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Radiative Corrections to the Goldberger-Treiman Relation: High-Frequency Contributions

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The second-order electromagnetic corrections to the Goldberger-Treiman relation are discussed. It is shown that when this relation is expressed in terms of physical coupling constants and masses, renormalized by electromagnetism, the residual electromagnetic corrections are finite. The derivation assumes, among other things, the existence of operator-product expansions at short distances and Wilson's enumeration of the fields of low dimensionality. Furthermore, the hadronic matrix elements in the corrections of order α are treated in the PCAC (partial conservation of axial-vector current) approximation. Some observations are made concerning the applicability of PCAC to the study of radiative corrections. A new physical decay constant, f_π', renormalized by electromagnetism, is introduced and determined from experiment. Some salient features of the theory of β decay to first order in α are reviewed, with particular emphasis on the role played by the renormalized constants.

I. INTRODUCTION

In this paper we discuss the high-frequency contributions to the electromagnetic corrections of order α to the Goldberger-Treiman relation.[1,2] As in this problem the axial-vector coupling constant is involved in a fundamental way, it is necessary to review some salient features of the radiative corrections to neutron β decay.

It has been known for some time that if the contributions of order $\alpha l/M$ and $\alpha q/M$ are neglected (l is the electron momentum, q is the total momentum transfer to the leptons, and M is a generic hadronic mass), the sum of the diagrams of zeroth order in α and the virtual corrections depicted in Fig. 1 can be written in the form[3,4]

$$M(n\to p+e^-+\bar\nu)=(G_V/\sqrt{2})[(\mathcal{V}_\lambda-\mathcal{A}_\lambda)L^\lambda+\alpha M^f], \tag{1a}$$

$$\mathcal{V}_\lambda=\bar u_p[f_1'(q^2)\gamma_\lambda+if_2(q^2)\sigma_{\lambda\nu}q^\nu]u_n, \tag{1b}$$

$$\mathcal{A}_\lambda=\bar u_p[g_1'(q^2)\gamma_\lambda+g_2(q^2)q_\lambda]\gamma_5 u_n, \tag{1c}$$

where $L_\lambda=\bar u_l\gamma_\lambda(1-\gamma_5)v_\nu$ is the lepton current, the bare vector coupling constant G_V is to be identified with $G_\mu\cos\theta$ in the Cabibbo theory, $f_1'(q^2)$ and $g_1'(q^2)$ are vector and axial-vector form factors which contain contributions of zero and first order in α, and $f_2(q^2)$ and $g_2(q^2)$ are the usual "weak magnetism" and "induced pseudoscalar" form factors.[5] The quantity αM^f is a known amplitude which is free from ultraviolet divergences and involves complicated functions of the invariants q^2 and $(p\cdot l)$ (p is the nucleon momentum).[6] Furthermore, αM^f contains all the contributions from infrared virtual photons and is independent of the choice of gauge in the covariant photon propagator. In many cases such as in neutron decay or in allowed β decays, it is clearly a good approximation to further neglect the q^2 dependence of the form factors and the terms linear in q in which case Eq. (1) reads[7]

$$M(n\to p+e^-+\bar\nu)=(G_V/\sqrt{2})[\bar u_p(f_V'\gamma_\lambda-g_A'\gamma_\lambda\gamma_5)u_nL^\lambda+\alpha M^f], \tag{2}$$

where $f_V'=f_1'(0)$ and $g_A'=g_1'(0)$ are constants "renormalized" by electromagnetism. The quantities $G_V'\equiv G_Vf_V'$ and $G_A'\equiv G_Vg_A'$ can then be regarded as the basic parameters to be determined by experiments in allowed β decays. In fact, the phenomenological determination of the real parts of G_V' and G_A' has been recently discussed by Blin-Stoyle and Freeman[8] and by Shann.[9,10]

As we show in detail in Sec. IV, a similar analysis can be made in the case of $\pi^-\to\mu^-+\bar\nu_\mu$ decay. The sum of the diagrams of zeroth order in α and the corrections depicted in Fig. 2 can be written as

$$M(\pi^-\to\mu^-+\bar\nu_\mu)=(G_V/\sqrt{2})(-if_\pi'P_\alpha L^\alpha+\alpha N^f), \tag{3}$$

where P_α is the pion four-momentum, f_π' is a coupling constant "renormalized" by electromagnetism, and αN^f is a known quantity which contains all the infrared divergences and which is free from ultraviolet divergences. The precise definition of αN^f and the determination of $G_V|f_\pi'|$ from experiments is analyzed in Sec. IV. We will therefore regard g_A' and f_π' as the physical decay constants when electromagnetism is included to order α.

In Secs. II and III we study several theoretical aspects of the electromagnetic corrections to the Goldberger-Treiman relation.[11] In this analysis we assume the existence of operator-product expansions at short distances, as well as Wilson's enumeration of the fields of low dimensionality.[12] Furthermore, we treat the hadronic matrix elements which control the contributions of high-frequency photons in the PCAC (partial conservation of axial-vector current) approximation; that is, matrix elements involving the divergence of the axial-vector current at $q^2 \approx 0$ are approximated by the corresponding "pion pole" contributions. We give, in passing, arguments that indicate that the naive application of PCAC to study the contributions from low-frequency photons to the radiative corrections is, at best, very delicate and may not be a useful approximation. We point out that these arguments, which are connected with the vanishing photon mass, do not affect the discussion of the high-frequency contributions under rather general assumptions.

Our main theoretical result can then be stated as follows: Subject to the above assumptions and approximations, when the corrections of order α are included, the Goldberger-Treiman relation can be written in the form

$$g_A'(m_p + m_n) = \sqrt{2}\, f_\pi' g_{pn\pi^-} + \delta^h + \alpha C' , \tag{4}$$

where C' is a finite quantity, m_p and m_n are the physical proton and neutron masses, $g_{pn\pi^-}$ is the renormalized $pn\pi^-$ coupling constant in the presence of electromagnetism, and δ^h is a quantity of zeroth order in α which represents the purely hadronic corrections. Note that $G_V g_A'$ and $G_V f_\pi'$ are observable quantities as can be ascertained from Eqs. (2) and (3) or from the more detailed discussion in Sec. IV.[13] Thus, the above result essentially says that when the Goldberger-Treiman relation is expressed in terms of the physical masses and coupling constants, renormalized by electromagnetism, the residual corrections of order α are finite. This in turn implies that all the formally divergent contributions have been "absorbed" into the physical constants.

