}_\perp|^{-2}+a_2(x^2+y^2\cos W)|\vec{x}_\perp|^{-2}+ME^{-1}a_0y|\vec{x}_\perp|^{-1}\sin W]\exp\left(-ie\int_{-\infty}^{z}V(\vec{x}_\perp,z')dz'\right)-iM^{-1}\frac{\partial}{\partial x}\Phi_0^{(0)}. \tag{3.27}$$

Notice that $\lim_{|\vec{x}|\to\infty}\Phi_-^{(1)}=0$, since V' vanishes at infinity.

B. The Wave Function in the Next Order

We shall now give the equations which determine $\Phi_0^{(1)}$, $\Phi_-^{(2)}$, and $\vec{\Phi}_\perp^{(2)}$. These three equations are obtained by equating to zero, respectively, the following: (i) the terms which are $O(1)$ in (3.1); (ii) the terms which are $O(E)$ in (3.2); (iii) the terms which are $O(1)$ in (3.5). We get

$$2iM\mathfrak{D}\vec{\Phi}_\perp^{(2)}-\mathfrak{D}\vec{\nabla}_\perp\Phi_0^{(1)}+iM\vec{\nabla}_\perp\Phi_-^{(2)}=\left(2ieV\mathfrak{D}+ie\frac{\partial V}{\partial z}-\mathfrak{D}^2-\vec{\nabla}_\perp^{\,2}+M^2+\vec{\nabla}_\perp\vec{\nabla}_\perp\cdot\right)\vec{\Phi}_\perp^{(1)}+(ieV-\mathfrak{D})\vec{\nabla}_\perp\Phi_-^{(1)}, \tag{3.28}$$

$$i\mathfrak{D}\Phi_0^{(1)} - M\Phi_-^{(2)} = i\vec{\nabla}_\perp\cdot\vec{\Phi}_\perp^{(1)} + (ieV\mathfrak{D} - \vec{\nabla}_\perp{}^2 + M^2)M^{-1}\Phi_0^{(0)} - 2eV\Phi_-^{(1)}, \tag{3.29}$$

and

$$eM(\vec{\nabla}_\perp V)\cdot\vec{\Phi}_\perp^{(2)} + \left(ie\frac{\partial V}{\partial z}\mathfrak{D} + ie(\vec{\nabla}_\perp V)\cdot\vec{\nabla}_\perp + M^2\right)\Phi_0^{(1)} - eM\frac{\partial V}{\partial z}\Phi_-^{(2)}$$
$$= [e^2V(\vec{\nabla}_\perp V) - M^2\vec{\nabla}_\perp]\cdot\vec{\Phi}_\perp^{(1)} - \left[eV\left(e\frac{\partial V}{\partial z} + iM^2\right) - M^2\mathfrak{D}\right]\Phi_-^{(1)}. \tag{3.30}$$

Although Eqs. (3.28)–(3.30) are easy to solve, their solutions are quite complicated. Fortunately, for the purpose of obtaining the scattering amplitude, we need only the asymptotic form of the wave function in the limit $z\to\infty$. In particular, we shall need $\lim_{z\to\infty}\Phi_-^{(2)}$.

As $z\to\infty$, the potential V can be neglected, and (2.2) and (2.3) become

$$\left(\frac{\partial^2}{\partial x_\mu \partial x_\mu} - M^2\right)\phi_\nu \sim 0, \quad z\to\infty \tag{3.31}$$

and

$$\frac{\partial \phi_\nu}{\partial x_\nu} \sim 0, \quad z\to\infty. \tag{3.32}$$

Substituting (2.7) into (3.31) and (3.32), we get

$$\left(2iE\frac{\partial}{\partial z} + \frac{\partial^2}{\partial z^2} + \vec{\nabla}_\perp{}^2 - M^2\right)\Phi_\nu \sim 0, \quad z\to\infty \tag{3.33}$$

and

$$-iE\Phi_- + \frac{\partial\Phi_3}{\partial z} + \vec{\nabla}_\perp\cdot\vec{\Phi}_\perp \sim 0, \quad z\to\infty. \tag{3.34}$$

