Electromagnetic Excitations of the Nucleon in a Relativistic Quark Model*

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A variation of the relativistic quark model of Feynman, Ravndal, and Kislinger is considered that admits spatial wave functions normalizable in all components of the relative variables. With these wave functions, the matrix elements of the hadronic electromagnetic current between the nucleon and excited states are calculated for final states up to 1700 MeV in mass. Very good agreement between the predicted elastic nucleon electromagnetic form factors and experiment is obtained. Cross sections for the electroproduction of nucleon resonances through the third resonance are derived and compared with experiment. A discussion of the consistency and limitations of the model is presented.

I. INTRODUCTION

A recent formulation of a relativistic equation to represent the symmetric quark model was proposed by Feynman, Ravndal, and Kislinger.[1] Their encouraging results have led us to use this model with a different solution to the relativistic equation. This solution is required to vanish when any variable describing the relative position of two quarks becomes infinite and leads to normalizable wave functions. Such a solution was first considered by Fujimura, Kobayashi, and Namiki,[2] but their choice of the electromagnetic hadronic current differs from that of the present paper.

In Sec. II we develop the model. We derive the spatial wave function, consider a subsidiary to eliminate unphysical states, and present a summary of the calculations in Table I. The spin and unitary-spin wave functions are treated and presented in Table II. We discuss the over-all wave function and baryon spectrum of states with masses below about 1700 MeV. Finally, we conclude the section by giving a prescription for the electromagnetic hadronic current.

In Sec. III we give the results of the model. We discuss the elastic nucleon form factors for which agreement with experimental data is very good.[2] The photoelectric matrix elements for excitation of nucleon resonances are presented for proton and neutron targets and compared with experiment. These results are essentially the same as those of nonrelativistic quark models. We give results for electroproduction of the first, second, and third nucleon resonances which are in general agreement with experiment.

In Sec. IV we discuss two problems of the model. The solution that is given in this work to the first problem involving the mass spectrum will probably not be sufficient when the hadronic current acts more than once. The solution to the second problem involving handling large momentum transfers between baryons is probably correct and seems to be borne out by experiment. Finally, the successes of the model are summarized.

II. PRESENTATION OF THE MODEL

A. Space Wave Function

The basic assumption about the dynamics of the three-quark system is that the quarks effectively interact through a four-dimensional harmonic-oscillator potential.[1] This is motivated by the success of the symmetric nonrelativistic oscillator quark model,[3,4] and provides one way of extending those results to relativistic quark motion. The limitations and philosophy of this approach are discussed in Ref. 1.

We take the spatial wave function $\psi(r_1, r_2, r_3)$ to satisfy the equation

$$\{\Box_1+\Box_2+\Box_3+V_0-\tfrac{1}{27}\alpha^2[(x_1-x_2)^2+(x_1-x_3)^2+(x_2-x_3)^2]\}\psi(x_1,x_2,x_3)=0\,, \tag{1}$$

where $\Box_i=g^{\mu\nu}(\partial/\partial x_i^\mu)(\partial/\partial x_i^\nu)$ (the metric $g^{00}=+1$, $g^{11}=g^{22}=g^{33}=-1$ is used throughout the paper) and $(x_i-x_j)^2=g^{\mu\nu}(x_i-x_j)_\mu(x_i-x_j)_\nu$. The equation diagonalizes in the coordinates

$$\begin{aligned}\rho&=(\tfrac{1}{3})^{1/2}(x_1+x_2+x_3)\,,\\ \xi&=(\tfrac{1}{2})^{1/2}(x_1-x_2)\,,\\ \eta&=(\tfrac{1}{6})^{1/2}(x_1+x_2-2x_3)\,,\end{aligned} \tag{2}$$

and becomes

$$[\Box_\rho+\Box_\xi+\Box_\eta+V_0-(\tfrac{1}{9})\alpha^2(\xi^2+\eta^2)]\psi(\rho,\xi,n)=0\,.$$

The usual separation of variables $\psi(\rho,\xi,\eta)=U(\rho)V(\eta)W(\xi)$ gives

$$(\Box_\rho+c_1)U(\rho)=0\,, \tag{3a}$$

$$(\Box_\eta+c_2-\tfrac{1}{9}\alpha^2\eta^2)V(\eta)=0\,, \tag{3b}$$

$$(\Box_\xi+c^3-\tfrac{1}{9}\alpha^2\xi^2)W(\xi)=0\,, \tag{3c}$$

where $c_1 + c_2 + c_3 = V_0$, the depth of the potential. Equation (3a) has the familiar solution $U(\rho) = e^{-i(P\cdot\rho/\sqrt{3})}$, where P^μ is a four-vector satisfying $P^2 = 3c_1$. Equations (3b), (3c) are two independent four-dimensional harmonic-oscillator equations. We choose, as done in Ref. 2, solutions that are normalizable in all four variables, i.e., $\int_{-\infty}^{\infty}|V(\eta)|^2 \times d\eta_\mu < \infty$ and similarly for $W(\xi)$. This solution differs from the one chosen in Ref. 1, but with some modification (see the section on nucleon form factors) leads asymptotically to a dipole nucleon elastic form factor. Both (3b) and (3c) describe four independent quantum oscillators with the number of excitations satisfying a condition that leads to the mass spectrum

$$P^2 = M_0^2 + 2\alpha[n_x + n_y + n_z - n_t + l_x + l_y + l_z - l_t]$$
$$= M_0^2 + 2\alpha N, \tag{4}$$

where n_μ and l_μ are the number of excitations in the μth component of η and ξ, respectively, $M_0 = (4\alpha + 3V_0)^{1/2}$ is the mass of the system when all oscillators are in the ground state, and N is the total number of spacelike minus timelike excitations. In the rest frame $P^\mu = (M_0, 0, 0, 0)$, the ground-state wave function (normalized in relative coordinates ξ, η) is

$$\psi_0(\rho, \xi, \eta; \vec{0}) = e^{-iM_0\rho_t/\sqrt{3}}\left(\frac{\alpha}{3\pi}\right)^2 \times \exp[-\tfrac{1}{6}\alpha(\eta_t^2 + \vec{\eta}^2 + \xi_t^2 + \vec{\xi}^2)], \tag{5a}$$

which can be written covariantly as

$$\psi_0(\rho, \xi, \eta; \vec{\mathbf{P}}) = e^{-iP\cdot\rho/\sqrt{3}}\left(\frac{\alpha}{3\pi}\right)^2 \times \exp\left\{-\frac{\alpha}{6}\left[2\left(\frac{P\cdot\eta}{M_0}\right)^2 - \eta^2 + 2\left(\frac{P\cdot\xi}{M_0}\right)^2 - \xi^2\right]\right\}. \tag{5b}$$

The excited states differ from the ground state only by factors of products of Hermite polynomials in η_μ, ξ_μ. They have the form in their rest frame

$$\psi_{n_x n_y n_z, l_x l_y l_z} = N_{n_x n_y n_z, l_x l_y l_z} \times \left(\prod_{i=x,y,z} H_{n_i}(\eta_i) H_{l_i}(\xi_i)\right)\psi_0(\rho, \xi, \eta; \vec{0}), \tag{6}$$

where $N_{n_x n_y n_z, l_x l_y l_z}$ is the appropriate normalization constant. From the mass spectrum (4), we see that a sufficiently large number of timelike excitations implies states of imaginary mass. To avoid this catastrophe (for which we have no physical interpretation), we restrict ourselves as in Eq. (6) to spacelike excitations only in the system's rest frame. The price we pay is the loss of unitarity; various sum rules for the electromagnetic current cannot be satisfied unless the timelike excitations are included. To express the excited wave functions covariantly, the arguments of the Hermite polynomials become components of the spacelike four-vectors $\eta_\mu - (P\cdot\eta/M_0^2)P_\mu$, $\xi_\mu - (P\cdot\xi/M_0^2)P_\mu$ and $\psi_0(\rho, \xi, \eta; 0)$ becomes (5b).

The reader may wonder why these details of the spatial wave functions are given when one customarily describes such systems with creation and destruction operators acting on a ground state. We have found the algebraic techniques less useful since an orthogonality condition between states of different excitation in relative motion does not hold.[5]

A listing of the spatial wave functions through $N=2$ is given in Table I. These functions are states of definite angular momentum and symmetry type.[6]

B. Spin–Unitary-Spin Wave Functions

To obtain the spin–unitary-spin wave functions, we assume all baryon resonances are qqq composites of spin-$\frac{1}{2}$, SU(3)-triplet, positive-parity quarks, and that they fall into mass-degenerate representations of SU(6)⊗O(3), with SU(6) being the symmetry of spin and unitary spin and O(3) classifying the system's orbital angular momentum. The three quark states can be in the <u>56</u>-, <u>70</u>-, or <u>20</u>-dimensional representations of SU(6), while the spatial wave function of the previous section will determine the orbital angular momentum. In interactions, the baryons will generally reach relativistic motion, so we will use the relativistic generalization of SU(6).[7] There are well-known basic difficulties with this approach,[8] but electromagnetic interactions can always be described in a frame where the initial and final baryons and the photon (real or virtual) are colinear. So, the actual symmetry at the hadronic vertex is $SU(6)_W$, which is known to give good results,[9] and not the full $\tilde{U}(12)$ of Ref. 7.

The spin wave functions have a Dirac index $i = 1, \ldots, 4$ for each quark; furthermore, they are required to satisfy the Bargmann-Wigner[10] equations

$$(\not{P} - M)_{ii'}\psi_{i'jk} = (\not{P} - M)_{ii'}\psi_{ji'k} = (\not{P} - M)_{ii'}\psi_{jki'} = 0$$

for all i, j, k.