In Sec. IV we discuss the phenomenological determination of $G_V|f_\pi'|$ and $G_V|g_A'|$ and the degree of departure from an exact "Goldberger-Treiman" relation.

The following observations are useful for orientational purposes:

(i) The renormalized coupling constants f_V', g_A', and f_π' are not to be confused with the constants defined by the hadronic matrix elements of the vector and axial-vector currents in the presence of electromagnetism. The latter are not gauge-invariant, but are, in general, infrared-divergent and involve only the corrections of Figs. 1(a) and 2(a). Instead, the constants f_V', g_A', and f_π' involve contributions from all diagrams of Figs. 1 and 2. This will be explained in greater detail in Secs. II and III.

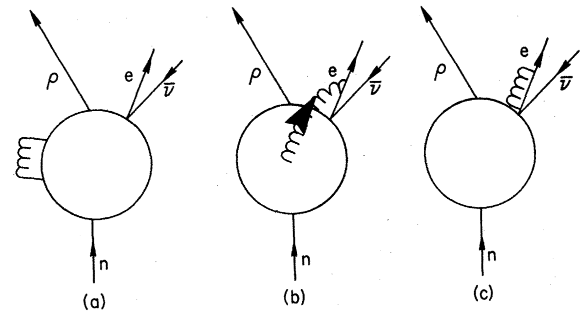

FIG. 1. Virtual radiative corrections to neutron β decay.

(ii) The relation between G_V' and the bare coupling constant $G_V = G_\mu \cos\theta$ is of course of great interest in discussing the universality of the weak interactions.[4] The reason for this is that the principle of universality is stated in terms of the bare couplings G_V and G_μ. In the formulation of the present paper, in which the focus of interest lies in the study of the relation between coupling constants such as $g_{pn\pi^-}$, m_p, m_n, g_A', f_π', the connection between these observable constants and the corresponding bare quantities will be completely bypassed.

II. FORMULATION

Including terms of order α the matrix element of the axial-vector current between n and p states can be written as

$$\langle p' | A^\lambda_{\pi^+}(0) | p \rangle = \bar{u}(p')[\tilde{g}_1(q^2)\gamma^\lambda + \tilde{g}_2(q^2) q^\lambda + \tilde{g}_3(q^2)\sigma^{\lambda\nu} q_\nu]\, \gamma_5 u(p) , \tag{5a}$$

where $A^\lambda_{\pi^+} \equiv A^\lambda_1 + i A^\lambda_2$, p and p' are the four-momenta of the neutron and proton, and $q = p' - p$. Note that Eq. (5a) includes only the contributions of zero order in α and those arising from the diagrams of Fig. 1(a). The latter are, in general, gauge-dependent and infrared-divergent. It should therefore be clear that when the corrections of order α are included, the coupling constants defined

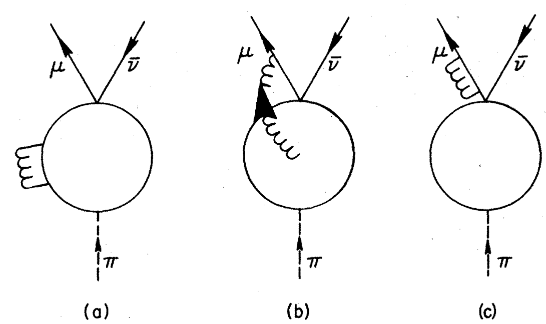

FIG. 2. Virtual radiative corrections to $\pi^- \to \mu^- + \bar{\nu}$ decay.

by the matrix elements of current operators such as $\tilde{g}_1(0)$ cannot correspond to observable quantities.

From Eq. (5a) it follows that

$$\langle p'|\partial_\lambda A^\lambda_{\pi^+}(0)|p\rangle = i\bar{u}(p')[\tilde{g}_1(q^2)(m_p+m_n) + \tilde{g}_2(q^2)q^2]\gamma_5 u(p). \quad (5b)$$

Next we assume that the Hamiltonian density of the strong and electromagnetic interactions is of the form

$$\mathcal{H} = \mathcal{H}_0 + \epsilon_0 u_0 + \epsilon_8 u_8 + \epsilon_3 u_3 + e j^\mu a_\mu , \quad (6)$$

where $\mathcal{H}_0$ is $SU_3 \times SU_3$-invariant, the u's belong to the $(3, 3^*)+(3^*, 3)$ representation of $SU_3\times SU_3$, and j^μ and a^μ are the electromagnetic current and the electromagnetic field, respectively. The term $\epsilon_3 u_3$ represents a violation of isospin of order α which is of the form suggested by the tadpole picture of electromagnetic mass splittings.[14] The possible connection of $\epsilon_3 u_3$ with the subtractions in the effective electromagnetic Lagrangian (to order e^2) has been recently emphasized by Wilson.[12]

We will also assume the validity of the divergence relation

$$\partial_\mu A^\mu_{\pi^+}(x) = -i[Q^A_{\pi^+}(x_0), \mathcal{H}(x)] = -\sqrt{2}\,\epsilon' v_{\pi^+}(x) + ieA^\mu_{\pi^+}(x)a_\mu(x) , \quad (7a)$$

where $Q^A_{\pi^+}(x_0) = \int d^3x\, A^0_{\pi^+}(x)$, $\epsilon' = (\sqrt{2}\,\epsilon_0 + \epsilon_8)/\sqrt{3}$, and $v_{\pi^+} = (v_1 + iv_2)/\sqrt{2}$ is the pseudoscalar counterpart of u_{π^+}. Note that Eq. (7a) corresponds formally to the prescription of "minimal electromagnetic coupling." In particular, the "tadpole" term $\epsilon_3 u_3$ commutes with $Q^A_{\pi^+}$ at equal times and does not affect the expression for $\partial_\mu A^\mu_{\pi^+}(x)$.

