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[9]We take $c=1$ unless specified otherwise; the metric signature is $(+---)$, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$; $F^*_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ with $\epsilon^{0123}=1$; if a and b are 4-vectors, then $(a\wedge b)_{\alpha\beta}=a_\alpha b_\beta-a_\beta b_\alpha$. When tensor indices are omitted in a product, it means that they are contracted according to the rules of tensor multiplication.

[10]The gradient operator ∇ only acts on $F_{\mu\nu}$.

[11]The same symbol will denote the frame (the set of four-vectors) and the square matrix formed by their components (a Lorentz transformation).

[12]Another fact in favor of Eq. (6) is that it is the BMT equation as it appears before any specific expression for $\ddot{x}$ is inserted. For the latter, we use Eq. (3) derived from the Lagrangian (2) or (taking $I\rightarrow 0$) the Lagrangian (5).

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Nonmeasurability of the Quantum Numbers of a Black Hole*

Claudio Teitelboim†
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540
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We consider neutral-meson (scalar and vector), electromagnetic, and neutrino quantum fields interacting with classical sources on a Schwarzschild background. The meson fields decouple from the source when it reaches $r=2m$. This means that the coupling constant is effectively set equal to zero at the horizon. Such a phenomenon does not happen for an electromagnetic field. The sharp difference between vector mesons of any mass and photons suggests that charged black holes can exist only if the photon mass is exactly zero. Neutrinos do not decouple from the source either, but the remaining coupling is given by an unobservable phase factor. These results provide further evidence in favor of Wheeler's conjecture that the only measurable quantum numbers of a black hole are mass, charge, and angular momentum and that, consequently, the laws of conservation of baryon and lepton number are "transcended" in black-hole physics.

I. INTRODUCTION

Active work is in progress towards the observation of the first black hole.[1] Today more than ever one should study the startling properties that standard physical theory predicts for such an object. Among them we shall concentrate in this paper on the presumed "ideal perfection" of this final state of gravitational collapse. All theoretical evidence favors the conjecture that the most general final configuration for a collapsed object is a Kerr-Newman black hole. If this conjecture is true then it follows that the only measurable quantum numbers of a black hole are mass, charge, and angular momentum – these three quantities being the only independently adjustable parameters appearing in the Kerr-Newman metric. Any other particularity that the collapsing matter had fades away. This state of affairs has been summarized by John A. Wheeler in the phrase "A black hole has no hair" and is represented pictorially in Fig. 1.[2]

Now baryon number and lepton number, key quantities of particle physics, are not in the list of allowed properties of a black hole and should therefore be unobservable. Consider, to fix the ideas, baryon number. In flat space the baryonic content of a collection of matter can be (ideally) measured by means of strong interactions. On account of meson exchange there is a Yukawa-like force between two baryons that can be used to determine baryon number in much the same way as the Coulomb force can be used to measure electric charge. One can also imagine more complicated experiments, as for example meson-nucleon scattering, and infer the baryonic content of the scatterer from the cross section. The essential point is the existence of a coupling between baryons and a meson field. Repeat now the experiment not in flat space but on the geometry of a black-hole exterior. Consider a body approaching the horizon at the boundary of the black hole. When such a body reaches the horizon it becomes an integral part of the black hole; it can never escape again, and consequently its baryon number should become unobservable. It should be clear from the above discussion that this will be the case if the baryon decouples from the meson field at the horizon, namely, if the coupling constant is effectively set equal to zero by the gravitational field. As we shall see below, this is exactly the sort of a phenomenon that happens at $r = 2m$ in a Schwarzschild black hole. (We restrict ourselves in this article to a Schwarzschild background. The ideas, however, are applicable to a Kerr-Newman geometry, though the formalism is likely to become more involved.)[3]

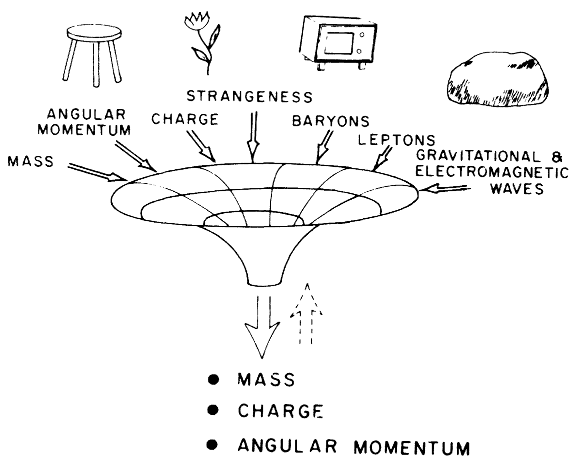

FIG. 1. Wheeler's characterization of a black hole: "Fire in neutrons, protons, antiprotons, particles and radiations of whatever kind one chooses. In the final object not one of these particularities remains. Make meticulous count of the leptons that one drops in. Check in the baryons with equal care. Compare the resulting black hole with another hole built from a very different number of baryons and leptons. Only require that the two objects have the same mass, charge and angular momentum. Then the one black hole can be distinguished from the other by no known means whatsoever. No measurable meaning of any kind do we know how to give to the baryon number and lepton number of a black hole of unknown provenance. Gravitational collapse deprives baryon number and lepton number of all significance . . . ." (Adapted from J. A. Wheeler, Ref. 2, with the kind permission of Professor Wheeler.)

Only quantities given by dynamically conserved surface integrals can be unambiguously associated to a black hole. If there were a (not yet observed) field associated to matter which would yield, say, baryon number as a dynamically conserved surface integral (for example, a field satisfying Maxwell's equations with the baryon number current as a source) then baryon number would be a measurable quantum number for a black hole.

In Sec. II we give a precise meaning to the notion of decoupling and we discuss also the important issue of boundary conditions. In Sec. III the formalism is applied to scalar and vector mesons and it is shown how the baryon number of a body that "falls quasistatically" into a Schwarzschild black hole becomes unobservable. Photons are considered in Sec. IV. They do not decouple from the source at $r = 2m$ as expected since electric charge is one of the measurable quantum numbers of a black hole. The sharp difference between the behavior of vector mesons of arbitrarily small but finite mass and photons is due to the presence of gauge invariance in the latter case. The electromagnetic potential A^μ is unobservable and consequently the restrictions of regularity at the horizon are less severe in that case. This suggests that charged black holes can be formed only if the mass of the photon is exactly zero.

Finally, neutrinos coupled by means of weak interactions to a lepton (electron or muon) source are treated in Sec. V. We find that they do not decouple totally from the source at $r = 2m$, but the remaining coupling is given by an unobservable phase factor.

II. FORMALISM

A. Decoupling

In trying to build a mathematical formalism for the concept of decoupling the first point that comes up is that the detachment of, say, the baryons from the meson field should happen *at* a given place, namely at $r = 2m$. This shows the need for a degree of spatial localizability in the formalism; not being too ambitious at the beginning, we restrict ourselves to quantum fields interacting with classical sources.[4] The formalism becomes then practically classical throughout – one only has to carry an incoming field ψ^{in} in the equations – but still one has the machinery of quantum field theory behind.

The notion of decoupling can be precisely formulated as follows: If $\psi(x)$ is the field operator in question, one can write

$$\psi(x) = \psi^{\text{in}}(x) + (\text{coupling constant})\, I(x)\,, \qquad (2.1)$$

where $I(x)$ is an integral containing the free-field retarded propagator ("free" here means free propagation on the given background), the world line of the source, and sometimes the ψ field itself [in this case (2.1) is not a solution of the field equations, but an integral equation]. If $I(x)$ vanishes when the source approaches the horizon, it is equivalent to setting, in that limit, the coupling constant equal to zero: The field $\psi(x)$ decouples from the source and no experiment whatsoever can detect such a coupling. In particular, the S matrix is unity[5] (the scattering by the gravitational field alone being described by the field ψ^{in} and considered as free propagation). On the other hand if the coupling integral does not vanish, one should still investigate the question of whether what is left contributes to processes which are observable from outside of the black hole.

What we will do then is to study the asymptotic behavior of the coupling integral when the source is very near to the horizon. This behavior will tell us not only whether decoupling happens or not, but will also provide precise formulas for the rate of extinction. In spite of the fact that it would be more natural to take for the world line of the source a geodesic corresponding to free fall, we will consider instead, for reasons of simplicity, a "quasistatic fall," namely a sequence of static problems. That is, we will evaluate the coupling integral for a source at rest at radius r' and we will examine the limit $r' \to 2m$. The formalism as it stands is, however, applicable to general dynamical problems.