From (3.33), we get

$$\frac{\partial\Phi_\nu}{\partial z} \sim i(2E)^{-1}(\vec{\nabla}_\perp{}^2 - M^2)\Phi_\nu, \quad z\to\infty, \tag{3.35}$$

the term $\partial^2\Phi_\nu/\partial z^2$ being negligible. From (3.34) and (3.35), we get

$$\Phi_- \sim -iE^{-1}\left(\vec{\nabla}_\perp\cdot\vec{\Phi}_\perp + \frac{\partial\Phi_0}{\partial z}\right) \sim -iE^{-1}\vec{\nabla}_\perp\cdot\vec{\Phi}_\perp + (2E^2)^{-1}(\vec{\nabla}_\perp{}^2 - M^2)\Phi_0, \quad z\to\infty. \tag{3.36}$$

Notice that Φ_- is of the order of E^{-1} in the limit $z\to\infty$. This is consistent with the earlier observation that $\lim_{z\to\infty}\Phi_-^{(1)} = 0$. Equations (3.35) and (3.36) can also be obtained from (3.28)–(3.30) by setting $V=0$.

Finally, let us write down explicitly the wave functions in the limit $z\to\infty$. We have from (3.10) and (3.24)–(3.27) that, as $z\to\infty$,

$$\Phi_0 \sim [-(xa_1 + ya_2)|\vec{x}_\perp|^{-1}EM^{-1}\sin U + a_0\cos U]\exp\left(-ie\int_{-\infty}^{\infty} V(\vec{x}_\perp, z')dz'\right) \tag{3.37}$$

and

$$\vec{\Phi}_\perp \sim [\vec{a}_\perp - \vec{x}_\perp(\vec{x}_\perp\cdot\vec{a}_\perp)(1-\cos U)|\vec{x}_\perp|^{-2} + ME^{-1}a_0\vec{x}_\perp|\vec{x}_\perp|^{-1}\sin U]\exp\left(-ie\int_{-\infty}^{\infty} V(\vec{x}_\perp, z')dz'\right) - iE^{-1}\vec{\nabla}_\perp\Phi_0, \tag{3.38}$$

where

$$U = \tfrac{1}{2}eM^{-1}|\vec{x}_\perp|\int_{-\infty}^{\infty}\frac{V'(\vec{x}_\perp, z')}{(z'^2 + |\vec{x}_\perp|^2)^{1/2}}dz'. \tag{3.39}$$

C. The Polarization Vectors

The incoming vector meson has three polarizations. They are, in the limit $E\to\infty$,

$$[1, 0, 0, 0], \tag{3.40}$$

$$[0, 1, 0, 0], \tag{3.41}$$

$$[0, 0, E/M, iE/M - i\tfrac{1}{2}M/E], \tag{3.42}$$

where we use the notation $[a_1, a_2, a_3, a_4]$ for a four-vector. The first two vectors, given by (3.40) and (3.41), respectively, are transverse, while the third vector, given by (3.42), is longitudinal.

Let us denote the momentum transfer as $\vec{\Delta}$, which is taken to be in the direction of the x axis. Then the outgoing vector meson can have the following three polarizations:

$$[1, 0, -\Delta E^{-1}, 0], \tag{3.43}$$

$$[0, 1, 0, 0], \tag{3.44}$$

and

$$[\Delta/M, 0, E/M - \tfrac{1}{2}\Delta^2/(EM), iE/M - \tfrac{1}{2}iM/E], \tag{3.45}$$

where $\Delta = |\vec{\Delta}|$. The polarization vectors given by (3.43) and (3.44) are transverse, while that given by (3.45) is longitudinal.

IV. THE SCATTERING AMPLITUDE

The scattering amplitude is equal to the integral of the right-hand side of (2.2) multiplied by the complex conjugate of the wave function of the outgoing plane wave. Thus

$$\mathfrak{M}_{fi} = b_\nu^* \int d^3x\, d^{-ip_f x}\left[ie\left(A_\mu \frac{\partial \phi_\nu}{\partial x_\mu} + \frac{\partial}{\partial x_\mu}(A_\mu\phi_\nu) - A_\mu \frac{\partial \phi_\mu}{\partial x_\nu} - \frac{\partial}{\partial x_\mu}(A_\nu\phi_\mu)\right) + e^2(A_\mu A_\mu \phi_\nu - A_\mu A_\nu \phi_\mu) - ie\kappa\phi_\mu F_{\mu\nu}\right]. \tag{4.1}$$

