Interaction of a Circularly Polarized Laser Pulse with Free Electrons*

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The interaction of an intense, circularly polarized, electromagnetic wave of finite duration with a free electron is analyzed classically. The effects of radiative reaction and intensity gradients in smoothly varying pulses are treated relativistically. It is shown that the effect of radiative reaction is to impart to the electron a final energy which depends only on the energy per unit area carried by the wave.

The importance of radiative reaction in determining the interaction of intense coherent radiation with a free electron has been pointed out by Sanderson.[1] By taking into account the rate at which Thomson scattering removes momentum from the beam, he estimates the mean acceleration of the electron in the direction of wave propagation. While several other authors[2,3] have commented on the effect, it appears that Sen Gupta[4] is the only investigator who has attempted a detailed calculation of the electron motion. He assumes a linearly polarized wave and employs an expansion technique the validity of which is limited to pulses of short duration. If the radiation pulse is instantly switched on or has a steeply rising leading edge, the longitudinal velocity acquired by the electron depends on the initial phase of the radiation.[5,6]

In the present article, we treat the interaction of circularly polarized radiation with free electrons including the effects of radiative reaction. We derive a relativistic equation of motion which is valid for smoothly varying pulses of arbitrary length. Neglecting radiative reaction, Kibble[2] has shown that for such pulses the electron orbit can be characterized as a helical curve of slowly varying radius and slowly varying slope. Because of this somewhat complicated electron orbit, it is difficult to derive in the laboratory frame, K_a, the relativistically correct expression for the longitudinal acceleration caused by the combined effects of radiative reaction and gradients of the laser beam intensity. However, in a Lorentz frame K_b, chosen at a given instant of (laboratory) time t_a such that the longitudinal velocity of the electron vanishes, it is possible to utilize a well-known, relativistically correct expression for the radiative reaction, and in this frame the longitudinal acceleration may be readily expressed.

The radiation-reaction force is essentially tangential to a circle of radius[3]

$$r_b = \frac{|e|E_b}{m\gamma_b\omega_b^2} = \frac{c}{\omega_b}\frac{\mu}{(1+\mu^2)^{1/2}}, \tag{1}$$

where γ_b is given by

$$\gamma_b = \mathcal{E}_b/mc^2 = (1+\mu^2)^{1/2}, \tag{2}$$

E_b, $\mathcal{E}_b$, and ω_b are the electric intensity, the total particle energy, and the frequency of the radiation, respectively, in the moving frame K_b, and μ is the Kibble parameter,[2] related to the radiation intensity by

$$\mu = \left(\frac{4\pi e^2 I_b}{m^2c^3\omega_b^2}\right)^{1/2} = \left(\frac{4\pi e^2 I_a}{m^2c^3\omega_a^2}\right)^{1/2}. \tag{3}$$

The radiation intensity and the square of the radiation frequency transform according to the relation

$$\frac{I_b}{I_a} = \frac{\omega_b^2}{\omega_a^2} = \frac{1-\beta_{ab}}{1+\beta_{ab}},$$

where $c\beta_{ab}$ denotes the velocity of the frame K_b relative to the frame K_a. The Kibble parameter μ therefore has the same value in both frames. The equations of motion to be derived are valid provided the radiation intensity in the neighborhood of the particle varies slowly and smoothly enough that

$$\left|\frac{d^n\mu(\eta)}{d\mu^n}\right| \ll 1, \quad n=1,2,3 \tag{4}$$

where $\eta = \omega_a t_a - k_a z_a = \omega_b t_b - k_b z_b$ is the Lorentz-invariant phase.

The power required to increase the particle energy $\mathcal{E}_b$ when the intensity is increasing, and to sustain the nonlinear inverse Compton radiation,[8] is given by

$$e\vec{E}_b \cdot \vec{v}_b = \frac{d\mathcal{E}_b}{dt_b} + \vec{F}_b \cdot \vec{v}_b , \tag{5}$$

where

$$\frac{d\mathcal{E}_b}{dt_b} = mc^2 \frac{d\gamma_b}{dt_b} = \frac{mc^2\mu}{(1+\mu^2)^{1/2}} \frac{d\mu}{dt_b} , \tag{6}$$

and where the radiation reaction force for motion on a circle is given by[3,9]

$$\vec{F}_b = -(2e^2/3c^3)\omega_b^2\gamma_b^4\vec{v}_b . \tag{7}$$

Associated with this power there is a longitudinal acceleration of the electron in the direction of the wave propagation vector $\vec{k}_b$. This acceleration is[3]

