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Bjorken Scaling For Inclusive and Quasi-Inclusive Processes in the Dual-Resonance Model

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From a dual-resonance model with currents included through the minimal gauge interaction, the deep-inelastic electron scattering is considered. It gives the Bjorken scaling for νW_2 while W_1 vanishes, a property which is common among the models where the current is coupled to bosons. Scaling occurs because of the existence of a current-algebra fixed pole. Deep-inelastic electron scattering with one detected final-state particle is also considered which, following Mueller, is connected with the discontinuity of a six-point amplitude. In a special kinematic region, three out of four structure functions scale because of a fixed pole, while outside this region the fixed pole cannot be responsible for scaling anymore. Then a speculation of Pomeranchukon assignment does show the scaling. Inelastic Compton scattering is also considered which, in the parton model of Bjorken and Paschos, scales and is proportional to νW_2. This property is satisfied in the present model. Electron-positron annihilation into hadrons is considered without renormalization whose cross section falls off as $s^{-5/2}$. It is suggested that a proper dual renormalization for the self-mass diagram of the photon may change this result.

I. INTRODUCTION

There exists the possibility that the dual-resonance model (DRM)[1,2] may finally provide us with a theory of hadronic processes. The unitarization program to treat the model as a Born term[3] puts it on the same footing as a field-theory expansion. Further, besides some quantitative agreements of the model with the data, it reproduces some qualitative features of the hadronic inclusive reactions[4,5] such as the Feynman scaling law,[6] pionization, limiting fragmentation,[7] small transverse momentum of the produced particles, etc.[8] However, notice that these limiting distributions are obtained when one puts the intercept of the relevant Regge trajectory α_0 equal to unity, i.e., the Pomeranchukon is exchanged, while in the case of the usual Regge trajectories with $\alpha_0 \neq 1$ exchanged, one gets scaling (generalized) only for the ratio of the differential cross section to the total cross section.[5,8] The above-mentioned successes of DRM in purely hadronic exclusive and inclusive reactions are certainly interesting both theoretically and phenomenologically.

In the processes where currents are involved there exists a "similar" kind of scaling behavior, namely the one originally predicted by Bjorken[9] for the deep-inelastic electroproduction structure functions for inclusive reactions and its generalization for quasi-inclusive ones.

Besides the parton model,[10] which has a transverse momentum cutoff as its ingredient, the only field-theoretical model which gives the scaling of at least the structure function νW_2 is the sum of ladder graphs in $\lambda\varphi^3$ theory.[11] But this is because the $\lambda\varphi^3$ model is a superrenomalizable theory and has good convergence properties. All the other models,[12] including the field-theoretical treatment of the parton model,[13] also need a transverse momentum cutoff in order to obtain scaling. When no cutoff is imposed Bjorken scaling breaks down. Even the sum of an infinite set of renormalized graphs in field theory[14] does not possess scaling. Another approach[15] based on a DRM point of view for the parton model has been used for the case of the deep-inelastic region. Here too, the normal-mode expansion for the hadronic string is cut off from the start. There exists, however, no theoretical background whatsoever for justifying the crucial assumption of a cutoff in the above-mentioned approaches.

On the other hand, it is known that the DRM has a cutoff of the exponential type in the transverse momentum[4,8] (see also Ref. 16). This fact already gives a hint that a current amplitude which has the same property as the DRM for its strong part may have a good chance to give scaling; in this case the analysis of Bloom and Gilman,[17] which indicates that the resonant component of the structure functions does show the scaling property, could be most naturally understood.

A vast number of prescriptions on how to include the currents in DRM has been proposed, each having its own shortcomings. Among them we intend to use the one which has the least freedom. One such model is the prescription of including the currents as the minimal gauge interactions $\partial_\mu \to \partial_\mu - ieA_\mu$, proposed by Kikkawa and Sato.[18] Once the minimal gauge prescription is accepted, in principle everything is fixed and there is no freedom left. The only ambiguity is the one due to the renormalization of dual loops.[19] Nevertheless, if one takes the model seriously then it may restrict the ambiguities in the dual-loop renormalization. Throughout the present paper we renormalize in the manner of Neveu and Scherk.[20]

One of the shortcomings of the present model used here is that its Born term has only one single pole, say in the s channel, and therefore has no duality property between s and t channels (see, e.g., Ref. 21). But since we are interested in inclusive reactions and therefore in the discontinuities of the graphs, we shall ignore the above shortcoming since those graphs simply do not give a contribution to the discontinuities, and therefore the whole treatment of the present paper is dual in this sense.

