meromorphic function in a domain enclosing the unitary cut with possible poles on the cut. The main point of the argument is that the discontinuity across these poles is irrelevant as far as the definition of $t^{(2)}(+)$ is concerned. As a consequence the two-particle Heitler equation (3.19) in the three-particle space is satisfied identically whether or not we take into account the Dirac-δ-function piece of these poles. Therefore, it appears to be entirely consistent to retain only the principal-value part of these poles when defining the two-body K matrices on the three-particle space.

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Some Aspects of Field Symmetries. III

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Some aspects of the connection between trilinear and bilinear equal-time commutation relations are clarified. Locality requirements satisfied by the generalized fields under discussion are examined in some detail. Using algebraic arguments it is shown that for representations of operators satisfying the generalized trilinear equal-time commutation relations either the half-integral- or the integral-spin fields or both satisfy conventional statistics. In connection with a question raised previously it is shown that the hadronic schemes considered suggest which fields should in each scheme be associated with the (hypothetical) intermediate bosons. It is shown that the discrepancy in physical content between the various particle classification schemes discussed can be substantially reduced, if not eliminated, by the introduction of a new selection rule in addition to the selection rules derived from locality and self-adjointness of the Lagrangian. The new selection rule is not unrelated to locality considerations and is also related to the metric. The question of the metric in the context of the generalized fields is briefly considered.

I. INTRODUCTION

It is well known that the bilinear equal-time commutation relations between distinct fields have implications bearing on the interactions of the fields concerned, on the selection rules they satisfy, and on their vacuum expectation values.[1-15] It is also known that TCP invariance requirements do not uniquely determine the bilinear equal-time commutation relations between distinct fields.[10,16,17] It is therefore of interest to inquire whether from first principles[18] it is possible to derive a set of fields which has the property that the bilinear equal-time commutation or anticommutation relations are specified for each pair of kinematically independent field variables contained in the set. Quite apart from any intrinsic interest, such a set of fields can conceivably serve as a classification scheme for elementary particles. It is assumed from the outset that all spinor, vector, etc., components of a field have identical equal-time commutation behavior with all other fields belonging to the set.[19]

In connection with the specification of bilinear equal-time commutation relations between distinct fields the following trilinear equal-time commutation relations[12–15, 20, 21] have been studied:

$$[\Psi_{\alpha,i}(\vec{x}), [\chi_{\beta,j}(\vec{y}), \chi'_{\gamma,k}(\vec{z})]_-]_- = \delta(\vec{x}-\vec{y})(\gamma'_4)_{\alpha\beta}M'\chi'_{\gamma,k}(\vec{z}) - \delta(\vec{x}-\vec{z})(\gamma'_4)_{\alpha\gamma}M\chi_{\beta,j}(\vec{y}), \tag{1a}$$

and

$$[\Phi_{\mu,l}(\vec{x}),[\chi_{\nu,m}(\vec{y}), \chi'_{\rho,n}(\vec{z})]_+]_- = i\tilde{\delta}_{\mu\nu}\,\delta(\vec{x}-\vec{y})N'\chi'_{\rho,n}(\vec{z}) + i\tilde{\delta}_{\mu\rho}\,\delta(\vec{x}-\vec{z})N\chi_{\nu,m}(\vec{y}), \tag{1b}$$

where Greek subscripts denote spinor indices and Latin subscripts $(i, \ldots, n)$ denote integer values from a finite range, which depends on the particular representation of "generalized fields" [operators satisfying Eqs. (1)] being studied. In the subsequent discussion Greek subscripts (spinor indices) will sometimes be suppressed when no misunderstanding is likely to occur.

Equations (1a) and (1b) are assumed to be simultaneously valid, and in keeping with a notation developed earlier[12, 20–22] each field operator in Eqs. (1) denotes, independently of the other fields, either a field variable or its canonical conjugate. Moreover, any two field variables in Eq. (1a) may be kinematically related or unrelated. The same applies to Eq. (1b).

The space-time independent "undetermined multipliers" M, M', N, and N', which implicitly depend on the field operators in whose trilinear commutation relations they occur, can be nonvanishing only when the fields being contracted are canonical conjugates of each other, i.e., they have the same Latin subscript i or l, as the case may be. The relevant Kronecker δ's are assumed to be absorbed in M, M', N, or N' or in γ'_4 and $\tilde{\delta}_{\mu\nu}$, respectively.[12, 20, 21]

In the derivation of the commutation relations (1) from the action principle,[12, 18] the fields Ψ_i differ from the fields Φ_j in that the generators of the infinitesimal transformations are antisymmetrized in the former and symmetrized in the latter case. The symmetrization of the generators of the χ fields is not specified in the derivation, and the generator of any particular χ may therefore be either symmetrized or antisymmetrized.

The commutation relations (1) are a natural generalization of a set of trilinear equal-time commutation relations first discussed by Green[23] in connection with an attempt to generalize quantum statistics. A related issue had earlier been raised by Wigner.[24] In recent years, particularly since the contributions of Volkov,[22] there has been a growing interest in these generalized statistics. A representative sampling from the literature is given below. Discussions of statistical-mechanical aspects of generalized statistics can be found in Refs. 25–32; Refs. 33–37 contain references to lecture notes and review articles; suggestions, pertinent comments, and references to generalized statistics can be found in Refs. 38–45; discussions of the subject in the context of second quantization, field theory, particle physics, quantum mechanics, and quantum statistics can be found in Refs. 20–23, 46–93; Refs. 94–113 contain references to discussions of possible applications of generalized statistics to quark and related models.

As far as is known at the present time there appears to be no compelling empirical evidence indicating that any elementary particle obeys statistics other than Bose-Einstein or Fermi-Dirac.[114]

In this discussion and in previous investigations on the subject[12–15, 20, 21] the point of view has been taken that the trilinear commutation relations should be generalized to the form (1), and that they contain information concerning the bilinear equal-time commutation relations between distinct fields satisfying ordinary statistics. There have been other attempts to deemphasize the generalized statistical aspects of higher-order commutation relations.[115]

II. TRILINEAR AND BILINEAR COMMUTATION RELATIONS

With the aid of trilinear algebraic identities it is possible to deduce from the commutation relation (1a) that it has symmetry properties in the canonically conjugate variables being contracted. These symmetry properties can symbolically be expressed as

$$M(\Psi_{\alpha,i}, \overline{\Psi}_{\beta,i}, \chi)(\gamma'_4)_{\alpha\beta} = M(\overline{\Psi}_{\beta,i}, \Psi_{\alpha,i}, \chi)(\gamma'_4)_{\beta\alpha} \tag{2a}$$

and

$$M'(\Psi_{\alpha,i}, \overline{\Psi}_{\beta,i}, \chi)(\gamma'_4)_{\alpha\beta} = M'(\overline{\Psi}_{\beta,i}, \Psi_{\alpha,i}, \chi)(\gamma'_4)_{\beta\alpha}, \tag{2b}$$

where χ is any field variable belonging to the representation being considered. Similarly, the commutation relation (1b) implies antisymmetry relations in canonical variables:

$$N(\Phi_{\mu,i}, \Pi_{\nu,i}, \chi)\tilde{\delta}_{\mu\nu} = -N(\Pi_{\nu,i}, \Phi_{\mu,i}, \chi)\tilde{\delta}_{\nu\mu} \quad (3a)$$

and

$$N'(\Phi_{\mu,i}, \Pi_{\nu,i}, \chi)\tilde{\delta}_{\mu\nu} = -N'(\Pi_{\nu,i}, \Phi_{\mu,i}, \chi)\tilde{\delta}_{\nu\mu}\,. \quad (3b)$$

Equations (2) and (3) of course, respectively, mean that the commutation relation (1a) is symmetric and the commutation relation (1b) is antisymmetric under exchange of canonical variables. The symmetry properties (2) and (3) also imply that, for each representation of the generalized fields, a generalized field and its canonically conjugate momentum have the same equal-time commutation behavior with respect to all the other generalized fields belonging to the representation. Thus, although this discussion is based on the Lagrangian formalism and not on the axiomatic formulation of quantum field theory, the commutation relations (1) imply that, for each representation of the generalized fields, Dell'Antonio's theorem[116] is satisfied *ab initio*.

With the aid of the symmetry properties (2) and (3), the following equal-time commutation relations are obtained from Eqs. (1):

$$[\Phi_i(\vec{x}), \Psi_j(\vec{y})\Psi_k'(\vec{z})]_- = 0 \quad (4a)$$

and

$$[\Psi_i(\vec{x}), \Phi_j(\vec{y})\Phi_k'(\vec{z})]_- = 0\,, \quad (4b)$$

where again each field operator denotes independently of the other fields either a field variable or its canonically conjugate momentum. For any representation Eqs. (4) are valid for all values of the subscripts i, j, and k.