$$w_{bz} = c\frac{d\beta_{bz}}{dt_b} = \frac{1}{\gamma_b m}\frac{e}{c}|\vec{v}_b \times \vec{B}_b| = \frac{1}{\gamma_b m}\frac{e}{c}[\vec{v}_b \times (\vec{k}_b \times \vec{E}_b)] \cdot \hat{k}_b , \tag{8}$$

where $\hat{k}_b = \vec{k}_b/|\vec{k}_b|$. Since $\vec{v}_b \cdot \hat{k}_b = 0$, it follows that

$$w_{bz} = \frac{1}{\gamma_b m}\frac{e}{c}\vec{v}_b \cdot \vec{E}_b . \tag{9}$$

The projection of $\vec{E}_b$ onto the direction of the velocity $\vec{v}_b$ is many orders of magnitude less than $|\vec{E}_b|$ itself, and Eq. (1) remains therefore essentially valid. By virtue of the equations (2), (5), (6), (7), and (9), the longitudinal component of acceleration may be expressed as

$$w_{bz} = c\frac{d\beta_{bz}}{dt_b} = \left(\frac{c\mu}{1+\mu^2}\right)\left(\frac{d\mu}{dt_b}\right) + \left(\frac{2e^2}{3c^3}\right)\left(\frac{c}{m}\right)\omega_b^2\mu^2(1+\mu^2)^{1/2} . \tag{10}$$

The corresponding expression in the frame K_a can be obtained from Eq. (10) by using the relations

$$\omega_b^2 = \omega_a^2\left(\frac{1-\beta_{ab}}{1+\beta_{ab}}\right) ,$$

$$d/dt_b = \gamma_{ab} d/dt_a ,$$

$$d\beta_{bz}/dt_b = \gamma_{ab}^3 d\beta_{az}/dt_a ,$$

$$\gamma_{ab} = \frac{1}{(1-\beta_{ab}^2)^{1/2}} ,$$

and $\beta_{ab} = \beta_{az}$. The longitudinal acceleration in the laboratory frame is then

$$w_{az} = c\frac{d\beta_{az}}{dt_a} = (1-\beta_{az}^2)\left(\frac{c\mu}{1+\mu^2}\right)\frac{d\mu}{dt_a} + \left(\frac{2e^2}{3c^3}\right)\left(\frac{c\omega_a^2}{m}\right)(1-\beta_{az})^2 \times (1-\beta_{az}^2)^{1/2}\mu^2(1+\mu^2)^{1/2} . \tag{11}$$

If the differentiation with respect to the time parameter t_a is replaced by the differentiation with respect to the phase η,

$$d/dt_a = \omega_a(1-\beta_{az})d/d\eta ,$$

and if the change of variable

$$\xi(\eta) = [1+\mu^2(\eta)]^{1/2}\left[\frac{1-\beta_{az}(\eta)}{1+\beta_{az}(\eta)}\right]^{1/2} \tag{12}$$

is introduced, Eq. (11) reduces to the differential equation

$$\frac{1}{\xi^2(\eta)}\frac{d\xi(\eta)}{d\eta} + \left(\frac{2e^2}{3c^3}\right)\left(\frac{\omega_a}{m}\right)\mu^2(\eta) = 0 . \tag{13}$$

For an electron at rest before the arrival of the pulse, the solution of Eq. (13) is

$$\xi(\eta) = \frac{1}{1+\epsilon(\eta)} , \tag{14}$$

where

$$\epsilon(\eta) = \left(\frac{2}{3}\frac{e^2}{c^3}\right)\left(\frac{\omega_a}{m}\right)\int_0^\eta \mu^2(\eta')d\eta' \tag{15a}$$

$$= \sigma\frac{1}{mc^2}\int_0^{\tau_a} I_a(\tau_a')d\tau_a' . \tag{15b}$$

In the last expression the Thomson cross section $\sigma = (8\pi/3)(e^2/mc^2)^2$, the beam intensity $I_a = m^2c^3\omega_a^2\mu^2/4\pi e^2$, as given by Eq. (3), and the proper time[7] of the wave $\tau_a = \eta/\omega_a$ have been introduced. Equation (15b) reveals that $\epsilon(\eta)/\sigma$ represents in units of mc^2 the energy content of the pulse per unit area. The phase η is a measure of the distance by which the electron lags behind the leading edge of the laser pulse. To this edge we have assigned the phase $\eta = 0$.