B. Boundary Conditions

The coupling integral contains the free-field retarded propagator which is invariantly defined as the Green's function of the corresponding wave equation that vanishes outside the future light cone of the source point. In flat space there exists an equivalent characterization in terms of the behavior of the Fourier transform in time $\Delta_R(\vec{x},\vec{x}\,';\omega)$ at large distances from the source, namely

$$\Delta_R(\vec{x},\vec{x}\,';\omega) \underset{\substack{r\to\infty\\ \omega>0}}{\sim} \begin{cases} \exp[i(\omega^2-\mu^2)^{1/2}r]\,, & \omega^2-\mu^2>0 \qquad (2.2\text{a})\\ \exp[-(\mu^2-\omega^2)^{1/2}r]\,, & \omega^2-\mu^2<0 \qquad (2.2\text{b})\end{cases}$$

(μ is the mass of the field) and

$$\Delta_R(\vec{x},\vec{x}\,';-\omega) = \Delta_R^*(\vec{x},\vec{x}\,';\omega)\,. \qquad (2.2\text{c})$$

Equation (2.2a) says that for frequencies such that wave propagation exists the waves are escaping towards infinity. For the other frequencies there is exponential damping instead. It should be stressed that the condition of exponential damping at infinity for $\omega^2<\mu^2$ is common to *all* the Green's functions (retarded, advanced, Feynman, etc.). Furthermore, condition (2.2b) follows directly from only the assumption that the Green's function has an integral representation – it is not imposed separately – and is therefore a condition inherent to the existence of the Green's function more than a boundary condition. The difference between the various boundary conditions is introduced only through (2.2a), which depends on the particular way of handling the poles in the integral representation of the propagator.[6]

In order to obtain a characterization analogous to (2.2) when working on a Schwarzschild background, one must go from the usual Schwarzschild coordinate r to the Wheeler "tortoise coordinate"[7] r^* by means of the transformation

$$r^* = r + 2m \ln\left(\frac{r}{2m} - 1\right), \qquad (2.3)$$

which pushes off the horizon $r = 2m$ to $r^* = -\infty$ and is such that the metric takes the form

$$ds^2 = -\left(1-\frac{2m}{r}\right)(dt^2 - dr^{*2}) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2). \qquad (2.4)$$

In terms of r^* there is a great symmetry between spatial infinity and the event horizon (a symmetry that has made John Wheeler call a black hole a "trapped infinity"). Both of them are the boundaries of the region of interest (the exterior Schwarzschild geometry). Both of them are at an infinite coordinate distance and, what is more important, the task of finding the retarded propagator reduces, after the angular dependence has been separated out, to solving an equation of the form

TABLE I. Effective potentials for various fields. The letter μ denotes the inverse Compton wavelength of the corresponding field quanta. The Schwarzschild radial coordinate r is to be understood as a function of r^* defined implicitly by $r^* = r + 2m\ln(r/2m - 1)$. All potentials listed below are strictly positive in the interval $-\infty < r^* < +\infty$.

Field	Effective potential	Value μ_-^2 of potential at the horizon ($r^* = -\infty$)	Value μ_+^2 of the potential at infinity ($r^* = +\infty$)
Scalar	$V_l(r^*) = \left(1 - \frac{2m}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2m}{r^3} + \mu^2\right)$, $l = 0, 1, 2, \ldots$	0	μ^2
Vector	$V_l(r^*) = \left(1 - \frac{2m}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2m}{r^3} + \mu^2\right) + \frac{m^2}{r^4}$, $l = 0, 1, 2, \ldots$	$(4m)^{-2}$	μ^2
Spinor (massless)	$V_k(r^*) = \left(1 - \frac{2m}{r}\right)^{1/2}\left[\frac{k^2}{r^2}\left(1 - \frac{2m}{r}\right)^{1/2} + \frac{k}{r^2}\left(1 - \frac{2m}{r}\right) + \frac{m}{r^3}\right]$, $k = \pm 1, \pm 2, \ldots$	0	0

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^{*2}} - V(r^*)\right]\Delta_R(r^*, r^{*\prime}; t - t') = \delta(r^* - r^{*\prime})\delta(t - t'), \tag{2.5}$$

the asymptotic forms of which at $r^* = \pm\infty$ are very similar.

In effect, the potential $V(r^*)$ appearing in (2.5) has the following characteristic behavior: (a) It is strictly positive for finite r^*. (b) For $r^* \to \pm\infty$ it tends to constant values $\mu_\pm^2$ (which are in general different from each other). The asymptotic value μ_+^2 at $r^* = +\infty$ is the square of the mass of the field in consideration, whereas μ_-^2 is a constant that depends, in general, on the mass of the black hole and on the particular wave equation being considered. As a consequence the potential $V(r^*)$ (which is shown for the various cases considered in this paper in Table I) does not admit permanently bound states for any value of the mass m of the black hole.

The behaviors of the propagator at $r^* = \pm\infty$ are therefore very similar: For some frequencies there will be wave propagation, for others exponential damping. The retarded propagator is then characterized by the following asymptotic behavior:

$$\Delta_R(\vec{x}, \vec{x}'; \omega) \underset{\substack{r^* \to \pm\infty \\ \omega > 0}}{\sim} \begin{cases} \exp[i(\omega^2 - \mu_\pm^2)^{1/2}|r^*|], & \omega^2 - \mu_\pm^2 > 0 \quad (2.6\text{a}) \\ \exp[-(\mu_\pm^2 - \omega^2)^{1/2}|r^*|], & \omega^2 - \mu_\pm^2 < 0 \quad (2.6\text{b}) \end{cases}$$

and

$$\Delta_R(\vec{x}, \vec{x}'; -\omega) = \Delta_R^*(\vec{x}, \vec{x}'; \omega), \tag{2.6c}$$

in total analogy with (2.2). It should be stressed again that the choice of boundary conditions resides in (2.6a). The requirement of exponential damping (2.6b) is common to all boundary conditions and is necessary to guarantee the existence of the Green's function as such. This condition ensures that invariants constructed from the field in consideration are bounded at the horizon. For the case of the electromagnetic field the condition (2.6b) has to be relaxed, as explained in Sec. IV.[8]

III. MESONS: BARYON NUMBER

A. Scalar Mesons[9]

We apply here the above formalism to the case of a scalar-meson field $\phi(x)$ interacting with a baryon source of world line $x'(\tau)$ according to the generally covariant interaction

$$(\Box^2 - \mu^2)\phi(x) = \lambda\int_{-\infty}^{+\infty} d\tau\, \delta^{(4)}(x - x'(\tau)). \tag{3.1}$$

Equation (3.1) has the solution

$$\phi(x) = \phi^{\rm in}(x) + \hat{\phi}(x), \tag{3.2a}$$

with $\hat{\phi}$ given by

$$\hat{\phi}(x) = \lambda \int_{-\infty}^{+\infty} d\tau\, \Delta_R(x, x'(\tau)), \tag{3.2b}$$

which reduces to

$$\hat{\phi}(x) = \lambda \left(1 - \frac{2m}{r'}\right)^{1/2} \Delta_R(\vec{x}, \vec{x}'; \omega = 0) \tag{3.3}$$

for a source at rest at the point $\vec{x}'$.

Now the retarded propagator appearing in (3.2b) is a solution of[10]

$$(\Box^2 - \mu^2)\Delta_R(x, x') \equiv \left[(-g)^{-1/2} \frac{\partial}{\partial x^\alpha}\left((-g)^{1/2} g^{\alpha\beta} \frac{\partial}{\partial x^\beta}\right) - \mu^2\right] \Delta_R(x, x') = \delta^{(4)}(x, x'), \tag{3.4}$$

subject to the conditions (2.6). On account of the spherical symmetry of the background each Fourier component of the propagator can be resolved in spherical harmonics as

$$\Delta_R(\vec{x}, \vec{x}'; \omega) = \sum_{l=0}^{\infty} (rr')^{-1} \Delta_R^{(l)}(r^*, r^{*\prime}; \omega) \sum_{m=-l}^{+l} Y_{lm}(\theta, \varphi) Y_{lm}^*(\theta', \varphi'). \tag{3.5}$$

Equation (3.4) translates then into the following equation for $\Delta_R^{(l)}(r^*, r^{*\prime}; \omega)$:

$$\left[\frac{\partial^2}{\partial r^{*2}} + \omega^2 - V_l(r^*)\right] \Delta_R^{(l)}(r^*, r^{*\prime}; \omega) = \delta(r^* - r^{*\prime}), \tag{3.6}$$

with

$$V_l(r^*) = \left(1 - \frac{2m}{r}\right)\left(\frac{2m}{r^3} + \frac{l(l+1)}{r^2} + \mu^2\right). \tag{3.7}$$

Equations (3.6) and (3.7) are in agreement with the general discussion in Sec. II. In particular, the constants $\mu_\pm$ are given in this case by $\mu_+ = \mu$ and $\mu_- = 0$.