The DRM with currents is the next step towards the construction of a theoretical frame where one, e.g., using Mueller's analysis,[22] can study all the relevant limits in the same manner as in 4, 5, and 8 for purely hadronic reactions, investigate the existence or absence of fixed poles in the amplitudes with currents, and, in case such amplitudes satisfy the Bjorken scaling, find its "dynamical" origin. It turns out, for instance, that such amplitudes satisfy Bjorken scaling and that the existence of a current-algebra fixed pole for the amplitudes with two currents is responsible for this scaling (see a similar situation in Landshoff and Polkinghorne, Ref. 12). This is contrary to a "similar" situation in purely hadronic processes where only Pomeranchuk exchange is responsible for the Feynman scaling law and the limiting distributions,[4,5,8] while the usual Regge trajectories give vanishing contributions in these limits. This fixed-pole responsibility for Bjorken scaling, in the language of light-cone expansion, would probably mean that the Regge trajectories have nothing to do with the degree of singularity on the light cone.[23]

In Sec. II we consider the two structure functions W_1 and νW_2 of inelastic electron scattering. In the model, νW_2 scales while W_1 vanishes. This is a property of all the other models where currents are coupled to spin-0 particles. When a proper DRM for fermions is constructed, we suggest that the same minimal gauge interaction which now would couple the current to the tower of fermions will restore the scaling for W_1.

In Sec. III we consider the inelastic electron scattering where a final-state hadron with momentum p' is detected, i.e.,

$$e + \text{hadron}\,(p) \to e' + \text{hadron}\,(p') + \text{anything}.$$

Following Mueller, we connect this process to the discontinuity of a forward six-point function, which we then study. It appears here, too, that in the Bjorken limit a fixed pole is responsible for the scaling behavior of three out of the four structure functions νW_2, νW_3, νW_4, and the vanishing of W_1, which is reminiscent of the same situation in Sec. II that was suggested to be due to the coupling of currents to the tower of bosons rather than to the tower of fermions. Notice, however, that this fixed pole can be responsible for the above scaling only in a special kinematical region of $p \cdot p' =$ fixed and not large, i.e., when the detected hadron is very near to the forward direction in the center-of-mass system or is slowly moving in the lab system. Beyond this kinematical region, i.e., where $p \cdot p'$ is large, the fixed pole can no longer be responsible for the scaling. In this case, only with the speculation of assigning the Regge trajectory to a Pomeranchukon with the intercept $\alpha_0 = 1$ does one get the above scaling. This is very similar to the purely hadronic case[4,5] where limiting distributions are obtained by putting $\alpha_0 = 1$.

In Sec. IV we study the inelastic Compton scattering

$$\text{photon } (k) + \text{hadron } (p) \to \text{photon } (k') + \text{anything}.$$

This reaction is interesting since, from the parton model of Bjorken and Paschos,[10] there should be a similar scaling law for the structure functions where the scaling variable now is $(k-k')\cdot p/k\cdot k'$, and, in addition, from the same parton model one concludes that the present reaction should be proportional to the electron scattering of Sec. II, for partons of unit charge and spin 0 or $\frac{1}{2}$. Both these results are also valid within the model of the present paper, which may suggest a deeper analogy with the parton model.

Finally, Sec. V is devoted to the study of the high-energy behavior of electron-positron total annihilation into hadrons. It turns out that this cross section falls off like $s^{-5/2}$. This result is incompatible with the results of other models. In evaluating the above high-energy behavior we have not renormalized the amplitude for the self-energy of the virtual photon which has the exponential divergence of the dual loops. It may happen that a dual renormalization changes the above high-energy behavior. We hope to study this question further.

Throughout the paper only one leading diagram for each process is written down and discussed and dots mean nonleading diagrams.

II. DEEP-INELASTIC ELECTRON SCATTERING

Consider the virtual Compton scattering averaged and summed over the spins of hadrons corresponding to Fig. 1,

$$k_1 + p_1 \to k_2 + p_2 . \tag{2.1}$$

The notation is the usual one[24]:

$$\begin{gathered} M_{\mu\nu}(\nu, t; k_1^2, k_2^2) = P_\mu P_\nu A_1 + \cdots + g_{\mu\nu} A_{10}, \\ W_1 = \frac{1}{\pi}\,\mathrm{Im}A_{10}(\nu, k_1 = k_2 = k), \qquad W_2 = \frac{m^2}{\pi}\,\mathrm{Im}\,A_1(\nu, k_1 = k_2 = k), \\ t = (k_1 - k_2)^2, \qquad \nu = \tfrac{1}{4}(s-u), \quad P = \tfrac{1}{2}(p_1 + p_2), \quad p_1^2 = p_2^2 = m^2, \\ \alpha(s) = \alpha' s + \alpha_0, \quad \alpha_0 < 0, \quad \alpha(m^2) = 0, \quad \alpha' = \tfrac{1}{2}. \end{gathered} \tag{2.2}$$