For quark spin $\frac{3}{2}$, the wave function D_{ijk} must be totally symmetric in all indices, which leads to [7]

$$D_{ijk} = \left(\frac{\not{P} + M}{2M}\gamma^\mu C\right)_{ij}(\psi_\mu(P))_k, \tag{7}$$

where $C\gamma_\mu C^{-1} = -\gamma_\mu^T$, all Γ matrices are in the metric of Ref. 11, and $\psi_\mu(P)$ is the Rarita-Schwinger spin-$\frac{3}{2}$ wave function of momentum P and mass M.

For quark spin $\frac{1}{2}$, two independent wave functions exist, depending upon the symmetry under quark interchange. The one that is antisymmetric under the interchange of quarks 1 and 2 must satisfy $\chi^{(\xi)}_{ijk} = -\chi^{(\xi)}_{jik}$ and $\chi^{(\xi)}_{ijk} + \chi^{(\xi)}_{jki} + \chi^{(\xi)}_{kij} = 0$, which leads to[7]

$$\chi^{(\xi)}_{ijk} = \left(\frac{\not{P}+M}{2M}\gamma_5 C\right)_{ij}(U(P))_k\,, \tag{8a}$$

where $U(p)$ is the usual Dirac spinor of momentum P and mass M. We use as the other independent spin-$\frac{1}{2}$ wave function, one that is symmetric under the interchange of quarks 1 and 2:

$$\chi^{(\eta)}_{ijk} = (\tfrac{1}{3})^{1/2}(\chi^{(\xi)}_{jki} - \chi^{(\xi)}_{kij})\,, \tag{8b}$$

where the superscript η just indicates it has the same symmetry under quark interchange that the variable η of Eq. (2) does, while $\chi^{(\xi)}_{ijk}$ has the symmetry of the variable ξ, and D_{ijk} has the symmetry of ρ.

The unitary-spin wave functions are familiar from SU(3). Each quark has a triplet index $\alpha = 1, 2, 3$. The state symmetric under interchange of any two quarks is the $\underline{10}$-dimensional representation $\Delta_{\alpha\beta\gamma}$. There are two independent $\underline{8}$-dimensional representations, just as there are two independent wave functions for quark spin $\frac{1}{2}$. The one antisymmetric under interchange of quarks 1 and 2 we will write as $U^{(\xi)}_{\alpha\beta\gamma} = \epsilon_{\alpha\beta\delta}B^\delta_\gamma$, where $\epsilon_{\alpha\beta\delta}$ is the Levi-Civita symbol and B^δ_γ is the usual baryon-octet matrix. The one symmetric under interchange of quark 1 and 2 we will write as $U^{(\eta)}_{\alpha\beta\gamma} = (\frac{1}{3})^{1/2}(U^{(\xi)}_{\beta\gamma\alpha} - U^{(\xi)}_{\gamma\alpha\beta})$. Finally, there is a one-dimensional singlet $\epsilon_{\alpha\beta\gamma}S$, where S is the particle state.

The spin and unitary-spin wave functions are combined into states of definite symmetry type to form the irreducible SU(6) representations in the rest frame of the system. We use the notation ${}^{2S+1}\underline{B}$ for a multiplet of quark spin S and SU(3) representation of dimension B. The multiplets ${}^4\underline{10}$ and ${}^2\underline{8}$ comprise the $\underline{56}$, the multiplets ${}^4\underline{8}$, ${}^2\underline{8}$, ${}^2\underline{10}$, and ${}^2\underline{1}$ the $\underline{70}$, and the multiplets ${}^4\underline{1}$ and ${}^2\underline{8}$ the $\underline{20}$. The combinations of wave functions that produce the SU(6) irreducible representations in the particle rest frame are displayed in Table II.

C. Baryon Spectrum and Over-all Wave Function

We assume the over-all baryon wave functions are symmetric under the interchange of any two

TABLE I. Spatial wave functions. The ground-state wave functions $\psi_0(\rho,\eta,\xi;\vec{P})$ and $\psi_0(\rho,\eta,\xi;\vec{0})$ are given in Eq. (5). All states are normalized to unity in their rest frame.

Excitations	Symmetry	Angular momentum	Wave function
$N=0$	Symmetric	$L=0$	$\psi_0(\rho,\eta,\xi;\vec{P})$
$N=1$	Mixed η	$L=1$	$\left(\frac{2\alpha}{3}\right)^{1/2}\left(\eta_\mu - \frac{P\cdot\eta}{M^2}P_\mu\right)\psi_0(\rho,\eta,\xi;\vec{P})$
	Mixed ξ	$L=1$	$\left(\frac{2\alpha}{3}\right)^{1/2}\left(\xi_\mu - \frac{P\cdot\xi}{M^2}P_\mu\right)\psi_0(\rho,\eta,\xi;\vec{P})$
$N=2$	Symmetric	$L=0$	$(\frac{1}{3})^{1/2}\left\{3+\frac{\alpha}{3}\left[\eta^2-\left(\frac{P\cdot\eta}{M}\right)^2+\xi^2-\left(\frac{P\cdot\xi}{M}\right)^2\right]\right\}\psi_0(\rho,\eta,\xi;\vec{P})$
		$L=2$ [a]	$\frac{4\alpha}{3}\left(\frac{\pi}{30}\right)^{1/2}[\vec{\eta}^2Y_2(\Omega_\eta)+\vec{\xi}^2Y_2(\Omega_\xi)]\psi_0(\rho,\eta,\xi;\vec{0})$
	Mixed η	$L=0$	$\frac{\alpha}{3\sqrt{3}}\left[\xi^2-\left(\frac{P\cdot\xi}{M}\right)^2-\eta^2+\left(\frac{P\cdot\eta}{M}\right)^2\right]\psi_0(\rho,\eta,\xi;\vec{P})$
	Mixed ξ		$\frac{2\alpha}{3\sqrt{3}}\left(\eta\cdot\xi-\frac{P\cdot\xi P\cdot\eta}{M^2}\right)\psi_0(\rho,\eta,\xi;\vec{P})$
	Mixed η	$L=2$ [a]	$\frac{4\alpha}{3}\left(\frac{\pi}{30}\right)^{1/2}[-\vec{\eta}^2Y_2(\Omega_\eta)+\vec{\xi}^2Y_2(\Omega_\xi)]\psi_0(\rho,\eta,\xi;\vec{0})$
	Mixed ξ		$\frac{1}{9}8\pi\alpha[\eta\xi Y_2(\Omega_\eta\Omega_\xi)]\psi_0(\rho,\eta,\xi;\vec{0})$ [b]
	Antisymm.	$L=1$	$\frac{\sqrt{2}\alpha}{3M}\epsilon_{\mu\nu\rho\sigma}\xi^\mu\eta^\nu P^\sigma\psi_0(\rho,\eta,\xi;\vec{P})$

[a] The $L=2$ states are expressed in the rest frame. In a moving frame $\eta_\mu \to \eta_\mu - (P\cdot\eta/M^2)P_\mu$ and similarly for ξ_μ.
[b] $Y_2(\Omega_\eta\Omega_\xi)$ is the angular part obtained by combining $\vec{\eta}$ and $\vec{\xi}$ to form $L=2$ and normalizing to unity over Ω_η, Ω_ξ.

quarks, but avoid the question of what type of statistics the quarks obey.[12,13] To discuss the supermultiplets we use the notation $[\underline{A}, L^P]_N$, where $\underline{A}$ is the SU(6) multiplet of the particle at rest, $\bar{L}$ is the internal angular momentum, N is the number of excitations of the spatial wave functions, and $P=(-1)^N$ is the parity of the system. The quark spin can combine with the orbital angular momentum to form the particles' total angular momentum J which we indicate by placing a suffix J on the $^{2S+1}\underline{B}_J$ notation for the constituent multiplets.

The ground state $[\underline{56}, 0^+]_0$ is that which contains the famous $^4\underline{10}_{3/2}$ and $^2\underline{8}_{1/2}$. The over-all wave function is the $N=0$ function from Table I multiplied by the $\underline{56}$ states of Table II.

The first excited state is the $[\underline{70}, 1^-]$, since the spatial wave function necessarily has a mixed symmetry which means the spin–unitary-spin state must also. The single excitation states have $L=1$, so the supermultiplet contains the nucleon states $^2\underline{8}_{1/2}$, $^2\underline{8}_{3/2}$; $^4\underline{8}_{1/2}$, $^4\underline{8}_{3/2}$, $^4\underline{8}_{5/2}$; and $^2\underline{10}_{1/2}$, $^2\underline{10}_{3/2}$. To form the over-all wave function of definite angular momentum J and projection J_z, we combine the space part with $L=1$ to the spin-unitary spin part which has $S=\frac{1}{2}$ or $\frac{3}{2}$ with the appropriate Clebsch-Gordan coefficients.

With a total of two excitations, we can combine the individual symmetry types[6] to form spatial states that are symmetric with $L=0$ and $L=2$, mixed with $L=0$ and $L=2$, and antisymmetric with $L=1$. The symmetric and mixed spatial states lie in the $[\underline{56}, 0^+]_2$, $[\underline{56}, 2^+]_2$, $[\underline{70}, 0^+]_2$, and $[\underline{70}, 2^+]_2$ which can accommodate all the nucleon resonances not assigned to the lower lying $[\underline{56}, 0^+]_0$, $[\underline{70}, 1^-]_1$. The antisymmetric spatial states belong in the $[\underline{20}, 1^+]_2$ which does not seem to be needed to classify presently observed resonances. This is interesting since models of interactions with perturbations that act on only one quark predict that the antisymmetric $\underline{20}$ could not be excited from states in the symmetric $\underline{56}$.

