Coulomb Scattering of a Charged Vector Meson with an Electric Dipole Moment*

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We generalize, by including an electric dipole moment that leads to parity nonconservation, our previous investigation of the scattering of a charged vector meson by a static Yukawa potential, or, in particular, a static Coulomb field. The existence of a solution that satisfies Meixner's corner condition requires the following inequality between the anomalous magnetic moment κ and the electric dipole moment w: $\kappa > 1 + w^2$. In the special case where the electric dipole moment is absent, this condition reduces to the one obtained previously.

I. INTRODUCTION

In connection with the charged intermediate boson[1] for weak interactions, we have recently reinvestigated[2] the Corben-Schwinger problem[3] of the scattering of a charged vector meson with anomalous magnetic moment by an external static Yukawa potential, or in particular a static Coulomb field. It is found that, in this example of a nonrenormalizable theory, a finite solution that satisfies Meixner's corner condition[4] of finite integrated energy in every bounded region can be found if and only if the anomalous magnetic moment κ is larger than 1:

$$\kappa > 1. \tag{1.1}$$

If parity need not be conserved, then the charged intermediate boson may have in addition an electric dipole moment.[5,6] It is the purpose of the present paper to show that the condition (1.1) can be generalized in a completely straightforward manner to

$$\kappa > 1 + w^2, \tag{1.2}$$

where w is the electric dipole moment. Furthermore, all the miracles that happen in the previous case of $w=0$ to make an explicit perturbation expansion possible for small coupling constants also happen in the present more general case. In fact, the underlying fourth-order ordinary differential equations are altered only by the redefinition of one constant.

Since the development follows very closely the investigation[2] of the special case $w=0$, we shall emphasize the differences and be rather sketchy otherwise.

II. FIELD EQUATIONS

With the presence of an electric dipole moment w, the Lagrangian density for the interacting charged vector-meson field and electromagnetic field is

$$\mathcal{L} = -\frac{1}{2}\frac{\partial A_\mu}{\partial x_\nu}\frac{\partial A_\mu}{\partial x_\nu} - \tfrac{1}{2} G^\star_{\mu\nu} G_{\mu\nu} - \phi^\star_\mu \phi_\mu - ie\kappa F_{\mu\nu}\phi^\star_\mu\phi_\nu - \tfrac{1}{4} iw\epsilon_{\mu\nu\sigma\rho} G^\star_{\mu\nu} G_{\sigma\rho}, \tag{2.1}$$

where the notation is the same as that of Ref. 2. As discussed further in the Appendix, this quantity w does not appear explicitly in the energy density $\mathcal{E}_V$ for the vector-meson field:

$$\begin{aligned}\mathcal{E}_V = &\frac{1}{2}\left[\left(\frac{\partial}{\partial x_i}+ieA_i\right)\phi_j^* - \left(\frac{\partial}{\partial x_j}+ieA_j\right)\phi_i^*\right]\left[\left(\frac{\partial}{\partial x_i}-ieA_i\right)\phi_j - \left(\frac{\partial}{\partial x_j}-ieA_j\right)\phi_i\right]\\ &+[(\nabla+ie\vec{A})\phi_0^* + (\partial/\partial t - ieA_0)\vec{\phi}^*]\cdot[(\nabla - ie\vec{A})\phi_0 + (\partial/\partial t + ieA_0)\vec{\phi}]\\ &+\phi_0^*\phi_0 + \vec{\phi}^*\cdot\vec{\phi} + ie\kappa(\nabla A_0 + \partial\vec{A}/\partial t)\cdot(\vec{\phi}^*\phi_0 - \phi_0^*\vec{\phi}) + ie\kappa(\nabla\times\vec{A})\cdot(\vec{\phi}^*\times\vec{\phi}).\end{aligned} \tag{2.2}$$