For a representation of the generalized fields for which there is one integral-spin field Φ_i which, for equal times, commutes with one half-integral-spin field Ψ_j belonging to the same representation, i.e.,

$$[\Phi_i(\vec{x}), \Psi_j(\vec{y})]_- = 0\,, \quad (5)$$

Eqs. (4) imply that the commutation relation (5) is valid for all fields Φ_i and Ψ_j of the representation considered.

Equation (5) together with the commutation relation (1b) then implies that

$$[\Phi_i(\vec{x}), \Phi_j'(\vec{y})]_- = 0\,, \quad \Phi_j' \neq \Pi_i \quad (6)$$

and

$$[\Phi_{\mu,i}(\vec{x}), \Pi_{\nu,i}(\vec{y})]_- \chi(\vec{z}) = \tfrac{1}{2} i\tilde{\delta}_{\mu\nu}\, \delta(\vec{x}-\vec{y}) N'(\Phi_i, \Pi_i, \chi)\chi(\vec{z})\,. \quad (7)$$

For any representation of the generalized fields and for any value of the subscript i, Eq. (7) is, as a consequence of Eqs. (1) and (5), valid for all field variables χ kinematically unrelated to the integral-spin field Φ_i. Hence Eq. (7) suggests that if Eq. (5) applies, one should look for representations for which for all values of the subscript i

$$[\Phi_{\mu,i}(\vec{x}), \Pi_{\nu,i}(\vec{y})]_- = \tfrac{1}{2} i\tilde{\delta}_{\mu\nu}\, \delta(\vec{x}-\vec{y}) N'(\Phi_i, \Pi_i)\,. \quad (8)$$

If instead of Eq. (5) the two fields concerned satisfy equal-time anticommutation relations,

$$[\Phi_i(\vec{x}), \Psi_j(\vec{y})]_+ = 0\,, \quad (9)$$

Eqs. (4) imply that for the representation under consideration, Eq. (9) is valid for all values of the subscripts i and j.

In analogy to Eqs. (6), (7), and (8) one then obtains from the commutation relations (1) and (9) the following three equations:

$$[\Psi_i(\vec{x}), \Psi_j'(\vec{y})]_+ = 0\,, \quad \Psi_j' \neq \bar{\Psi}_i \quad (10)$$

$$[\Psi_{\alpha,i}(\vec{x}), \bar{\Psi}_{\beta,i}(\vec{y})]_+ \chi(\vec{z}) = \tfrac{1}{2}\delta(\vec{x}-\vec{y})(\gamma_4')_{\alpha\beta} M'(\Psi_i, \bar{\Psi}_i, \chi)\chi(\vec{z})\,, \quad (11)$$

and

$$[\Psi_{\alpha,i}(\vec{x}), \bar{\Psi}_{\beta,i}(\vec{y})]_+ = \tfrac{1}{2}\delta(\vec{x}-\vec{y})(\gamma_4')_{\alpha\beta} M'(\Psi_i, \bar{\Psi}_i)\,. \quad (12)$$

The quantization scheme (8) is, of course, also compatible with the trilinear commutation relations obtained when in Eq. (1b) all fields are assumed to be kinematically related. The same applies to Eq. (12) and the commutation relation (1a). However, the commutation relations (1) and their derivation from the action principle preclude the possibility of quantizing the Ψ fields by means of commutators and the Φ fields by means of anticommutators.

For representations for which the generalized fields satisfy either the condition (5) or (9) some of the multipliers M or N are, of course, automatically determined.

In the above an attempt has been made to show that the commutation relations between distinct fields [Eqs. (1), (5), or (9)] imply that the Ψ fields are to be associated with fermions and the Φ fields with bosons. Actually it is possible to arrive at these conclusions under assumptions somewhat less restrictive than Eqs. (5) or (9).

The commutation relations (1) imply that

$$[\Psi_{\alpha,i}(\vec{x})\bar{\Psi}_{\beta,i}(\vec{y}) + \bar{\Psi}_{\beta,i}(\vec{y})\Psi_{\alpha,i}(\vec{x})]\Phi(\vec{z}) - [\Psi_{\alpha,i}(\vec{x})\Phi(\vec{z})\bar{\Psi}_{\beta,i}(\vec{y}) + \bar{\Psi}_{\beta,i}(\vec{y})\Phi(\vec{z})\Psi_{\alpha,i}(\vec{x})] = \delta(\vec{x}-\vec{y})(\gamma_4)_{\alpha\beta} M\Phi(\vec{z}) \quad (13a)$$

and

$$[\Phi_{\mu,i}(\vec{x})\Pi_{\nu,i}(\vec{y}) - \Pi_{\nu,i}(\vec{y})\Phi_{\mu,i}(\vec{x})]\Psi(\vec{z}) + [\Phi_{\mu,i}(\vec{x})\Psi(\vec{z})\Pi_{\nu,i}(\vec{y}) - \Pi_{\nu,i}(\vec{y})\Psi(\vec{z})\Phi_{\mu,i}(\vec{x})] = i\tilde{\delta}_{\mu\nu}\delta(\vec{x}-\vec{y})N\Psi(\vec{z}). \tag{13b}$$

The two bracketed expressions on the left-hand side of Eq. (13a) are separately symmetric, and the two bracketed expressions on the left-hand side of Eq. (13b) are separately antisymmetric under exchange of canonical variables.

Disregarding the possibly singular character of quadrilinear products of generalized field operators, Eqs. (13a) and (13b) are, respectively, multiplied by generalized field variables as follows:

$$[\Psi_{\alpha,i}(\vec{x})\bar{\Psi}_{\beta,i}(\vec{y}) + \bar{\Psi}_{\beta,i}(\vec{y})\Psi_{\alpha,i}(\vec{x})]\Phi(\vec{z})\Psi'(\vec{u}) - [\Psi_{\alpha,i}(\vec{x})\Phi(\vec{z})\bar{\Psi}_{\beta,i}(\vec{y}) + \bar{\Psi}_{\beta,i}(\vec{y})\Phi(\vec{z})\Psi_{\alpha,i}(\vec{x})]\Psi'(\vec{u}) = \delta(\vec{x}-\vec{y})(\gamma_4)_{\alpha\beta}M\Phi(\vec{z})\Psi'(\vec{u}) \tag{14a}$$

and

$$[\Phi_{\mu,i}(\vec{x})\Pi_{\nu,i}(\vec{y}) - \Pi_{\nu,i}(\vec{y})\Phi_{\mu,i}(\vec{x})]\Psi(\vec{z})\Phi'(\vec{u}) + [\Phi_{\mu,i}(\vec{x})\Psi(\vec{z})\Pi_{\nu,i}(\vec{y}) - \Pi_{\nu,i}(\vec{y})\Psi(\vec{z})\Phi_{\mu,i}(\vec{x})]\Phi'(\vec{u}) = i\tilde{\delta}_{\mu\nu}\delta(\vec{x}-\vec{y})N\Psi(\vec{z})\Phi'(\vec{u}). \tag{14b}$$

Using the trilinear commutation relations, Eqs. (14) imply that

$$[\Psi_{\alpha,i}(\vec{x})\bar{\Psi}_{\beta,i}(\vec{y}) + \bar{\Psi}_{\beta,i}(\vec{y})\Psi_{\alpha,i}(\vec{x})][\Phi(\vec{z}), \Psi'(\vec{u})]_- = \delta(\vec{x}-\vec{y})(\gamma_4)_{\alpha\beta}M\Phi(\vec{z})\Psi'(\vec{u}), \tag{15a}$$

$$[\Phi_{\mu,i}(\vec{x})\Pi_{\nu,i}(\vec{y}) - \Pi_{\nu,i}(\vec{y})\Phi_{\mu,i}(\vec{x})][\Psi(\vec{z}), \Phi'(\vec{u})]_+ = i\tilde{\delta}_{\mu\nu}\delta(\vec{x}-\vec{y})N\Psi(\vec{z})\Phi'(\vec{u}). \tag{15b}$$

Since M and N are space-time-independent factors one can conclude from Eqs. (15) that

$$\Psi_{\alpha,i}(\vec{x})\bar{\Psi}_{\beta,i}(\vec{y}) + \bar{\Psi}_{\beta,i}(\vec{y})\Psi_{\alpha,i}(\vec{x}) = \delta(\vec{x}-\vec{y})(\gamma_4)_{\alpha\beta}\times(\text{space-time-independent factor}) \tag{16a}$$

and

$$\Phi_{\mu,i}(\vec{x})\Pi_{\nu,i}(\vec{y}) - \Pi_{\nu,i}(\vec{y})\Phi_{\mu,i}(\vec{x}) = i\tilde{\delta}_{\mu\nu}\,\delta(\vec{x}-\vec{y})\times(\text{space-time-independent factor}), \tag{16b}$$

unless Φ and Ψ' commute or Ψ and Φ' anticommute for equal times. However, these two cases were discussed previously.