The radial Green's function $\Delta_R^{(l)}$ can be written as

$$\Delta_R^{(l)}(r^*, r^{*\prime}; \omega) = W^{-1} \begin{cases} f_2(r^*) f_1(r^{*\prime}), & r^* < r^{*\prime} \\ f_1(r^*) f_2(r^{*\prime}), & r^* > r^{*\prime}. \end{cases} \tag{3.8a}$$

Here f_1 and f_2 are two linearly independent solutions of the homogeneous radial equation [i.e., (3.6) without the δ function on the right-hand side] and W is their Wronskian, which is independent of r^*. In order to satisfy the conditions (2.6) we take for f_1 a plane wave of unit amplitude that comes from $r^* = -\infty$, gets reflected, and for $r^* \to +\infty$ becomes either partially transmitted (when $\omega^2 > \mu^2$) or exponentially damped (when $\omega^2 < \mu^2$). On the other hand, f_2 behaves for $r^* \to -\infty$ as $e^{-i\omega r^*}$. Expressed formally,

$$f_1(r^*) \sim e^{i\omega r^*} + R^{(l)}(\omega) e^{-i\omega r^*} \qquad (r^* \to -\infty),$$
$$f_2(r^*) \sim e^{-i\omega r^*} \qquad (r^* \to -\infty). \tag{3.8b}$$

We get then

$$\Delta_R^{(l)}(r^*, r^{*\prime}; \omega) = (2i\omega)^{-1}[e^{i\omega r^*} + R^{(l)}(\omega) e^{-i\omega r^*}] e^{-i\omega r^{*\prime}} \qquad (r^{*\prime} < r^* \ll 0). \tag{3.9}$$

Now, for low frequencies the reflection amplitude $R^{(l)}(\omega)$ behaves as

$$R^{(l)}(\omega) \underset{\omega\to 0}{\sim} -1 - 2i\omega a, \quad 0 \neq a \in \Re \tag{3.10}$$

and it follows that

$$\lim_{r^{*\prime}\to -\infty} \Delta_R^{(l)}(r^*, r^{*\prime}; \omega = 0) = f_0^{(l)}(r^*), \tag{3.11}$$

where $f_0^{(l)}(r^*)$ is the solution of the zero-energy homogeneous radial equation that behaves for large negative r^* as $(r^* - a)$, where a is such that the function goes like $\exp(-\mu r^*)$, when $r^* \to +\infty$. The function f_0 is everywhere regular and, of course, not identically zero.

It follows, therefore, from (3.3) that the coupling extinguishes as $(1 - 2m/r')^{1/2}$, when $r' \to 2m$. This enables one to read off immediately the extinction factors for various physical quantities by merely counting powers of the coupling constant. For example, the Yukawa-like force between two baryons, which is proportional to λ^2, vanishes as $(1 - 2m/r')^{1/2}$ when one of the baryons approaches the black hole, because only one of the λ's comes from the "collapsing baryon."

This decoupling seems at first sight to lead to a paradoxical situation. The preceding result applies for any value to the mass μ; in particular, it holds in a massless scalar field. Now for a general time-dependent situation there is nothing like a Gauss law associated to the equation

$$\Box^2\hat{\phi}(x)=\rho(x). \tag{3.12a}$$

However, for a static regime (3.12a) reduces to

$$\frac{\partial}{\partial x^i}\left[(-g)^{1/2}g^{ij}\frac{\partial\hat{\phi}}{\partial x^j}(\vec{x})\right]=(-g)^{1/2}\rho(\vec{x}), \tag{3.12b}$$

which does lead to the integral law

$$\oint_{\partial V} d^2\sigma^i(-g_{00})^{1/2}\frac{\partial\hat{\phi}}{\partial x^i}=\int_V d^3x(-g)^{1/2}\rho. \tag{3.13a}$$

The integral on the left-hand side can be taken, for example, over a sphere at spatial infinity, where it reduces to the usual flat-space form. But we have seen that, in the limit $r'\to 2m$, $\hat{\phi}(\vec{x})\to 0$ and (3.13a) seems to indicate that the scalar charge λ of the source is equal to zero. The failure resides of course in that the volume integral in (3.13a) is not the scalar charge of the source, but rather

$$\int_V d^3x(-g)^{1/2}\rho=\lambda\left(1-\frac{2m}{r'}\right)^{1/2}, \tag{3.13b}$$

which indeed vanishes as when $r'\to 2m$ in the right manner.

As a matter of fact, (3.13a) combined with (3.13b) could have been used (in the zero-mass case) not only to show that decoupling arises, but also to obtain the extinction rate. We prefer, however, the line of attack based on the effective potential approach because it applies also to a dynamical situation.

We see, therefore, that the field can decouple from the source in spite of the existence of a Gauss-type law because the surface integral is not conserved during the "virtual motion" of the source. This state of affairs is to be contrasted with the situation for the electromagnetic field, where the surface integral gives precisely what is conserved during the motion of the source, namely, its electric charge.[11]

B. Vector Mesons

We consider now a vector-meson field $\varphi^\mu(x)$ coupled to a classical current $j^\mu(x)$ by means of the equation

$$\varphi^{\mu\nu}{}_{;\nu}+\mu^2\varphi^\mu=4\pi j^\mu, \tag{3.14a}$$

where

$$\varphi_{\mu\nu}\equiv\varphi_{\nu;\mu}-\varphi_{\mu;\nu}=\varphi_{\nu,\mu}-\varphi_{\mu,\nu}. \tag{3.14b}$$

The solution of (3.14) can be exhibited in terms of the retarded propagator $\Delta^\mu_{R\nu'}(x,x')$ as

$$\varphi^\mu(x)=\varphi^{\text{in}\,\mu}(x)+\hat{\varphi}^\mu(x), \tag{3.15a}$$

with

$$\hat{\varphi}^\mu(x)=\int d^4x'[-g(x')]^{1/2}\Delta^\mu_{R\nu'}(x,x')j^{\nu'}(x'). \tag{3.15b}$$

The propagator $\Delta^\mu_{R\nu'}(x,x')$ is a bitensor [it transforms as a covariant vector at x' (index ν') and as a contravariant vector at x (index μ)] which solves the equation

$$\begin{aligned}(\Delta^{\alpha}_{R\nu'};{}^{\mu}-\Delta^{\mu}_{R\nu'};{}^{\alpha})_{;\alpha}+\mu^2\Delta^\mu_{R\nu'}&\equiv(-g)^{-1/2}\frac{\partial}{\partial x^\alpha}\left[(-g)^{1/2}g^{\alpha\beta}g^{\mu\rho}\left(\frac{\partial\Delta_{R\beta\nu'}}{\partial x^\rho}-\frac{\partial\Delta_{R\rho\nu'}}{\partial x^\beta}\right)\right]+\mu^2\Delta^\mu_{R\nu'}\\&=\delta^\mu_{\nu'}\delta^{(4)}(x,x')\end{aligned} \tag{3.16}$$

subject to the conditions (2.6).

If we wanted to attack the problem in full generality, we should need [in analogy with (3.5)] an expansion of $\Delta^\mu_{R\nu}(\vec{x},\vec{x}';\omega)$ in vector spherical harmonics.[12] However, we are interested in the case of a static source, and we need, therefore, only the zero-frequency component. Furthermore, in this case, only the $\mu=0$ component of the coupling integral survives. Equation (3.15) collapses then to

$$\varphi_i(x)=\varphi_i^{\text{in}}(x);\quad \varphi_0(x)=\varphi_0^{\text{in}}(x)+\hat{\varphi}_0(\vec{x}), \tag{3.17a}$$

with

$$\hat{\varphi}_0(\vec{x})=\int d^3x'(-g(\vec{x}'))^{1/2}\Delta_{R00}(\vec{x},\vec{x}';\omega=0)j^0(\vec{x}'). \tag{3.17b}$$

For the case of a point source of strength λ at rest at the point $\vec{x}'$, j^0 is given by

$$j^0(\vec{x})=\lambda(r^2\sin\theta)^{-1}\delta(r-r')\delta(\theta-\theta')\delta(\varphi-\varphi') \tag{3.18}$$

(which is correctly normalized as $\int_{t=t_0}d^3\sigma_\mu j^\mu=-\lambda$, since $j^i=0$), and consequently Eq. (3.17b) takes finally the form

$$\hat{\varphi}_0(\vec{x})=\lambda\Delta_{R00}(\vec{x},\vec{x}';\omega=0), \tag{3.19}$$

which should be compared with (3.3).