By making use of Eq. (2.2), (4.1) is reduced to

$$\mathfrak{M}_{fi} = b_\nu^* \int d^3x\, e^{-ip_f x}\left[\frac{\partial^2}{\partial x_\mu \partial x_\mu}\phi_\nu - \frac{\partial}{\partial x_\nu}\left(\frac{\partial \phi_\mu}{\partial x_\mu}\right) - M^2\phi_\nu\right]. \tag{4.2}$$

In the above, b_ν and p_f are, respectively, the polarization vector and the four-momentum of the outgoing vector meson.

Since

$$\left(\frac{\partial^2}{\partial x_\mu \partial x_\mu} - M^2\right)\phi_\nu \quad \text{and} \quad \frac{\partial \phi_\mu}{\partial x_\mu}$$

are both zero if ϕ_μ is a free field, we may rewrite (4.2) as

$$\mathfrak{M}_{fi} = b_\nu^* \int d^3x\, e^{-ip_f x}\left(\frac{\partial^2}{\partial x_\mu \partial x_\mu}\overline{\phi}_\nu - \frac{\partial^2}{\partial x_\nu \partial x_\mu}\overline{\phi}_\mu - M^2\overline{\phi}_\nu\right), \tag{4.3}$$

where

$$\overline{\phi}_\nu = \phi_\nu - e^{-ip_i x}a_\nu, \tag{4.4}$$

with a_ν and p_i the polarization vector and the four-momentum, respectively, of the incoming vector meson. Notice that $\overline{\phi}_\nu$ vanishes at infinity.

In order to make use of the results in Sec. III on the asymptotic form of the wave function in the limit $z \to \infty$, we write (4.3) as

$$\mathfrak{M}_{fi} = \lim_{L\to\infty} b_\nu^* \int_{|z|<L} d^3x\, e^{-ip_f x}\left(\frac{\partial^2}{\partial x_\mu \partial x_\mu}\overline{\phi}_\nu - \frac{\partial^2}{\partial x_\nu \partial x_\mu}\overline{\phi}_\mu - M^2\phi_\nu\right). \tag{4.5}$$

In fact, if the external field vanishes outside of a finite region, we only need to take L large enough so that the external field vanishes for $|z|>L$. This is because (2.2) becomes the free wave equation in the region where A_ν vanishes, and the integrand in (4.5) is equal to zero. For clarity of argument it is helpful to imagine that L can be so chosen and the limiting process $L \to \infty$ is unnecessary.

We also observe that since $\phi_{f\nu} = e^{ip_f x}b_\nu$ is a free field, we have

$$\frac{\partial \phi_{f\nu}}{\partial x_\nu} = 0 \quad \text{and} \quad \left(\frac{\partial^2}{\partial x_\mu \partial x_\mu} - M^2\right)\phi_{f\nu} = 0.$$

Therefore, if we perform integration by parts to get the differential operators in (4.5) to apply on the final-state wave function $\phi_{f\nu}$, the integrand vanishes and we are left with only the surface terms. Furthermore, the integrated terms at the surfaces $|x|=\infty$ and $|y|=\infty$ vanish as $\overline{\phi}_\nu$ vanishes at infinity, and the integrated terms at the surface $z=-L$ vanishes because of the boundary condition. Thus we are left with only those terms contributed by the surface $z=L$ and originating from integration by parts with respect to z. Thus (4.5) is reduced to

$$\mathfrak{M}_{fi}=\lim_{L\to\infty}\int d^2x_\perp \left(\Phi^*_{f\nu}\frac{\partial\overline{\phi}_\nu}{\partial z}-\overline{\phi}_\nu\frac{\partial\phi^*_{f\nu}}{\partial z}+\phi^*_{f3}\frac{\partial\overline{\phi}_\mu}{\partial x_\mu}\right)\bigg|_{z=L}. \tag{4.6}$$

Since $\partial\overline{\Phi}_\mu/\partial x_\mu$ vanishes if the external field vanishes, we can drop it from (4.6). Thus (4.6) is finally reduced to

$$\mathfrak{M}_{fi}=\lim_{L\to\infty}\int d^2x_\perp \left(\Phi^*_{f\nu}\frac{\partial\overline{\phi}_\nu}{\partial z}-\overline{\phi}_\nu\frac{\partial\Phi^*_{f\nu}}{\partial z}\right)\bigg|_{z=L}. \tag{4.7}$$

So far no approximation has been made and (4.7) is exact.