In an entirely analogous manner one obtains the relations

$$\Psi_i(\vec{x})\Psi_j'(\vec{y}) + \Psi_j'(\vec{y})\Psi_i(\vec{x}) = 0, \quad \Psi_j' \neq \bar{\Psi}_i \tag{17a}$$

and

$$\Phi_k(\vec{x})\Phi_l'(\vec{y}) - \Phi_l'(\vec{y})\Phi_k(\vec{x}) = 0, \quad \Phi_l' \neq \Pi_k. \tag{17b}$$

Equation (17a) follows as above unless Φ and Ψ' commute, while Eq. (17b) follows unless Ψ and Φ' anticommute.

The conclusion therefore is that the commutation relations (1) preclude any esoteric statistics except in the following three special cases:

(1) Only fields of one type (i.e., only Ψ's or only Φ's) are considered in a particular theory.

(2) All Ψ's commute for equal times with all Φ's, in which case all the Φ's of the representation considered are bosons, but a separate argument has to be made to exclude generalized statistics for the Ψ fields. Distinct Φ's commute for equal times but distinct Ψ's do not necessarily have to satisfy bilinear commutation relations with each other.

(3) All Ψ's anticommute for equal times with all Φ's, in which case all the Ψ's are fermions, but a separate argument has to be made to exclude generalized statistics for the Φ fields. Distinct Ψ's anticommute for equal times but distinct Φ's do not necessarily have to satisfy bilinear commutation relations with each other.

These conclusions are reasonable because it is known that representations of the generalized fields that give rise to generalized statistics are compatible with Eqs. (1) when the validity of each one of these commutation relations is restricted to the kinematically related case.

If none of the above three special cases apply, i.e., for a representation for which no Ψ satisfies a bilinear equal-time commutation relation with a Φ, all the generalized fields obey ordinary statistics, distinct Ψ's anticommute and distinct Φ's commute for equal times.

Thus, with the possible exception of the above-mentioned three possibilities, one is justified in referring to the Ψ fields as fermions and to the Φ fields as bosons. Taking the conventional connection between spin and statistics for granted, the Ψ's are half-integral-spin and the Φ's are integral-spin fields, as has been anticipated above.

As is well known,[3-6] locality requirements and right-left symmetry of the positions of the variations obtained in the process of deriving the equations of motion from Lagrangians imply that two fermions entering into a trilinear interaction with a boson field must both commute with the boson field if they anticommute with each other for equal times, and they must both anticommute with the boson field if they commute with each other. Analogous selection rules are obtained for higher-order interactions.[4,17]

The commutation relations (1) permit another possibility for trilinear interactions, or more generally for interactions of even order in fermion fields, provided the interactions are suitably symmetrized. Suitable symmetrization means, for example, that all interactions should be antisymmetrized pairwise in fermion fields, as indicated by Eq. (1a). With such a symmetrization it is apparently possible to satisfy all locality requirements and right-left symmetry, even if distinct fermions neither commute nor anticommute for equal times, provided Eqs. (1) and (5) are satisfied. Indeed, with these assumptions the commutation relations (1) permit not only the "weak" locality requirement[117]

$$[H_1(x), H_2(y)]_- = 0 \quad \text{for} \quad x-y \quad \text{spacelike} \tag{18}$$

to be satisfied, where H_1 and H_2 are any two (suitably symmetrized) Hamiltonian densities of generalized fields, but also the stronger locality statement

$$[\chi(x), H(y)]_- = 0 \quad \text{for} \quad x-y \quad \text{spacelike}, \tag{19}$$

where χ is any generalized field of a representation under consideration.

It is, of course, possible to make Klein transformations[17,118] from one set of fields to another set satisfying "normal"[17] or some other equal-time commutation relations. However, one must be mindful of the fact that any physics contained in the equal-time commutation relations should be Klein-transformation-invariant.[9] The normal case appears to be distinguished from other cases by the complete absence of selection rules or "symmetries" due only to equal-time commutation relations (assuming "reasonable" interactions, i.e., interactions of even order in half-integral-spin fields).

III. REPRESENTATIONS

The representations of the generalized fields of interest in this discussion are of the form[12-15,21]

$$\Psi_i(x) = A_i \times \psi(x), \quad \overline{\Psi}_i(x) = A_i' \times \overline{\psi}(x) \tag{20a}$$

and

$$\Phi_i(x) = B_i \times \phi(x), \quad \Pi_i(x) = B_i' \times \Pi(x), \tag{20b}$$

where the A_i's and B_i's are numerical space-time-independent matrices. These matrices and the bilinear equal-time commutation relations between the various "component fields"[119] ψ and ϕ (as far as they necessarily satisfy bilinear commutation relations) are determined for any particular (irreducible) matrix representation by the commutation relations (1) for suitable choices of the undetermined multipliers M and N. The subscripts of the component fields are assumed to be implied by the matrix with which each component field is associated. Some algebraic aspects of the numerical matrix equations, obtained from Eqs. (1) under the assumption that any two component fields satisfy bilinear equal-time commutation relations, have been studied previously.[12]

If the generalized fields (20) satisfy ordinary statistics, then

$$\Psi_{\alpha,i}(\vec{x})\overline{\Psi}_{\beta,i}(\vec{y}) + \overline{\Psi}_{\beta,i}(\vec{y})\Psi_{\alpha,i}(\vec{x}) = \tfrac{1}{2}\,\delta(\vec{x}-\vec{y})(\gamma_4')_{\alpha\beta}A_i A_i' \times \delta(\Psi_i, \overline{\Psi}_i) \tag{21a}$$

and

$$\Phi_{\mu,i}(\vec{x})\Pi_{\nu,i}(\vec{y}) - \Pi_{\nu,i}(\vec{y})\Phi_{\mu,i}(\vec{x}) = \tfrac{1}{2}\, i\,\tilde{\delta}_{\mu\nu}\,\delta(\vec{x}-\vec{y})B_i B_i' \times \delta(\Phi_i, \Pi_i), \tag{21b}$$

i.e., A_i and A_i' commute and so do B_i and B_i'. The operator Kronecker δ's $\delta(\Psi_i, \overline{\Psi}_i)$ and $\delta(\Phi_i, \Pi_i)$ have been introduced previously[13] in order to ensure that in the quantization scheme presented here both sides of Eqs. (21) have the same equal-time commutation behavior with respect to all generalized fields of a representation being considered.

Representations for which A_i and A_i' (and B_i and B_i') anticommute would give rise to a paradoxical situation where a generalized field and its associated component field would satisfy opposite statistics. Such representations will not be considered in this discussion.

As a trivial example one may consider a representation based on the unit matrix. Another simple representation can be obtained from the Pauli matrices. The simplest representation of possible physical significance can be obtained from the Dirac γ matrices or more conveniently from the three generating matrices[12]

$$C_1 = \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_y \end{pmatrix}, \quad C_2 = \begin{pmatrix} \sigma_y & 0 \\ 0 & \sigma_x \end{pmatrix}, \quad C_3 = \begin{pmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{pmatrix}. \tag{22}$$

There is then the problem of the division of the matrices obtained from the generators (22) into sets (A's and B's). In this connection it is expedient to consider the four sets of linearly independent 4×4 matrices,

$$\begin{aligned} G &= (I,\ C_1C_3C_1C_3,\ C_1C_2,\ C_1C_3C_2C_3), \\ K &= ((C_1C_3 \pm C_3C_1),\ (C_2C_3 \pm C_3C_2)), \\ A &= (C_1,\ C_2,\ C_3C_1C_3,\ C_3C_2C_3), \\ B &= (C_3,\ C_1C_3C_1,\ C_1C_2C_3,\ C_1C_3C_2). \end{aligned} \tag{23}$$

An example of a representation for which no Ψ satisfies a bilinear commutation relation with a Φ is obtained when the matrices in set A are associated with fermions and those in set B with bosons or vice versa. For such a representation all bilinear commutation relations between the component fields can be determined if two component fields ψ and ϕ either commute or anticommute for equal times.