To go further, we expand the propagator in (3.19) as

$$\Delta_{R\,00}(\vec{x},\vec{x}\,';\omega=0)=(rr')^{-1}\left[\left(1-\frac{2m}{r}\right)\left(1-\frac{2m}{r'}\right)\right]^{1/2}\sum_{l=0}^{\infty}\Delta_R^{(l)}(r^*,r^{*\prime})\sum_{m=-l}^{+l}Y_{lm}(\theta,\varphi)Y^*_{lm}(\theta',\varphi'). \quad (3.20)$$

Equation (3.16) translates then into the following equation for $\Delta_R^{(l)}(r^*, r^{*\prime})$:

$$\left[\frac{\partial^2}{\partial r^{*2}}-V_l(r^*)\right]\Delta_R^{(l)}(r^*,r^{*\prime})=\delta(r^*-r^{*\prime}), \quad (3.21)$$

with

$$V_l(r^*)=\left(1-\frac{2m}{r}\right)\left(\frac{l(l+1)}{r^2}+\frac{2m}{r^3}+\mu^2\right)+\frac{m^2}{r^4}. \quad (3.22)$$

This potential presents the general features discussed in Sec. II, and it has the important property of tending to a *nonvanishing constant,*

$$\mu_-^{\,2}=(4m)^{-2}, \quad (3.23)$$

for $r^*\to-\infty$.

We can write then, in total analogy with (3.8a),

$$\Delta_R(r^*,r^{*\prime})=W^{-1}\begin{cases}f_2(r^*)f_1(r^{*\prime}), & r^*<r^{*\prime}\\ f_1(r^*)f_2(r^{*\prime}), & r^*>r^{*\prime}\end{cases} \quad (3.24a)$$

And on account of the conditions (2.6), we select this time f_1 and f_2 as

$$f_1(r^*)=e^{-\mu r^*}\quad(r^*\to+\infty),$$
$$f_2(r^*)=e^{r^*/4m}\quad(r^*\to-\infty). \quad (3.24b)$$

The crucial point to realize now is that f_1 and f_2 defined by (3.24b) are indeed linearly independent. This can be seen without any calculation by noticing that if it were not the case, namely, if one were a multiple of the other, there would exist a nonzero solution of the homogeneous version of Eq. (3.21) decaying exponentially for *both* $r^*\to\pm\infty$. This solution would be a zero-energy bound state in a potential that is always positive, but this implies a negative mean kinetic energy, which is impossible.[13]

We get, therefore, the following asymptotic form for $\Delta_R^{(l)}(r^*, r^{*\prime})$ when $r^*>r^{*\prime}$:

$$\Delta_R^{(l)}(r^*,r^{*\prime})=\frac{(1-2m/r')^{1/2}f_1^{(l)}(r^*)}{W^{(l)}}. \quad (3.25)$$

When this result is inserted back into (3.20), we see from (3.19) that the coupling fades away this time not as $(1-2m/r')^{1/2}$, but twice as fast, namely as $1-2m/r'$.

Before leaving this section we would like to point out that if $f_2(r^*)$ is replaced by $\tilde{f}_2(r^*)$ which goes as $\exp(-r^*/4m)$ for $r^*\to-\infty$, one can still make $\tilde{f}_2$ and f_1 linearly independent. (If, by accident, he picks up $\tilde{f}_2$ proportional to f_1, then the addition of a multiple of f_2 lifts the coincidence without altering the asymptotic behavior.) With this choice the coupling would not vanish. However, such an $\tilde{f}_2$, apart from not being uniquely defined, causes the invariant $\hat{\varphi}_\mu\hat{\varphi}^\mu$ to blow up at the horizon as $(1-2m/r)^{-1}$ for any finite r'^*, and has therefore to be rejected. This argument does not apply for the case of an electromagnetic field, because then $\hat{\varphi}^\mu=\hat{A}^\mu$ is not observable due to gauge invariance, as emphasized by Bekenstein.[3] This issue is discussed in detail in Sec. IV.

IV. PHOTONS: ELECTRIC CHARGE

We have seen in Sec. III B that vector mesons of any nonzero rest mass decouple from the source at the horizon. Now the electromagnetic field which can be regarded as a vector-meson field of zero mass should *not* decouple because electric charge is one of the measurable quantum numbers of a black hole – as it can be defined unambiguously by means of the flux of a remote electric field. There is then a sharp difference between both cases: For $\mu>0$ the field decouples; for $\mu=0$ it does not. There are other more familiar differences between both fields: A vector meson has three degrees of polarization – a photon has only two. The trace of the energy-momentum tensor is not zero for the vector-meson field, but it is zero for the electromagnetic field. Proca's equations are not gauge-invariant, but Maxwell's equations are. There is no Gauss's law for a vector-meson field, but there is one for an electromagnetic field. All these features are actually related to each other.

The fact that Maxwell's equations are gauge-invariant means that the potential A^μ is not physically observable, but only its curl, $F_{\mu\nu}$, is. This situation becomes particularly evident and illuminating when one looks at the energy-momentum tensor of the vector-meson field,

$$T_{\mu\nu}=\frac{1}{4\pi}\left[\,(\varphi_\mu{}^\alpha\varphi_{\alpha\nu}+\tfrac14 g_{\mu\nu}\varphi^{\alpha\beta}\varphi_{\alpha\beta})-\mu^2(\varphi_\mu\varphi_\nu-\tfrac12 g_{\mu\nu}\varphi^\alpha\varphi_\alpha)\right]. \quad (4.1)$$

One sees that the gravitational field, the source of which is the energy tensor, is not influenced by φ^μ but only by its curl $\varphi_{\mu\nu}$, when the mass goes to zero. If the mass is not zero, φ^μ itself acts directly as a source of curvature. In particular, for a gravitational field that has a vector-meson field as its only source, it is a consequence of Einstein's equations that the scalar curvature of space-time is proportional to $\mu^2\varphi^\alpha\varphi_\alpha$.

It follows from the above discussion that, for an electromagnetic field on a Schwarzschild background, one does not have the right to require that the invariant $\hat{A}_\mu\hat{A}^\mu$ be bounded on the horizon.[14] As we shall see below, it is this fact which enables the electromagnetic field to persist instead of decoupling itself from the source at the horizon.

In the zero-mass case the asymptotic behaviors $e^{-\mu r}$ and $e^{+\mu r}$ of the solutions of the homogeneous radial equation at spatial infinity have to be replaced by r^{-l} and r^{l+1}, respectively. If, following Cohen and Wald,[15] one requires that the observable invariant

$$\tfrac{1}{2}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}=\left(\frac{\partial\hat{A}_0}{\partial r}\right)^2+\frac{1}{r^2}\frac{1}{1-2m/r}\left(\frac{\partial\hat{A}_0}{\partial\gamma}\right)^2 \tag{4.2}$$

(γ = angle between the directions θ, φ and θ', φ'), which is a source of curvature according to (4.1), should be bounded at the horizon, then it follows that one has to make, for $l\neq 0$, the same choice for f_2 as in (3.24b). As a consequence, all the results for vector mesons translate unchanged: All multipoles of order $l\neq 0$ fade away as $1-2m/r'$ when the source approaches the horizon. We recover in this way, thanks to the use of the variable r^*, and without detailed knowledge of f_1 and f_2, a result that was obtained in Ref. 15 after solving exactly the radial equation in terms of Legendre functions.

The whole difference between the zero-mass case and the massive case resides then in the monopole term. For $l=0$ one can find two linearly independent solutions of the homogeneous equation which are well behaved at the horizon in the sense of leaving the observable invariants bounded, namely

$$h_1(r^*)=\left(1-\frac{2m}{r'}\right)^{-1/2}, \tag{4.3a}$$

$$h_2(r^*)=r\left(1-\frac{2m}{r}\right)^{-1/2} \tag{4.3b}$$

[note that $W(h_1,h_2)=1$].