In the high-energy limit $E\to\infty$ (4.7) is reduced to

$$\mathfrak{M}_{fi}\sim 2Ei\int d^2x_\perp \lim_{z\to\infty}\phi^*_{f\nu}\overline{\phi}_\nu=2Ei\int d^2x_\perp e^{-i\Delta x}\lim_{z\to\infty}\left[b^*_\nu\Phi_\nu-b^*_\nu a_\nu\right], \tag{4.8}$$

where (2.7) has been used. We may, alternatively, write (4.8) as

$$\mathfrak{M}_{fi}\sim 2Ei\int d^2x_\perp e^{-i\Delta x}\lim_{z\to\infty}\left[-b^*_3(\Phi_- - a_-)-b^*_-(\Phi_0-a_0)+\vec{b}^*_\perp\cdot(\vec{\Phi}_\perp-\vec{a}_\perp)\right], \tag{4.9}$$

where $b_-=b_0-b_3$.

In the following, we shall explicitly calculate $\mathfrak{M}_{fi}$ from (4.9), (3.36)–(3.38), and (3.31). We shall consider only the case in which the potential is central so that the wave function is given by (3.36)–(3.38).

A. Transverse to Transverse

If both the incoming and the outgoing vector mesons are of transverse polarization, then b_3 and b_- are both $O(E^{-1})$ and $a_0=a_3=0$. Equation (4.9) becomes

$$\mathfrak{M}_{fi}\sim 2Ei\int d^2x_\perp e^{-i\Delta x}\lim_{z\to\infty}\left[-b^*_-(\Phi_0-a_0)+\vec{b}^*_\perp\cdot(\vec{\Phi}_\perp-\vec{a}_\perp)\right]. \tag{4.10}$$

Substituting (3.37) and (3.38) into (4.10), we get

$$\mathfrak{M}_{fi}\sim 2Ei\int d^2x_\perp e^{-i\Delta x}\left[\left[\vec{b}^*_\perp\cdot\vec{a}_\perp-(\vec{x}_\perp\cdot\vec{b}^*_\perp)(\vec{x}_\perp\cdot\vec{a}_\perp)(1-\cos U)|\vec{x}_\perp|^{-2}\right]\exp\left(-ie\int_{-\infty}^{\infty}V(\vec{x}_\perp,z)dz\right)-\vec{b}^*_\perp\cdot\vec{a}_\perp\right]. \tag{4.11}$$

In obtaining (4.11), we have set

$$\int d^2x_\perp e^{-i\Delta x}[-b^*_-(\Phi_0-a_0)-iE^{-1}\vec{b}^*_\perp\cdot\vec{\nabla}_\perp\Phi_0]=0. \tag{4.12}$$

Equation (4.12) is true because

$$\begin{aligned} iE^{-1}\int d^2x_\perp e^{-i\Delta x}\vec{b}^*_\perp\cdot\vec{\nabla}_\perp\Phi_0 &= iE^{-1}\int d^2x_\perp e^{-i\Delta x}\vec{b}^*_\perp\cdot\vec{\nabla}_\perp(\Phi_0-a_0)\\ &=-E^{-1}\int d^2x_\perp e^{-i\Delta x}(\vec{b}^*_\perp\cdot\vec{\Delta})(\Phi_0-a_0), \end{aligned} \tag{4.13}$$

the last step following an integration by parts. Furthermore, from (3.36) or (3.37), it is easily shown that

$$b_-=E^{-1}(\vec{b}_\perp\cdot\vec{\Delta})$$

if b_μ is transverse. Thus (4.12) is verified.