If it is desired to consider a representation which utilizes all the matrices (23), it is obvious that the matrices in sets G and K must be associated with integral-spin and those in sets A and B with half-integral-spin fields or vice versa, if distinct component fields either commute or anticommute for equal times, the reason being that no matrix in set A commutes or anticommutes with a matrix in set B. These examples show that the use of matrices which neither commute nor anticommute renders the division of the matrices into A's and B's more or less unique.

For a possibly physically relevant representation it is desirable to satisfy the locality requirements (18) and (19). This means that, as has been discussed above, Eq. (5) should apply, and that not only for reasons of locality but also because of phenomenological considerations (cf. below) the matrices in sets G and K should be associated with bosons.

If Eq. (5) applies, separate arguments have to be made to determine the statistics of and the bilinear commutation relations between distinct Ψ fields. If it is assumed that the Ψ's satisfy Fermi-Dirac statistics, a component field with matrix in set A commutes or anticommutes for equal times with component fields whose associated matrix is in set B. Vice versa, if component fields with matrix in set A satisfy bilinear commutation relations with component fields whose associated matrix is in set B, the Ψ's, as a consequence of Eqs. (1), satisfy Fermi-Dirac statistics.

A self-consistent set of bilinear equal-time commutation relations for the Ψ's can be obtained if it is assumed that they are fermions and if, furthermore, distinct Ψ's with matrix in the same set, A or B, anticommute for equal times. Though this may not be the only way of determining the bilinear commutation relations for the component fermions, it will be adopted for the representation to be considered.

If the bilinear commutation relations are determined as outlined above, then the symmetry of the component bosons, as expressed in their bilinear commutation relations, is different from the symmetry of the component fermions.[14]

The structure of the Lagrange function determines which generalized fields are related by canonical conjugation. In this discussion the customary relationship between a field variable and its canonical momentum is maintained. This presentation departs from the conventional Lagrangian formalism in that, for the representation discussed, each term in the Lagrangian density is the direct product of a 4×4 matrix and component fields.

It is instructive to consider a Klein transformation of the form

$$\begin{aligned} \Phi_i' &= iC_1C_3C_1C_3\Phi_i\,, \\ \Psi_i' &= \Psi_i\,, \end{aligned} \tag{24}$$

where Φ_i is a field of the form (20b) with matrix coefficient either in set G or K, and Ψ_i is a field of the form (20a) with matrix coefficient either in set A or B. For a representation for which Eq. (5) applies the bilinear commutation relations of any boson component field with all the other component fields are determined. It is understood that these bilinear commutation relations are not affected by the transformation (24). The sets (23) are individually invariant under multiplication by the Hermitian matrix $i\,C_1C_3C_1C_3$ (up to possible phase factors). The transformed fields satisfy the commutation relations

$$[\Phi_i'(\vec{x}), \Phi_j'(\vec{y})]_- = [\Phi_i'(\vec{x}), \Psi_j'(\vec{y})]_+ = 0, \quad \Phi_j' \neq \Pi_i'. \tag{25}$$

The transformed fields satisfy the condition (9) but not generally the condition (10), because a matrix in set A neither commutes nor anticommutes with a matrix in set B. Hence the transformed fields do not constitute a representation of Eqs. (1) if distinct component fermions satisfy bilinear equal-time commutation relations. The conclusion is that for the transformed fields the matrices to be properly considered instead of (23) are, for example,

$$\begin{aligned} &G' = G, \quad K' = K, \quad A' = A, \\ &B' = ((C_3 \pm C_1C_3C_1),\ (C_1C_2C_3 \pm C_1C_3C_2))\,. \end{aligned} \tag{26}$$

As far as bilinear equal-time commutation relations are concerned, the same results are obtained whether one uses the matrices (23) together with the above prescription for determining the bilinear commutation relations, or whether one uses the matrices (26) together with

$$\begin{aligned} &[\Phi_i(\vec{x}), \Phi_j'(\vec{y})]_- = [\Phi_i(\vec{x}), \Psi_j(\vec{y})]_- = [\Psi_i(\vec{x}), \Psi_j'(\vec{y})]_+ = 0, \\ &\Phi_j' \neq \Pi_i\,, \quad \Psi_j' \neq \overline{\Psi}_i\,. \end{aligned} \tag{27}$$

Since it is more convenient to work with matrices any two of which satisfy bilinear commutation relations, the matrices (26) instead of (23) will be used in the subsequent discussion. The bilinear commutation relations between distinct component fields are determined by means of Eq. (27).

It is a simple matter to construct higher-order representations of generalized fields.[12]

IV. FIELDS AND PARTICLES

Only couplings which are at least formally local will be considered in this discussion. For a coupling to be allowed it is required that not only the generalized fields entering the interaction but also the associated component fields, disregarding the matrix structure of the Lagrangian, have equal-time commutation behavior consistent with locality, Eqs. (18) and (19). Deferring the discussion of the metric, it is also required that all Lagrangian densities be self-adjoint. The self-adjointness requirements apply to both factors of the direct-product densities, i.e., to the matrix factors and to the products of the component fields.

Using these selection rules and relying heavily on phenomenological considerations, such as the knowledge of the existence of certain particles and their observed interactions, the representation of the generalized fields under consideration has been used in an attempt to classify the elementary particles.[15] In this connection it is desirable to remove some ambiguities[15] and to clarify some considerations and to motivate the introduction of another selection rule in addition to the selection rules derived from locality and self-adjointness.

In the present attempt to construct a classification scheme, observed processes should in general be consistent with and unobserved processes should in general be inconsistent with locality and/or self-adjointness requirements ("or" applies only to inconsistency).

TABLE I. Particles and fields (scheme A).[a] This table summarizes a particular correspondence between fields and particles. To each particle corresponds a generalized field operator, which consists of the direct product of a numerical (space-time-independent) matrix and a component-field operator. E.g. "$C_2C_1\times\phi$: $\pi^\pm$" means that in scheme A the positive (negative) pion is described by the generalized field $C_2C_1\times\phi$ ($C_2C_1\times\phi\dagger$). The bilinear equal-time commutation relations between ϕ ($\phi\dagger$) and other component fields occurring in the table are obtained from Eq. (27). As usual a free-field generalized operator is assumed to annihilate particles and create antiparticles or vice versa. It is understood that the same numerical matrix is associated with a particle and all its higher-spin recurrences.