If we neglect terms of order α^2 and higher, we can write

$$ie\langle p'|A^\mu_{\pi^+}(0)a_\mu(0)|p\rangle = e^2\int d^4x\,\langle p'|T(A^\mu_{\pi^+}(0)j^\lambda(x))|p\rangle\langle 0|T(a_\mu(0)a_\lambda(x))|0\rangle . \quad (7b)$$

Therefore, Eq. (7a) leads to

$$\langle p'|\partial_\mu A^\mu_{\pi^+}(0)|p\rangle = -\sqrt{2}\,\epsilon'\langle p'|v_{\pi^+}(0)|p\rangle + \frac{2\alpha}{(2\pi)^3 i}\int\frac{d^4k}{k^2+i\epsilon}\int d^4x\, e^{ik\cdot x}\langle p'|T(A^\mu_{\pi^+}(0)j_\mu(x))|p\rangle . \quad (7c)$$

Alternatively, Eq. (7c) can be derived by making use of Ward-Takahashi relations rather than Eq. (7a). Next we note that each of the three terms in Eq. (7c) contains π pole contributions.[15] Separating out these terms and recalling Eq. (5b), we obtain

$$\begin{aligned}\langle p'|\partial_\lambda A^\lambda_{\pi^+}(0)|p\rangle &= i\bar{u}(p')[\tilde{g}_1(q^2)(m_p+m_n) + \tilde{g}_2(q^2)q^2]\gamma_5 u(p)\\ &= \tilde{f}_\pi m_{\pi^+}{}^2\frac{1}{m_{\pi^+}{}^2 - q^2}\sqrt{2}\, g_{pn\pi^-}\bar{u}(p')i\gamma_5 u(p) - \sqrt{2}\,\epsilon'\langle p'|v_{\pi^+}(0)|p\rangle^{\text{no }\pi\text{ p.}}\\ &\quad + \frac{2\alpha}{(2\pi)^3 i}\int\frac{d^4k}{k^2+i\epsilon}\int d^4x\, e^{ik\cdot x}\langle p'|T(A^\mu_{\pi^+}(0)j_\mu(x))|p\rangle^{\text{no }\pi\text{ p.}} ,\end{aligned} \quad (8a)$$

where $\tilde{f}_\pi$ is defined by[16]

$$\langle 0|A^\lambda_{\pi^+}(0)|\pi^-(q)\rangle = i\tilde{f}_\pi q^\lambda . \quad (8b)$$

$g_{pn\pi^-}$ is the $pn\pi^-$ coupling constant renormalized by electromagnetism and the superscript no πp. (no π poles) means that the π pole contributions have been subtracted from the corresponding invariant amplitudes. To see quickly how Eq. (8a) follows from Eq. (7c) note that the first term on the right-hand side of Eq. (8a) [which is the π pole contribution from $\langle p'|\partial_\lambda A^\lambda_{\pi^+}(0)|p\rangle$] simply cancels the π pole contributions that have been subtracted from the second and third terms on the right-hand side of Eq. (8a). Observe also that $\tilde{f}_\pi$ includes only contributions of zeroth order in α and those arising from Fig. 2(a). Thus $\tilde{f}_\pi$ will be, in general, infrared divergent and gauge-dependent in analogy with $\tilde{g}_A$.

Next we take the limit $q^2\to 0$ in Eq. (8a) to obtain

$$\begin{aligned}\tilde{g}_A(m_p+m_n)\bar{u}(p')i\gamma_5 u(p) &= \tilde{f}_\pi\sqrt{2}\, g_{pn\pi^-}\bar{u}(p')i\gamma_5 u(p) - \sqrt{2}\,\epsilon'\langle p'|v_{\pi^+}(0)|p\rangle^{\text{no }\pi\text{ p.}}\\ &\quad + \frac{2\alpha}{(2\pi)^3 i}\int\frac{d^4k}{k^2+i\epsilon}\int d^4x\, e^{ik\cdot x}\langle p'|T(A^\mu_{\pi^+}(0)j_\mu(x))|p\rangle^{\text{no }\pi\text{ p.}} ,\end{aligned} \quad (9)$$

where $\tilde{g}_A = \tilde{g}_1(0)$, and it is understood that all terms are evaluated at $q^2 = 0$. Observe that in this approximation the two last terms of Eq. (9) are constant multiples of $\bar{u}(p')i\gamma_5 u(p)$.

The left-hand side and the first term on the right-hand side of Eq. (9) look very much like the contributions obtained in the usual derivation of the Goldberger-Treiman relation, to zeroth order in α. There is, however, the difference that in Eq. (9) m_p , m_n , $g_{pn\pi^-}$, $\tilde{g}_A$, and $\tilde{f}_\pi$ are quantities renormalized by electromagnetism.

Next we discuss the last term on the right-hand side of Eq. (9). We analyze the high-frequency behavior of the integrand using the operator-product expansion of $T(A^\mu_{\pi^+}(0)j_\mu(x))$ at short distances in conjunction with Wilson's enumeration of the fields of low dimensionality (i.e., dimensionality ≤ 4).[12] Note that only pseudoscalar fields need be retained in the expansion: The contributions from other fields will automatically vanish when the eight-dimensional integral over x and k is performed. Therefore, we write

$$T(A^\mu_{\pi^+}(0)j_\mu(x)) = c(x)\partial_\mu A^\mu_{\pi^+}(0) + \cdots \qquad (10)$$

as $\partial_\mu A^\mu_{\pi^+}(0)$ [or equivalently, $v_{\pi^+}(0)$] is assumed to be the only pseudoscalar field of dimensionality ≤ 4 with the quantum numbers of π^+.

The short-distance behavior of $c(x)$ can be obtained by means of a spurion analysis completely analogous to that developed by Wilson.[12] The operator $\partial_\mu A^\mu_{\pi^+}(0)$ belongs to the (2, 2) representation of $SU_2 \times SU_2$ while j_μ and $A^{\pi^+}_\mu$ belong to the representations $(1,3)+(3,1)$ and $(1,1)$. Thus, in the limit of $SU_2 \times SU_2$ symmetry $c(x)$ must vanish. If Δ is the dimensionality of the u's and v's, to first order in the breaking of $SU_2 \times SU_2$, $c(x)$ must scale as $x^{4-\Delta}/x^{6-\Delta} = x^{-2}$. If $\Delta<4$, higher contributions in the perturbation expansion give rise to milder singularities. Barring unforeseen cancellations, this leads to a logarithmically divergent integral proportional to the matrix element $\langle p'|\partial_\mu A^\mu_{\pi^+}(0)|p\rangle^{\text{no }\pi\text{ p.}}$ in the last term of Eq. (9).

As is well known, several points of view are possible regarding the mechanism that transforms formally divergent quantities into the finite results which are relevant in the real world.[17] If we adopt the "renormalization point of view," the formally divergent part of the integral in Eq. (9) should be subtracted and a contribution $(\alpha/2\pi)f \times \langle p'|\partial_\mu A^\mu_{\pi^+}(0)|p\rangle^{\text{no }\pi\text{ p.}}$, where f is an undetermined but finite constant, should be added.[18] However, because the π pole has been subtracted, the hadronic matrix element vanishes in the PCAC approximation and will be, therefore, neglected in the spirit of this work. It is crucial for this argument that the formally divergent contributions are controlled by $\partial_\mu A^\mu_{\pi^+}(0)$. Clearly, an identical analysis can be put forward for any of the other "solutions" of the divergence problem, with the important qualification that the mechanism that renders the answer finite should not alter the operator structure which controls the high-energy behavior.