We can take at this stage for $f_2(r^*)$ in (3.24a) any linear combination of h_1 and h_2. The Green's function is not uniquely determined by the conditions at the horizon and at spatial infinity. This new degree of freedom comes from the fact that the geometry *can house a test electromagnetic field;* in fact, h_1 given by (4.3a) gives a field that is well behaved not only at the horizon, but also at infinity.

It becomes apparent at this point that the very fact that introduced the ambiguity in the choice of the propagator provides also the way of lifting this indeterminacy: Due to the gauge invariance, the geometry can house a test field, but due to the same gauge invariance, there exists Gauss's law which can be used to fix the value of the test field. We require the electric flux through a sphere of radius r that lies between $2m$ and r' to be zero. This amounts to requiring that the black hole be a Schwarzschild black hole and not a Reissner-Nordström one, since in our approximation of a fixed background the charge of the black hole manifests itself not in the metric, but only through the presence of an attached electric field. To ensure then that we are dealing with a neutral black hole, no admixture of h_1 into f_2 should be allowed. We take then $f_2=h_2$ and, of course, $f_1=h_1$ (in order to ensure regular behavior at $r=\infty$ for fixed r'). When these choices are inserted into Eqs. (3.24a) and (3.20), Eq. (3.19) becomes

$$\hat{A}_0(\vec{x})=e/r \quad (r'=2m), \tag{4.4}$$

and we see that the electromagnetic field does not decouple from the source at $r'=2m$, but rather remains coupled to the black hole, converting a Schwarzschild black hole into a Reissner-Nordström black hole of charge e (Refs. 15, 16).

To the standard differences between massive and massless electromagnetism mentioned at the beginning of this section, gravitational collapse adds a new and dramatic one: A charged black hole can exist only if the mass of the photon is *exactly* zero. In fact, the above results [and the validity of the hair conjecture in general (in particular Ref. 3)] suggest that a collection of charged matter that would collapse, say, to a Reissner-Nordström black hole if the photon mass is zero, would instead collapse to a Schwarzschild black hole if the photon mass is arbitrarily small but different from zero. This state of affairs is quite peculiar because the zero-mass limit of massive electromagnetism is Maxwell's theory,[17] which means that all physical consequences of the $\mu\neq 0$ theory should coincide with the Maxwellian predictions in the $\mu\to 0$ limit. To give a simple example, the Yukawa potential $\varphi(r)=er^{-1}e^{-\mu r}$ becomes in the $\mu\to 0$ limit the Coulomb potential $\varphi(r)=er^{-1}$. What a sharp contrast between this situation and the case of a spherically symmetric black hole where $\varphi(r)=0$ if $\mu\neq 0$, and $\varphi(r)=er^{-1}$ if $\mu=0$!

Clearly, the discontinuity does not come from the equations themselves, which depend smoothly on μ. It has been, rather, introduced from outside when prescribing the boundary conditions on the horizon: The boundary condition for the massive case does not go in the $\mu \to 0$ limit into the one adopted for the massless case; it rather reduces to the situation discussed in Ref. 16. A similar situation will arise if one considers the more realistic case of a freely falling particle, since he would also need, in that case, to prescribe the behavior of the propagator on the horizon for $\omega^2 < \mu_-^2 = (4m)^{-2}$ (note that this problem does not arise for the scalar field, because there $\mu_-^2 = 0$). We cannot avoid this jump in the boundary condition unless we are willing to admit that, if the photon mass is not zero, the presence of a test charge on a Schwarzschild background would destroy the horizon no matter how far away it is placed. A better understanding of this issue could be obtained by considering a fully dynamical situation in the case where the photon mass is not zero. A conveniently simple problem to study in this context would be the collapse of a charged shell.

V. NEUTRINOS: LEPTON NUMBER[18]

A. Generalities

The possibility of measuring the lepton number of a black hole by means of weak interactions was originally suggested by Hartle,[19] who recalled that in flat space there is a long-range neutrino force with a potential $V(r) = G_w^2 L_1 L_2 / r^5$ between two lumps of matter having lepton numbers L_1 and L_2 (G_w here is the weak coupling constant), and that this force could be used in principle to measure the leptonic content of a black hole. Later Hartle himself[20] showed that the potential in question vanishes when one of the bodies is a Schwarzschild or Kerr black hole. This result strongly suggests that a black hole should not have any weak-interaction properties at all. Not only the particular process (exchange of a virtual neutrino-antineutrino pair between two static leptons) responsible for the r^{-5} interaction should disappear, but it should also be impossible to measure the lepton number of a black hole by a scattering experiment involving real neutrinos. For example, the cross section for the scattering of neutrinos by a lepton source at rest at $r = r'$ should reduce to the cross section for scattering by the gravitational field alone in the limit $r' \to 2m$. In order to see whether this is indeed the case, we investigate below whether the neutrino field decouples from the source at the horizon or does not.

We consider a quantum neutrino field $\psi(x)$ interacting with a classical lepton current $N^\mu(x)$ according to[21]

$$[\gamma^\mu(x)\nabla_\mu + iG_w 2^{-1/2}\gamma^\mu(x)N_\mu(x)(1+\gamma^5)]\psi = 0 \,. \quad (5.1)$$

Here ∇_μ is a covariant spinor derivative[22] and the Dirac matrices satisfy

$$\{\gamma^\mu(x), \gamma^\nu(x)\} = 2g^{\mu\nu}(x) \,. \quad (5.2)$$

The retarded propagator S_R is a solution of

$$\gamma^\mu(x)\nabla_\mu S_R(x, x') = \delta^{(4)}(x, x')\underline{1} \quad (5.3)$$

subject to the boundary conditions (2.6).

In terms of S_R, Eq. (5.1) can be rewritten as an integral equation

$$\psi(x) = \psi^{\text{in}}(x) - iG_w 2^{-1/2}\hat{\psi}(x) \,, \quad (5.4a)$$

with

$$\hat{\psi}(x) = \int d^4x'(-g(x'))^{1/2} S_R(x, x') \times \gamma^\mu(x')N_\mu(x')(1+\gamma^5)\psi(x') \,. \quad (5.4b)$$

We will limit ourselves to first order in perturbation theory both for simplicity reasons and because only to that order has the theory received experimental verification. That is to say we will consider only the first iteration step of (5.4b), namely

$$\hat{\psi}_1(x) = \int d^4x'(-g(x'))^{1/2} S_R(x, x') \times \gamma^\mu(x')N_\mu(x')(1+\gamma^5)\psi^{\text{in}}(x') \,. \quad (5.5)$$

Furthermore, we shall take for $N^\mu(x)$ not a point source, but a spherical shell – in order to be able to exhibit the result in a form as transparent and compact as possible. This we shall do, however, at the very end, all the previous work applying to a point source.

Since the background is time-independent, both $S_R(x, x')$ and $\psi^{\text{in}}(x)$ can be Fourier analyzed in time as

$$S_R(x, x') = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\, e^{-i\omega(t-t')} S_R(\vec{x}, \vec{x}'; \omega) \,, \quad (5.6a)$$

$$\psi^{\text{in}}(x) = \int_{-\infty}^{+\infty} d\omega\, e^{-i\omega t}\psi_\omega^{\text{in}}(\vec{x}) \,. \quad (5.6b)$$

For the case of a point source with L leptons at rest at $\vec{x}'$ the number current $N^\mu(x)$ has only one nonvanishing component:

$$N^0(x) = \frac{L\delta(r-r')\delta(\theta-\theta')\delta(\varphi-\varphi')}{r^2 \sin\theta} \,.$$

If this value of N^0 is introduced into (5.5) one gets, taking into account (5.6), that

$$\hat{\psi}_1(x) = L \int_{-\infty}^{+\infty} d\omega\, e^{-i\omega t} S_R(\vec{x}, \vec{x}\,'; \omega)\gamma_0(\vec{x}\,')(1+\gamma^5)\psi_\omega^{\rm in}(\vec{x}\,')\,. \tag{5.7}$$

If, instead of a point source, one considers a spherical shell of leptonic content L, he has still to integrate (5.7) over all primed angles.

When evaluating $\hat{\psi}_1(x)$ we will expand both $S_R(\vec{x}, \vec{x}\,'; \omega)$ and $\psi^{\rm in}(x)$ appearing in (5.7) in terms of a complete set of stationary-state solutions of the free Dirac equation [i.e., (5.1) with $N^\mu = 0$]. An account of the properties of such solutions follows.