In Table III we assign the nucleon resonances to these supermultiplets. Most of the resonances have been observed; those that are speculative have an estimate of the mass and a "?" following the assignment.

D. Prescription for the Electromagnetic Current

The dynamical Eq. (1) is supposed to describe a composite fermion, yet it contains only Klein-Gordon operators and no Dirac matrices. This means if we assume minimal electromagnetic coupling (via $\partial_\mu \to \partial_\mu + ieA_\mu$), we will produce a current operator that cannot mix upper and lower components as does the operator in the usual spinor electrodynamics. A solution to this dilemma was proposed in Ref. 1, whereby the Klein-Gordon operators were written as $\Box_i = \gamma_\mu \partial_i^\mu \gamma_\nu \partial_i^\nu$, $i=1$, 2, or 3. As stated there, this does not affect the dynamical equation, but only defines the perturbational effects of the electromagnetic potential. We can construct the following Lagrangian density that leads to the dynamical Eq. (1):

TABLE II. Spin–unitary-spin wave functions. The functions are normalized to $2\bar{\Delta}^{\alpha\beta\gamma}\Delta_{\alpha\beta\gamma}$, $2\mathrm{Tr}(\bar{B}B)$, and $2\bar{S}S$ for the decimet, octet, and singlet, respectively.

SU(6) representation	SU(2)⊗ SU(3) substate	Wave function
$\underline{56}$	$^4\underline{10}$	$D_{ijk}\Delta_{\alpha\beta\gamma}$
	$^2\underline{8}$	$\frac{1}{2}[\chi_{ijk}^{(\xi)}U_{\alpha\beta\gamma}^{(\xi)} + \chi_{ijk}^{(\eta)}U_{\alpha\beta\gamma}^{(\eta)}]$
$\underline{70}$ η type	$^2\underline{10}$	$\chi_{ijk}^{(\eta)}\Delta_{\alpha\beta\gamma}$
	$^4\underline{8}$	$(\frac{1}{2})^{1/2}D_{ijk}U_{\alpha\beta\gamma}^{(\eta)}$
	$^2\underline{8}$	$\frac{1}{2}[\chi_{ijk}^{(\xi)}U_{\alpha\beta\gamma}^{(\xi)} - \chi_{ijk}^{(\eta)}U_{\alpha\beta\gamma}^{(\eta)}]$
	$^2\underline{1}$	$(\frac{1}{6})^{1/2}\chi_{ijk}^{(\eta)}\epsilon_{\alpha\beta\gamma}S$
$\underline{70}$ ξ type	$^2\underline{10}$	$\chi_{ijk}^{(\xi)}\Delta_{\alpha\beta\gamma}$
	$^4\underline{8}$	$(\frac{1}{2})^{1/2}D_{ijk}U_{\alpha\beta\gamma}^{(\xi)}$
	$^2\underline{8}$	$\frac{1}{2}[\chi_{ijk}^{(\xi)}U_{\alpha\beta\gamma}^{(\eta)} + \chi_{ijk}^{(\eta)}U_{\alpha\beta\gamma}^{(\xi)}]$
	$^2\underline{1}$	$(\frac{1}{6})^{1/2}\chi_{ijk}^{(\xi)}\epsilon_{\alpha\beta\gamma}S$
$\underline{20}$	$^2\underline{8}$	$\frac{1}{2}[\chi_{ijk}^{(\xi)}U_{\alpha\beta\gamma}^{(\eta)} - \chi_{ijk}^{(\eta)}U_{\alpha\beta\gamma}^{(\xi)}]$
	$^4\underline{1}$	$(\frac{1}{6})^{1/2}D_{ijk}\epsilon_{\alpha\beta\gamma}S$

$$L=\sum_{i=1}^{3}\left(\frac{\partial}{\partial x_i^{\mu}}\gamma_{(i)}^{\mu}\psi\right)^{\dagger}(\gamma_0\otimes\gamma_0\otimes\gamma_0)\left(\frac{\partial}{\partial x_i^{\nu}}\gamma_{(i)}^{\nu}\psi\right) - \bar{\psi}\{V_0-\tfrac{1}{27}\alpha^2[(x_1-x_2)^2+(x_1-x_3)^2+(x_2-x_3)^2]\}\psi, \tag{9}$$

where the index i refers to the quark on which the operator it labels acts, $\gamma_0\otimes\gamma_0\otimes\gamma_0$ is the direct product of matrices acting on the three quarks simultaneously, and $\bar{\psi}=\psi^{\dagger}(\gamma_0\otimes\gamma_0\otimes\gamma_0)$. Minimal electromagnetic coupling adds a piece to the Lagrangian, which to first order in the potential A_μ is

$$\int d^4x_1 d^4x_2 d^4x_3\, i\sum_{i=1}^{3} A_\mu(x_i)\{\bar{\psi}[\overleftarrow{\partial}_{\nu i}\gamma_{(i)}^{\nu}Q_i\gamma_{(i)}^{\mu} - \gamma_{(i)}^{\mu}Q_i\gamma_{(i)}^{\nu}\overrightarrow{\partial}_{\nu i}]\psi\}, \tag{10}$$

where Q is the SU(3) charge operator $[Q=\frac{1}{2}\lambda_3+(1/2\sqrt{3})\lambda_8$ with Gell-Mann's λ matrices$]$ measured in units of the electron charge. Each term in the sum is the same because of the symmetry of the wave function, so we need only consider one term, say for the third quark. The effective current acting at the position x_3 of the third quark is

$$j^{\mu}(x_3)=3i\int d^4x_1 d^4x_2\, \bar{\psi}[\overleftarrow{\partial}_{\nu 3}\gamma^{\nu}Q\gamma^{\mu}-\gamma^{\mu}Q\gamma^{\nu}\overrightarrow{\partial}_{\nu 3}]\psi, \tag{11}$$

where we have dropped the label 3 in the SU(2) ⊗ SU(3) space. This current differs from the one of Ref. 2 because of basic differences in the models. Although the same spatial wave functions are used, the authors of Ref. 2 do not use the relativistic SU(6) wave function of Sec. II B. Instead they take the nonrelativistic spin–unitary-spin wave func-

TABLE III. Quark-model nucleon resonance assignments and helicity amplitudes for photon excitation. The quantities A, B, Γ, and λ are defined by the equations at the end of Sec. II E.

Supermultiplet	Substate	Nucleon resonance	Width (MeV)	Target	F_+	F_-	F_0
$[\underline{56},0^+]_0$	$^2\underline{8}_{1/2}$	$P_{11}(938)$	···	p	0	$\sqrt{6}B$	$-\sqrt{6}\,\Gamma$
				n	0	$-\frac{2}{3}\sqrt{6}B$	0
	$^4\underline{10}_{3/2}$	$P_{33}(1236)$	120		$2B$	$-(2/\sqrt{3})B$	0
$[\underline{70},1^-]_1$	$^2\underline{8}_{1/2}$	$S_{11}(1535)$	120	p	0	$A+\lambda B$	$\lambda\Gamma$
				n	0	$-A-\frac{1}{3}\lambda B$	$-\lambda\Gamma$
	$^2\underline{8}_{3/2}$	$D_{13}(1520)$	120	p	$-(\frac{3}{2})^{1/2}A$	$-(\frac{1}{2})^{1/2}A+\sqrt{2}\,\lambda B$	$-\sqrt{2}\lambda\Gamma$
				n	$(\frac{3}{2})^{1/2}A$	$(\frac{1}{2})^{1/2}A-(\sqrt{2}/3)\lambda B$	$\sqrt{2}\lambda\Gamma$
	$^4\underline{8}_{1/2}$	$S_{11}(1700)$	250	p	0	0	0
				n	0	$-\frac{1}{3}\lambda B$	0
	$^4\underline{8}_{3/2}$	$D_{13}(1700)$?	150?	p	0	0	0
				n	$-(\frac{3}{5})^{1/2}\lambda B$	$-\frac{1}{3}(\frac{1}{5})^{1/2}\lambda B$	0
	$^4\underline{8}_{5/2}$	$D_{15}(1670)$	140	p	0	0	0
				n	$-(\frac{2}{5})^{1/2}\lambda B$	$(\frac{1}{5})^{1/2}\lambda B$	0
	$^2\underline{10}_{1/2}$	$S_{31}(1650)$	150		0	$A-\frac{1}{3}\lambda B$	$\lambda\Gamma$
	$^2\underline{10}_{3/2}$	$D_{33}(1670)$	240		$-(\frac{3}{2})^{1/2}A$	$-(\frac{1}{2})^{1/2}A-(\sqrt{2}/3)\lambda B$	$-\sqrt{2}\lambda\Gamma$
$[\underline{56},0^+]_2$	$^2\underline{8}_{1/2}$	$P_{11}(1470)$	250	p	0	$-(\frac{1}{2})^{1/2}\lambda^2 B$	$(\frac{1}{2})^{1/2}\lambda^2\Gamma$
				n	0	$(\sqrt{2}/3)\lambda^2 B$	0
	$^4\underline{10}_{3/2}$	$P_{33}(1690)$?	250?		$-(\frac{1}{3})^{1/2}\lambda^2 B$	$\frac{1}{3}\lambda^2 B$	0
$[\underline{56},2^+]_2$	$^2\underline{8}_{3/2}$	$P_{13}(1700)$?	200?	p	$-\frac{1}{2}(\frac{3}{10})^{1/2}\lambda A$	$(\frac{2}{5})^{1/2}\lambda(\frac{3}{4}A+\lambda B)$	$(\frac{2}{5})^{1/2}\lambda^2\Gamma$
				n	0	$-\frac{2}{3}(\frac{2}{5})^{1/2}\lambda^2 B$	0
	$^2\underline{8}_{5/2}$	$F_{15}(1688)$	125	p	$-(\frac{3}{10})^{1/2}\lambda A$	$(\frac{3}{5})^{1/2}\lambda(-\frac{1}{2}A+\lambda B)$	$-(\frac{3}{5})^{1/2}\lambda^2\Gamma$
				n	0	$-\frac{2}{3}(\frac{3}{5})^{1/2}\lambda^2 B$	0