The contribution of the last diagram of Fig. 1, after the dual-loop renormalization in the manner of Neveu and Scherk,[20] is

$$\begin{aligned} M_{\mu\nu} = e^2 g^2 \int d^4Q \exp(-\tfrac{1}{2}\ln W\, Q^2) \int_0^1 dx dy dz\, & W^{-\alpha_0 - 1}(1-u)^{\alpha_0-1}(1-V)^{\alpha_0 - 1}(1-W) \\ & \times [f(W)]^{-4}\{[\psi(u,W)]^{m^2} - [\tilde{\psi}(u,W)]^{m^2}\} \\ & \times \exp\left\{\frac{t}{2}\left[\frac{-\ln x \ln z}{\ln W} + \frac{\ln y \ln u}{2\ln W} + \sum_{n=1}^{\infty} \frac{V^n + u^n}{n(1-W^n)}\right]\right\} \\ & \times \exp\left(-\nu\,\frac{\ln y \ln u}{\ln W} - \frac{k_1^2 + k_2^2}{2}\,\frac{\ln y \ln xzu}{2\ln W} + \frac{k_1^2 - k_1^2}{2}\ln y\,\frac{\ln z^2 u - \ln xzu}{2\ln W}\right) \\ & \times \left(\frac{\ln^2 u}{\ln^2 W} P_\mu P_\nu + \cdots + \frac{1}{\ln W} g_{\mu\nu}\right), \end{aligned} \tag{2.3}$$

where

$$\begin{gathered} V = xyz, \quad W = xyzu, \\ \psi(u, W) = \exp\left[-\frac{\ln u \ln(W/u)}{2\ln W} + \sum_{n=1}^{\infty} \frac{2W^n - u^n - (W/u)^n}{n(1-W^n)}\right], \\ \tilde{\psi}(u,W) = -\frac{\ln W}{\pi}\sin\left(\pi\frac{\ln u}{\ln W}\right), \qquad f(W) = \prod_{n=1}^{\infty}(1-W^n). \end{gathered} \tag{2.4}$$

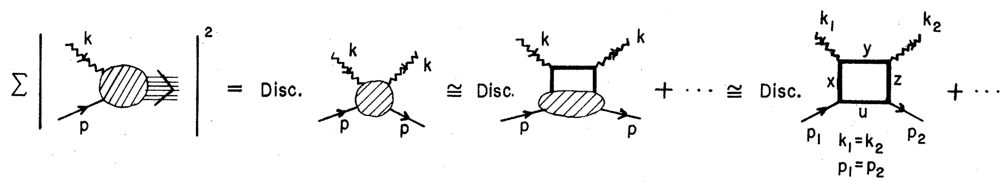

FIG. 1. The dual diagram used for the study of Bjorken scaling. Thick lines are the tower of boson resonances; dots mean nonleading diagrams.

Whenever needed, we can use for the loop integrations (with the Wick rotation) the following formula:

$$\int d^4Q \exp(-\tfrac{1}{2}\ln W\, Q^2) = (\pi/\tfrac{1}{2}\ln W)^2 .$$

For the large values of a variable s of a function $\varphi(s,\ldots)$ we use the Mellin-transform technique

$$\tilde{\varphi}(\beta,\ldots) = \int_0^\infty \varphi(s,\ldots)s^{-\beta-1}ds .$$

Then the right-most singularity of $\tilde{\varphi}(\beta,\ldots)$ in the plane of the Mellin transform variable β defines the leading high-s behavior of the function $\varphi(s,\ldots)$.

From (2.3) for the $\nu \to -\infty$, t = fixed behavior of the invariant amplitudes defined in (2.2) after the use of the Mellin transform, we find for the A_1 amplitude a singularity at $\beta = -1$ (coming from $y \approx 1$) corresponding to a fixed pole at $J = 1$ in the angular momentum plane, and another at $\beta = \alpha(t) - 2$ (coming from the $u \approx 1$ region) corresponding to the usual Regge pole. Hence

$$A_1 \underset{\nu \to -\infty}{\sim} \frac{-eF(t)}{\nu} + \beta(t)\nu^{\alpha(t)-2} . \tag{2.5}$$