Next we discuss the term involving $\epsilon' \times \langle p'|v_{\pi^+}(0)|p\rangle^{\text{no }\pi\text{ p.}}$ in Eq. (9). To zeroth order in α this term represents the hadronic corrections to the Goldberger-Treiman relation. It also contains contributions of order α proportional to

$$\alpha \int d^4y\, e^{-iq\cdot y} \langle p'|T(\partial_\mu A^\mu_{\pi^+}(y)\mathcal{L}(0))|p\rangle^{\text{no }\pi\text{ p.}}, \qquad (11a)$$

where $\mathcal{L}(0)$ is the effective second-order electromagnetic Lagrangian. [As Eq. (11a) involves α explicitly, $\partial_\mu A^\mu_{\pi^+}(y)$ may be regarded here as the divergence operator in the absence of electromagnetism.] At first hand it may appear that we can altogether neglect this contribution by invoking PCAC as the momentum transferred by $\partial_\mu A^\lambda_{\pi^+}$ is $q^2 \approx 0$ and the pion pole contribution has been subtracted. However, the situation is more complicated: Because $\mathcal{L}(0)$ involves a photon propagator, the amplitude of Eq. (11a) will not only contain a pion pole, but will also exhibit a branch cut starting at a value of $q^2 = (m_\pi + \lambda_{\min})^2$, arising from intermediate states involving a pion and a photon. Under these circumstances the use of PCAC is at best very delicate, and, in fact, may not be useful for the full amplitude.[19] However, as we are interested here in the high-frequency contributions, we can bypass the problem by separating the photon propagator as follows:

$$\frac{1}{k^2} = \frac{1}{k^2}\frac{M^2}{M^2-k^2} + \frac{1}{k^2-M^2} , \qquad (11b)$$

where M may be chosen to be a few times the pion mass. The first term on the right-hand side will still give rise to the troublesome branch cut, but its high-frequency contributions have been dampened by the convergence factor $M^2/(M^2-k^2)$. If the k integration in Eq. (11a) with the original propagator $1/k^2$ is at most linearly or logarithmically divergent, this contribution is convergent. The high-frequency behavior of the second term on the right-hand side of Eq. (11b) is the same as that of the original propagator, but it behaves as a "massive photon" so that the corresponding branch cut in Eq. (11a) is moved far away from the pion pole. For this part, which contains the high-frequency contributions, we can invoke PCAC to argue that its contribution may be neglected as the pion pole terms have been subtracted. If the k integration in Eq. (11a) lead, say, to quadratic divergences in the ultraviolet region, one can repeat essentially the same argument with a simple modification of Eq. (11b).

In summary, we reach the conclusion that the high-frequency contributions to the second and third terms on the right-hand side of Eq. (9) involve vanishing hadronic matrix elements in the PCAC approximation. This means that if we write Eq. (9) as

$$\tilde{g}_A(m_p+m_n)=\tilde{f}_\pi\sqrt{2}\,g_{pn\pi^-}+\delta^h+\alpha\tilde{C}, \tag{12}$$

where δ^h are the purely hadronic corrections to the Goldberger-Treiman relation, then, subject to the assumptions of this paper, the constant $\tilde{C}$ is free from ultraviolet divergences in the PCAC approximation. All such divergences have been absorbed in the renormalized couplings and masses that appear in Eq. (12).

Note that this result depends critically on the basic assumption that $\partial_\mu A^\mu_{\pi^+}(0)$ is the only pseudoscalar operator that occurs in the operator-product expansion of Eq. (10). For example, the presence of hypothetical pseudoscalar fields with dimensionality ≤ 4 belonging, say, to the $(1,8)+(8,1)$ representation of $SU_3\times SU_3$ would lead to formally divergent answers even in the PCAC approximation.

We must now consider the couplings $\tilde{g}_A$ and $\tilde{f}_\pi$ and express them in terms of the observable constants g'_A and f'_π. The question of whether or not such replacement introduces ultraviolet divergences in Eqs. (9) and (12) is studied in Sec. III.

III. MORE ABOUT HIGH-FREQUENCY CONTRIBUTIONS

It was pointed out in Secs. I and II that the coupling constants $\tilde{g}_A$ and $\tilde{f}_\pi$ which occur in Eqs. (9) and (12) are infrared-divergent and gauge-dependent and, therefore, unobservable. Our task in this section is to show that when these quantities are expressed in terms of the physical constants g'_A and f'_π, no ultraviolet divergences of order α are introduced in Eqs. (9) and (12), in the PCAC approximation.

This result is closely connected with the following properties of the radiative corrections which will be obtained in this section: (i) The formally divergent parts of diagrams (b) and (c) in Figs. 1 and 2 are proportional to $\langle p'|A^\lambda_{\pi^+}(0)|p\rangle L_\lambda$ in the case of n decay and to $\langle 0|A^\lambda_{\pi^+}(0)|P\rangle L_\lambda$ in the case of π decay. (ii) The divergent integrals of order α which multiply these matrix elements are controlled by unknown functions of k^2, which are however the same for n and π decays. As we will see, point (ii) is again a consequence of the existence of operator-product expansions and the enumeration of fields of dimensionality ≤ 4.