tions[3,4] for the composite nucleon, add the two lower components by which Dirac and Pauli spinors at rest differ, and boost to relativistic motion. An analogous procedure is used for the excited final states. The nonrelativistic electromagnetic current operator is generalized in a corresponding manner, but requires further modification for current conservation when initial and final states differ in mass. In contrast, the current (11) is divergenceless as can be explicitly demonstrated since $\bar{\psi}$, ψ satisfy Eq. (1):

$$\partial_{\mu 3} j^{\mu}(x_3) = 3i \int d^4x_1 d^4x_2 [(\Box_3 \bar{\psi}) Q \psi - \bar{\psi} Q (\Box_3 \psi)]$$

$$= -3i \int d^4x_1 d^4x_2 \{[(\Box_1 + \Box_2)\bar{\psi}] Q\psi - \bar{\psi} Q [(\Box_1 + \Box_2)\psi]\} = 0 ,$$

where the last step follows from integration by parts. This result holds for any initial and final states that satisfy Eq. (1). Consequently, current conservation will hold only if Eq. (4) is satisfied, i.e., only in the limit of strictly degenerate multiplets. The empirical masses of different particles assigned to the same multiplets, of course, differ. We will maintain current conservation by allowing the α in Eq. (4) to depend on the experimental resonance mass.

Let us now derive a more convenient expression for the current. Expression (11) is for a configuration-space current, but we would like a momentum-space expression to utilize the conventional formulas of quantum electrodynamics. Expression (11) is the matrix element of the hadronic current at point x_3 between an initial baryon with momentum P and a final (possibly excited) baryon of momentum P'. Taking the four-dimensional Fourier transform gives

$$\int d^4x_3 \, e^{-iq\cdot x_3} j^{\mu}(x_3) = (2\pi)^4 \delta^4(P' - P - q)(-27)\{\bar{\psi}_{f,2\times 3} Q \psi_{i,2\times 3} [(P' - \tfrac{1}{3}P)^{\mu} I - 2i(\tfrac{2}{3})^{1/2} I^{\mu}] + \bar{\psi}_{f,2\times 3} Q i\sigma^{\mu\nu} q_{\nu} \psi_{i,2\times 3} I\} , \tag{12}$$

with

$$I = \int d^4\eta \, d^4\xi \, \psi_f(\xi, \eta; \vec{P}')\psi_i(\xi, \eta; \vec{P}) e^{i(2/3)^{1/2} q\cdot\eta} ,$$

$$I^{\mu} = \int d^4\eta \, d^4\xi \, \psi_f(\xi, \eta; \vec{P}') \left(\frac{\partial}{\partial \eta_{\mu}} \psi_i(\xi, \eta; \vec{P})\right) e^{i(2/3)^{1/2} q\cdot\eta} ,$$

where the $\psi_{2\times 3}$ are the SU(2)⊗SU(3) spin–unitary-spin wave functions and the $\psi(\xi, \eta; \vec{P})$ are the parts of the spatial wave functions that depend only on the relative coordinates since the center-of-mass motion factor $e^{-iP\cdot\rho(1/3)^{1/2}}$ has been integrated out. The momentum-space expression for the current is obtained by removing the $(2\pi)^4\delta^4(P' - P - q)$:

$$j^{\mu}(P', P) = 27 \bar{\psi}_{f,2\times 3} Q \{(P' - \tfrac{1}{3}P)^{\mu} I - 2i(\tfrac{2}{3})^{1/2} I^{\mu} + i\sigma^{\mu\nu} q_{\nu}\} \psi_{i,2\times 3} . \tag{13a}$$

We normalize the current by considering the time component for identical initial and final states. We replace Q by the identity operator since we are counting particles and not their charge and multiply by $\frac{1}{3}$ since we are not concerned with a superposition of ways in which the individual quarks can interact with the electromagnetic potential. This gives $6E_P \bar{\psi}_{i,2\times 3}(P)\psi_{i,2\times 3}(P)$, so the matrix element of the hadronic current normalized to one baryon per unit volume is

$$J^{\mu}(P', P) = j^{\mu}(P', P)[36 E_P E_{P'} \bar{\psi}_{f,2\times 3}\psi_{f,2\times 3} \times \bar{\psi}_{i,2\times 3}\psi_{i,2\times 3}]^{-1/2} , \tag{13b}$$

where E_P, $E_{P'}$ are the energies of the initial and final baryons, respectively.

In the Appendix we evaluate a useful integral from which the overlap integrals I, I^{μ} in Eq. (12) can be determined. As we shall see, these overlap integrals are closely related to the nucleon excitation form factors.

E. Matrix Elements of Hadronic Current

To study the electromagnetic excitation of the nucleon, we need to evaluate the hadronic current (13b) for excited final states. This is conveniently done in the rest frame of the excited resonance in a coordinate system with the photon (real or virtual) and nucleon momenta defining the z axis. The transverse photons couple to $J^{\pm} = \mp(\frac{1}{2})^{1/2}(J_x \pm iJ_y)$, the scalar photons couple to J_0, and we need not consider the longitudinal photons separately since current conservation requires $q_{\mu}J^{\mu} = 0 = q_0 J_0 - q_z J_z$. We assume the initial nucleon has spin projection $+\frac{1}{2}$ along the z axis, so J^+ produces a final state with projection $+\frac{3}{2}$, J^- one with $-\frac{1}{2}$, and J^0 one with $+\frac{1}{2}$. These specific matrix elements for the normalized current of Eq. (13b) multiplied by $E_P E_{P'}/2M'^2$ are given in Table III for experimentally observed resonances and some speculative ones. A factor $H/2$ has been removed from all matrix elements for reasons discussed in Sec. III A. They are expressed as functions of the invariants M^2, which is the initial mass squared, M'^2, which is the final mass squared, and $P\cdot P'$ for ease in presentation and not, of course, to suggest that they are Lorentz invariants. The quantities A, B, Γ, and λ that appear in this table are defined by the following equations:

$$A=\frac{G}{4i}\left(\frac{H}{3\alpha}\right)^{1/2}\frac{M'^2-M^2}{M'},$$

$$B=\frac{\sqrt{3}\,G}{4}\left(\frac{P\cdot P'-MM'}{MM'}\right)^{1/2}\frac{M+M'}{M'},$$

$$\Gamma=\frac{G}{4M'^2}\left(\frac{3H}{2}\right)^{1/2}\left(-\frac{2}{3}\frac{M'^2M^2}{P\cdot P'}+\frac{1}{3}M^2+MM'\right),$$

$$\lambda=\frac{M'^2}{P\cdot P'\sqrt{\alpha}\,i}\left(\frac{(P\cdot P')^2-M^2M'^2}{M'^2}\right)^{1/2},$$

with $H=(P\cdot P'+MM')/MM'$ and G is the function evaluated in the Appendix.

III. RESULTS OF THE MODEL

A. Nucleon Elastic Form Factors

For proton states of momenta P' and P, the hadronic current of Eq. (13b) becomes

$$J^\mu(P',P)=\frac{1}{(4E_PE_{P'})^{1/2}}G(q^2)\left(1-\frac{q^2}{4M^2}\right)\bar{u}_{P',\sigma'}\left[\left(\frac{2M^2-3q^2}{2M^2-q^2}\right)(P+P')^\mu+3i\sigma^{\mu\nu}q_\nu\right]u_{P,\sigma},\qquad(14\text{a})$$

with

$$G(q^2)=\left(1-\frac{q^2}{2M^2}\right)^{-2}\exp\left[\frac{1}{2\alpha}q^2\left(1-\frac{q^2}{2M^2}\right)^{-1}\right],$$

where the spinors are normalized so that $\bar{u}u=1$. If we normalize the spinors to $\bar{u}u=2M$ and perform a Gordon decomposition, we obtain the form

$$J^\mu(P',P)=\frac{1}{(4E_PE_{P'})^{1/2}}G(q^2)\left(1-\frac{q^2}{4M^2}\right)\bar{u}_{P',\sigma'}\left[\left(\frac{2M^2-3q^2}{2M^2-q^2}\right)\gamma^\mu+\left(\frac{4M^2}{2M^2-q^2}\right)\frac{i}{2M}\sigma^{\mu\nu}q_\nu\right]u_{P,\sigma},\qquad(14\text{b})$$

which allows us to read off the nucleon form factors[14]

$$F_{1p}(q^2)=G(q^2)\left(1-\frac{q^2}{4M^2}\right)\frac{2M^2-3q^2}{2M^2-q^2},$$

$$\kappa F_{2p}(q^2)=G(q^2)\left(1-\frac{q^2}{4M^2}\right)\frac{4M^2}{2M^2-q^2},$$

and determine charge and magnetic form factors

$$G_{Ep}(q^2)=G(q^2)\left(1-\frac{q^2}{4M^2}\right)\frac{2(M^2-q^2)}{2M^2-q^2},$$

$$G_{Mp}(q^2)=G(q^2)\left(1-\frac{q^2}{4M^2}\right)3.$$

So, as pointed out in Ref. 1, this model predicts the magnetic moment $G_{Mp}(0)=\mu_p$ for the proton to be 3 nuclear magnetons, when experimentally it is 2.79. If, for the moment, we neglect the factor $1-q^2/4M^2$, the model predicts $G_{Mp}(q^2)\sim q^{-4}$ as $q^2\to\infty$ which seems to be the case experimentally. In fact, the factor $G(q^2)$ provides a fit[2] to the experimental data for $G_{Mp}(q^2)/\mu_p$. This comparison is shown in Fig. 1. The fitted value of α is 0.39 GeV^2. When the Regge recurrence $F_{15}(1688)$ of the nucleon is identified as a state having two spatial excitations, we see from Eq. (4) that α should be approximately 0.5 GeV^2, so the fitted value is consistent to within about 20%.