By substituting in Eq. (14) the definition (12) for $\xi(\eta)$, we obtain

$$\beta_{az}(\eta) = \frac{[1+\epsilon(\eta)]^2[1+\mu^2(\eta)]-1}{[1+\epsilon(\eta)]^2[1+\mu^2(\eta)]+1} . \tag{16}$$

In the absence of radiative reaction this expression reduces to the well-known result[1,7]

$$\beta_{az}(\eta) = \frac{\frac{1}{2}\mu^2(\eta)}{1+\frac{1}{2}\mu^2(\eta)} .$$

TABLE I. The final kinetic energy in eV (upper entries) and the corresponding longitudinal displacement in cm (lower entries) of an electron which has interacted with a laser pulse. The frequency of the radiation is assumed to be $\omega_a = 1.78\times 10^{15}$ sec^{-1}, corresponding to a neodymium-glass laser.

Pulse duration (sec) \ Intensity (W/cm^2)	10^{17}	10^{18}	10^{19}	10^{20}
10^{-12}	1.7×10^{-7} 6.2×10^{-4}	1.7×10^{-5} 6.2×10^{-3}	1.7×10^{-3} 6.2×10^{-2}	1.7×10^{-1} 6.2×10^{-1}
10^{-11}	1.7×10^{-5} 6.2×10^{-3}	1.7×10^{-3} 6.2×10^{-2}	1.7×10^{-1} 6.2×10^{-1}	1.6×10^{1} 6.2
10^{-10}	1.7×10^{-3} 6.2×10^{-2}	1.7×10^{-1} 6.2×10^{-1}	1.6×10^{1} 6.2	1.5×10^{3} 6.7×10^{1}
10^{-9}	1.7×10^{-1} 6.3×10^{-1}	1.6×10^{1} 6.3	1.5×10^{3} 6.8×10^{1}	9.3×10^{4} 1.3×10^{3}

The kinetic energy of the electron is $\mathcal{E}_{\text{kin}} = \mathcal{E}_a - mc^2 = mc^2(\gamma_a - 1)$, where γ_a is found to be $\gamma_a = \gamma_{ab}\gamma_b$. Using these relations together with Eqs. (2) and (16), we obtain

$$\mathcal{E}_{\text{kin}}(\eta) = mc^2 \frac{\epsilon^2(\eta) + [1+\epsilon(\eta)]^2 \mu^2(\eta)}{2[1+\epsilon(\eta)]} . \quad (17)$$

The final value of the kinetic energy, which the electron has acquired after interaction with the entire pulse, is then

$$\mathcal{E}_{\text{kin}}(\eta_f) = mc^2 \frac{\epsilon_f^2}{2(1+\epsilon_f)} , \quad (18)$$

where ϵ_f is given by Eqs. (15) with the integration extending over the phase interval $0 \leq \eta \leq \eta_f$. The novelty of this result lies in the fact that the final energy imparted to the electron depends only on the energy content per unit cross-sectional area of the pulse.

For the longitudinal displacement of the electron in the laboratory frame, we find

$$z_a(\eta) = c \int_0^{t_a} \beta_{az}(t_a')\, dt_a' .$$

$$= \frac{c}{\omega_a} \int_0^{\eta} \frac{\beta_{az}(\eta')}{1-\beta_{az}(\eta')} d\eta'$$

$$= \frac{3c^4 m}{4e^2\omega_a^2} \epsilon(\eta) + \frac{c}{2\omega_a} \int_0^{\eta} \epsilon(\eta')[2+\epsilon(\eta')] \times [1+\mu^2(\eta')]\, d\eta' , \quad (19)$$

with $\beta_{az}(\eta)$ determined by Eq. (16).

Let us consider the interaction with an electron of a pulse characterized by its duration τ and its average intensity $I = (mc^2/\sigma)(1/\tau)\epsilon(\eta_f)$. For the frequency $\omega_a = 1.78 \times 10^{15}$ sec^{-1}, corresponding to a neodymium-glass laser, we list in Table I as upper entries some values of the final kinetic energy in electron volts given by Eq. (18). Determination of the corresponding longitudinal electron displacement requires specification of the pulse profile. As lower entries in Table I we have presented values of the final electron displacement determined from Eq. (19) with $I_a(\eta) = I$.

Laser pulses of very high intensity are realized within a focal region. The electron may leave this region before having interacted with the entire pulse, unless the focused beam can be made parallel by means of (possibly expendable) refractive or reflective optical devices.

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