Consider the contributions to the matrix elements of the axial-vector current in neutron β decay. We will denote by $M^{(0)}_{n\to p}$ the contributions of zeroth order in α and by $M^{(a)}_{n\to p}$, $M^{(b)}_{n\to p}$, $M^{(c)}_{n\to p}$ those arising from the diagrams of Fig. 1(a), 1(b), and 1(c), evaluated in the Feynman gauge.

$$\begin{aligned}(M^{(0)}+M^{(a)})^A_{n\to p}&=-(G_V/\sqrt{2})\langle p'|A^\lambda_{\pi^+}(0)|p\rangle L_\lambda\\&=-(G_V/\sqrt{2})\tilde{g}_A\bar{u}(p')\gamma^\lambda\gamma_5u(p)L_\lambda\,,\end{aligned} \tag{13}$$

where the superscript A means that we have retained only the contribution of the axial-vector current. In Eq. (13) we have set $q^2=0$ and have neglected the induced couplings as these play no role in the present argument.

$$M^{(b)}_{n\to p}=-\frac{G_V}{\sqrt{2}}\frac{i\alpha}{4\pi^3}\times\int\frac{d^4k}{k^2+i\epsilon}\left[\bar{u}_l\frac{2l_\mu-\gamma_\mu k\!\!\!/}{k^2-2l\cdot k+i\epsilon}\gamma_\rho(1-\gamma_5)v_\nu\right]T^{\mu\rho}\,, \tag{14a}$$

where l is the lepton momentum and

$$T^{\mu\rho}=i\int d^4x\,e^{ik\cdot x}\langle p'|T(j^\mu(x)[V^\rho_{\pi^+}(0)-A^\rho_{\pi^+}(0)])|p\rangle\,. \tag{14b}$$

Finally

$$M^{(c)A}_{n\to p}=-\frac{Z_e-1}{2}\frac{G_V}{\sqrt{2}}\langle p'|A^\lambda_{\pi^+}(0)|p\rangle L_\lambda\,, \tag{15}$$

where Z_e is the electron renormalization constant.

To study the ultraviolet divergences in Eq. (14a), we need the asymptotic behavior of $T^{\mu\rho}$ for large k. To be more precise, we need the contributions $\geq 1/k$ as $k\to\infty$. This in turn is controlled by the operators of dimensionality ≤ 3 in the operator-product expansion of $T(j^\mu(x)[V^\rho_{\pi^+}(0)-A^\rho_{\pi^+}(0)])$. The most singular contribution in Eq. (14a) comes from the term involving $\gamma_\mu k\!\!\!/\gamma_\rho$ in the lepton covariant. Setting $l=0$ in the denominator and using the identity

$$\gamma_\mu\gamma_\sigma\gamma_\rho=g_{\mu\sigma}\gamma_\rho-g_{\mu\rho}\gamma_\sigma+g_{\sigma\rho}\gamma_\mu+i\epsilon_{\mu\sigma\rho\alpha}\gamma^\alpha\gamma_5\,, \tag{16a}$$

one finds that only vector and axial-vector operators need be considered. We therefore write

$$\begin{aligned}T(j^\mu(x)[V^\rho_{\pi^+}(0)-A^\rho_{\pi^+}(0)])&=d_1(x^2)x^\mu A^\rho_{\pi^+}(0)+d_2(x^2)x^\rho A^\mu_{\pi^+}(0)\\&\quad+id_3(x^2)\epsilon^{\mu\rho\alpha\beta}x_\alpha A^{\pi^+}_\beta(0)+\cdots\,.\end{aligned} \tag{16b}$$

Following Wilson we have assumed that $A^\rho_{\pi^+}(0)$ is the only axial-vector operator of dimensionality ≤ 3 with the appropriate quantum numbers. We have not included the terms involving $V^\mu_{\pi^+}(0)$ because we are only interested here in the matrix elements of the axial-vector current. In the limit of scale invariance, the $d_j(x^2)$ $(j=1,\,2,\,3)$ scale as $1/x^4$ and lead to logarithmic divergences in Eq. (14a).

From Eq. (16b) we obtain

$$i\int d^4x\, e^{ik\cdot x}T(j^\mu(x)[V^\rho_{\pi^+}(0)-A^\rho_{\pi^+}(0)])$$
$$=E_1(k^2)k^\mu A^\rho_{\pi^+}(0)+E_2(k^2)k^\rho A^\mu_{\pi^+}(0)+iE_3(k^2)\epsilon^{\mu\rho\alpha\beta}k_\alpha A^{\pi^+}_\beta(0)+\cdots\,, \tag{16c}$$

where the functions

$$E_j(k^2)\equiv\frac{i}{k^2}\int e^{ik\cdot x}(k\cdot x)d_j(x^2)d^4x \qquad (j=1,\ 2,\ 3) \tag{16d}$$

scale as $1/k^2$.

Insertion of Eq. (16c) into Eq. (14a) shows after some elementary algebra that the divergent part of $M^{(b)A}_{n\to p}$ (in the ultraviolet region) is given by

$$M^{(b)A\,\mathrm{div}}_{n\to p}=\frac{G_V}{\sqrt{2}}\langle p'|A^\lambda_{\pi^+}(0)|p\rangle L_\lambda\left(\frac{i\alpha}{4\pi^3}\right)\times\int\frac{d^4k}{k^2}(E_1+E_2+\tfrac{3}{2}E_3). \tag{16e}$$

We recall from the general discussion in Sec. I and Ref. 3 that we can write

$$(M^{(0)}+M^{(a)}+M^{(b)}+M^{(c)})^A_{n\to p}=\frac{G_V}{\sqrt{2}}(-g'_A\bar{u}_p\gamma_\lambda\gamma_5u_nL^\lambda+\alpha M^f), \tag{17a}$$

where M^f is free from ultraviolet singularities, and we have neglected terms of order $\alpha l/M$ and $\alpha q/M$. As we pointed out g'_A (or rather $G'_A=G_Vg'_A$)[13] can be regarded as the observable coupling constant in neutron β decay. Let us now write

$$g'_A=\tilde{g}_A(1+\alpha C_{n\to p})\,, \tag{17b}$$

where $C_{n\to p}$ is a constant.

Then comparison of Eqs. (13), (14a), (15), (16e), (17a), and (17b) shows that the divergent part of $\alpha C_{n\to p}$ is given by

$$\alpha C^{\mathrm{div}}_{n\to p}=\frac{[Z(e)-1]^{\mathrm{div}}}{2}-\frac{i\alpha}{4\pi^3}\int\frac{d^4k}{k^2}[E_1(k^2)+E_2(k^2)+\tfrac{3}{2}E_3(k^2)]\,. \tag{17c}$$

A moment's thought shows that we can carry verbatim an identical argument for $\pi\to\mu+\nu$ decay. We obtain in that case

$$f'_\pi=\tilde{f}_\pi(1+\alpha C_{\pi\to}) \tag{18a}$$

and

$$\alpha C^{\mathrm{div}}_{\pi\to}=\frac{(Z_\mu-1)^{\mathrm{div}}}{2}-\frac{i\alpha}{4\pi^3}\int\frac{d^4k}{k^2}[E_1(k^2)+E_2(k^2)+\tfrac{3}{2}E_3(k^2)]\,, \tag{18b}$$

where Z_μ is the muon renormalization constant. As the divergent parts of Z_e-1 and $Z_\mu-1$ are identical, we conclude that

$$C^{\mathrm{div}}_{n\to p}=C^{\mathrm{div}}_{\pi\to}\,. \tag{18c}$$

Inserting Eqs. (17b) and (18a) into Eq. (12) and noting Eq. (18c), one sees immediately that the formally divergent contributions of order α from $C_{n\to p}$ and $C_{\pi\to}$ cancel each other in the PCAC approximation.