The experimental situation[15,16] concerning the ratio $G_{Ep}/(G_{Mp}/\mu_p)$ is not clear, but the most recent measurements indicate that the ratio decreases with $-q^2$ in the region $1\lesssim -q^2\lesssim 3$ GeV^2. The model predicts an increase consistent with only a couple of experimental points[16] having large error bars. It, thus, appears that the model fails for this ratio. An extension of experimental measurements to larger q^2 would be very helpful in clarifying the extent of failure.

The hadronic current between neutron states of momentum P' and P is

$$J^\mu(P',P)=\frac{1}{(4E_PE_{P'})^{1/2}}G(q^2)\left(1-\frac{q^2}{4M^2}\right)\bar{u}_{P',\sigma'}\times\left[\left(\frac{2q^2}{4M^2-q^2}\right)(P'+P)^\mu-2i\sigma^{\mu\nu}q_\nu\right]u_{P,\sigma},\qquad(15)$$

which has charge and magnetic form factors

$$G_{En}(q^2)=0,$$

$$G_{Mn}(q^2)=G(q^2)\left(1-\frac{q^2}{4M^2}\right)(-2).$$

Experimentally, the charge form factor departs slightly from zero, but is certainly consistent with zero to within the accuracy of the model. The neutron magnetic moment $G_{Mn}(0)=\mu_n$ from the well-known SU(6) result is predicted to be $-\frac{2}{3}\mu_p$. The model gives −2 nuclear magnetons while experiment gives −1.91. With neglect of the factor $1-q^2/4M^2$, the scaling result is predicted in the form $G_{Mp}(q^2)/\mu_p = G_{Mn}(q^2)/\mu_n = G(q^2)$ which gives a good fit to the data.[2]

Let us now comment on the neglect of $1-q^2/4M^2$. When a nucleon absorbs a photon and changes its four-momentum, it also changes its spin wave function. A change in the composite spin wave function implies a change in the individual spin wave functions of all three identical constituent quarks. Yet, our model hypothesizes an explicit perturbation of only one quark. We can give no satisfactory solution to the problem of how the two unperturbed quarks change their spin wave functions. Instead, we propose the following approximate treatment: Discard the factor $H/2 = (P\cdot P' + MM')/2MM'$ present in most of the SU(2)⊗SU(3) matrix elements which roughly represents the overlap between the spin wave functions of the two unperturbed quarks. It is just this factor ($1-q^2/4M^2$ in the equal-mass case) that we have neglected above. We can give no other justification except that the correct asymptotic q^2 dependence seems to result from this procedure.

Other models[3,4,17] involving a harmonic interaction between quarks predict an exponential falloff in q^2 for the form factors. The wave function of Ref. 1 causes an overlap that exceeds unity and actually diverges as an exponential for large q^2; this forces the introduction of an adjustment factor for comparison with experimental data. In contrast, this model overcomes both defects by a choice of spatial wave functions that couples the space and time components of the relative variables η, ξ when the particle is in motion [Eq. (5b)]. This feature reduces the form factor falloff in large q^2 from an exponential to a power that is observed experimentally.

B. Photoproduction Matrix Elements

An experimental analysis of photoproduction matrix elements has been made by Walker[18,19] in which numerical values of helicity amplitudes are determined. To compare with Walker's results, we must multiply the normalized current (13b) by

$$\left(\frac{E_P}{M}\frac{E_{P'}}{M'}\frac{4\pi\alpha}{2\omega}\right)^{1/2},$$

where α is the fine-structure constant, and ω is the energy of the photon, since he uses a different normalization. In terms of the $F_\pm$ of Table III, the relationship to within a sign is

$$A_\pm = \left(\frac{4\pi\alpha M'}{M\omega}\right)^{1/2} F_\pm, \tag{16}$$

where A_+, A_- are Walker's $A_{3/2}$, $A_{1/2}$, respectively. Since we have not calculated the amplitude for the resonance to decay to a pion and nucleon, we cannot determine the sign of $A_{3/2}$, $A_{1/2}$. In Table IV we present a comparison of the predicted and experimental magnitudes for these amplitudes.

The principal experimental features of photoproduction discussed by Walker in Ref. 19 are reproduced by the model, but these are almost independent of the details of the spatial wave functions. Such features include (i) a dominant magnetic dipole amplitude in the first resonance, (ii) a small $A_{1/2}$ amplitude for the $D_{13}(1520)$, (iii) a small $A_{1/2}$ amplitude for the $F_{15}(1688)$, and (iv) small excitation of the $F_{15}(1688)$ from a neutron target. More quantitatively, the predictions for the first resonance are good to within 20%. Those for the

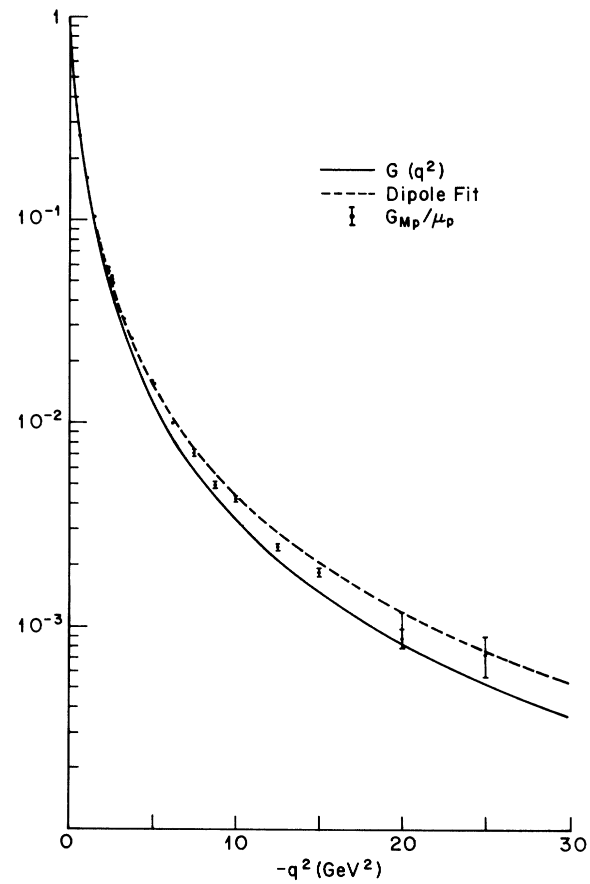

FIG. 1. Comparison of experimental G_{Mp}/μ_p with the empirical dipole fit $[(1-q^2/0.71)^{-2}]$ and $G(q^2)$ with $\alpha = 0.39$ GeV^2. Experimental data from Ref. 33.

second are good to within 40% for the $S_{11}(1535)$, are very close for the $A_{1/2}$ amplitude of the $D_{13}(1520)$, but off by a factor of 1.8 for the $A_{3/2}$ amplitude. The predictions for the third resonance are too small by a factor of 4.2 for the $F_{15}(1688)$ $A_{3/2}$ amplitude, but it should be noted that Walker's fit assumed only that the $F_{15}(1688)$ and the $D_{15}(1670)$ contributed to this resonance. However, other resonance contributions are quite possible, and, if present, would probably diminish the $F_{15}(1688)$ $A_{3/2}$ value. The $F_{15}(1688)$ $A_{1/2}$ amplitude is small as experimentally required.

C. Electroproduction of Nucleon Resonances

We have already seen that the model as modified in Sec. III A can successfully determine nucleon elastic form factors even at large momentum transfers. Considering the enormous effort in particle physics that has been made to understand these form factors, we view this success as quite significant. In this section, we shall extend the model to treat electroproduction of nucleon resonances.[20]

The reaction consists of incoming electron and nucleon of four-momenta $P_e=(E,\vec{P}_e)$ and $P=(E_1,\vec{P})$, respectively, scattering to outgoing electron and baryon system of four-momenta $P_e'=(E',\vec{P}_e)$ and $P'=(E_2,\vec{P}')$, respectively. We assume the reaction proceeds through one-photon exchange. Because the final baryon system can have any mass, the hadron vertex depends on two invariants which we can take as the momentum transfer or virtual-photon mass squared

$$q^2=(P'-P)^2=(P_e'-P_e)^2=-4EE'\sin^2(\tfrac{1}{2}\theta) \qquad (17)$$

and this final baryon mass M'. Equation (17) neglects the electron mass as we will do throughout.