The residue of the fixed pole at $J = 1$, i.e., $F(t)$, coincides with the expression for the form factor corresponding to Fig. 2, and hence the Fubini–Dashen–Gell-Mann sum rule is satisfied. The expression for the form factor of Fig. 2 is

$$\begin{aligned} F(t) = eg^2 \int d^4Q \exp(-\tfrac{1}{2}\ln W Q^2) \int_0^1 dx du dz\, W^{-\alpha_0 -1}(1-xz)^{\alpha_0-1}(1-u)^{\alpha_0-1} \\ \times [f(W)]^{-4}\{[\psi(u,W)]^{m^2} - [\tilde{\psi}]^{m^2}\}(1-W)(\ln u/\ln W) \\ \times \exp\left\{t\left[-\frac{\ln x \ln z}{2\ln W} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{(xz)^n + u^n}{n(1-W^n)}\right]\right\} , \end{aligned} \tag{2.6}$$

with $W = xuz$, and its high-t behavior is $\sim t^{-1-3m^2/4}\ln^{-2}t$, which is the singularity in the Mellin transform variable coming from the region of integration $(1-x)z(1-u) \approx 1$ or $x(1-z)(1-u) \approx 1$. Analogously, the A_{10} amplitude has $\beta = -1$ and $\beta = \alpha(t)$ singularities corresponding to a fixed $J = -1$ pole and the usual moving pole.

In the Bjorken scaling limit we put $k_1 = k_2 = k$ and find the Mellin transform with respect to $-k^2$, keeping $2\nu/-k^2 = \omega$ fixed. The right-most singularity for both A_1 and A_{10} is at $\beta = -1$, coming from the region $y \approx 1$, i.e., exactly where the current-algebra fixed pole in (2.5) came from. Therefore we get

$$\begin{aligned} \nu A_1(\nu,k^2) \underset{\mathrm{Bj}}{\to} e^2g^2 \int d^4Q \exp(-\tfrac{1}{2}\ln W Q^2)\int_0^1 dx dz du\, W^{-\alpha_0-1}(1-u)^{\alpha_0-1}(1-xz)^{\alpha_0-1}(1-W)[f(W)]^{-4} \\ \times\{[\psi(u,W)]^{m^2} - [\tilde{\psi}]^{m^2}\}\frac{\ln u}{\ln W}\frac{1}{(2\nu/-k^2) - \ln W/\ln u} , \end{aligned} \tag{2.7}$$

where $W = xzu$, and an analogous expression for A_{10}. Finally, for the structure functions we get

$$\begin{aligned} \nu W_2(\nu,k^2) &\underset{\mathrm{Bj}}{\to} F_2(\omega) , \\ W_1(\nu,k^2) &\underset{\mathrm{Bj}}{\to} \frac{1}{k^2}F_1(\omega) , \end{aligned} \tag{2.8}$$

where

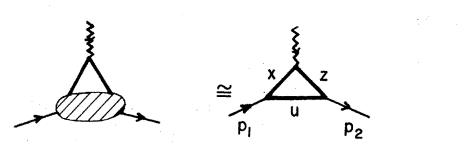

FIG. 2. Form factor corresponding to the last diagram of Fig. 1 through the Fubini–Dashen–Gell-Mann sum rule.

$$F_2(\omega)=e^2g^2m^2\omega\int d^4Q\,\exp(-\tfrac{1}{2}\ln WQ^2)\int_0^1 dx\,dz\,du\,W^{-\alpha_0-1}(1-u)^{\alpha_0-1}(1-xz)^{\alpha_0-1}(1-W)[f(W)]^{-4}$$
$$\times\{[\psi(u,W)]^{m^2}-[\bar\psi]^{m^2}\}\left(\frac{\ln u}{\ln W}\right)\delta\left(\omega-\frac{\ln W}{\ln u}\right),\quad W=xzu \tag{2.9}$$

$$F_1(\omega)=e^2g^2\int\cdots\left(\frac{1}{\ln u}\right)\delta\left(\omega-\frac{\ln W}{\ln u}\right). \tag{2.10}$$

There seems to be a similarity with the parton model if $X=1/\omega=\ln u/\ln(xzu)$ ($\leqslant 1$) is interpreted as the fraction of longitudinal momentum carried by a parton in an infinite-momentum frame. As was already mentioned in the Introduction, the scaling of νW_2 and vanishing of W_1 is typical for the models where the current is attached to bosons rather than to fermions.

III. BJORKEN SCALING OF QUASI-INCLUSIVE PROCESSES

Consider reactions like

$$\text{hadron}+\text{hadron}\to\mu^-\mu^+ +\text{anything} \tag{3.1}$$

or

$$e+\text{hadron}\to e'+\text{hadron}+\text{anything}. \tag{3.2}$$

Using Mueller's analysis, we connect these reactions to the discontinuity of a forward six-point reaction as shown in Fig. 3. For these processes there are four structure functions W_1, W_2, W_3, W_4 analogous to the two functions in deep-inelastic electron scattering. The differential cross sections for the above reactions are proportional to the tensor $W_{\mu\nu}$,[25] where

$$W_{\mu\nu}=-\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)W_1+\frac{1}{m_2}\left(p-\frac{p\cdot k}{k^2}k\right)_\mu\left(p-\frac{p\cdot k}{k^2}k\right)_\nu W_2$$
$$+\frac{1}{m^2}\left(p'-\frac{p'\cdot k}{k^2}k\right)_\mu\left(p'-\frac{p'\cdot k}{k^2}k\right)_\nu W_3+\frac{1}{m^2}\left[\left(p-\frac{p\cdot k}{k^2}k\right)_\mu\left(p'-\frac{p'\cdot k}{k^2}k\right)_\nu+p\leftrightarrow p'\right]W_4, \tag{3.3}$$

and $W_{\mu\nu}=\text{Disc}\,M_{\mu\nu}$, where $M_{\mu\nu}$ is the forward six-point amplitude of Fig. 3.