On the basis of the results of Secs. II and III we conclude that if we write the Goldberger-Treiman relation in the form of Eq. (4), then subject to our assumptions, C' is a finite constant in the PCAC approximation. The phenomenological situation regarding f'_π and g'_A is examined in Sec. IV.

IV. THE CONSTANT f'_π AND ITS DETERMINATION

The constant f'_π is defined by the equation

$$N^{(0)}+N^{(a)}+N^{(b)}+N^{(c)}=(G_V/\sqrt{2})(-if'_\pi P_\alpha L^\alpha+\alpha N^f)\,, \tag{19a}$$

where $N^{(0)}$ is the matrix element for $\pi^-\to\mu^-+\bar{\nu}_\mu$ to zeroth order in α, $N^{(a)}$, $N^{(b)}$, and $N^{(c)}$ are the contributions of the diagrams of Figs. 2(a)–2(c) and αN_f is given by[20]

$$\alpha N_f=-\frac{\alpha}{8\pi^3i}(-if'_\pi P^\lambda L_\lambda)\int d^4k\frac{(2P-k)_\mu(2P-k)_\nu D^{\mu\nu}(k)}{(k^2-2P\cdot k+i\epsilon)^2}+\frac{\alpha}{4\pi^3i}\int\frac{d^4k\,D^{\mu\nu}(k)[\bar{u}_l(2l_\nu-\gamma_\nu\not{k})\gamma_\lambda(1-\gamma_5)v_\nu]}{(k^2-2l\cdot k+i\epsilon)(k^2-2P\cdot k+i\epsilon)}(2P-k)_\mu(-if'_\pi P^\lambda)+(-if'_\pi P^\lambda L_\lambda)\tfrac{1}{2}(Z_\mu-1)\,. \tag{19b}$$

The function αN_f possesses the following basic properties: (i) It is free from ultraviolet divergences. (ii) It contains all the infrared and soft-photon contributions of Figs. 2(a)–2(c). (iii) It is independent of the gauge adopted for the covariant photon propagator $D^{\mu\nu}(k)$.[21] Explicit evaluation of the integrals in Eq. (19b) gives the result

$$\alpha N_f = i f_\pi' P^\lambda L_\lambda \frac{\alpha}{2\pi} \times \left\{ \left[\frac{x^2+1}{x^2-1}\ln x - 1\right]\left[2\ln\left(\frac{m_\mu}{\lambda_{\min}}\right) + \ln x\right] + \tfrac{1}{2}\ln x + \tfrac{3}{8}\right\}, \tag{19c}$$

where $x = m_\pi / m_\mu$ and $\lambda_{\min}$ is the photon mass.

Note that the two terms in the right-hand side of Eq. (19a) are constants multiplying the matrix element $P^\lambda L_\lambda$. In order for this separation to be consistent, it is necessary that the left-hand side in Eq. (19a) should be of the same form, after the integration over the virtual photon momenta is performed. This is obvious for $N^{(0)}$, $N^{(a)}$, and $N^{(c)}$. That it also holds for $N^{(b)}$ can be readily proved by studying the equations analogous to Eqs. (14a) and (14b) in the case of $\pi^- \to \mu^- + \bar{\nu}_\mu$ decay.

The separation described in Eq. (19a) is analogous to that carried out in Sec. II of Ref. 3. There is, however, the difference that Eq. (19a) is exact while the work of Ref. 1 involved some approximations (however good), such as the neglect of terms of order $\alpha l/M$, $\alpha q/M$, etc. This simplifying feature of Eq. (19a) is due essentially to the fact that we are dealing here with a two-body decay, rather than the more complex situation of β decay.

The separation of Eq. (19a) is useful for the following reason: In studying the probability for the decay $\pi^- \to \mu^- + \bar{\nu}_\mu$, one must include the contributions of the inner bremsstrahlung (I.B.) in order to eliminate the infrared divergences. However, the properties of the function αN_f described after Eq. (19b) tells us that f_π' is free from such singularities. Therefore, the infrared divergences of the real quanta must cancel those of αN^f, and the decay probabilities for $\pi \to \mu + \nu$ plus $\pi \to \mu + \nu + \gamma$ should determine $|f_\pi'|^2$. Note also that f_π' is independent of the choice of the gauge adopted for the covariant propagator $D_{\mu\nu}(k)$ in the evaluation of the virtual radiative corrections.

In considering the contributions of real quanta, one may envisage two different types of realistic experiments: (a) The muon energy is restricted to the range $E_{\max} - \Delta E \leq E \leq E_{\max}$. (b) The muon energy is unrestricted. Assuming $\Delta E \ll E_{\max}$ one finds for the transition probability for $\pi^- \to \mu^- + \bar{\nu}_\mu + \gamma$ in case (a)[22]:

$$P_{\text{I.B.}}^{(\Delta E)} = \frac{\alpha}{\pi} P^{(0)} \left[\frac{x^2+1}{x^2-1}\ln x - 1\right] \times \left[2\ln\left(\frac{2\Delta E}{\lambda_{\min}}\right) + 3\ln x - 2\ln(x^2-1)\right]. \tag{20}$$

Adding this result to the contributions of Eqs. (19a) and (19c) to the transition probability for $\pi^- \to \mu^- + \bar{\nu}_\mu$, we obtain for the decay rate $\pi^- \to \mu^- + \bar{\nu}_\mu$ plus $\pi^- \to \mu^- + \bar{\nu}_\mu + \gamma$ in the case $\Delta E \ll E_{\max}$:

$$P_{\text{tot}}^{(\Delta E)} = \frac{G_\mu^2 \cos^2\theta}{8\pi} |f_\pi'|^2 m_\mu^2 m_{\pi^+} \times \left(1 - \frac{m_\mu^2}{m_{\pi^+}^2}\right)^2 \left(1 + \frac{\alpha}{2\pi} A^{(\Delta E)}\right), \tag{21a}$$

$$A^{(\Delta E)} = 4\left[\frac{x^2+1}{x^2-1}\ln x - 1\right]\left[\ln\left(\frac{2\Delta E}{m_\mu}\right) + \ln x - \ln(x^2-1)\right] - \ln x - \tfrac{3}{4}, \tag{21b}$$

where we have set $G_V = G_\mu \cos\theta$.