As Hand showed,[21] we can express the doubly differential cross section for this reaction in the laboratory frame as

$$\frac{d\sigma}{dE'd\Omega}=\frac{\alpha^2}{(-q^2)}\frac{E'}{E}\frac{K}{4\pi^2\alpha}\left[\left(2+\frac{\cot^2(\frac{1}{2}\theta)}{1-q_0{}^2/q^2}\right)\sigma_t(M',q^2)+\frac{\cot^2(\frac{1}{2}\theta)}{1-q_0{}^2/q^2}\sigma_0(M',q^2)\right], \qquad (18)$$

where $q_0=E-E'$ and $K=(M'^2-M^2)/2M$ is the equivalent energy for a real photon to photoproduce the final state of mass M'. $\sigma_t(M',q^2)$ is the cross section for nucleon absorption of transverse photons of mass$^2=-q^2$ at energy M'. In terms of the hadronic current (13b), we have when the final state is a single particle

$$\begin{aligned}\sigma_t(M',q^2)&=\frac{4\pi^2\alpha}{K}\frac{E_P}{2M}\sum_{\sigma,\sigma'}\int d^3P'\,\delta^4(P'-P-q)\tfrac{1}{2}(|J_x|^2+|J_y|^2)\\&=\frac{4\pi^2\alpha}{K}\frac{E_PE_{P'}}{M}\delta((P+q)^2-M'^2)\sum_{\sigma,\sigma'}\tfrac{1}{2}[|J_x|^2+|J_y|^2]\\&=\frac{4\pi^2\alpha}{K}\delta(M'^2-(P+q)^2)\frac{2M'^2}{M}\sum_{\sigma,\sigma'}\tfrac{1}{2}[|F_+|^2+|F_-|^2],\end{aligned} \qquad (19a)$$

where the $F_\pm$ come from Table III. $\sigma_0(M',q^2)$ is a combination of cross sections for nucleon absorption of longitudinal and scalar photons which has the same value, because of current conservation, in all frames reached from the laboratory frame by a boost along the final-baryon momentum direc-

TABLE IV. Photoproduction matrix elements. The quantities A, B, and λ are defined by the equations at the end of Sec. II E.

State	Target	Model $A_{3/2}$	Exp $A_{3/2}$	Model $A_{1/2}$	Exp $A_{1/2}$
$P_{33}(1236)$	p	0.202	0.244	0.117	0.138
$S_{11}(1535)$	p			0.122	0.096
	n			0.085	0.118
$D_{13}(1520)$	p	0.082	0.151	0.031	0.026
	n	0.082	0.132	0.021	
$D_{15}(1670)$	p	0	0.040	0	~0
	n	0.036		0.026	
$S_{31}(1650)$	p			0.040	
$D_{33}(1670)$	p	0.071		0.068	
$P_{11}(1470)$	p			0.039	
	n			0.026	
$F_{15}(1688)$	p	0.033	0.139	0.024	~0
	n	0	~0	0.032	

tion. In terms of the hadronic current

$$\sigma_0(M', q^2) = \frac{4\pi^2\alpha}{K}\frac{E_p}{2M}\sum_{\sigma,\sigma'}\int d^3p'\,\delta^4(P'-p-q)\,|J_0|^2\left(\frac{-q^2}{q^{*2}}\right)$$

$$= \frac{4\pi^2\alpha}{K}\delta(M'^2-(P+q)^2)\frac{2M'^2}{M}\left(\frac{-q^2}{q^{*2}}\right)\sum_{\sigma,\sigma'}|F_0|^2, \quad (19b)$$

where the photon's momentum q^* and the amplitude F_0, given by Table III, are evaluated in the final-baryon rest frame. For single-particle resonance production, we see from Eqs. (19) that both σ_t and σ_0 have a factor $\delta(M'^2-(P+q)^2)$. To compare with experiment, we approximate this by a Breit-Wigner form

$$\delta(M'^2-(P+q)^2) \approx \frac{M'\Gamma/\pi}{[(P+q)^2-M'^2]^2+M'^2\Gamma^2} \xrightarrow[(P+q)^2=M'^2]{} \frac{1}{\pi M'\Gamma}, \quad (20)$$

where Γ is the width of the resonance, and the last expression is the approximate form of the δ function at the peak cross section. So the theoretical σ_t and σ_0 are given by

$$\sigma_t = \frac{4\pi\alpha}{K}\frac{2M'}{M\Gamma}[|F_+|^2+|F_-|^2],$$
$$\sigma_0 = \frac{4\pi\alpha}{K}\frac{2M'}{M\Gamma}|F_0|^2\left(\frac{-2q^2}{q^{*2}}\right). \quad (21)$$

Most experiments detect only the final electron, so the cross section of Eq. (18) is measured, and not σ_t or σ_0 individually. Consequently, we will compare the cross sections (21) with the quantity

$$\Sigma_t \equiv \sigma_t + \epsilon\sigma_0 = \frac{1}{\Gamma_t}\frac{d\sigma}{dE'd\Omega}, \quad (22a)$$

with

$$\epsilon = \cot^2(\tfrac{1}{2}\theta)\left[2\left(1-\frac{q_0^2}{q^2}\right)+\cot^2(\tfrac{1}{2}\theta)\right]^{-1}, \quad (22b)$$

$$\Gamma_t = \frac{\alpha K E'}{4\pi^2 E(-q^2)}\left[2+\left(1-\frac{q_0^2}{q^2}\right)^{-1}\cot^2(\tfrac{1}{2}\theta)\right], \quad (22c)$$

as measured at the value of E' and q^2 corresponding to the resonant peak. These comparisons are shown in Figs. 2 through 6. We have evaluated σ_t, σ_0 for all resonances predicted by the quark model that could contribute to the first, second, or third peaks. This includes some unobserved states having the same quark spin and SU(3) rep-

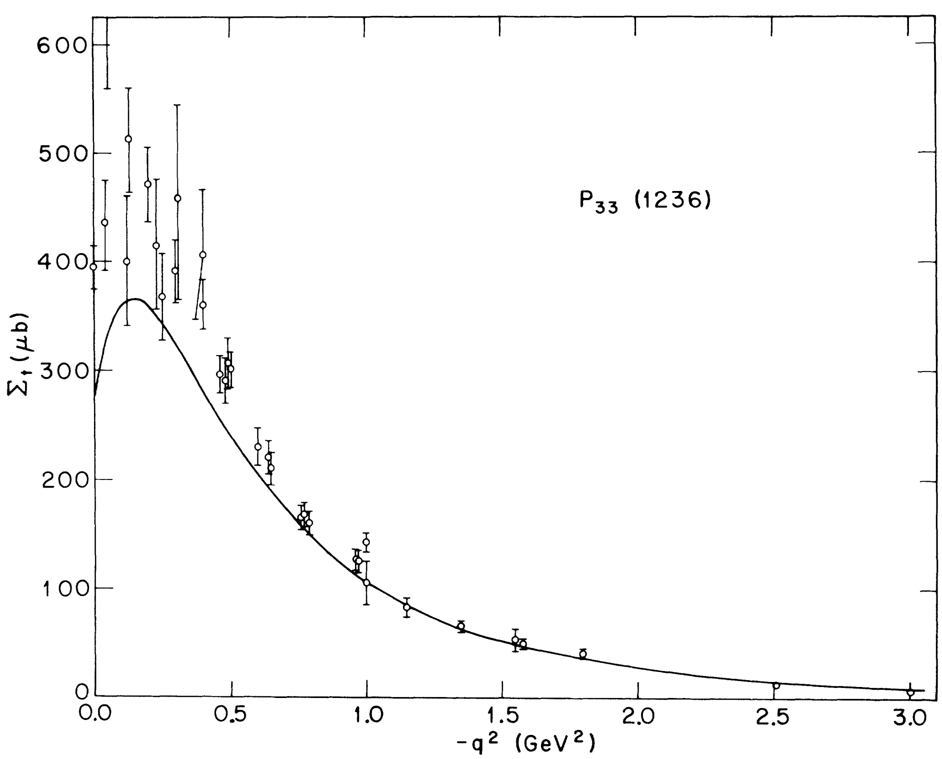

FIG. 2. Experimental Σ_t and theoretical σ_t and σ_0 for $P_{33}(1236)$ from proton target. Experimental data from Refs. 31 and 32.

resentation as observed states, so we have assumed they have roughly the same masses.[22]

The model has three types of selection rules. First, the time component of the current cannot couple states of differing quark spin, so σ_0 vanishes for all final states of quark spin $\frac{3}{2}$ produced from nucleon targets. Second, σ_0 vanishes for the production of a $^2\underline{8}$ of a $\underline{56}$ from a neutron target. In the model this selection rule is a generalization of the usual quark-model prediction that the charge form factor G_{En} of the neutron vanish. Third, the Moorhouse[23] selection rule given for the nonrelativistic quark model holds since the relativistic model has the same symmetries under quark interchange and corresponding unitary-spin matrix elements. This means the $^4\underline{8}_{1/2}$, $^4\underline{8}_{3/2}$, and $^4\underline{8}_{5/2}$ of the $[\underline{70}, 1^-]_1$ are not electroproduced from the proton.