We are here only interested in the case of a static external electric field described by $\vec{A}=0$ and $A_0 = V$. When the time dependence of ϕ_μ is e^{-iEt}, the field equations from the Lagrangian density (2.1) are explicitly

$$(\nabla^2 - 1)\phi_0 - i\nabla\cdot(E - eV)\vec{\phi} + ie\kappa(\nabla V)\cdot\vec{\phi} = 0, \tag{2.3}$$

and

$$[(E-eV)^2+\nabla^2-1]\vec{\phi}-\nabla(\nabla\cdot\vec{\phi})+i(E-eV)\nabla\phi_0+ie\kappa\phi_0\nabla V-iew\vec{\phi}\times\nabla V=0. \tag{2.4}$$

The Lorentz condition that follows from (2.3) and (2.4) is

$$\nabla\cdot\vec{\phi}-i(E-eV)\phi_0+e(1-\kappa)\nabla V\cdot[(E-eV)\vec{\phi}+i\nabla\phi_0]-ie\kappa\phi_0\nabla^2V+iew(\nabla V)\cdot(\nabla\times\vec{\phi})=0. \tag{2.5}$$

From the Corben-Schwinger case[3] $w=0$, we learn that the result $\kappa>1$ can be obtained without considering the situation where the incident plane wave is transversely polarized. We therefore restrict ourselves to the case of longitudinal polarization. However, when $w\neq 0$, all three components of $\vec{\phi}$ in spherical coordinates are present, i.e., for $w\neq 0$ the simplification $\phi_\phi=0$ no longer holds. The five field equations are therefore explicitly

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial\phi_0}{\partial r}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial\phi_0}{\partial\theta}-\phi_0-\frac{i}{r^2}\frac{\partial}{\partial r}r^2(E-eV)\phi_r-\frac{i}{r\sin\theta}(E-eV)\frac{\partial}{\partial\theta}\sin\theta\,\phi_\theta+ie\kappa\frac{dV}{dr}\phi_r=0, \tag{2.6}$$

$$[(E-eV)^2-1]\phi_r+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial\phi_r}{\partial\theta}-\frac{2}{r^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\,\phi_\theta-\frac{\partial}{\partial r}\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\,\phi_\theta+i(E-eV)\frac{\partial\phi_0}{\partial r}+ie\kappa\phi_0\frac{dV}{dr}=0, \tag{2.7}$$

$$[(E-eV)^2-1]\phi_\theta+\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}\phi_\theta-\frac{1}{r}\frac{\partial^2\phi_r}{\partial r\partial\theta}+\frac{i}{r}(E-eV)\frac{\partial\phi_0}{\partial\theta}-iew\phi_\phi\frac{dV}{dr}=0, \tag{2.8}$$

$$[(E-eV)^2-1]\phi_\phi+\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}\phi_\phi+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial\phi_\phi}{\partial\theta}-\frac{\phi_\phi}{r^2\sin^2\theta}+iew\phi_\theta\frac{dV}{dr}=0, \tag{2.9}$$

and

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2\phi_r+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\,\phi_\theta-i(E-eV)\phi_0+e(1-\kappa)\frac{dV}{dr}\left[(E-eV)\phi_r+i\frac{\partial\phi_0}{\partial r}\right]+ie\kappa\phi_0\frac{1}{r^2}\frac{d}{dr}r^2\frac{dV}{dr}+iew\frac{dV}{dr}\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\,\phi_\phi=0. \tag{2.10}$$

The partial-wave expansion takes the form

$$\phi_0=\sum_{n=0}^{\infty}(2n+1)i^n\Phi_{0n}(r)P_n(\cos\theta),$$

$$\phi_r=\sum_{n=0}^{\infty}(2n+1)i^{n-1}\Phi_{rn}(r)P_n(\cos\theta),$$

$$\phi_\theta=\sum_{n=1}^{\infty}(2n+1)i^{n-1}\Phi_{\theta n}(r)P_n^1(\cos\theta), \tag{2.11}$$

and

$$\phi_\phi=\sum_{n=1}^{\infty}(2n+1)i^n\Phi_{\phi n}(r)P_n^1(\cos\theta).$$

It follows from (2.6)–(2.11) that the radial equations for the partial waves are as follows:

$$\left[r^{-2}\frac{d}{dr}r^2\frac{d}{dr}-n(n+1)r^{-2}-1\right]\Phi_{0n}-\left[r^{-2}\frac{d}{dr}r^2(E-eV)-e(1+\eta)\frac{dV}{dr}\right]\Phi_{rn}+n(n+1)r^{-1}(E-eV)\Phi_{\theta n}=0 \tag{2.12}$$

for $n\geq 0$;

$$-\left[(E-eV)\frac{d}{dr}+e(1+\eta)\frac{dV}{dr}\right]\Phi_{0n}+[(E-eV)^2-1-n(n+1)r^{-2}]\Phi_{rn}+n(n+1)r^{-2}\frac{d}{dr}r\Phi_{\theta n}=0 \tag{2.13}$$

for $n\geq 0$;

$$-r^{-1}(E-eV)\Phi_{0n}-r^{-1}\frac{d}{dr}\Phi_{rn}+\left[r^{-2}\frac{d}{dr}r^2\frac{d}{dr}+(E-eV)^2-1\right]\Phi_{\theta n}+ew\frac{dV}{dr}\Phi_{\phi n}=0 \tag{2.14}$$

for $n\geq 1$;

$$ew\frac{dV}{dr}\Phi_{\theta n}+\left[r^{-2}\frac{d}{dr}r^2\frac{d}{dr}+(E-eV)^2-1-n(n+1)r^{-2}\right]\Phi_{\phi n}=0 \tag{2.15}$$

for $n \geq 0$;

$$\left[(E-eV)+e\eta\frac{dV}{dr}\frac{d}{dr}+e(1+\eta)r^{-2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right)\right]\Phi_{0n}+\left[r^{-2}\frac{d}{dr}r^2-e\eta\frac{dV}{dr}(E-eV)\right]\Phi_{rn}-n(n+1)r^{-1}\Phi_{\theta n}+ewn(n+1)r^{-1}\frac{dV}{dr}\Phi_{\phi n}=0 \tag{2.16}$$

for $n \geq 0$. We have again used the notation $\eta = \kappa - 1$. Note that (2.16) can be obtained from

$$-(E-eV)[(2.12)]-r^{-2}\frac{d}{dr}r^2[(2.13)]+n(n+1)r^{-1}[(2.14)]$$

without using (2.15).

Although this set of radial equations looks much more complicated than the corresponding set for $w=0$, we show in Sec. III that the behaviors for small r are very similar for these two sets of equations.

III. BEHAVIOR FOR SMALL DISTANCES

As before,[2] let $eV = ge^{-\mu r}/r$, $R = r/g$ so that

$$E - eV \sim E' - R^{-1}, \tag{3.1}$$

where $E' = E + \mu g$, and approximate (2.12)–(2.16) by

$$\left[R^{-2}\frac{d}{dR}R^2\frac{d}{dR}-n(n+1)R^{-2}\right]\Phi_{\text{l}n}-\left[R^{-2}\frac{d}{dR}R^2(E'-R^{-1})+(1+\eta)R^{-2}\right]\Phi_{rn}+n(n+1)R^{-1}(E'-R^{-1})\Phi_{\theta n}=0, \tag{3.2}$$

$$-n(n+1)R^{-2}\Phi_{rn}+n(n+1)R^{-2}\frac{d}{dR}R\Phi_{\theta n}=0, \tag{3.3}$$

$$-R^{-1}\frac{d}{dR}\Phi_{rn}+R^{-2}\frac{d}{dR}R^2\frac{d}{dR}\Phi_{\theta n}=0, \tag{3.4}$$

$$-wR^{-2}\Phi_{\theta n}+\left[R^{-2}\frac{d}{dR}R^2\frac{d}{dR}-n(n+1)R^{-2}\right]\Phi_{wn}=0, \tag{3.5}$$

and

$$\left[-\eta R^{-2}\frac{d}{dR}-4\pi(1+\eta)\delta(\vec{R})\right]\Phi_{\text{l}n}+\left[R^{-2}\frac{d}{dR}R^2+\eta R^{-2}(E'-R^{-1})\right]\Phi_{rn}-n(n+1)R^{-1}\Phi_{\theta n}-wn(n+1)R^{-3}\Phi_{wn}=0. \tag{3.6}$$