Since we have taken V to be central, we may make in (4.11) the replacement

$$(\vec{x}_\perp\cdot\vec{b}^*_\perp)(\vec{x}_\perp\cdot\vec{a})=(xb^*_1+yb^*_2)(xa_1+ya_2)\to x^2a_1b^*_1+y^2a_2b^*_2. \tag{4.14}$$

This is because the terms linear in y vanish upon integration as they are odd functions of y. Thus if $\vec{a}_\perp$ and $\vec{b}_\perp$ are both in the scattering plane, we have

$$\mathfrak{M}_\parallel \sim 2Ei \int d^2x_\perp e^{-i\Delta x}\left[[1 - x^2|\vec{x}_\perp|^{-2}(1-\cos U)] \exp\left(-ie\int_{-\infty}^{\infty} V(\vec{x}_\perp, z)dz\right) - 1\right]. \tag{4.15}$$

If $\vec{a}_\perp$ and $\vec{b}_\perp$ are both perpendicular to the scattering plane, we have

$$\mathfrak{M}_\perp \sim 2Ei \int d^2x_\perp e^{-i\Delta x}\left[[1 - y^2|\vec{x}_\perp|^{-2}(1-\cos U)] \exp\left(-ie\int_{-\infty}^{\infty} V(\vec{x}_\perp, z)dz\right) - 1\right]. \tag{4.16}$$

If one of $\vec{a}_\perp$ and $\vec{b}_\perp$ is parallel to the scattering plane and the other is perpendicular to the scattering plane, the scattering amplitude vanishes.

In terms of the helicity states, we have

$$\mathfrak{M}_{++} = \mathfrak{M}_{--} \sim 2Ei \int d^2x_\perp e^{-i\Delta x}\left[\tfrac{1}{2}(1+\cos U) \exp\left(-ie\int_{-\infty}^{\infty} V(\vec{x}_\perp, z)dz\right) - 1\right], \tag{4.17}$$

and

$$\mathfrak{M}_{+-} = \mathfrak{M}_{-+} \sim Ei \int d^2x_\perp e^{-i\Delta x}(y^2 - x^2)|\vec{x}_\perp|^{-2}(1-\cos U) \exp\left(-ie\int_{-\infty}^{\infty} V(\vec{x}_\perp, z)dz\right). \tag{4.18}$$

In the above $\mathfrak{M}_{+-}$ designates the scattering amplitude for the case in which the incoming vector meson is of helicity -1 and the outgoing vector meson is of helicity $+1$. The other notations should be obvious. Notice that at the forward direction $\Delta = 0$, we have $\mathfrak{M}_{+-} = \mathfrak{M}_{-+} = 0$.

B. Longitudinal to Transverse

As long as the outgoing vector meson is of transverse polarization, (4.10) always holds. Now if the incoming vector meson is of longitudinal polarization, we have from (3.37)–(3.38) that, as $z \to \infty$,

$$\Phi_0 \sim EM^{-1}\cos U \exp\left(-ie\int_{-\infty}^{\infty} V(\vec{x}_\perp, z')dz'\right), \tag{4.19}$$

$$\vec{\Phi}_\perp \sim \vec{x}_\perp|\vec{x}_\perp|^{-1}\sin U \exp\left(-ie\int_{-\infty}^{\infty} V(\vec{x}_\perp, z')dz'\right) - iE^{-1}\vec{\nabla}_\perp\Phi_0. \tag{4.20}$$

Substituting (4.19) and (4.20) into (4.10) and remembering (4.12), we get

$$\mathfrak{M}_{fi} \sim 2Ei \int d^2x_\perp e^{-i\Delta x}(\vec{b}_\perp^* \cdot \vec{x}_\perp)|\vec{x}_\perp|^{-1}\sin U \exp\left(-ie\int_{-\infty}^{\infty} V(\vec{x}_\perp, z)dz\right). \tag{4.21}$$

Thus, if the polarization vector of the outgoing vector meson lies in the scattering plane, the scattering amplitude is given by

$$2Ei \int d^2x_\perp e^{-i\Delta x} x|\vec{x}_\perp|^{-1}\sin U \exp\left(-ie\int_{-\infty}^{\infty} V(\vec{x}_\perp, z)dz\right). \tag{4.22}$$

And if the polarization vector of the outgoing vector meson is perpendicular to the scattering plane, the scattering amplitude vanishes.