The first resonance is entirely due to the $P_{33}(1236)$ which is in the $[\underline{56}, 0^+]_0$ with the nucleon $P_{11}(938)$. In the model which has no mass-splitting mechanism their masses are degenerate, but the kinematic factors in Eq. (21) require the experimental masses. We treat this problem by calculating the form factor G of Table III and the Appendix as if no mass splitting were present which means $M = M'$. We take M to be the proton mass and $\alpha = 0.39$ GeV^2 as in the elastic form-factor fit, which assumes that the elastic form factor is appropriate for electromagnetic transitions in the $[\underline{56}, 0^+]_0$. In the remaining terms in Eq. (21) we use the experimental masses $M' = 1.236$ GeV, $M = 0.938$ GeV to handle the kinematic factors properly. The comparison of σ_t with the experimental Σ_t is shown in Fig. 2. σ_t in the low $-q^2$ region ($\lesssim 0.5$ GeV^2) is too low by the order of 30%, but experimental agreement is good for $-q^2 \gtrsim 0.5$ GeV^2.

The Roper "resonance" $P_{11}(1470)$ of pion-nucleon phase shifts does not appear experimentally in electroproduction. Although its status as a resonance is not settled, we can assign it to a quark-model classification to derive a prediction. If we assign it to the $[\underline{20}, 1^+]_2$, then we can explain its absence in electroproduction. This follows from the fact that interactions involving only one quark cannot connect a $\underline{56}$ and a $\underline{20}$. The more conventional assignment to the $[\underline{56}, 0^+]_2$ gives the result of Fig. 5. In this and all the following calculations we have used $M = 0.938$ GeV^2, M' equals the resonance mass, and $\alpha = (M'^2 - M^2)/2N$ with N the number of spatial excitations. This very small cross

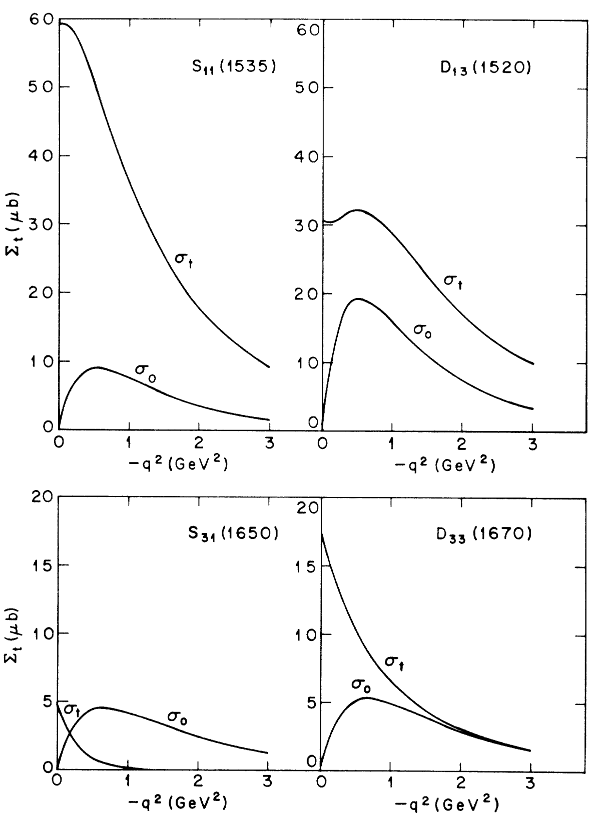

FIG. 3. Theoretical σ_t and σ_0 for the individual states $S_{11}(1535)$, $D_{13}(1520)$, $S_{31}(1650)$, and $D_{33}(1670)$ from a proton target.

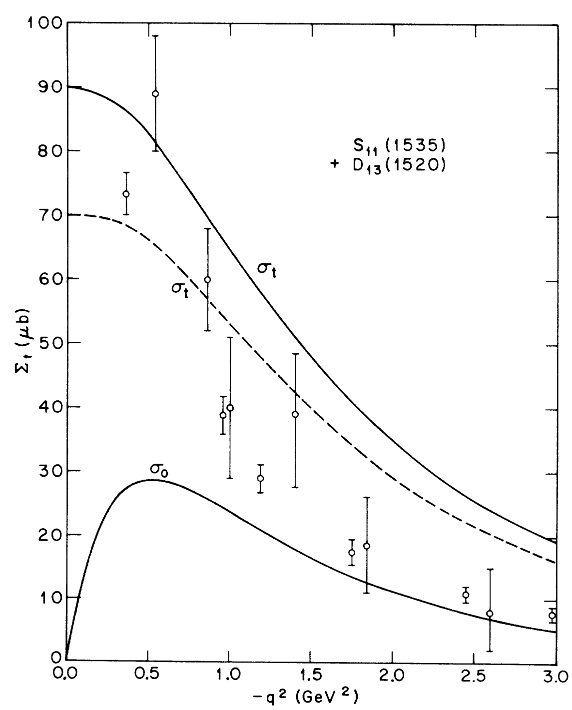

FIG. 4. Experimental Σ_t and theoretical σ_t and σ_0 for the second resonance peak from a proton target. The dashed curve is for S_{11}-state mixing with an angle of 35°. Experimental data from Refs. 31 and 32.

section would not produce a resonant "bump" unless accompanied by other resonances with approximately the same mass.

The second peak receives contributions from the $S_{11}(1535)$ and the $D_{13}(1520)$, individually shown in Fig. 3 and collectively compared to Σ_t in Fig. 4. The theoretical values are too large for $-q^2 \gtrsim 0.5$ GeV^2, but the shape of the falloff appears correct. Also, the contribution to σ_t from the $S_{11}(1535)$ is about twice that of the $D_{13}(1535)$ for low q^2, contrary to results of photoproduction analyses.[17, 24] If the physical states $S_{11}(1535)$ and $S_{11}(1700)$ were mixtures of the $^2\underline{8}_{1/2}$ and $^4\underline{8}_{1/2}$ in the $[\underline{70}, 1^-]_1$, since the $^4\underline{8}_{1/2}$ cannot contribute, the cross section for $S_{11}(1535)$ would be decreased by $\cos^2\theta$, where θ is the mixing angle. Qualitatively, this decreases both the theoretical σ_t and the ratio of the $S_{11}(1535)$ to $D_{13}(1520)$ production as called for by experiment. To give some quantitative feeling for the effect of mixing, we present the dashed curves for σ_t in Fig. 4 calculated for the mixing angle of 35° determined by Faiman and Hendry.[25] The mixing hardly affects σ_0.

The third peak has many contributors in the quark model. These are the $S_{31}(1650)$, $D_{33}(1670)$, $F_{15}(1688)$, $P_{33}(1685)$, and $P_{13}(1700)$ whose individual cross sections are shown in Figs. 3 and 5. The speculative $P_{33}(1685)$ is the partner of the $P_{11}(1470)$ in the $[\underline{56}, 0^+]_2$ as the $P_{33}(1236)$ is the partner of the nucleon in $[\underline{56}, 0^+]_0$. We arrive at this mass by assuming the same splitting in mass squared, i.e., 0.65 GeV^2. Actually, its inclusion is unimportant since, as shown in Fig. 5, its contribution to the resonant peak is negligibly small. The unobserved $P_{13}(1700)$ is the $^2\underline{8}_{3/2}$ in the $[\underline{56}, 2^+]_2$ and should be roughly degenerate in mass with the $^2\underline{8}_{5/2}$ $F_{15}(1688)$ as Ravndal points out in Ref. 20 (see also Ref. 22). Conventionally, the $P_{13}(1860)$ is assigned to the $^2\underline{8}_{3/2}$ at the loss of approximate mass degeneracy. The $P_{13}(1700)$ is an important contributor to the third peak, accounting for about one-fourth of the low q^2 cross section. Photoproduction analysis for the third resonance[18] revealed a dominant $F_{15}(1688)$ contribution, not borne out by this model, but it assumed that the $D_{15}(1670)$ was the only other contributor. Theoretically, at least, the $D_{15}(1670)$ is not produced, so the analysis may have been too restrictive. For the third peak, comparison with experiment is shown in Fig. 6. The theoretical value is too small at low q^2, but looks good for $-q^2 \gtrsim 0.8$ GeV^2. If mixing is considered, the situation improves as shown by the

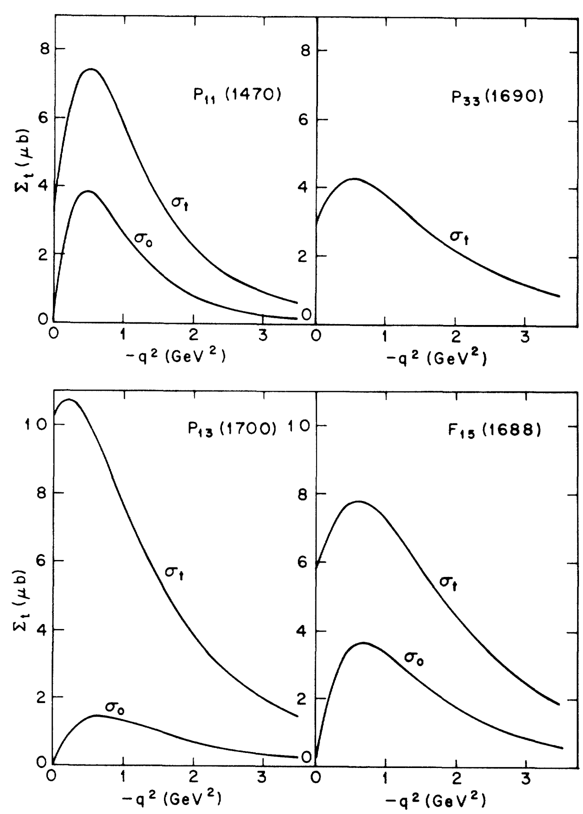

FIG. 5. Theoretical σ_t and σ_0 for the individual states $P_{11}(1470)$, $P_{33}(1690)$, $P_{13}(1700)$, and $F_{15}(1688)$ from a proton target.