In (3.2), (3.5), and (3.6), we have used the notation

$$\Phi_{\text{l}n}=\Phi_{0n}/g$$

and (3.7)

$$\Phi_{wn}=\Phi_{\phi n}/g.$$

Let us consider the case $n \geq 1$. It then follows from (3.3) that the relation

$$\Phi_{rn}=\frac{d}{dR}R\Phi_{\theta n} \tag{3.8}$$

still holds even for $w \neq 0$. From (3.2) and (3.6) with $\delta(\vec{R})$ deleted, we get

$$\Phi_{\text{l}n}=\left\{[n(n+1)\eta]^{-1}\frac{d}{dR}R^2\frac{d}{dR}R^2\frac{d}{dR}-(1+\eta)[n(n+1)]^{-1}\frac{d}{dR}-\eta^{-1}\frac{d}{dR}R^2+(E'-R^{-1})\right\}f-w\eta^{-1}\frac{d}{dR}R\Phi_{wn}, \tag{3.9}$$

when $f = R\Phi_{\theta n}$. The substitution of (3.9) back to (3.6) then gives

$$\left\{\frac{d^2}{dR^2}R^2\frac{d}{dR}R^2\frac{d}{dR}-n(n+1)\left[\frac{d^2}{dR^2}R^2+\frac{d}{dR}R^2\frac{d}{dR}-\eta R^{-2}\right]-\eta(1+\eta)\frac{d^2}{dR^2}+n^2(n+1)^2\right\}f=wn(n+1)R\left[R^{-2}\frac{d}{dR}R^2\frac{d}{dR}-n(n+1)R^{-2}\right]\Phi_{wn}. \tag{3.10}$$

It is miraculous that by (3.5) the right-hand side of (3.10) is simply $w^2 n(n+1)R^{-2}\Phi_{\theta n}$. We therefore get the following fourth-order ordinary differential equation for f:

$$\left\{\frac{d^2}{dR^2}R^2\frac{d}{dR}R^2\frac{d}{dR}-n(n+1)\left[\frac{d^2}{dR^2}R^2+\frac{d}{dR}R^2\frac{d}{dR}-(\eta-w^2)R^{-2}\right]-\eta(1+\eta)\frac{d^2}{dR^2}+n^2(n+1)^2\right\}f=0. \quad (3.11)$$

In (3.11), not only E does not appear, but also w merely modifies η.

The method of solution given in Appendix A of Ref. 2 applies step by step here. Alternatively, we may define

$$\eta'=(\eta^2+w^2)/(\eta-w^2) \quad (3.12)$$

and

$$R'=(\eta^2+w^2)^{1/2}R/|\eta-w^2|. \quad (3.13)$$

Thus

$$1+\eta'=\eta(1+\eta)/(\eta-w^2), \quad (3.14)$$

and, in terms of R', (3.11) is

$$\left\{\frac{d^2}{dR'^2}R'^2\frac{d}{dR'}R'^2\frac{d}{dR'}-n(n+1)\left[\frac{d^2}{dR'^2}R'^2+\frac{d}{dR'}R'^2\frac{d}{dR'}-\eta'R'^{-2}\right]-\eta'(1+\eta')\frac{d^2}{dR'^2}+n^2(n+1)^2\right\}f=0. \quad (3.15)$$

In other words, w can be transformed away.

IV. CONDITION ON THE MOMENTS

Since w does not appear in the expression (2.2) for the energy density E_V or in the field equations (2.12) and (2.13), the situation for $n=0$ is completely independent of w. This merely reflects the fact that an electric dipole moment has no effect on a spherically symmetrical wave. Thus[2] η must be either positive or less than 1. Next, we apply the previous considerations for $n\geq 1$ step by step. Instead of $\eta>0$ for the special case $w=0$, we get more generally

$$\eta'>0. \quad (4.1)$$

By (3.12), (4.1) means that

$$\eta>w^2, \quad (4.2)$$

which is identical to (1.2). This is the desired result.