From (4.22), the helicity amplitudes are given by

$$\mathfrak{M}_{+0} = \mathfrak{M}_{-0}$$
$$= \sqrt{2}Ei \int d^2x_\perp e^{-i\Delta x} x|\vec{x}_\perp|^{-1}\sin U \exp\left(-ie\int_{-\infty}^{\infty} V(\vec{x}_\perp, z)dz\right), \tag{4.23}$$

where $\mathfrak{M}_{+0}$ ($\mathfrak{M}_{-0}$) is the amplitude for the transition from the helicity-zero state to the helicity-plus-(minus-) one state.

Notice that (4.22) and the right-hand side of (4.23) both vanish at $\Delta = 0$.

C. Transverse to Longitudinal

If the outgoing vector meson is longitudinal and the incoming vector meson is transverse, we have from

(3.38) and (4.9) that

$$\mathfrak{M}_{fi} \sim 2Ei \int d^2x_\perp e^{-i\Delta x} \lim_{z\to\infty} [-EM^{-1}\Phi_- + (2EM)^{-1}(M^2 - \Delta^2)\Phi_0 + \Delta M^{-1}(\Phi_x - a_x)], \tag{4.24}$$

a_0 and a_- being zero. From (3.36), we have

$$\int d^2x_\perp e^{-i\Delta x} \lim_{z\to\infty}(-EM^{-1}\Phi_-) = \int d^2x_\perp e^{-i\Delta x} \lim_{z\to\infty} [iM^{-1}\vec{\nabla}_\perp \cdot (\vec{\Phi}_\perp - \vec{a}_\perp) - (2EM)^{-1}(\vec{\nabla}_\perp{}^2 - M^2)\Phi_0]$$

$$= \int d^2x_\perp e^{-i\Delta x} \lim_{z\to\infty} [-\Delta M^{-1}(\Phi_x - a_x) + (2EM)^{-1}(\Delta^2 + M^2)\Phi_0]. \tag{4.25}$$

The last step is a result of integration by parts. Combining (4.24), (4.25), and (3.37), we get

$$\mathfrak{M}_{fi} \sim 2Mi \int d^2x_\perp e^{-i\Delta x} \lim \Phi_0$$

$$\sim -2Ei \int d^2x_\perp e^{-i\Delta x}(xa_1)\, |\vec{x}_\perp|^{-1} \sin U \exp\left(-ie \int_{-\infty}^{\infty} V(\vec{x}_\perp, z)dz\right). \tag{4.26}$$

Thus, if the polarization vector of the incoming vector meson is in the scattering plane, the scattering amplitude is equal to

$$-2Ei \int d^2x_\perp e^{-i\Delta x}\; x|\vec{x}_\perp|^{-1} \sin U \exp\left(-ie \int_{-\infty}^{\infty} V(\vec{x}_\perp, z)dz\right). \tag{4.27}$$

And if the polarization vector of the incoming vector meson is perpendicular to the scattering plane, the scattering amplitude vanishes. We also have

$$\mathfrak{M}_{0+} = \mathfrak{M}_{0-} = -\sqrt{2}Ei \int d^2x_\perp e^{-i\Delta x}\; x|\vec{x}_\perp|^{-1} \sin U \exp\left(-ie \int_{-\infty}^{\infty} V(\vec{x}_\perp, z)dz\right). \tag{4.28}$$

Notice that, aside from the sign, (4.27) and (4.28) are, respectively, identical to (4.22) and (4.23).

D. Longitudinal to Longitudinal

If both the incoming and the outgoing vector mesons are longitudinal, we have from (3.35), (3.38), and (4.9), that

$$\mathfrak{M}_{fi} \sim 2Ei \int d^2x_\perp e^{-i\Delta x} \lim_{z\to\infty} [-EM^{-1}\Phi_- + (2ME)^{-1}(M^2 - \Delta^2)\Phi_0 + \Delta M^{-1}\Phi_x - 1 + \tfrac{1}{2}\Delta^2 M^{-2}]. \tag{4.29}$$

Now, similar to (4.25), we may derive the equality

$$\int d^2x_\perp e^{-i\Delta x} \lim_{z\to\infty} (-EM^{-1}\Phi_-) = \int d^2x_\perp e^{-i\Delta x} \lim_{z\to\infty} [-\Delta M^{-1}\Phi_x + (2EM)^{-1}(\Delta^2 + M^2)\Phi_0 - \tfrac{1}{2}\Delta^2 M^{-2}]. \tag{4.30}$$