Octet		Decimet	
$-C_3C_1C_3\times\overline{\Psi}$: $\overline{\Xi}^0$ $C_2\times\Psi$: Ξ^0 $i(C_1C_2C_3-C_1C_3C_2)\times\overline{\Psi}$: $\overline{Y}$ $(C_3-C_1C_3C_1)\times\Psi$: Y $(C_3+C_1C_3C_1)\times\overline{\Psi}$: $\overline{\Sigma}^-$ $i(C_1C_2C_3+C_1C_3C_2)\times\Psi$: Σ^+ $C_3C_2C_3\times\overline{\Psi}$: $\overline{p}$ $C_1\times\Psi$: p	$-C_3C_2C_3\times\overline{\Psi}$: $\overline{\Xi}^+$ $C_1\times\Psi$: Ξ^- $(C_3-C_1C_3C_1)\times\overline{\Psi}$: $\overline{\Sigma}^+$ $-i(C_1C_2C_3-C_1C_3C_2)\times\Psi$: Σ^- $-i(C_1C_2C_3+C_1C_3C_2)\times\overline{\Psi}$: $\overline{Z}$ $(C_3+C_1C_3C_1)\times\Psi$: Z $C_3C_1C_3\times\overline{\Psi}$: $\overline{n}$ $C_2\times\Psi$: n	$(C_3-C_1C_3C_1)\times\overline{\Psi}$: $\overline{\Omega}_1^+$ $i(C_1C_2C_3-C_1C_3C_2)\times\Psi$: Ω_1^- $C_2\times\overline{\Psi}$: $\overline{\Xi}_1^{+*}$ $-C_3C_1C_3\times\Psi$: Ξ_1^{-*} $-C_3C_1C_3\times\overline{\Psi}$: $\overline{\Xi}_1^{0*}$ $C_2\times\Psi$: Ξ_1^{0*} $(C_3+C_1C_3C_1)\times\overline{\Psi}$: $\overline{\Sigma}_1^{+*}$ $i(C_1C_2C_3+C_1C_3C_2)\times\Psi$: Σ_1^{-*} $i(C_1C_2C_3-C_1C_3C_2)\times\overline{\Psi}$: $\overline{\Sigma}_1^{0*}$ $(C_3-C_1C_3C_1)\times\Psi$: Σ_1^{0*} $(C_3+C_1C_3C_1)\times\overline{\Psi}$: $\overline{\Sigma}_1^{-*}$ $i(C_1C_2C_3+C_1C_3C_2)\times\Psi$: Σ_1^{+*} $C_1\times\overline{\Psi}$: $\overline{\Delta}_1^+$ $C_3C_2C_3\times\Psi$: Δ_1^- $\begin{pmatrix} C_3C_2C_3\times\overline{\Psi}: \overline{\Delta}_1^0 \\ C_1\times\Psi: \Delta_1^0 \end{pmatrix}$ $C_1\times\overline{\Psi}$: $\overline{\Delta}_1^-$ $C_3C_2C_3\times\Psi$: Δ_1^+ $\begin{pmatrix} C_3C_2C_3\times\overline{\Psi}: \overline{\Delta}_1^{--} \\ C_1\times\Psi: \Delta_1^{++} \end{pmatrix}$	$(C_3+C_1C_3C_1)\times\overline{\Psi}$: $\overline{\Omega}_2^+$ $-i(C_1C_2C_3+C_1C_3C_2)\times\Psi$: Ω_2^- $-C_3C_2C_3\times\overline{\Psi}$: $\overline{\Xi}_2^{+*}$ $C_1\times\Psi$: Ξ_2^{-*} $C_1\times\overline{\Psi}$: $\overline{\Xi}_2^{0*}$ $-C_3C_2C_3\times\Psi$: Ξ_2^{0*} $(C_3-C_1C_3C_1)\times\overline{\Psi}$: $\overline{\Sigma}_2^{+*}$ $-i(C_1C_2C_3-C_1C_3C_2)\times\Psi$: Σ_2^{-*} $-i(C_1C_2C_3+C_1C_3C_2)\times\overline{\Psi}$: $\overline{\Sigma}_2^{0*}$ $(C_3+C_1C_3C_1)\times\Psi$: Σ_2^{0*} $(C_3-C_1C_3C_1)\times\overline{\Psi}$: $\overline{\Sigma}_2^{-*}$ $-i(C_1C_2C_3-C_1C_3C_2)\times\Psi$: Σ_2^{+*} $\begin{pmatrix} C_3C_1C_3\times\overline{\Psi}: \overline{\Delta}_2^+ \\ C_2\times\Psi: \Delta_2^- \end{pmatrix}$ $C_2\times\overline{\Psi}$: $\overline{\Delta}_2^0$ $C_3C_1C_3\times\Psi$: Δ_2^0 $\begin{pmatrix} C_3C_1C_3\times\overline{\Psi}: \overline{\Delta}_2^- \\ C_2\times\Psi: \Delta_2^+ \end{pmatrix}$ $C_2\times\overline{\Psi}$: $\overline{\Delta}_2^{--}$ $C_3C_1C_3\times\Psi$: Δ_2^{++}
Mesons	**Intermediate bosons**	**Other nonstrange bosons**	**Leptons**
$i(C_2C_3-C_3C_2)\times\phi^\dagger$: K^- $(C_2C_3+C_3C_2)\times\phi$: K^+ $(C_1C_3+C_3C_1)\times\phi^\dagger$: $\overline{K}^0$ $i(C_1C_3-C_3C_1)\times\phi$: K^0 $C_2C_1\times\phi$: $\pi^\pm$ $I\times\phi$: π^0	$(C_2C_3+C_3C_2)\times\phi^\dagger$: W^- $-i(C_2C_3-C_3C_2)\times\phi$: W^+ $-i(C_1C_3-C_3C_1)\times\phi^\dagger$: $\overline{W}^0$ $(C_1C_3+C_3C_1)\times\phi$: W^0 $C_1C_3C_2C_3\times\phi$: $W'^\pm$ $C_1C_3C_1C_3\times\phi^\dagger$: $\overline{W}'^0$ $C_1C_3C_1C_3\times\phi$: W'^0	$(C_1C_3+C_3C_1)\times\phi$: B_1 $i(C_1C_3-C_3C_1)\times\phi$: B_2 $(C_2C_3+C_3C_2)\times\phi$: B_3 $i(C_2C_3-C_3C_2)\times\phi$: B_4 $I\times\phi$: γ, g	$C_1\times\Psi$: 1_1 $C_1\times\overline{\Psi}$: $\overline{1}_1$ $C_3C_2C_3\times\Psi$: $1_1'$ $C_3C_2C_3\times\overline{\Psi}$: $\overline{1}_1'$ $C_2\times\Psi$: 1_2 $C_2\times\overline{\Psi}$: $\overline{1}_2$ $C_3C_1C_3\times\Psi$: $1_2'$ $C_3C_1C_3\times\overline{\Psi}$: $\overline{1}_2'$

[a] Some spinor and vector indices and normalization factors $1/\sqrt{2}$ have for simplicity been omitted.

Using the representation of the generalized fields under discussion, a set of fields to represent particles can be constructed, essentially by matrix multiplication, using phenomenology as a guide, by starting from a suitable choice for the generalized fields representing the neutral and charged kaons, the proton, neutron, and their antiparticles.[15]

In order to observe regularities, similarities, and differences, if any, it is instructive to compare two schemes A and B (Tables I and II) which are constructed in the same manner, as outlined above, but which differ in some of the basic fields just mentioned, i.e., in the generalized fields representing the neutron and the charged kaons. Positive-strangeness baryon resonances, for whose existence there is inconclusive empirical evidence,[120] have not been included in Tables I and II, but the possibility of including such resonances in the schemes presented here has been discussed previously.[15]

Generalizing a definition of Gell-Mann[121] the Y and Z fields occurring in Tables I and II are defined as

$$Y \equiv a\Lambda + b\Sigma^0 \tag{28a}$$

and

$$Z \equiv a'\Lambda + b'\Sigma^0 , \tag{28b}$$

where a, b, a', and b' are suitably normalized complex numbers. A reasonable correspondence with the Prentki-d'Espagnat Hamiltonian can be obtained if particular values are assigned to these numbers.[15]

In the multiplet construction, as outlined in Tables I and II, it is tacitly assumed that the mesonic interactions of baryons are trilinear, but the exact form of the couplings is not specified. The same statement applies to interactions of higher-spin recurrences of baryons and mesons. While four-fermion couplings cannot in general be ruled out on the basis of the selection rules discussed, an examination of Tables I and II indicates that it appears impossible to devise a set of al-

TABLE II. Particles and fields (scheme B).[a] See the caption of Table I for an explanation of the symbols.