In a certain region of kinematical variables

$$k\cdot p\to\infty;\quad \frac{-k^2}{k\cdot p},\quad\frac{k\cdot p'}{k\cdot p},\quad\frac{p\cdot p'}{k\cdot p}=\text{fixed}, \tag{3.4}$$

from the analogous original considerations of Bjorken,[25] one would expect to have scaling for all four structure functions, namely, that W_1, $(k\cdot p)W_2$, $(k\cdot p')W_3$, and $[(k\cdot p)(k\cdot p')]^{1/2}W_4$ should all become functions of the ratios $-k^2/k\cdot p$, $k\cdot p'/k\cdot p$, and $p\cdot p'/k\cdot p$. The purpose of the Section is to study the above scaling in the aspect of DRM considered in the present paper.

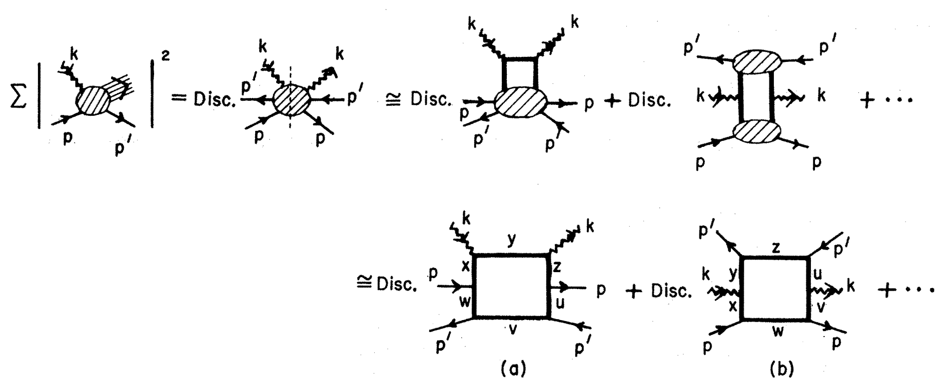

FIG. 3. Diagrams considered for the study of Bjorken scaling in deep-inelastic electron scattering with one final-state hadron detected.

The dual amplitude of Fig. 3(a) gives the following contribution:

$$M_{\mu\nu}^{(a)}(k^2,k\cdot p,k\cdot p',p\cdot p')=\tfrac{1}{4}e^2g^4\int d^4Q\int_0^1 dxdydzdudvdw\,\exp(-\tfrac{1}{2}\ln WQ^2)W^{-\alpha_0-1}[(1-xyz)(1-u)(1-v)(1-w)]^{\alpha_0-1}$$
$$\times(1-W)[f(W)]^{-4}\{[\psi(uvw,W)]^{p^2}-[\tilde{\psi}]^{p^2}\}\{[\psi(v,W)]^{p'^2}-[\tilde{\psi}(v,W)]^{p'^2}\}$$
$$\times\exp\left\{k^2\left[-\frac{\ln y\ln(W/y)}{2\ln W}\right]+k\cdot p\left[-\frac{\ln y\ln uvw}{\ln W}\right]+k\cdot p'\left[\frac{\ln y\ln v}{\ln W}\right]\right.$$
$$\left.+p\cdot p'\left[\frac{\ln v\ln(W/uvw)}{\ln W}-\sum_{n=1}^{\infty}\frac{(1-v^n)[1-(xyz)^n](u^n+w^n)}{n(1-W^n)}\right]\right\}$$
$$\times\left\{\left(\frac{-4}{\ln W}\right)g_{\mu\nu}+\left[\frac{4\ln^2(uvw)}{-\ln^2 W}\right]p_\mu p_\nu+\left[\frac{4\ln^2 v}{-\ln^2 W}\right]p'_\mu p'_\nu\right.$$
$$\left.+\left[\frac{4\ln v\ln(uvw)}{\ln^2 W}\right](p_\mu p'_\nu+p_\nu p'_\mu)+\left[1+\frac{4\ln y}{\ln W}+\frac{4\ln^2 y}{\ln^2 W}\right]k_\mu k_\nu+\cdots\right\}, \qquad (3.5)$$

where $W=xyzuvw$.