Substituting (4.30) into (4.29), we get

$$\mathfrak{M}_{fi} \sim 2Ei \int d^2x_\perp e^{-i\Delta x} \lim_{z\to\infty} (ME^{-1}\Phi_0 - 1)$$

$$\sim 2Ei \int d^2x_\perp e^{-i\Delta x}\left[\cos U \exp\left(-ie \int_{-\infty}^{\infty} V(\vec{x}_\perp, z')dz'\right) - 1\right]. \tag{4.31}$$

In deriving (4.31), we have made use of (3.37) and (3.35).

V. SUMMARY

For the convenience of later uses, we list together here all the helicity amplitudes obtained in See. IV:

$$\mathfrak{M}_{++} \sim 2Ei \int d^2x_\perp e^{-i\Delta x}\left[\tfrac{1}{2}(1 + \cos U) \exp\left(-ie \int_{-\infty}^{\infty} V(\vec{x}_\perp, z)dz\right) - 1\right],$$

$$\mathfrak{M}_{--} = \mathfrak{M}_{++},$$

$$\mathfrak{M}_{00} \sim 2Ei \int d^2x_\perp e^{-i\Delta x}\left[\cos U \exp\left(-ie\int_{-\infty}^{\infty} V(\vec{x}_\perp, z)dz\right) - 1\right],$$

$$\mathfrak{M}_{+0} \sim \sqrt{2}Ei \int d^2x_\perp e^{-i\Delta x} x|\vec{x}_\perp|^{-1} \sin U \exp\left(-ie\int_{-\infty}^{\infty} V(\vec{x}_\perp, z)dz\right), \tag{5.1}$$

$$\mathfrak{M}_{-0} = \mathfrak{M}_{+0},$$

$$\mathfrak{M}_{0+} = -\mathfrak{M}_{+0},$$

$$\mathfrak{M}_{0-} = -\mathfrak{M}_{+0},$$

$$\mathfrak{M}_{+-} \sim Ei \int d^2x_\perp e^{-i\Delta x}(y^2 - x^2)|\vec{x}_\perp|^{-2}(1-\cos U) \exp\left(-ie\int_{-\infty}^{\infty} V(\vec{x}_\perp, z)dz\right),$$

$$\mathfrak{M}_{-+} = \mathfrak{M}_{+-},$$

with U given by (3.39).

As is seen from (5.1), all of the helicity amplitudes fail to exhibit the simple-exponentiation form of Molière.[1] This failure is due to the coupling of the three helicity states of a vector meson. Thus we have once again verified, in a special example, that the failure of simple exponentiation always occurs if a particle has several degrees of freedom and they couple together. We shall next discuss the implications of (5.1).

A. Born Term

It is probably worthwhile to list the Born terms of the scattering amplitudes below. They are

$$\mathfrak{M}_{++}^{(B)} \sim 2Ee\tilde{V}(\vec{\Delta}),$$

$$\mathfrak{M}_{--}^{(B)} = \mathfrak{M}_{++}^{(B)},$$

$$\mathfrak{M}_{00}^{(B)} \sim \mathfrak{M}_{++}^{(B)},$$

$$\mathfrak{M}_{+0}^{(B)} \sim (2)^{-1/2}EeM^{-1}i\int d^3x\, e^{-i\Delta x} x|\vec{x}|^{-1}V'(x) = (2)^{-1/2}EeM^{-1}i\int d^3x\, e^{-i\Delta x}\frac{\partial V}{\partial x} = -(2)^{-1/2}EM^{-1}\Delta e\tilde{V}(\vec{\Delta}),$$

$$\mathfrak{M}_{-0}^{(B)} = \mathfrak{M}_{+0}^{(B)}, \tag{5.2}$$

$$\mathfrak{M}_{0+}^{(B)} = -\mathfrak{M}_{+0}^{(B)},$$

$$\mathfrak{M}_{0-}^{(B)} = -\mathfrak{M}_{+0}^{(B)}.$$

$$\mathfrak{M}_{+-}^{(B)} \sim 0,$$

and

$$\mathfrak{M}_{-+}^{(B)} \sim 0.$$

In the above, $\tilde{V}(\vec{\Delta})$ is the Fourier transform of $V(x)$:

$$\tilde{V}(\vec{\Delta}) = \int d^3x\, e^{-i\vec{\Delta}\cdot\vec{x}}V(\vec{x}).$$

Note that (5.2) implies that the helicity of a vector meson can change by one unit by interacting once with the external field. It also suggests a simple way to test whether simple exponentiation can be expected: We study the Born terms in the high-energy limit. If coupling occurs in more than one channel, simple exponentiation cannot occur for an arbitrary potential.