Octet		Decimet	
		$(C_3 - C_1C_3C_1)\times\bar{\Psi}$: $\bar{\Omega}_1^+$	$(C_3 - C_1C_3C_1)\times\bar{\Psi}$: $\bar{\Omega}_2^+$
		$i(C_1C_2C_3 - C_1C_3C_2)\times\Psi$: Ω_1^-	$-i(C_1C_2C_3 - C_1C_3C_2)\times\Psi$: Ω_2^-
	$C_2\times\bar{\Psi}$: $\bar{\Xi}^+$	$C_2\times\bar{\Psi}$: $\bar{\Xi}_1^{+*}$	$C_2\times\bar{\Psi}$: $\bar{\Xi}_2^{+*}$
	$C_3C_1C_3\times\Psi$: Ξ^-	$-C_3C_1C_3\times\Psi$: Ξ_1^{-*}	$C_3C_1C_3\times\Psi$: Ξ_2^{-*}
$C_3C_2C_3\times\bar{\Psi}$: $\bar{\Xi}^0$		$C_3C_2C_3\times\bar{\Psi}$: $\bar{\Xi}_1^{0*}$	$-C_3C_2C_3\times\bar{\Psi}$: $\bar{\Xi}_2^{0*}$
$C_1\times\Psi$: Ξ^0		$C_1\times\Psi$: Ξ_1^{0*}	$C_1\times\Psi$: Ξ_2^{0*}
	$(C_3 + C_1C_3C_1)\times\bar{\Psi}$: $\bar{\Sigma}^+$	$(C_3 + C_1C_3C_1)\times\bar{\Psi}$: $\bar{\Sigma}_1^{+*}$	$(C_3 + C_1C_3C_1)\times\bar{\Psi}$: $\bar{\Sigma}_2^{+*}$
	$-i(C_1C_2C_3 + C_1C_3C_2)\times\Psi$: Σ^-	$i(C_1C_2C_3 + C_1C_3C_2)\times\Psi$: Σ_1^{-*}	$-i(C_1C_2C_3 + C_1C_3C_2)\times\Psi$: Σ_2^{-*}
$(C_3 - C_1C_3C_1)\times\bar{\Psi}$: $\bar{Y}$	$(C_3 - C_1C_3C_1)\times\bar{\Psi}$: $\bar{Z}$	$(C_3 - C_1C_3C_1)\times\bar{\Psi}$: $\bar{\Sigma}_1^{0*}$	$(C_3 - C_1C_3C_1)\times\bar{\Psi}$: $\bar{\Sigma}_2^{0*}$
$i(C_1C_2C_3 - C_1C_3C_2)\times\Psi$: Y	$-i(C_1C_2C_3 - C_1C_3C_2)\times\Psi$: Z	$i(C_1C_2C_3 - C_1C_3C_2)\times\Psi$: Σ_1^{0*}	$-i(C_1C_2C_3 - C_1C_3C_2)\times\Psi$: Σ_2^{0*}
$(C_3 + C_1C_3C_1)\times\bar{\Psi}$: $\bar{\Sigma}^-$		$(C_3 + C_1C_3C_1)\times\bar{\Psi}$: $\bar{\Sigma}_1^{-*}$	$(C_3 + C_1C_3C_1)\times\bar{\Psi}$: $\bar{\Sigma}_2^{-*}$
$i(C_1C_2C_3 + C_1C_3C_2)\times\Psi$: Σ^+		$i(C_1C_2C_3 + C_1C_3C_2)\times\Psi$: Σ_1^{+*}	$-i(C_1C_2C_3 + C_1C_3C_2)\times\Psi$: Σ_2^{+*}
		$C_1\times\bar{\Psi}$: $\bar{\Delta}_1^+$	($C_1\times\bar{\Psi}$: $\bar{\Delta}_2^+$
		$C_3C_2C_3\times\Psi$: Δ_1^-	$-C_3C_2C_3\times\Psi$: Δ_2^-)
	$C_2\times\bar{\Psi}$: $\bar{n}$	($-C_3C_1C_3\times\bar{\Psi}$: $\bar{\Delta}_1^0$	$C_3C_1C_3\times\bar{\Psi}$: $\bar{\Delta}_2^0$
	$C_3C_1C_3\times\Psi$: n	$C_2\times\Psi$: Δ_1^0)	$C_2\times\Psi$: Δ_2^0
$C_3C_2C_3\times\bar{\Psi}$: $\bar{p}$		$C_1\times\bar{\Psi}$: $\bar{\Delta}_1^-$	($C_1\times\bar{\Psi}$: $\bar{\Delta}_2^-$
$C_1\times\Psi$: p		$C_3C_2C_3\times\Psi$: Δ_1^+	$-C_3C_2C_3\times\Psi$: Δ_2^+)
		($-C_3C_1C_3\times\bar{\Psi}$: $\bar{\Delta}_1^{--}$	$C_3C_1C_3\times\bar{\Psi}$: $\bar{\Delta}_2^{--}$
		$C_2\times\Psi$: Δ_1^{++})	$C_2\times\Psi$: Δ_2^{++}

Mesons	Intermediate bosons	Other nonstrange bosons	Leptons
$i(C_1C_3 - C_3C_1)\times\phi^\dagger$: K^-	$-i(C_1C_3 - C_3C_1)\times\phi^\dagger$: W^-	$(C_1C_3 + C_3C_1)\times\phi$: B_1	$C_1\times\Psi$: 1_1
$(C_1C_3 + C_3C_1)\times\phi$: K^+	$(C_1C_3 + C_3C_1)\times\phi$: W^+	$i(C_1C_3 - C_3C_1)\times\phi$: B_2	$C_1\times\bar{\Psi}$: $\bar{1}_1$
			$C_3C_1C_3\times\Psi$: $1_1'$
$(C_1C_3 + C_3C_1)\times\phi^\dagger$: $\bar{K}^0$	$(C_1C_3 + C_3C_1)\times\phi^\dagger$: $\bar{W}^0$	$(C_2C_3 + C_3C_2)\times\phi$: B_3	$C_3C_1C_3\times\bar{\Psi}$: $\bar{1}_1'$
$i(C_1C_3 - C_3C_1)\times\phi$: K^0	$-i(C_1C_3 - C_3C_1)\times\phi$: W^0	$i(C_2C_3 - C_3C_2)\times\phi$: B_4	$C_2\times\Psi$: 1_2
		$I\times\phi$: γ, g	$C_2\times\bar{\Psi}$: $\bar{1}_2$
$C_1C_3C_1C_3\times\phi$: $\pi^\pm$	$iC_1C_3C_1C_3\times\phi$: $W'^\pm$		$C_3C_2C_3\times\Psi$: $1_2'$
$I\times\phi$: π^0	$-I\times\phi^\dagger$: $\bar{W}'^0$		$C_3C_2C_3\times\bar{\Psi}$: $\bar{1}_2'$
	$I\times\phi$: W'^0		

[a] Some spinor and vector indices and normalization factors $1/\sqrt{2}$ have for simplicity been omitted.

lowed four-fermion couplings which could even qualitatively account for the symmetries exhibited by weak interactions.

An inspection of the tables shows that there are in addition to the pions and kaons (and their higher-spin recurrences) four other pairs of bosons which can cause $|\Delta S| \leq 1$ transitions among the baryons by means of trilinear interactions.

For scheme A, these fields are

$$\left.\begin{aligned} W'^0 &= C_1C_3C_1C_3\times\phi \\ \bar{W}'^0 &= C_1C_3C_1C_3\times\phi^\dagger \end{aligned}\right\}, \quad \Delta S = 0 \tag{29a}$$

$$W'^\pm = C_1C_3C_2C_3\times\phi, \quad \Delta S = 0 \tag{29b}$$

$$\left.\begin{aligned} W^0 &= (C_1C_3 + C_3C_1)\times\phi/\sqrt{2} \\ \bar{W}^0 &= -i(C_1C_3 - C_3C_1)\times\phi^\dagger/\sqrt{2} \end{aligned}\right\}, \quad \Delta S = \pm 1 \tag{29c}$$

$$\left.\begin{aligned} W^+ &= -i(C_2C_3 - C_3C_2)\times\phi/\sqrt{2} \\ W^- &= (C_2C_3 + C_3C_2)\times\phi^\dagger/\sqrt{2} \end{aligned}\right\}, \quad \Delta S = \pm 1, \quad \Delta S = \Delta Q\,. \tag{29d}$$

The corresponding fields for scheme B are

$$\left.\begin{aligned} W'^0 &= I\times\phi \\ \bar{W}'^0 &= -I\times\phi^\dagger \end{aligned}\right\}, \quad \Delta S = 0 \tag{30a}$$

$$W'^\pm = iC_1C_3C_1C_3\times\phi, \quad \Delta S = 0 \tag{30b}$$

$$\left.\begin{aligned} W^0 &= -i(C_1C_3 - C_3C_1)\times\phi/\sqrt{2} \\ \bar{W}^0 &= (C_1C_3 + C_3C_1)\times\phi^\dagger/\sqrt{2} \end{aligned}\right\}, \quad \Delta S = \pm 1 \tag{30c}$$

$$\left.\begin{aligned} W^+ &= (C_1C_3 + C_3C_1)\times\phi/\sqrt{2} \\ W^- &= -i(C_1C_3 - C_3C_1)\times\phi^\dagger/\sqrt{2} \end{aligned}\right\}, \quad \Delta S = \pm 1, \quad \Delta S = \Delta Q\,. \tag{30d}$$

In Eqs. (29) and (30) the changes in strangeness ΔS refer to the baryons among which transitions are induced by the W fields.

If the classification schemes discussed are even remotely related to physical reality, it is legitimate to inquire whether there are any particles which on the basis of phenomenology and selection rules could conceivably correspond to the fields (29) and (30). The answer appears to be negative.

Leaving the question of the possible decays of the W bosons open, all first-order (virtual) baryonic transitions induced by the strangeness-changing charged W fields in both schemes A and B [fields (29d) and (30d), respectively],

$$B_1 \to B_2 + W^\pm\,, \tag{31}$$

satisfy the $\Delta S = \Delta Q$ rule as far as the baryons B_1 and B_2 are concerned. At least for strangeness-changing leptonic decays of baryons this rule appears to be well satisfied.

Disregarding such considerations as universality, the spin of the W fields (0 or 1), the relative magnitude of their possible couplings, and the precise form of any trilinear interactions with baryons and trilinear or higher-order couplings to bosons the W fields may enter into, it is tempting to associate the W fields with the hypothetical intermediate bosons, though the question as to why they have not been observed is left open.