Consider the Regge limit $\nu=k\cdot p$, $k\cdot p'\to-\infty$, and the other variables fixed. After a Mellin transform we get a $\beta=-1$ singularity coming from the region $y\approx1$ of integration for all four invariant amplitudes:

$$M_{\mu\nu}^{(a)}\xrightarrow[\text{Regge}]{} g_{\mu\nu}\nu^{-1}+p_\mu p_\nu \nu^{-1}+p'_\mu p'_\nu \nu^{-1}+(p_\mu p'_\nu+p_\nu p'_\mu)\nu^{-1}, \qquad (3.6)$$

i.e., there is a fixed pole in the angular momentum J plane.

For the special case of a scaling limit, namely

$$\text{Bj}':\ -k^2\to\infty;\quad \frac{k\cdot p}{-k^2},\ \frac{k\cdot p'}{-k^2},\ p\cdot p'=\text{fixed}, \qquad (3.7)$$

again the same region $y\approx1$ as in (3.6) gives the singularity $\beta=-1$ and one gets the behavior

$$M_{\mu\nu}^{(a)}\xrightarrow[\text{Bj}']{} g_{\mu\nu}(k^2)^{-1}+p_\mu p_\nu(k^2)^{-1}+p'_\mu p'_\nu(k^2)^{-1}+(p_\mu p'_\nu+p_\nu p'_\mu)(k^2)^{-1}, \qquad (3.8)$$

exactly because of the same fixed pole as in the Regge limit. Therefore, in the limit of (3.7),

$$\begin{aligned}&W_1\to(k^2)^{-1}F_1(\omega,\omega'),\\&\nu W_2\to F_2(\omega,\omega'),\\&\nu' W_3\to F_3(\omega,\omega'),\\&(\nu\nu')^{\frac{1}{2}}W_4\to F_4(\omega,\omega'),\end{aligned} \qquad (3.9)$$

where

$$\nu=k\cdot p,\quad \nu'=k\cdot p',\quad \omega=\frac{2\nu}{-k^2},\quad \omega'=\frac{2\nu'}{-k^2}.$$

In the scaling limit of (3.4) or

$$\text{Bj}:\ -k^2\to\infty,\quad \omega,\omega',\quad\text{and}\quad \frac{p\cdot p'}{-k^2}=\kappa\ \text{fixed}, \qquad (3.10)$$

the above fixed pole is no longer responsible for the scaling and the region of integration $xyz\approx1$ becomes important; one gets

$$M_{\mu\nu}^{(a)}\xrightarrow[\text{Bj}]{} g_{\mu\nu}(k^2)^{\alpha_0-2}M_1+p_\mu p_\nu(k^2)^{\alpha_0-2}M_2+p'_\mu p'_\nu(k^2)^{\alpha_0-2}M_3+(p_\mu p'_\nu+p_\nu p'_\mu)(k^2)^{\alpha_0-2}M_4. \qquad (3.11)$$

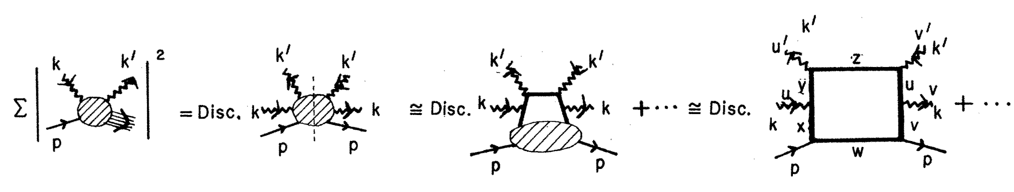

FIG. 4. Diagram for inelastic Compton scattering.

Expression (3.11) shows that only putting $\alpha_0=1$, i.e., the assignment of the Regge trajectory with a Pomeranchukon, can give the scaling of the structure functions.

The responsibility of the fixed pole for scaling in the region (3.7) and the disappearance of its effect in the region (3.10) are interesting both theoretically and perhaps experimentally. First of all, since the fixed pole is due to existence of the currents in the amplitude, it is not clear why its effect should depend on the variable $p\cdot p'$, which is entirely in the strong part of the amplitude. Other more detailed models such as the parton model or even the light-cone expansion technique may shed some light on this question, and then perhaps the origin of fixed poles will become more clear. Experimentally it may be interesting, since it predicts that if some analysis similar to the one done by Bloom and Gilman[17] will become possible in the future, then in the region (3.10) the resonance component of the structure functions will not show the scaling property, contrary to the case of deep-inelastic electron scattering.