To solve the differential equation (3.15) when (4.1) and (4.2) are satisfied, define[2]

$$\nu=\tfrac{1}{2}[1+4n(n+1)(1+\eta')^{-1}]^{1/2} \quad (4.3)$$

and

$$x=2^{1/2}[\eta'(1+\eta')]^{1/4}R'^{-1/2}. \quad (4.4)$$

By (3.12) and (3.14),

$$x=2^{1/2}[\eta(1+\eta)]^{1/4}R^{-1/2} \quad (4.5)$$

is actually independent of w; furthermore

$$\nu=\tfrac{1}{2}[1+4n(n+1)\eta(1+\eta)/(\eta-w^2)]^{1/2}. \quad (4.6)$$

Therefore, this redefinition of ν is the only change necessary to generalize to $w\neq 0$ all the results given in the appendixes of Ref. 2.

V. DISCUSSIONS

We plot in Fig. 1 the region in the κ-w plane where the condition (1.2) is satisfied. Note first that the admissible region is two-dimensional, not one-dimensional. Therefore, it is not possible, even in the present exactly solvable model, to obtain our result by the procedure of studying certain low-order terms in the perturbation series and requiring some nonrenormalizable quantities to be multiplied by a coefficient which is zero. The information about restrictions on coupling constants for nonrenormalizable theories is not contained in the first few terms of the perturbation series.

For the sake of comparison, we also show in Fig. 1 the circle $w^2+\kappa^2+3\kappa+1=0$, which is the condition obtained by Wellner.[7] This circle happens not to intersect our admissible region.

By this example, we have shown that in general there are conditions between the various coupling constants in a nonrenormalizable theory. The next question is: Does this condition $\kappa>1+w^2$ have any quantitative validity for charged intermediate bosons, if they do occur in nature? We believe that the answer is no. In order to get an explicit answer, we have here studied a problem in potential scattering. This implies, among other things, that every photon emitted by the vector meson is absorbed by the static charge, and vice versa. In other words, all radiative effects in which the vector meson emits and reabsorbs photons are neglected. Since these neglected terms are in no sense smaller, this approximation is not justified for intermediate bosons in nature. It is thus highly unlikely that our inequality (1.2) is quantitatively correct in such cases. Rather, we have given evidences by example that such conditions should exist.

The admissible region in Fig. 1 does not intersect the line $\kappa=0$. Thus, if (1.2) is correct, a vector meson without strong interactions cannot have electromagnetic interactions through the charge only, or through the charge and the electric dipole moment only. However, there does not seem to be any reason why this qualitative feature of the present result should hold. Let us therefore consider the possibility that the admissible region does intersect the line $\kappa=0$. For example, suppose that the condition (1.2) is instead $\kappa>1-w^2$. Then this condition can in particular be satisfied by $\kappa=0$ and $|w|>1$. With this possibility in mind, we conclude that, if *the charged intermediate boson* does not have strong interactions, then it is *expected to have either a sizable anomalous magnetic moment or a sizable electric dipole moment*. If $\kappa=0$,[8] this conclusion is very similar to the one previously reached[6] through the ξ-limiting formalism of Lee and Yang.[9] So far as we know, this remains the only proposal to the question *why CP* is violated.

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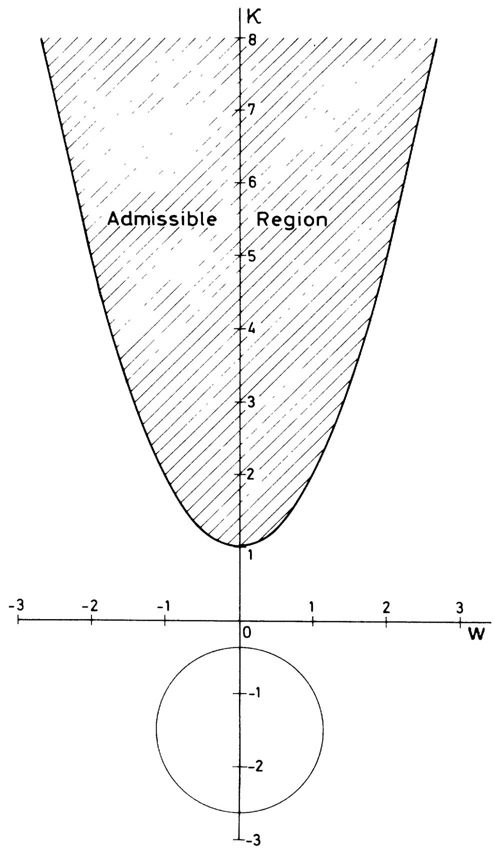

FIG. 1. Admissible values of the coupling constants κ and w.