B. Free Solutions

Due to the stationarity of the background one can look for free solutions of the Dirac equation of the form $e^{-i\omega t}\psi(\vec{x})$. Since the background is also spherically symmetric these solutions can be labeled by angular momentum quantum numbers j and m. Parity is also a good quantum number for the free equation, but it is no longer conserved when the interaction is taken into account, due to the factor $1+\gamma^5$. One considers, instead, the chirality $\lambda = \pm 1$, i.e., the eigenvalue of γ^5. Nevertheless, we shall use parity as a label for our eigenfunctions and only at the end shall we pass to a chirality representation. This we do only for a technical reason (intermediate calculations are simpler) and not because it has any physical relevance. The technical reason being that the total angular momentum j and the parity can be summarized into a useful single quantum number k which takes all positive and negative integer values, as[23]

$$j = |k| - \tfrac{1}{2}; \quad \text{parity} = (-1)^{j + (\mathrm{sgn}\, k)/2}\,. \tag{5.8}$$

All the above discussion applies without change in flat space, but in the case of a Schwarzschild background a new quantum number α has to be introduced, as emphasized by Hartle.[20]

Such a quantum number α distinguishes between neutrinos coming from the horizon ($\alpha = +$) and neutrinos coming from spatial infinity ($\alpha = -$). In flat space the condition of regularity at the origin allows only one particular combination of both solutions: The net neutrino flux across a small sphere around the origin is zero. No restriction of that kind exists in the Schwarzschild geometry; the radial problem is a Schrödinger problem for an infinite line ($-\infty < r^* < +\infty$) in contrast with the half-line character of its flat-space counterpart ($0 \leq r < \infty$). Note in this context that for fixed $km\omega$ the two states $\alpha = \pm$ are orthogonal because they correspond to different "in states" of a one-dimensional scattering problem.[24]

We write our stationary wave functions, following Brill and Wheeler,[22] as

$$\psi_{km\omega}^{(\alpha)}(\vec{x}) = \left(1 - \frac{2m}{r}\right)^{-1/4} r^{-1} \begin{pmatrix} f_{k\omega}^{(\alpha)}(r^*)\chi_k^m(\theta, \varphi) \\ i g_{k\omega}^{(\alpha)}(r^*)\chi_{-k}^m(\theta, \varphi) \end{pmatrix}. \tag{5.9}$$

Here, the $\chi_k^m(\theta, \varphi)$ are the standard two-component spinors[23] arising from adding spin $\frac{1}{2}$ and orbital angular momentum l [$l = k$ if $k > 0$; $l = -(k+1)$ if $k < 0$] to give total angular momentum $j = |k| - \frac{1}{2}$.

The radial functions f and g satisfy the coupled equations

$$\frac{df}{dr^*} = -\frac{k}{r}\left(1 - \frac{2m}{r}\right)^{1/2} f + \omega g\,, \tag{5.10a}$$

$$\frac{dg}{dr^*} = -\omega f + \frac{k}{r}\left(1 - \frac{2m}{r}\right)^{1/2} g\,, \tag{5.10b}$$

which are equivalent to the uncoupled second-order equations

$$\frac{d^2 f}{dr^{*2}} + [\omega^2 - V_k(r^*)] f = 0\,, \tag{5.11a}$$

$$\frac{d^2 g}{dr^{*2}} + [\omega^2 - V_{-k}(r^*)] f = 0\,, \tag{5.11b}$$

$$g(r^*) = \frac{1}{\omega}\left[\frac{df}{dr^*} + \frac{k}{r}\left(1 - \frac{2m}{r}\right)^{1/2} f\right]. \tag{5.11c}$$

The effective potential $V_k(r^*)$ appearing in (5.11) is given by

$$V_k(r^*) = \left(1 - \frac{2m}{r}\right)^{1/2} \left[\frac{k^2}{r^2}\left(1 - \frac{2m}{r}\right)^{1/2} + \frac{k}{r^2}\left(1 - \frac{2m}{r}\right) + \frac{m}{r^3}\right] \tag{5.12}$$

and has the general characteristics discussed in Sec. II. The constant μ_-^2 is zero and μ_+^2 also vanishes, as should be the case since neutrinos are massless.

For any given k and ω we select a basis of solutions of (5.11) by

$$f_{k\omega}^{(+)}(r^*) = \begin{cases} e^{i\omega r^*} + R_k^{(+)}(\omega)\, e^{-i\omega r^*} & (r^* \to -\infty)\,, \\ T_k^{(+)}(\omega)\, e^{i\omega r^*} & (r^* \to +\infty)\,, \end{cases} \tag{5.13a}$$

$$f_{k\omega}^{(-)}(r^*) = \begin{cases} T_k^{(-)}(\omega)\, e^{-i\omega r^*} & (r^* \to -\infty)\,, \\ e^{-i\omega r^*} + R_k^{(-)}(\omega) e^{i\omega r^*} & (r^* \to +\infty)\,. \end{cases} \tag{5.13b}$$

In each case the lower components are obtained from (5.11c), and one easily shows that

$$ig^{(\alpha)}_{k\omega} = -\alpha f^{(\alpha)}_{-k\omega} \tag{5.14a}$$

and

$$T^{(+)}_k(\omega) = T^{(-)}_k(\omega) = T^{(\pm)}_{-k}(\omega) \equiv T_k(\omega), \tag{5.14b}$$

$$R^{(\alpha)}_k(\omega) = -R^{(\alpha)}_{-k}(\omega). \tag{5.14c}$$

On account of (5.14a), Eq. (5.9) takes the form

$$\psi^{(\alpha)}_{km\omega}(\vec{x}) = \left(1-\frac{2m}{r}\right)^{-1/4} r^{-1}\begin{pmatrix} f^{(\alpha)}_{k\omega}\chi^m_k \\ -\alpha f^{(\alpha)}_{-k\omega}\chi^m_{-k}\end{pmatrix}, \tag{5.15}$$

and using (5.14b) and (5.14c) one obtains the following orthogonality and completeness relations:

$$\int d^3x\, ({}^{(3)}g)^{1/2}\psi^{(\alpha)\dagger}_{km\omega}\psi^{(\alpha')}_{k'm'\omega'} = 4\pi\delta_{\alpha\alpha'}\delta_{kk'}\delta_{mm'}\delta(\omega-\omega'), \tag{5.16a}$$

$$\sum_{km\alpha}\int_0^\infty d\omega\, \psi^{(\alpha)}_{km\pm\omega}(\vec{x})\psi^{\dagger(\alpha)}_{km\pm\omega}(\vec{x}') = \frac{4\pi\delta(r-r')\delta(\theta-\theta')\delta(\varphi-\varphi')}{({}^{(3)}g)^{1/2}}, \tag{5.16b}$$

where

$$ {}^{(3)}g \equiv \det \|g_{ij}\| = \left(1-\frac{2m}{r}\right)^{-1} r^4\sin^2\theta .$$

The integration over the energy ω in (5.16b) runs from 0 to ∞ only because the solutions $f^{(\alpha)}_{k-\omega}$ are linear combinations of the $f^{(\alpha)}_{k\omega}$.

C. Expansions

The incoming neutrino field $\psi^{\text{in}}(x)$, being a free field, can be expanded as

$$\psi^{\text{in}}(x) = (8\pi)^{-1}\sum_{km\alpha}\int_0^\infty d\omega\left[e^{-i\omega t}\psi^{(\alpha)}_{km\omega}(\vec{x})b^{(\alpha)}_{km\omega} + e^{i\omega t}\psi^{(\alpha)}_{km,-\omega}d^{\dagger(\alpha)}_{km\omega}\right]. \tag{5.17}$$

The operators b and d annihilate a neutrino and an antineutrino, respectively. They satisfy the usual anticommutation relations

$$\{b^{(\alpha)}_{km\omega}, b^{(\alpha')\dagger}_{k'm'\omega'}\} = \{d^{(\alpha)}_{km\omega}, d^{(\alpha')\dagger}_{k'm'\omega'}\} = \delta_{\alpha\alpha'}\delta_{kk'}\delta_{mm'}\delta(\omega-\omega') \tag{5.18a}$$

(all other anticommutators vanish), which ensure, on account of (5.16), that the free neutrino field satisfies a natural generalization

$$\{\psi_a(\vec{x},t), \psi^\dagger_b(\vec{x}',t)\} = \delta_{ab}({}^{(3)}g)^{-1/2}\delta(x^1-x^{1\prime})\delta(x^2-x^{2\prime})\delta(x^3-x^{3\prime}) \tag{5.18b}$$

of the flat-space equal-time anticommutation rules.