The diagram of Fig. 3(b) has no fixed pole in the Regge limit and, correspondingly, no scaling property coming from the fixed pole. In the Regge limit, say $\nu=k\cdot p\to-\infty$, other variables fixed, we get

$$M^{(b)}_{\mu\nu}\xrightarrow[\text{Regge}]{} g_{\mu\nu}\nu^{\alpha_0}+p_\mu p_\nu\nu^{\alpha_0-2}+p'_\mu p'_\nu\nu^{\alpha_0}+(p_\mu p'_\nu+p_\nu p'_\mu)\nu^{\alpha_0-1}\,. \tag{3.12}$$

In the scaling limit of both (3.7) and (3.10) we get

$$M^{(b)}_{\mu\nu}\xrightarrow[\text{Bj or Bj}']{} g_{\mu\nu}(k^2)^{\alpha_0-2}M_1+p_\mu p_\nu(k^2)^{\alpha_0-2}M_2+p'_\mu p'_\nu(k^2)^{\alpha_0-2}M_3+(p_\mu p'_\nu+p_\nu p'_\mu)(k^2)^{\alpha_0-3}M_4\,. \tag{3.13}$$

Again, with $\alpha_0=1$ we get scaling for νW_2 and $\nu' W_3$ and vanishing of W_1 and $(\nu\nu')^{1/2}W_4$.

IV. INELASTIC COMPTON SCATTERING

Consider the reaction

$$\text{photon }(k)+\text{hadron }(p)\to\text{photon }(k')+\text{anything}. \tag{4.1}$$

The expression from the dual diagram of Fig. 4 in the general case of k^2, $k'^2\neq 0$, is the following:

$$\begin{aligned}M_{\mu\mu'\nu\nu'}(k^2,k'^2,\dots)={}&e^4g^2\int d^4Q\,\exp(-\tfrac{1}{2}\ln WQ^2)\int_0^1 dx\,dy\,dz\,du\,dv\,dw\,W^{-\alpha_0-1}\\&\times(1-W/w)^{\alpha_0-1}(1-w)^{\alpha_0-1}(1-W)[f(W)]^{-4}\{[\psi(w,W)]^{m^2}-[\tilde\psi]^{m^2}\}\\&\times\exp\left\{k^2\left[-\frac{\ln yzu\ln(W/yzu)}{2\ln W}\right]+k'^2\left[-\frac{\ln z\ln(W/z)}{2\ln W}\right]+(k\cdot p)\left[-\frac{\ln w\ln yzu}{\ln W}\right]\right.\\&\left.+(k'\cdot p)\left[\frac{\ln z\ln w}{\ln W}\right]+(k\cdot k')\left[\frac{\ln z\ln(W/yzu)}{\ln W}\right]\right\}\\&\times\frac{1}{\ln^2W}\left[\frac{\ln^4 w}{\ln^2 W}p_\mu p_\nu p_{\mu'}p_{\nu'}+(g_{\mu\nu}g_{\mu'\nu'}+g_{\nu\mu'}g_{\mu\nu'}+g_{\mu\mu'}g_{\nu\nu'})+\cdots\right],\end{aligned} \tag{4.2}$$

where $W=xyzuvw$. If we define the decomposition of the amplitudes as

$$M_{\mu\mu'\nu\nu'}=p_\mu p_\nu p_{\mu'}p_{\nu'}\tilde M_2+\cdots \tag{4.3}$$

and

$$\tilde W_2=\mathrm{Disc}\tilde M_2\,,$$

from the parton model of Bjorken and Paschos,[10] it is seen that (for the case of real photons $k^2=k'^2=0$)

$$\tilde\nu^3\tilde W_2=(\text{a factor not depending on }\tilde x)\sum_N P(N)\tilde x f_N(\tilde x)\left\langle\sum_i Q_i^{\,4}\right\rangle_N\,, \tag{4.4}$$

where

$$\tilde x=-t/2\tilde\nu,\quad t=(k-k')^2=-2k\cdot k',\quad \tilde\nu=(k-k')\cdot p\,, \tag{4.5}$$

is satisfied for large values of t, $\tilde\nu$, and $k\cdot p$ with their ratios fixed. In other words, for large values a scaling law is satisfied for the structure fuction $\tilde\nu^3\tilde W_2$ with the scaling variable $\tilde x=-t/2\tilde\nu$, where $\tilde x$ is the longitudinal fraction of the parton momentum in the infinite-momentum frame. From the same parton model[10] one gets for the νW_2 structure function of deep-inelastic electron scattering an expression

$$\nu W_2(\nu, k^2) = \sum_N P(N) x f_N(x) \Big\langle \sum_i Q_i^{\,2} \Big\rangle_N , \tag{4.6}$$

with the scaling variable $x = 1/\omega = -k^2/2\nu$. By looking at expressions (4.4) and (4.6) one sees that the structure functions of inelastic Compton scattering have the same analytic dependence, as a function of $\tilde{x}$, that the structure function of deep-inelastic electron scattering has as a function of x in the case of parton charge $Q_i = 0, 1$.