APPENDIX

We discuss here briefly the derivation of (2.2) for the energy density $\mathcal{E}_V$. From the Lagrangian density (2.1), we get the Hamiltonian density

$$\mathcal{H}=\mathcal{H}_{em}+\mathcal{H}_1\,, \tag{A1}$$

where

$$\mathcal{H}_{em}=-\tfrac{1}{2}(\partial A_0/\partial t)^2+\tfrac{1}{2}(\partial\cdot\vec{A}/\partial t)^2-\tfrac{1}{2}(\nabla A_0)^2+\tfrac{1}{2}(\vec{\nabla}A)^2 \tag{A2}$$

is the electromagnetic part of the Hamiltonian density and

$$\begin{aligned}\mathcal{H}_1=&[(\nabla+ie\vec{A})\phi_0^*+(\partial/\partial t-ieA_0)\vec{\phi}^*+w(\nabla\times\vec{\phi}^*+ie\vec{A}\times\vec{\phi}^*)]\cdot(\partial\vec{\phi}/\partial t)\\&+(\partial\vec{\phi}^*/\partial t)\cdot[(\nabla-ie\vec{A})\phi_0+(\partial/\partial t+ieA_0)\vec{\phi}+w(\nabla\times\vec{\phi}-ie\vec{A}\times\vec{\phi})]+ie\kappa(\partial\vec{A}/\partial t)\cdot(\vec{\phi}^*\phi_0-\phi_0^*\vec{\phi})-\mathcal{L}_1,\end{aligned} \tag{A3}$$

with

$$\begin{aligned}\mathcal{L}_1=&-\tfrac{1}{2}(1+w^2)[(\partial/\partial x_i+ieA_i)\phi_j^*-(\partial/\partial x_j+ieA_j)\phi_i^*][(\partial/\partial x_i-ieA_i)\phi_j-(\partial/\partial x_j-ieA_j)\phi_i]\\&+[(\nabla+ie\vec{A})\phi_0^*+(\partial/\partial t-ieA_0)\vec{\phi}^*+w(\nabla\times\vec{\phi}^*+ie\vec{A}\times\vec{\phi}^*)]\cdot[(\nabla-ie\vec{A})\phi_0+(\partial/\partial t+ieA_0)\vec{\phi}+w(\nabla\times\vec{\phi}-ie\vec{A}\times\vec{\phi})]\\&+\phi_0^*\phi_0-\vec{\phi}^*\cdot\vec{\phi}+ie\kappa(\nabla A_0+\partial\vec{A}/\partial t)\cdot(\vec{\phi}^*\phi_0-\phi_0^*\vec{\phi})-ie\kappa(\nabla\times\vec{A})\cdot(\vec{\phi}^*\times\vec{\phi}).\end{aligned} \tag{A4}$$

Since

$$\begin{aligned}\Box A_0=&ie\kappa\nabla\cdot(\vec{\phi}^*\phi_0-\phi_0^*\vec{\phi})-ie\{\vec{\phi}^*\cdot[(\nabla-ie\vec{A})\phi_0+(\partial/\partial t+ieA_0)\vec{\phi}+w(\nabla\times\vec{\phi}-ie\vec{A}\times\vec{\phi})]\\&-[(\nabla+ie\vec{A})\phi_0^*+(\partial/\partial t-ieA_0)\vec{\phi}^*+w(\nabla\times\vec{\phi}^*+ie\vec{A}\times\vec{\phi}^*)]\cdot\vec{\phi}\},\end{aligned} \tag{A5}$$