Besides the expansion for ψ^{in}, we need an expansion of $S_R(\vec{x},\vec{x}';\omega)$ in terms of the free solutions (5.15). Equation (5.3) implies that the free retarded propagator at energy ω satisfies

$$(\omega - \mathcal{H}_{\vec{x}})\left[\left(1-\frac{2m}{r}\right)^{1/4}\left(1-\frac{2m}{r'}\right)^{1/4} rr'S_R(\vec{x},\vec{x}';\omega)\right] = -\delta(r^*-r^{*\prime})\delta(\theta-\theta')\delta(\varphi-\varphi')\beta, \tag{5.19}$$

where β is the usual Dirac matrix and where the Hamiltonian $\mathcal{H}$ acts on a spinor of the form (5.9) as

$$\mathcal{H}\begin{pmatrix} f\chi^m_k \\ ig\chi^m_{-k}\end{pmatrix} = \begin{pmatrix} -\left[\dfrac{dg}{dr^*} - \dfrac{k}{r}\left(1-\dfrac{2m}{r}\right)^{1/2} g\right]\chi^m_k \\ i\left[\dfrac{df}{dr^*} + \dfrac{k}{r}\left(1-\dfrac{2m}{r}\right)^{1/2} f\right]\chi^m_{-k}\end{pmatrix} \tag{5.20}$$

[note that $\mathcal{H}\psi = \omega\psi$ reproduces (5.10)].

It can be verified by integration through the singularity at $r^* = r^{*\prime}$ that the solution of (5.19) with the boundary conditions (2.6) is given by

$$S_R(\vec{x},\vec{x}';\omega) = \sum_{km}\begin{cases}\psi^{(+)}_{km\omega}(\vec{x})\,\phi^{(-)\dagger}_{km\omega}(\vec{x}'), & r^* > r^{*\prime} \\ \psi^{(-)}_{km\omega}(\vec{x})\,\phi^{(+)\dagger}_{km\omega}(\vec{x}'), & r^* < r^{*\prime}\end{cases} \tag{5.21a}$$

with the spinor ϕ being defined as

$$\phi^{(\alpha)}_{km\omega}=\left(1-\frac{2m}{r}\right)^{-1/4} r^{-1}[2iT^*_k(\omega)]^{-1}\begin{pmatrix} f^{(\alpha)*}_{k\omega}\chi^m_k \\ -\alpha f^{(\alpha)*}_{-k\omega}\chi^m_{-k}\end{pmatrix} \tag{5.21b}$$

$[T_k(\omega)$ is the transmission amplitude appearing in (5.13)$]$.

Inserting (5.17) and (5.21) into (5.7), integrating in $d\Omega'$, and using the orthogonality relation

$$\int d\Omega\chi^{m\dagger}_k\chi^{m'}_{k'}=\delta_{kk'}\delta_{mm'} \tag{5.22a}$$

for the angular spinors, one obtains

$$\hat\psi_1(x)=L(16\pi r'^2)^{-1}\int_0^\infty d\omega\sum_{k\,m\,\alpha}[T_k(\omega)]^{-1}[f^{(-)}_{k\omega}(r^{*\prime})f^{(\alpha)}_{k\omega}(r^{*\prime})+\alpha f^{(-)}_{-k\omega}(r^{*\prime})f^{(\alpha)}_{k\omega}(r^{*\prime})]$$
$$\times\{e^{-i\omega t}[\psi^{(\alpha)}_{km\omega}(\vec{x})-\psi^{(\alpha)}_{-km\omega}(\vec{x})]\,b^{(\alpha)}_{km\omega}+e^{i\omega t}[\psi^{(\alpha)}_{km-\omega}(\vec{x})-\psi^{(\alpha)}_{-km-\omega}(\vec{x})]\,d^{(\alpha)\dagger}_{km\omega}\}\,. \tag{5.22b}$$

Now Hartle[20] has shown that in the limit $r^{*\prime}\to-\infty$ one has the behavior

$$f^{(-)}_{k\omega}(r^{*\prime})\sim T_k(\omega)\,e^{-i\omega r^{*\prime}}[1+O(e^{r^{*\prime}/4m})]\,. \tag{5.23}$$

It follows then that the $\alpha=-$ term in (5.22) dies as $(1-2m/r')^{1/2}$, when $r'\to 2m$. On the other hand, on account of (5.13a) the $\alpha=+$ part of (5.23) does not vanish, but causes the field $\hat\psi_1(x)$ to take on the residual value

$$\lim_{r'\to 2m}\hat\psi_1(x)=(8\pi m^2)^{-1}\sum_{km}\int_0^\infty d\omega[e^{-i\omega t}(\psi^{(+)}_{km\omega}-\psi^{(+)}_{-km\omega})b^{(+)}_{km\omega}+e^{i\omega t}(\psi^{(+)}_{km-\omega}-\psi^{(+)}_{-km-\omega})d^{(+)\dagger}_{km\omega}]\,. \tag{5.24}$$

At this stage we can rewrite (5.24) in terms of chiralities. In fact, it follows from (5.15) that for $\alpha=+$ the state $2^{-1/2}(\psi_k-\psi_{-k})$ is a normalized state of positive chirality. The corresponding annihilation operators are $2^{-1/2}(b_k-b_{-k})$ and $2^{-1/2}(d_k-d_{-k})$. Equation (5.24) takes then the form

$$\lim_{r'\to 2m}\hat\psi_1(x)=L(4\pi m^2)^{-1}\sum_{\substack{jm\\(\lambda=+1)}}\int_0^\infty d\omega\,[e^{-i\omega t}\psi^{(+)}_{jm\lambda\omega}b^{(+)}_{jm\lambda\omega}+e^{i\omega t}\psi^{(+)}_{jm\lambda-\omega}d^{(+)\dagger}_{jm\lambda\omega}]\,. \tag{5.25}$$

Only positive chirality states enter in (5.25) as should be the case.

Equation (5.25) shows that the effect of the weak interaction is (to first order in the weak coupling[25]) merely to multiply the $\alpha=+$, $\lambda=+$ part of the incoming field by a constant phase factor

$$\exp\left[-i\frac{LG_w}{\sqrt{2}\,m^2}\right]\sim\exp\left[-3\times10^{-5}iL\left(\frac{\text{proton Compton wavelength}}{\text{Schwarzschild radius of black hole}}\right)^2\right]$$

which is not observable.

In particular, such a phase factor does not contribute to Hartle's potential

$$B^\mu(x)=\langle 0|\bar\psi(x)\gamma^\mu(x)(1+\gamma^5)\psi(x)|0\rangle\,,$$

and therefore our result is not in contradiction with what was shown in Ref. 20, but on the contrary is a confirmation of it.

We conclude therefore that a Schwarzschild black hole does not acquire any weak-interaction properties by the addition of a spherical shell of weakly interacting matter and that as a result its lepton number cannot be measured by means of weak interactions.

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†On leave from Facultad de Ciencias, Universidad de Chile, Santiago, Chile.

[1]See for example P. J. E. Peebles, Gen. Rel. J. (to be published), and references therein.

[2]For work on the "no hair" hypothesis see V. Ginzburg and L. Orzenoi, Zh. Eksp. Teor. Fiz. 47, 1030, (1964) [Sov. Phys.–JETP 20, 689 (1965)]; A. Doroshkevich, Ya. Zeldovich, and I. Novikov, Zh. Eksp. Teor. Fiz. 49, 170 (1965) [Sov. Phys.–JETP 22, 122 (1966)]; W. Israel, Phys. Rev. 164, 1776 (1967), Comm. Math. Phys. 8, 245 (1968); R. Price, Ph.D. thesis, California Institute of Technology, 1971 (unpublished). For a review and a complete list of references see K. S. Thorne, Caltech Report No. OAP-236, 1971 (unpublished). Figure 1 appears in J. A. Wheeler, Atti del Convegno Mendeleeviano, Accademia delle scienze di Torino, Accademia Nazionale dei Lincei, Torino-Roma, 1969, and also in an article by R. Ruffini and J. A. Wheeler, in *The Significance of Space Research for Fundamental Physics*, edited by A. F. Moore and V. Hardy (European Space Research Organization, Paris, 1970), and is reprinted here with their kind permission.