Note that when $\Delta E \ll E_{\max}$ only soft real photons contribute to $P_{\text{I.B.}}^{(\Delta E)}$. These contributions, given in Eq. (20), are independent of the details of the strong interactions. Thus, in principle, Eq. (21) allows the determination of $G_\mu^2 \cos^2\theta |f_\pi'|^2$ up to terms of order α in a model-independent manner.

As far as we know, however, detailed experiments with $\Delta E \ll E_{\max}$ have not been performed. For this reason, it seems that at the present time the best determination of $G_V^2 |f_\pi'|^2$ can be obtained from the π^- lifetime which is accurately known. To carry out this determination one needs the theoretical expression for the total decay probability for $\pi^- \to \mu^- + \bar{\nu}_\mu + \gamma$ corresponding to a photon of infinitesimal mass $\lambda_{\min}$, with the muon energy unrestricted. Such a calculation exists in the literature, done by Kinoshita.[23] Adding this calculation to the contributions of Eqs. (19a) and (19c) to the transition probability for $\pi^- \to \mu^- + \bar{\nu}_\mu$, we obtain

$$P_{\text{tot}} = \frac{G_\mu^2 \cos^2\theta}{8\pi} |f_\pi'|^2 m_\mu^2 m_{\pi^+} \times \left(1 - \frac{m_\mu^2}{m_{\pi^+}^2}\right)^2 \left(1 + \frac{\alpha}{2\pi} B\right), \tag{22a}$$

$$B = 4\left[\frac{x^2+1}{x^2-1}\ln x - 1\right]\left[\ln(x^2-1) - 2\ln x - \tfrac{3}{4}\right] + 4\frac{x^2+1}{x^2-1} L\left(1 - \frac{1}{x^2}\right) - \ln x - \tfrac{3}{4} + \frac{10x^2-7}{(x^2-1)^2}\ln x + \frac{15x^2-21}{4(x^2-1)}, \tag{22b}$$

$$L(z) = \int_0^z \ln(1-t)\frac{dt}{t}. \tag{22c}$$

Equation (22) gives the total decay probability for $\pi^- \to \mu^- + \bar{\nu}_\mu$ plus $\pi^- \to \mu^- + \bar{\nu}_\mu + \gamma$. Numerically $B = -1.4$ so that $(\alpha/2\pi)B$ represents a relative correction of -1.6×10^{-3}, which is quite small. It should be pointed out that the calculation of the inner bremsstrahlung in Ref. 22 was done for point particles, ignoring the structure effects of the strong interactions. Thus, unlike Eq. (21), Eq.

(22) is not model-independent. However, order of magnitude estimates indicate that the structure effects in the inner bremsstrahlung will also give very small contributions to Eq. (22).[24] Comparing Eq. (22) with the π^- lifetime, we obtain

$$G_\mu \cos\theta |f'_\pi| = \frac{1.497\times 10^{-6}}{\text{BeV}} . \tag{23a}$$

Using the values $G_\mu = 1.1660\times 10^{-5}$ BeV^{-2} (Ref. 25) and $\sin\theta = 0.221 \pm 0.004$,[8] Eq. (23a) gives

$$|f'_\pi| = 131.7 \text{ MeV} = 0.943\ m_{\pi^+} . \tag{23b}$$

Turning our attention to $|G'_A|$, we note that this renormalized constant can be obtained from the recent phenomenological analysis of Blin-Stoyle and Freeman[8] and of Shann.[9] These authors obtain $\lambda' = |G'_A/G'_V| = 1.226 \pm 0.011$ from the ft values of Al^{26} and n decays and $\lambda' = 1.26 \pm 0.02$ from the electron asymmetry in neutron β decay. Combining these values with the determination of $|G'_V|$ given by the first two authors on the basis of the Al^{26} data, one finds

$$|G'_A| = \begin{cases} (1.41 \pm 0.01)\times 10^{-5}\ \text{BeV}^{-2} & (\text{Al}^{26} \text{ and } n \text{ decays}) \\ (1.45 \pm 0.02)\times 10^{-5}\ \text{BeV}^{-2} & (\text{Al}^{26} \text{ and electron symmetry}). \end{cases} \tag{24}$$

It is interesting to point out that the radiative corrections affect more the determination of $|g'_A|$ than that of $|f'_\pi|$. The reason is easy to understand: As the corrections to $|g'_A|$ and $|f'_\pi|$ are finite by definition, those affecting $|f'_\pi|$ should be expected to be small as there are no large logarithmic coefficients depending on the pion and muon masses. This is reflected in the smallness of the correction $(\alpha/2\pi)B$. Instead, in the case of n decay large logarithmic terms proportional to $\ln(m_p/E_{\max})$ ($E_{\max}$ is the maximum electron energy) appear and the corrections decrease the value of $|g'_A|$ by about 1%.

Using $g_{pn\pi^-}{}^2/4\pi = 14.6$,[26] Eqs. (23a) and (24) lead to

$$\frac{|g'_A|(m_p + m_n)}{\sqrt{2}\,|f'_\pi| g_{pn\pi^-}} = \begin{cases} 0.92 & (\text{Al}^{26} \text{ and } n \text{ decays}) \\ 0.94 & (\text{Al}^{26} \text{ and electron asymmetry}). \end{cases} \tag{25}$$

This equation summarizes the degree of departure from an exact "Goldberger-Treiman" relation. One naturally expects that the bulk of this difference is accounted for by the hadronic corrections. We would like to point out that the error of the recommended experimental value of $g_{pn\pi^-}$ is roughly 2.5%.[26] Thus, the intriguing possibility exists that the Goldberger-Treiman relation may be, in fact, quite accurate.

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[1]M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958).

[2]For a recent exposition of the Goldberger-Treiman relation and its significance, see, for example, S. L. Adler and R. Dashen, *Current Algebras* (Benjamin, New York, 1968).

[3]A. Sirlin, Phys. Rev. 164, 1767 (1967).

[4]A. Sirlin, in *Proceedings of the Topical Conference on Weak Interactions, CERN, 1969*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1969), p. 408.