In both schemes A and B no baryonic first-order transitions mediated by pions or kaons can be mediated in first order by any of the W fields. For example, the fields in Tables I and II enclosed in large parentheses cannot be reached from the octet by pionic, kaonic, or electromagnetic transitions, but they can be reached from the octet by first-order transitions mediated by suitable W fields. In spite of the difference in interactions mediated by them, there is a close correspondence between the matrix structure of the mesons (pions and kaons) and that of the W fields, as indicated by the tables. Moreover, there is also a correspondence between baryonic transitions respectively mediated by mesons and by W fields.

The observations made above imply that the (virtual) transition

$$n \to p + W'^- \text{ (or } W^-) \tag{32}$$

cannot occur in first order but must, if possible, proceed via suitable intermediate processes.

The requirement that in the context of the selection rules discussed baryons and leptons, and similarly electronic and muonic leptons, cannot be directly (trilinearly) coupled does not specify uniquely the generalized fields selected in each scheme to represent the leptons.[15]

The fields in Tables I and II representing the leptons have been chosen in such a manner that the nonstrange charged W fields [Eqs. (29b) and (30b), respectively] can decay either into electronic or into muonic leptons:

$$W'^\pm \to \begin{cases} l_1(\bar{l}_1) + \bar{l}_1'(l_1') \\ l_2(\bar{l}_2) + \bar{l}_2'(l_2'). \end{cases} \tag{33}$$

Instead of selecting the generalized fields representing the leptons as in Tables I and II, it is *a priori* possible to choose these fields in such a manner that instead of the process (33) the strange charged W fields [Eqs. (29d) and (30d), respectively] can decay either into electronic or into muonic leptons. This can be accomplished for scheme A if the fields representing the leptons are, for example, chosen as follows:

$$
\begin{aligned}
&l_1 = C_1 \times \psi\,, \quad l_1' = C_3 C_2 C_3 \times \psi\,,\\
&\bar{l}_1 = (C_3 + C_1 C_3 C_1) \times \bar{\psi}/\sqrt{2}\,,\\
&\bar{l}_1' = i(C_1 C_2 C_3 + C_1 C_3 C_2) \times \bar{\psi}/\sqrt{2}\,,\\
&l_2 = i(C_1 C_2 C_3 - C_1 C_3 C_2) \times \psi/\sqrt{2}\,,\\
&l_2' = (C_3 - C_1 C_3 C_1) \times \psi/\sqrt{2}\,,\\
&\bar{l}_2 = C_2 \times \bar{\psi}\,, \quad \bar{l}_2' = -C_3 C_1 C_3 \times \bar{\psi}.
\end{aligned}
\qquad (34)
$$

An analogous assignment can be made in scheme B.

Some arguments as to why a choice for the fields representing the leptons as given in Tables I and II appears to be more reasonable than the one offered in Eq. (34) will be outlined below.

Regardless of whether one considers for the leptons the assignment given in Tables I and II or in Eq. (34), the leptons differ in their allowed interactions with the B_i fields (and their higher-spin recurrences). The conjecture has been advanced[15] that this possible difference of leptonic interactions with nonstrange massive bosons could conceivably remove the electron-muon degeneracy. In this connection it is of interest to note that there has been a report raising in a very tentative way the possibility of electron-muon universality breakdown at the ϕ mass.[122]

According to Tables I and II the B_i bosons cannot be coupled trilinearly to baryons. If one of these bosons represents the η resonance, for example, the classification scheme proposed in this discussion would differ from SU(3). Different model-dependent attempts[123] to fit the observed differential and total η-production cross sections by partial-wave analysis, for example, give estimates for the magnitude of the η-nucleon coupling constants varying over a considerable range. In a pure SU(3) scheme this coupling constant depends of course on the D/F ratio. In any event the empirical indications are that the η-baryon coupling constants are considerably smaller than the pion-nucleon coupling constant. There also are indications that the ϕ meson is decoupled from nonstrange hadrons.[124]

As indicated in Tables I and II there are for each possible set of quantum numbers two decimet fields, differing in their interactions and therefore conceivably also in their respective masses. No claim is made that for any decimet of baryons of particular half-integral-spin value (disregarding the complications encountered in the quantization of higher-spin fields) and parity all decimet fields given in Tables I or II should be physically realized or observable. However, in this connection it is of interest to note that in recent years there have been persistent reports in the literature about baryonic (or more generally hadronic) resonances with identical quantum numbers but differing in mass and decay modes.[125]

In Tables I and II there are some bosons for which the field and its charge conjugate are associated with the same anti-Hermitian matrix. For these non-Hermitian fields, particles and antiparticles can be expected to be distinguishable. Yet, since the same matrix is associated with the field and its charge conjugate, any transition mediated by the relevant particle can also be mediated by the antiparticle, unless the field concerned is endowed with a physical characteristic which is conserved and which distinguishes particle from antiparticle. The fields associated with the charged pions in Tables I and II and in schemes previously considered[15] are an example of the case being discussed: In all schemes the matrices respectively associated with the charged-pion field and its charge conjugate are equal, and they always turn out to be anti-Hermitian. In the case of charged pions, particle and antiparticle are, of course, distinguished by a conserved quantity (charge). This distinction is lacking in the case of the W'^0 and $\overline{W}'^0$ field variables [Eqs. (29a) and (30a) and Tables I and II]. In their interactions with baryons and bosons the nonstrange W'^0 and $\overline{W}'^0$ fields may therefore conceivably enter as linear combinations. This is analogous to the previously discussed[14,15] interaction of the K^0-$\overline{K}^0$ system with other bosons. The same consideration applies to the possible coupling of the W^0-$\overline{W}^0$ system [Eqs. (29c) and (30c), respectively] to other bosons. As has been pointed out,[14,15] such linear combinations may turn out to be singular if the relative phase of particle and antiparticle is 0 or π. In addition to the problem of the relative phase there is also the problem of the relative normalization of the fields entering the superposition. In any particular case the linear superposition may consist only of particle and antiparticle or conceivably of other particles as well.[126] These questions and more generally the allowed interactions into which only bosons enter, particularly in connection with weak decays, are presently being studied.

V. DIAGONALIZABLE LAGRANGIANS

As is to be expected for a reasonable classification scheme, Tables I and II are very similar in physical content, although the matrices associated with corresponding fields are not necessarily identical: For each field in scheme A there is a corresponding field in scheme B and vice versa. Moreover, the selection rules derived from locality and self-adjointness of the Lagrangian generally allow or forbid corresponding transitions. There are some exceptions: Disregarding other possible inhibiting factors, a trilinear coupling of charged pions to any of the B_i fields, for example, is allowed in scheme B but forbidden in scheme A by the selection rules under discussion. In order to remove this discrepancy between the two schemes and for other reasons to be mentioned below, it is expedient to introduce another selection rule. A set of interactions is allowed if in addition to locality and self-adjointness requirements the following condition is satisfied: For each field the free-field matrix (i.e., the matrix associated with the free-field Lagrangian density) and all the interaction matrices (i.e., matrices associated with the interaction Lagrangian densities) of interactions into which the field enters should all commute with each other, i.e., they should form a commuting set of matrices. Although this condition is required to be satisfied by each field, the condition is weaker than the requirement that all free-field and interaction matrices of all fields form a commuting set. In the case of 4×4 matrices there are, of course, four nonsingular, linearly independent matrices contained in any maximal commuting set.[127]

Assuming that pions can be directly coupled to baryons, the "diagonalizability rule" introduced precludes trilinear couplings of charged pions to the B_i fields also in scheme B. Morever, the rule precludes the direct decays of B_i bosons into only neutral pions in both schemes.[124] However, a trilinear coupling of charged kaons to some of the B_i fields is consistent with the new rule in both schemes.[124] Generally the diagonalizability condition is useful in sorting out allowed and forbidden couplings of only boson fields. For example, in both schemes this rule would forbid a direct trilinear coupling of only pions to the K^0-$\bar{K}^0$ systems.

Applying the new rule to the electromagnetic field, assuming minimal coupling, and drawing upon the empirical knowledge that some baryons and some leptons carry charge, it becomes obvious that the leptonic assignments given in Tables I and II are consistent with and the leptonic assignment (34) is inconsistent with the diagonalizability condition.

This condition therefore not only substantially reduces, if it does not actually eliminate, the discrepancy in physical content between the two schemes A and B, but it is also of heuristic value in selecting fields to represent particles, and it reduces the *a priori* ambiguity in the construction of allowed interactions. For example, according to the diagonalizability condition the nonstrange charged W fields, $W'^{\pm}$, and not the strange charged W fields, $W^{\pm}$, can, in the present context, be directly trilinearly coupled to leptons.