It would be interesting to study the same question in the DRM of the present paper in order to reveal a deeper analogy with the parton model. By putting $k^2 = k'^2 = 0$ in (4.2) and finding $\tilde{M}_2$ from the definition (4.3), we find the Mellin transform with respect to $k \cdot k'$, keeping $k \cdot p/k \cdot k'$ and $k' \cdot p/k \cdot k'$ fixed. There is a pole at $\beta = -3$ coming from $yzu \approx 1$, and we get the limit

$$\tilde{M}_2 \to e^4 g^2 (k \cdot k')^{-3} \int d^4Q \exp(-\tfrac{1}{2} \ln W Q^2) \int_0^1 dx dv dw\, W^{-\alpha_0 - 1} (1 - xv)^{\alpha_0 - 1} (1 - w)^{\alpha_0 - 1}$$
$$\times (1 - W)[f(W)]^{-4} \{[\psi(w, W)]^{m^2} - [\tilde{\psi}]^{m^2}\} \frac{\ln^4 w}{\ln^4 W} \left[\frac{2\tilde{\nu}}{t} \frac{\ln w}{\ln W} + 1 \right]^{-3} , \tag{4.7}$$

where $W = xvw$. Surprisingly enough, the variables $k \cdot p$ and $k' \cdot p$ in the complicated expression (4.2) combine in just such a manner as to give (4.7) depending only on $2\tilde{\nu}/(-t)$ with $\tilde{\nu} = (k - k')p$. By looking at (2.7) and (4.7) one sees that the two expressions are indeed very similar, provided one changes $v \leftrightarrow z$ and $w \leftrightarrow u$. Furthermore, assuming the analyticity of the discontinuity of $\tilde{M}_2$ in a proper region of variables $\tilde{\nu}$ and excluding the end points of the scaling variable $\tilde{x} = -t/2\tilde{\nu} = \ln w/\ln xvw$, i.e., $\tilde{x} \neq 0$ and 1 (where the δ function of these end-point values and their derivatives appear), and with the use of

$$\tilde{W}_2 = \mathrm{Disc}\tilde{M}_2 = \left[\mathrm{Disc} \frac{\partial^2}{\partial \nu^2} A_1 \right] = \left[\frac{\partial^2}{\partial \nu^2} \mathrm{Disc} A_1 \right]_{x \to \tilde{x};\, \nu \to \tilde{\nu}} = \left[\frac{\partial^2}{\partial \nu^2} W_2 \right]_{x \to \tilde{x};\, \nu \to \tilde{\nu}} ,$$

one can convince himself that the two structure functions are indeed proportional to each other.

V. e^-e^+ ANNIHILATION INTO HADRONS

The total cross section of e^-e^+ annihilation into hadrons is[26]

$$\sigma(k^2) = \frac{1}{2} \frac{16\pi^2 \alpha^2}{(k^2)^2} \rho(k^2) , \tag{5.1}$$

where

$$M_{\mu\nu}(k^2) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \rho(k^2) .$$

In the dual model of the present paper the calculation of $M_{\mu\nu}(k^2)$ is approximated by the discontinuity of the self-energy diagram of the virtual photon, as illustrated in Fig. 5, and its contribution without renormalization is

$$M_{\mu\nu}(k^2) = e^2 \int d^4Q \exp(-\tfrac{1}{2} \ln W Q^2) \int_0^1 dx dy\, W^{-\alpha_0 - 1} (1 - W)^{\alpha_0 - 1} [f(W)]^{-4} \exp\left(k^2 \frac{\ln^2 x}{2 \ln W} \right) \left(\frac{1}{\ln W} g_{\mu\nu} + \cdots \right) , \tag{5.2}$$

$W = xy$.

The singularity of the Mellin variable is at $\beta = -\frac{1}{2}$ and, therefore,

$$\rho(k^2) \to (k^2)^{-1/2} ,$$
$$\sigma(k^2) \to 1/(k^2)^{5/2} . \tag{5.3}$$

This is a kind of fixed-pole behavior independent of the trajectories exchanged. The result (5.3) is incompatible with the results of the other models.[26] Notice, however, that the expression (5.2) has the

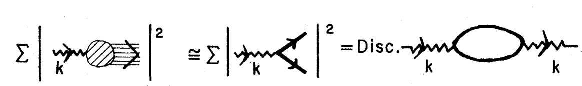

FIG. 5. Diagram for the electron-positron annihilation into hadrons.

exponential divergence of the dual loops and has not been renormalized. In the evaluation of (5.3) we have, in fact, applied a simple cutoff of the type $\Theta(2-\epsilon-x-y)$. We suggest that a proper dual renormalization of the self-mass diagram of the photon may change the result (5.3). We hope to study this question further.

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