[3]The approach in this paper is complementary to the one of J. D. Bekenstein [Phys. Rev. D 5, 1239 (1972)], who considers an otherwise unspecified static geometry that has a horizon on which all physical observables are bounded and outside of which there are no matter or sources of fields but only static classical meson fields. He concludes then that such meson fields have to be identically zero. On the other hand we consider the less general situation of a given Schwarzschild background with test fields on it and exhibit the rates at which the possible "hairs" of a black hole die. Our methods can also be used in a dynamical situation, although such a treatment can prove rather difficult since massive fields do not propagate on light cones and the radiation emitted is consequently rather difficult to handle. See in this context R. Cawley and E. Marx, Int. J. Theor. Phys. 1, 153 (1968); R. G. Cawley, Ann. Phys. (N.Y.) 54, 122 (1969).

[4]This means, for example, that we cannot study such processes as pair production or annihilation.

[5]It is well known that when treating the nucleon as fixed one gets no meson-nucleon scattering. This can be understood without recourse to quantum field theory by recalling that for an equation of the Klein-Gordon type with a source the mechanism for scattering is the familiar one from Thomson scattering: The incoming wave shakes the source, causing it to radiate. If the source is fixed there is no response to the incoming wave and consequently no scattering. Therefore if the world line of the source is considered prescribed, we get no scattering at all, irrespective of whether or not the field decouples from the source at the horizon. However, it is most plausible that even if the source is allowed to oscillate under the influence of the incoming field the scattering will not differ from the scattering by the black hole alone in the limit where the average position of the source approaches the horizon.

[6]Consider, for example, scalar mesons in flat space. The free-field retarded propagator is given by

$$\Delta_R(\vec{x},\vec{x}';t-t')=\int\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\Delta_R(\vec{x},\vec{x}';\omega)\,,$$

with

$$\Delta_R(\vec{x},\vec{x}';\omega)=-\int\frac{d^3k}{(2\pi)^3}\,e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}\,[k^2-(\omega^2-\mu^2+i\epsilon\omega)]^{-1}\,.$$

For $\omega^2-\mu^2<0$ there is no pole in the integral over $\vec{k}$, and the prescription $\omega\to\omega+i\epsilon\omega$ is superfluous and can be dropped. On the contrary for $\omega^2-\mu^2>0$ the added $i\epsilon\omega$ ensures that the Green's function is retarded. The relation $\Delta_R(\vec{x},\vec{x}';-\omega)=\Delta_R^*(\vec{x},\vec{x}';\omega)$ becomes evident after changing the variable of integration from $\vec{k}$ to $-\vec{k}$. Furthermore the integral can be evaluated to give, for $\omega>0$,

$$\Delta_R(\vec{x},\vec{x}';\omega)=-(4\pi r)^{-1}\begin{cases}\exp[i(\omega^2-\mu^2)^{1/2}|\vec{x}-\vec{x}'|]\,, & \omega^2-\mu^2>0\\ \exp[-(\mu^2-\omega^2)^{1/2}|\vec{x}-\vec{x}'|]\,, & \omega^2-\mu^2<0\,.\end{cases}$$

In the limit $r=|\vec{x}|\to\infty$ ($\vec{x}'$ fixed) this gives the behavior (2.2).

[7]J. A. Wheeler, Phys. Rev. 97, 511 (1955).

[8]Strictly speaking it should still be proved that (2.6) implies that the retarded propagator vanishes outside of the future light cone with vertex at x'. One becomes convinced that this is indeed the case after looking at an r^* t diagram where light cones are 45° lines. In effect (2.6b) says that when one looks at the wave field in the regions where it is not backscattered by the gravitational field [i.e., where $V(r^*)$ becomes constant, namely at $r^*\to\pm\infty$] there are only waves propagating away from the source. For $t<t'$ the source does not exist yet; hence the amplitude of the waves is zero in this case. The above argument shows that the Green's function vanishes inside of the past light cone. That it vanishes outside of the light cone follows from the fact that elementary disturbances do not propagate faster than light. A formal proof would, however, be desirable.

[9]A partial report of these results has appeared in C. Teitelboim, Lett. Nuovo Cimento 3, 326 (1972).

[10]We treat $\delta^{(4)}(x,x')$ as a biscalar defined by

$$\int d^4x'(-g)^{1/2}\delta^{(4)}(x,x')f(x')=f(x)$$

for any scalar testing function f.

[11]One would be tempted to argue that the result is being fed into the problem by the choice of the interaction. Why not choose $\rho(\vec{x})=\lambda\delta(\vec{x}-\vec{x}')$ instead of $\rho(\vec{x})=\lambda(1-2m/r')^{1/2}\delta(\vec{x}-\vec{x}')$? Answer: It would not be relativistically invariant.

[12]See in this context R. Ruffini, J. Tiomno, and C. V. Vishveshwara, Lett. Nuovo Cimento 3, 211 (1972).

[13]The argument can be put more formally as follows: Consider the equation

$$\frac{d^2f}{dr^{*2}}=Vf\,.$$

Multiply both sides by f and integrate by parts to get

$$\int_{-\infty}^{+\infty}dr^*\left[\left(\frac{df}{dr^*}\right)^2+Vf^2\right]=f\,\frac{df}{dr^*}\bigg|_{-\infty}^{+\infty}\,.$$

It follows that if $f\,df/dr^*$ vanishes at $r^*=\pm\infty$ then f has to vanish identically because the integrand in the left-hand side is positive definite.

[14]We replace, in Sec. IV, φ^μ by the standard notation A^μ to denote the electromagnetic vector potential. The

"strong coupling constant" λ is replaced by the symbol e to denote electric charge.

[15]J. M. Cohen and R. M. Wald, J. Math. Phys. 12, 1845 (1971).

[16]Had we overlooked the problem of gauge invariance and required that f_2 should behave as $e^{r^*/4m}$ as in the vector-meson case [cf. (3.24b)], we would have obtained that the electromagnetic field decouples as $(1-2m/r')$. This possibility is perfectly allowed but it corresponds to quite a different problem, namely to a test charge e lowered into a black hole of "image charge" $-2me/r'$ so that when $r'\to 2m$ one gets an uncharged black hole. This can be seen by calculating the flux of the corresponding electric field through a sphere of radius r, $2m<r<r'$. One gets

$$(\text{flux}) = 4\pi r^2\left(-\frac{\partial \hat{A}_0}{\partial r}\right) = -4\pi r^2 \frac{\partial}{\partial r}\left[\frac{1}{r'}\left(1-\frac{2m}{r}\right)\right] = 4\pi\left(-\frac{2me}{r'}\right).$$

[17]See in this context the classical discussion of L. Bass and E. Schrödinger, Proc. Roy. Soc. (London) A232, 1 (1955), and also K. Symanzik, DESY Report No. DESY T-71/1. 1971 (unpublished).

[18]A brief account of these results has appeared in C. Teitelboim, Lett. Nuovo Cimento 3, 397 (1972)

[19]J. B. Hartle, private communication to J. A. Wheeler.

[20]J. B. Hartle, in *Magic Without Magic: John Archibald Wheeler*, edited by John R. Klauder (Freeman, San Francisco, 1972). See also J. B. Hartle, Phys. Rev. D 3, 2938 (1971). I am greatly indebted to Professor Hartle for letting me know about his work prior to publication and also for illuminating discussions. His methods were of great value in obtaining the results presented in Sec. V.

[21]J. B. Hartle, Phys. Rev. D 1, 394 (1970).

[22]For the theory of the Dirac equation in curved space see V. Bargmann, Sitzber. Preuss. Akad. Wiss., Phys. Math. Kl., 346 (1932). For a lucid review and applications to neutrinos, see D. R. Brill and J. A. Wheeler, Rev. Mod. Phys. 29, 465 (1957). In order to parallel as closely as possible the usual flat-space procedure for separating the Dirac equation in spherical coordinates we use a Cartesian *vierbein*. Our angular quantum number k is the negative of the one used by Brill and Wheeler and our radial equations differ from theirs in the exchange $\omega\to-\omega$, which causes equations (5.10) and (5.11) to go over, in the $m\to 0$ limit, into the standard flat-space equations of Ref. 23.

[23]See for example M. E. Rose, *Relativistic Electron Theory* (Wiley, New York, 1961).

[24]I am indebted to Professor Barry Simon and Professor Arthur S. Wightman for an illuminating discussion about this point.

[25]See Ref. 18 for a discussion that applies in the eventuality that higher-order corrections could give a nontrivial contribution such as a frequency-dependent phase factor, which would cause a time delay in neutrino scattering. The conclusions are not changed significantly.