An inspection of Tables I and II shows that in each scheme all free-field matrices and all interaction matrices of allowed trilinear baryonic couplings form a commuting set. Because of the commutativity of these matrices, the locality condition (18) is applicable not only to the corresponding generalized Hamiltonian densities, but also to the component-field densities obtained from the generalized Hamiltonian density when the free-field and interaction matrices are disregarded. Allowed direct, trilinear couplings of the B_i bosons to kaons or to leptons, for example, if such couplings in fact occur, would give rise to interaction matrices which do not commute with all free-field and interaction matrices of the other fields in the tables. Thus, when the interaction matrices of the massive, nonstrange bosons are also considered, the free-field and interaction matrices of all fields included in each scheme no longer form a commuting set, although the diagonalizability condition is still satisfied for each field.

To summarize: For the representation of the generalized fields considered the locality conditions (18) and (19) are satisfied by all the generalized fields and all their free-field and interaction densities. The conditions (18) and (19) are also satisfied by each component-field variable and the component free-field and interaction Hamiltonian densities into which the component field enters. The diagonalizability condition is therefore related to the locality requirement (18), when this requirement is applied to component-field Hamiltonian densities.

VI. METRIC AND RELATED CONSIDERATIONS

The generalized canonical commutation relations (21) differ from the conventional commutation relations formally only by the factors $\frac{1}{2}A_iA_i'\times\delta(\Psi_i,\bar{\Psi}_i)$ and $\frac{1}{2}B_iB_i'\times\delta(\Phi_i,\Pi_i)$, which, respectively, modify the right-hand sides of Eqs. (21a) and (21b). The formalism permits A_i and A_i' to be different, although they are required to commute. The same is true for B_i and B_i'. It turns out that in order to phenomenologically match generalized fields with particles (Tables I and II) it is necessary to rely on the freedom offered by the formalism and to choose $A_i\neq A_i'$ for each baryon and $B_i\neq B_i'$ for the strange bosons. The question then arises how this procedure affects the metric. Although in the present context the modification of the right-hand sides of the canonical equal-time commutation relations arises naturally, this is of course not the first attempt at such a modification. Since the investigations by Dirac[128] there has been a growing literature on the subject of the indefinite metric, particularly in connection with efforts to eliminate the divergences plaguing quantum field theory. The modification of the right-hand sides of the canonical commutation relations as given in Eqs. (21) is, of course, different from mere multiplication by a factor of −1.

For the correspondence between fields and particles considered in Tables I or II all the factors $A_iA_i' \times \delta(\Psi_i, \overline{\Psi}_i)$ and $B_iB_i' \times \delta(\Phi_i, \Pi_i)$ commute not only with each other but with all the generalized fields associated with the representation being discussed. The factors A_iA_i' and B_iB_i' all turn out to be Hermitian and diagonal, and their squares are equal to the unit matrix. Hence the eigenvalues of all these matrices are ±1.

It seems reasonable to require that for the incoming and outgoing stable particles it should be possible to simultaneously choose corresponding positive eigenvalues of all the matrices associated with their generalized free-field energy densities. Furthermore, for all stable particles the component free-field energy densities should commute with each other for spacelike separations. It turns out that these requirements are satisfied for all known stable particles included in Table I or II. More than that: For all the stable particles Dell'Antonio's[116] theorem, as applied to canonically conjugate field variables, is satisfied not only by the corresponding generalized fields but also by their component fields, i.e., two component fields respectively associated with canonically conjugate generalized field variables of any stable particle have the same bilinear equal-time commutation behavior with respect to any component field associated with another stable particle. The statements just made concerning simultaneous positive eigenvalues of the generalized free-field energy densities of stable particles, the commutativity of their component free-field energy densities for spacelike separations, and the applicability of Dell'Antonio's theorem to component fields associated with generalized fields representing stable particles, are not valid for scheme A if instead of the assignment for the leptons made in Table I the generalized fields representing the leptons were chosen as in Eq. (34), for example.

The diagonalizability condition introduced in Sec. V does not only guarantee that appropriate locality conditions are satisfied, but also ensures that for each field it is possible to simultaneously diagonalize (by means of a unitary transformation, if necessary) its free-field matrix and all the interaction matrices of allowed interactions into which the field enters. Hence for each field these matrices have simultaneous sets of eigenvalues.

The question of the connection between Heisenberg's equations of motion and equal-time commutation relations between fields was already raised by Wigner.[24] This question was in fact the original motivation for the consideration of trilinear commutation relations.[23] In the present context, Heisenberg's equations of motion can be obtained for each component field by forming its commutator with the free-field and interaction component-field Hamiltonians into which the field concerned enters, with the understanding that a simultaneous set of eigenvalues of the free-field and interaction matrices concerned is being considered.

In the previous paper of this series[15] a preliminary application of the S-matrix formalism to generalized fields was outlined. In this connection it was demonstrated that essentially because of considerations related to the metric there is a possibility of canceling out some divergences occurring in quantum field theory, and in the process to obtain relationships involving masses and coupling constants. A more detailed investigation of perturbation theory and S-matrix formalism as applied to generalized fields, in particular in connection with problems related to the metric, will be considered in a future discussion.

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One-Loop Diagrams in Yang-Mills Theory*

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In this paper we discuss the problem of covariance of one-loop diagrams in a canonical formulation of Yang-Mills theory. We show that one-loop diagrams to all orders can be made covariant provided they satisfy certain Ward identities. Verification of these Ward identities to second order in the coupling constant shows that one must introduce renormalization counterterms into the theory.

I. INTRODUCTION

Using the methods of canonical quantization, the present author has recently obtained[1] the noncovariant Feynman rules for the Yang-Mills field in both radiation ($\partial_i \vec{b}_i = 0$) and axial ($\vec{b}_3 = 0$) gauges. It has also been proved[1] that in both cases the tree diagrams to all orders and one-loop diagrams to order g^2 can be described by a covariant set of rules that follow from the above-mentioned noncovariant rules through use of Ward-type identities on tree graphs. Remarkably enough, the covariant scalar loop discovered by many authors[1] (Feynman, DeWitt, Faddeev and Popov, Mandelstam, Fradkin and Tyutin, and others) using other approaches to quantum field theory, was also found to exist within the canonical formulation (at least to lowest order). The aim of the present paper is to show that the one-loop diagrams to all orders can also be made covariant, provided they satisfy certain generalized Ward identities described in the text. The scalar loop is found to exist to all orders. We further investigate whether the Ward identities are really satisfied and, to lowest order (g^2), we find that they are violated by a quadratically divergent term, and one has to renormalize the theory to satisfy the identities. Just such a program for the one-loop case has been recently carried out by t'Hooft[2] and therefore, using t'Hooft's result, we can prove the covariance of one-loop diagrams to all orders.

II. PROOF

We will work in the axial gauge where the propagator is given by[1]

$$D_{\mu\nu}^{ab}(k) = -\frac{i\delta_{ab}}{k^2 - i\epsilon}\left(\delta_{\mu\nu} - \frac{k\cdot\xi(k_\mu\xi_\nu + k_\nu\xi_\mu) - k_\mu k_\nu}{(k\cdot\xi)^2}\right), \tag{1}$$

where $\xi_\mu = \delta_{\mu 3}$ in the frame in which we are working and vertices are given by the following interaction Lagrangian:

$$H_I = -L_I = \tfrac{1}{2}g\vec{g}_{\mu\nu}\cdot(\vec{b}_\mu \times \vec{b}_\nu) + \tfrac{1}{4}g^2(\vec{b}_\mu \times \vec{b}_\nu)\cdot(\vec{b}_\mu \times \vec{b}_\nu), \tag{2}$$

where $\vec{b}_\mu$ denotes the isovector gauge field and

$$\vec{g}_{\mu\nu} = \partial_\mu \vec{b}_\nu - \partial_\nu \vec{b}_\mu. \tag{3}$$

If we can show that for the one-loop diagram all ξ_μ-dependent terms can be dropped, then we have proved our assertion. We will do this in three steps.

Step I. In this part, we will show that the $k_\mu k_\nu / k^2(k\cdot\xi)^2$ term in the propagator can be dropped, thereby making the effective propagator look like (for the one-loop case)

$$D_{\mu\nu}'^{ab}(k) = -\frac{i\delta_{ab}}{k^2 - i\epsilon}\left(\delta_{\mu\nu} - \frac{k_\mu\xi_\nu + k_\nu\xi_\mu}{k\cdot\xi}\right). \tag{4}$$