the Hamiltonian density $\mathcal{H}$ can be expressed alternatively as

$$\mathcal{H}=\mathcal{E}_{\mathrm{em}}+\mathcal{H}_2-\nabla\cdot\{A_0(\nabla A_0+\vec{A})+[(\nabla\cdot\vec{A})-(\vec{A}\cdot\nabla)]\vec{A}+ie\kappa A_0(\vec{\phi}^*\phi_0-\phi_0^*\vec{\phi})\}, \tag{A6}$$

where $\mathcal{E}_{\mathrm{em}}$ is the usual electromagnetic energy density, and

$$\begin{aligned}\mathcal{H}_2=&\ \tfrac{1}{2}(1+w^2)[(\partial/\partial x_i+ieA_i)\phi_j^*-(\partial/\partial x_j+ieA_j)\phi_i^*][(\partial/\partial x_i-ieA_i)\phi_j-(\partial/\partial x_j-ieA_j)\phi_i]\\ &+[(\partial/\partial t-ieA_0)\vec{\phi}^*]\cdot[(\partial/\partial t+ieA_0)\vec{\phi}]\\ &-[(\nabla+ie\vec{A})\phi_0^*+w(\nabla\times\vec{\phi}^*+ie\vec{A}\times\vec{\phi}^*)]\cdot[(\nabla-ie\vec{A})\phi_0+w(\nabla\times\vec{\phi}-ie\vec{A}\times\vec{\phi})]\\ &-\phi_0^*\phi_0+\vec{\phi}^*\cdot\vec{\phi}+ie\kappa(\nabla\times\vec{A})\cdot(\vec{\phi}^*\times\vec{\phi}).\end{aligned} \tag{A7}$$

Finally, the $\mathcal{E}_V$ of (2.2) differs from this $\mathcal{H}_2$ of (A7) by a divergence

$$\mathcal{E}_V=\mathcal{H}_2+2\nabla\cdot\mathrm{Re}\{\phi_0^*[(\nabla-ie\vec{A})\phi_0+(\partial/\partial t+ieA_0)\vec{\phi}+w(\nabla\times\vec{\phi}-ie\vec{A}\times\vec{\phi})]\}. \tag{A8}$$

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[1]T. D. Lee and C. N. Yang, Phys. Rev. 119, 1410 (1960).

[2]H. Cheng and T. T. Wu, this issue, Phys. Rev. D 5, 2978 (1972).

[3]H. C. Corben and J. Schwinger, Phys. Rev. 58, 953 (1940).

[4]J. Meixner, Ann. Physik 6, 1 (1949).

[5]F. Salzman and G. Salzman, Phys. Letters 15, 91 (1965).

[6]T. T. Wu, Phys. Rev. Letters 15, 718 (1965).

[7]M. Wellner, Phys. Rev. 141, 1497 (1966).

[8]When $\kappa\neq 0$, the degree of divergence for any given order is different for the scattering by a static Yukawa potential and for the quantum field theory of a charged vector particle.

[9]T. D. Lee and C. N. Yang, Phys. Rev. 128, 885 (1962).

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Unitary Models of Multiparticle Amplitudes*

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A class of models of multiparticle scattering and production amplitudes is constructed for which the S matrix is exactly unitary at high energies. Two specific models are studied in detail. One leads to a constant total cross section, the other to a logarithmically increasing one. Particle production in inclusive and exclusive experiments is considered for both models.

I. INTRODUCTION

One of the central problems in strong-interaction dynamics is to construct a realistic model of multiparticle scattering and production amplitudes. Certainly such a model must satisfy the constraints of multiparticle unitarity. Ideally one would like to construct models which automatically satisfy unitarity independent of any other physical input due to their structure.[1] As a first step in this direction we present a class of models for which the S-matrix elements satisfy all the multiparticle unitarity relations at high energies. To our knowledge this is the first example of a solvable multiparticle model with a unitary S matrix. Although in some respects the present model is quite crude we believe that the ideas discussed here can be used to construct more sophisticated and hopefully more realistic models.

In Sec. II we combined ideas from the multiperipheral and eikonal models to write down a general S-matrix element in our model. Each matrix