[5]In Eqs. (1b) and (1c) we have assumed that to zeroth order in α there are no second-class currents. Of course, these can be added if necessary. Electromagnetism induces, in general, second-class currents but these contributions are of order $\alpha q/M$ and have been neglected in Eqs. (1b) and (1c). As we have neglected terms of order $\alpha q/M$, we have $f'_1(q^2) = f_1(q^2) + (\alpha/2\pi)c$, $g'_1(q^2) = g_1(q^2) + (\alpha/2\pi)d$, where $f_1(q^2)$ and $g_1(q^2)$ are the usual vector and axial-vector form factors to zeroth order in α, and c and d are constants. Note that according to the conserved-vector-current hypothesis $f'_V \equiv f'_1(0) = 1 + (\alpha/2\pi)c$.

[6]More precisely, αM_f is the sum of the contributions of Eqs. (11) and (14) of Ref. 3 and the first term in the curly bracket of Eq. (9a) of that paper.

[7]Some of the notations of Ref. 3 are different from the present ones. The "bare" vector coupling constant was called G_V^0 rather than G_V. The renormalized constants $G'_V \equiv G_V f'_V$ and $G'_A \equiv G_V g'_A$ of the present paper were called $G_V^0[1 + (\alpha/2\pi)c]$ and $-G_A^0[1 + (\alpha/2\pi)d]$ and the sign of γ_5 was the opposite.

[8]R. J. Blin-Stoyle and J. M. Freeman, Nucl. Phys. A150, 369 (1970).

[9]R. T. Shann, University of Sussex report, 1971 (unpublished).

[10]In general, the constants G'_V and G'_A contain imaginary as well as real parts of order α. As pointed out by Shann, if one neglects the corrections of order α^2, the imaginary parts do not affect those observables which to zeroth order in α give no information about time-reversal invariance. In particular, the lifetime, the spec-

trum, and the electron asymmetry depend only on the real parts of G_V' and G_A'. In the present paper the possible effects of such imaginary parts are simply bypassed by taking the real parts or the absolute values of equations involving g_A' and f_π'. Note that to our approximation $|g_A'| = |\text{Re}\, g_A'|$.

[11]A recent and different discussion of this problem has been given by H. Pagels, Phys. Rev. D 3, 610 (1971); *ibid.* 4, 1932(E) (1971). We wish to thank Professor Pagels for informing us of his work and for an interesting discussion.

[12]K. Wilson, Phys. Rev. 179, 1499 (1969).

[13]In order to determine f_π' and g_A' from $G_V f_\pi'$ and $G_V g_A'$, one must use the value of $G_V = G_\mu \cos\theta$. Although G_μ is precisely known, the determination of $\cos\theta$ from either $\Delta S = 1$ or $\Delta S = 0$ decays involves some theoretical assumptions and approximations. Thus, strictly speaking, the basic observables are $G_V f_\pi'$ and $G_V g_A'$. However, we will not stress further this point and will frequently consider f_π' and g_A' as basic physical constants. In the Goldberger-Treiman relation the distinction is immaterial, as one only needs the ratio between these constants.

[14]S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964).

[15]For example, the π pole contribution of the last term in Eq. (7c) is $d(q^2)\bar{u}(p')i\gamma_5 u(p)$,

$$d(q^2) = \frac{2\alpha}{(2\pi)^3 i} \int \frac{d^4k}{k^2 + i\epsilon} \int d^4x\, e^{ik\cdot x} \times \langle 0|T(A^\mu_{\pi^+}(0)\, j_\mu(x))|\pi^-(P)\rangle \sqrt{2}\, g_{pn\pi^-}(m_{\pi^+}{}^2 - q^2)^{-1},$$

where $P^2 = m_{\pi^+}{}^2$. Note that after the k integration is performed the amplitude which multiplies $(m_{\pi^+}{}^2 - q^2)^{-1}$ in $d(q^2)$ depends only on P^2.

[16]Our f_π's have dimensions of mass. See also Sec. IV and, in particular, Eq. (23b).

[17]We have in mind any of the following possibilities: (i) Theories with indefinite metric, T. D. Lee and G. C. Wick, Phys. Rev. D 2, 1033 (1970). (ii) Theories in which the infinities are canceled by the interplay of various interactions and or by higher orders of the same interaction. A recent example is the work of A. Salam and J. Strathdee, International Centre for Theoretical Physics Report No. IC/70/38 (unpublished). (iii) The "renormalization approach." A recent example is given by Wilson's discussion of the infinities in the electromagnetic mass shifts (Ref. 12). The introduction of coupling constants renormalized by electromagnetism in Ref. 3 and in the present paper corresponds closely to the same method.

[18]Presumably, the undetermined coupling constants of the renormalization approach are of the same order of magnitude as the strength of the original interaction. Otherwise, one cannot make sense of the observable world. Hopefully, future and more profound developments of field theory and physics in general may lead to the theoretical determination of these constants.

[19]An interesting question is whether these arguments are relevant in the application of PCAC to other electromagnetic phenomena such as $\eta \to \pi^+ + \pi^- + \pi^0$. This point is being studied at present.

[20]The last term of Eq. (19b) is diagram 2(c) (i.e., the electron wave-function renormalization). The first and second terms contain all the infrared contributions of Figs. 2(a) and 2(b), respectively.

[21]This can be checked by replacing $D^{\mu\nu}(k) \to c(k^2)k^\mu k^\nu$, where $c(k^2)$ is arbitrary, and noting that with this substitution αN_f vanishes. The importance of this property for the physical effects was emphasized by L. D. Landau, in *Niels Bohr and the Development of Physics,* edited by W. Pauli (Pergamon, New York, 1955), p. 52.

[22]In computing Eq. (20a) one must take into account that although the energy of the muon is restricted, that of the $\bar{\nu}$ is not. In Eqs. (20a) and (20b) we have assumed that although $\Delta E \ll E_{\max}$, ΔE is not so small that one must also consider the contributions of order $[\alpha \ln(\Delta E/m_\mu)]^2$ and higher.

[23]T. Kinoshita, Phys. Rev. Letters 2, 477 (1959). The relevant result is given in Eq. (4) of that paper. Note that Eq. (4) minus Eq. (3) of Kinoshita's paper should coincide with Eq. (20) of the present paper which, in fact, occurs.

[24]Approximate calculations of the coefficients of the induced terms are given in R. E. Marshak, Riazuddin, and C. P. Ryan, *Weak Interaction of Elementary Particles* (Wiley, New York, 1969), p. 354 *et seq.*

[25]M. Ross and A. Sirlin, Nucl. Phys. B29, 296 (1971).

[26]G. Ebel *et al.*, in *Springer Tracts in Modern Physics,* edited by G. Höhler (Springer, Berlin, 1970), Vol. 55, p. 257.