B. Higher-Order Terms

It is also possible to obtain (5.2) easily from the Feynman rules. However, the calculation of the higher-order terms is rather difficult by the Feynman method. For example, in the second order, an individual Feynman diagram can give an amplitude of the order of s^2. We have explicitly checked that, upon adding up all second-order amplitudes, the s^2 terms cancel out. We have not even attempted, however, to check the cancellation of the $s(\ln s)^n$ terms, $n = 1, 2, \ldots,$ with the Feynman method. Our calculations in Sec. IV imply that all logarithmic factors indeed cancel in *all* orders of perturbation. In fact (5.1) is much more simple and elegant than what the perturbation calculations suggest.

C. Selection Rules

We note from (5.2) that, in the lowest order, the helicity can either remain the same or change by one unit, but it cannot change by two units. Thus all helicity states are coupled together, as we have mentioned before. In the second order, change of two helicity units is allowed, as is evidenced from (5.1).

In the forward direction $\Delta = 0$, all of the helicity-flip amplitudes vanish. This is a result of conservation of angular momentum.

From (5.1) we have

$$\mathfrak{M}_{++} = \mathfrak{M}_{--},$$
$$\mathfrak{M}_{+0} = \mathfrak{M}_{-0},$$
$$\mathfrak{M}_{0+} = \mathfrak{M}_{0-},$$
$$\mathfrak{M}_{+-} = \mathfrak{M}_{-+}. \tag{5.3}$$

Equations (5.3) are simply a consequence of invariance under space reflection. Another way to express (5.3) is that there are three polarization states: (i) the one with longitudinal polarization; (ii) the one with the polarization vector lying in the scattering plane; (iii) the one with the polarization vector perpendicular to the scattering plane. States (i) and (ii) couple together, while (iii) only couples to itself.

To conclude, there are no selection rules emerging in the limit of infinite energy. The only selection rules found are consequences of sacred laws such as the conservation of angular momentum or parity – and hold for low and intermediate, as well as high energy.

D. Ultraviolet Divergence

Let us consider the specific example

$$V(|\vec{x}|) = g(4\pi)^{-1}\frac{e^{-|\vec{x}|}}{|\vec{x}|}; \tag{5.4}$$

then

$$e\int_{-\infty}^{\infty} dz\; V(\vec{x}_\perp, z) = eg(2\pi)^{-1}K_0(|\vec{x}_\perp|) \tag{5.5}$$

and

$$U = \tfrac{1}{2}eM^{-1}|\vec{x}_\perp|\int_{-\infty}^{\infty} V'(\vec{x}_\perp, z)(\vec{x}_\perp^{\,2}+z^2)^{-1/2}dz$$
$$= \tfrac{1}{2}eM^{-1}|\vec{x}_\perp|\, x^{-1}\frac{\partial}{\partial x}\int_{-\infty}^{\infty} V(\vec{x}_\perp, z)dz$$
$$= \tfrac{1}{2}eM^{-1}g(2\pi)^{-1}K_0'(|\vec{x}_\perp|). \tag{5.6}$$

Thus, as $|\vec{x}_\perp| \to 0$, we have

$$U = O(|\vec{x}_\perp|^{-1}). \tag{5.7}$$

As a result of (5.7), the nth Born term, $n>1$, contains integrals of the form

$$\int d|\vec{x}_\perp||\vec{x}_\perp|^{-n+1}$$

which diverges at $|\vec{x}_\perp| = 0$. Thus if V is a single Yukawa potential, all the higher-order Born terms have ultraviolet divergences and are not defined. However, the sum of all these terms at high energies is formally given by (5.1), which is in the form of *convergent* integrals. The physical implications of this are most interesting and will be studied in a separate paper.

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