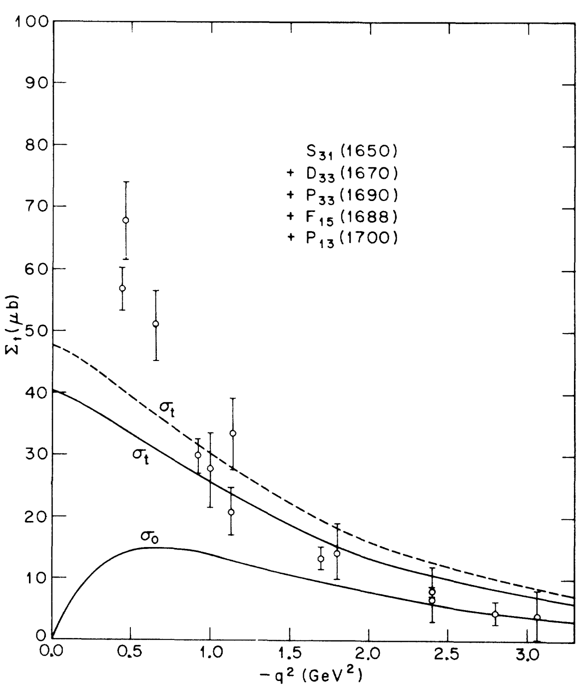

FIG. 6. Experimental Σ_t and theoretical σ_t and σ_0 for the third resonance peak from a proton target. The dashed curve is for S_{11}-state mixing with an angle of 35°. Experimental data from Refs. 31 and 32.

dashed curve for a mixing angle of 35°.

Brasse *et al.*[26] have found a fit to the ratio R of the scalar cross section σ_0 to the transverse cross section σ_t which satisfies $R \leqslant 0.2$ for $0.5 \leqslant -q^2 \leqslant 2.0$ and $R \leqslant 0.35$ for $2.0 < -q^2 \leqslant 4.0$. For the second resonance the model predicts an R which is compatible with this fit for $2.0 < -q^2 \leqslant 4.0$, but too large by, at most, a factor of 2 for $0.5 \leqslant -q^2 \leqslant 2.0$. For the third resonance the model predicts an R which is too large by, at most, a factor of 1.4 for $2.0 < -q^2 \leqslant 4.0$ and, at most, a factor of 2.8 for $0.5 \leqslant -q^2 \leqslant 2.0$. When S_{11} state mixing is considered, the model predictions are essentially unchanged.

IV. DISCUSSION

A glaring problem with the model is, of course, the mass spectrum which allows imaginary mass when the number of timelike excitations becomes sufficiently large. To eliminate the imaginary masses for which we have no interpretation, we restrict ourselves to only spacelike excitations in the rest frame of the particle. This procedure provides a partial solution in the sense that no difficulties arise if the current operator acts only once, as it does in all processes considered in this paper. We have not formulated a way to handle cases in which it acts more than once, but in such cases the timelike excited states will certainly appear in sums over a complete set of intermediate states unless unitarity is forfeited. These are problems we have not solved.

A second problem is the elimination of the factor $H/2$ from all matrix elements of the hadronic current. We can only justify this step by the successful fits to the elastic form factors and general good agreement with electroproduction data. In further defense of this procedure, it is interesting to notice the following: The structure function $W_2(\nu, q^2)$ of Bjorken and Walecka[27] can be expressed as

$$W_2(\nu, q^2) = \frac{K}{4\pi^2\alpha}\,\frac{q^2}{q^2-\nu^2}(\sigma_t + \sigma_0'), \tag{23}$$

where

$$\nu = P\cdot q/M = \frac{M'^2 - M^2 - q^2}{2M}.$$

When $\nu, -q^2 \to \infty$ as $\omega \equiv 2M\nu/q^2$ is held fixed, $\nu W_2(\nu, q^2)$ appears to "scale" and become a function of ω only.[28] Experimentally, this behavior appears for $-q^2 \gtrsim 1$ GeV^2, $M' \gtrsim 1.8$. Miller[29] determined empirically that the scaling function vanishes as $(\omega-1)^3$ when $\omega \to 1$. For fixed $M' \geqslant 1.8$, this limit is reached as $-q^2 \to \infty$, so Miller's result shows

$$\nu W_2 \underset{q\to-\infty,\, M'\text{fixed}}{\sim} \left(1 + \frac{M'^2 - M^2 - q^2}{q^2}\right)^3 \sim (q^2)^{-3}. \tag{24}$$

So, Eq. (24) shows in this limit that $\sigma_t + \sigma_0 \sim (q^2)^{-3}$. If we believe the duality of Bloom and Gilman,[30] the resonances ($M' \leqslant 1.8$) will have the same behavior in this limit. From Eq. (21) and Table III we see that, indeed, $\sigma_t + \sigma_0 \sim (q^2)^{-3}$ since $\sigma_t \sim (q^2)^{-3}$, $\sigma_0 \sim (q^2)^{-4}$. The $H/2$ elimination is consistent with all experimental information presently available.

On the positive side, the main successes of the model with this choice of spatial wave function are the prediction of the nucleon elastic form factors for all measured q^2 and the general agreement with the electroproduction data. The elastic form factor results are not new,[2] nor a complete success because of the trouble with the ratio $G_{Ep}(q^2)/G_{Mp}(q^2)$. Nevertheless, they are features of the model that require adjustment of only the parameter α to achieve a highly successful fit to $G_{Mp}(q^2)$; no other model can make such a claim and tremendous theoretical effort has been devoted to understanding nucleon form factors. This leads us to believe that the model is on the right track. The predictions for electroproduction are new and give good general agreement, although not as detailed as one might wish. The hope is, of course, that resolution of the two problems discussed above will produce closer agreement with experiment.

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APPENDIX

To determine the matrix elements of the hadronic current (13), we need to evaluate the integrals I and I^μ of Eq. (12). Both $\psi_f(\xi, \eta, P)$ and $\psi_i(\xi, \eta, P)$ are linear combinations of terms whose ξ and η dependence factors, so I, I^μ will be sums of integrals of the form

$$\int d^4\eta d^4\xi \exp\left\{-\frac{\alpha}{3}\left[\left(\frac{P'\cdot\eta}{M'}\right)^2+\left(\frac{P'\cdot\xi}{M'}\right)^2+\left(\frac{P\cdot\eta}{M}\right)^2+\left(\frac{P\cdot\xi}{M}\right)^2-\eta^2-\xi^2\right]\right\} e^{i(Q\cdot\eta+Q'\cdot\xi)}L_1(\eta)L_2(\xi),$$

where $L_1(\eta)$, $L_2(\xi)$ are polynomials in the components of η, ξ, respectively. Since $L_1(\eta)e^{iQ\cdot\eta}$ can be generated by a suitable linear combination of derivatives of $e^{iQ\cdot n}$ with respect to the components of Q [and analogously for $L_2(\xi)e^{iQ'\cdot\xi}$], we can easily calculate I, I^μ from a knowledge of

$$F(Q)\equiv\left(\frac{\alpha}{3\pi}\right)^2\int d^4\eta \exp\left\{-\frac{\alpha}{3}\left[\left(\frac{P'\cdot\eta}{M'}\right)^2+\left(\frac{P\cdot\eta}{M}\right)^2-\eta^2\right]\right\} e^{iQ\cdot\eta}. \tag{A1}$$

$F(Q)$ is a Lorentz invariant, so we will evaluate it in the convenient frame where $P=(M,0,0,0)$, $P'=(E,0,0,P)$, $Q=(Q_t,Q_x,0,Q_z)$. In this frame

$$\begin{aligned}F(Q)=&\left(\frac{\alpha}{3\pi}\right)^2\int d^4\eta \exp\left\{-\frac{\alpha}{3}\left[\eta_x^2+\frac{3i}{\alpha}Q_x\eta_x+\left(\frac{3i}{2\alpha}Q_x\right)^2-\left(\frac{3i}{2\alpha}Q_x\right)^2\right]\right\}e^{-\frac{\alpha}{3}\eta_y^2}\\&\times\exp\left\{-\frac{\alpha}{3}\left[\left(\frac{E}{M'}\right)^2\eta_t^2-2\left(\frac{PE}{M'^2}\eta_z+\frac{3i}{2\alpha}Q_t\right)\eta_t+\left(\frac{P}{M'}\eta_z+\frac{M'}{E}\frac{3i}{2\alpha}Q_t\right)^2\right]\right\}\\&\times\exp\left\{-\frac{\alpha}{3}\left[\eta_z^2+\frac{3i}{\alpha}\left(Q_z-\frac{P}{E}Q_t\right)+\left(\frac{3i}{2\alpha}\right)^2Q_z-\frac{P}{E}Q_t\right]^2-\left(\frac{3i}{2\alpha}\right)^2\left(Q_z^2-\frac{2P}{E}Q_zQ_t+Q_t^2\right)\right\}\\=&\frac{M'}{E}\exp\left[-\frac{3}{4\alpha}\left(Q_x^2+Q_z^2+Q_t^2-\frac{2P}{E}Q_zQ_t\right)\right].\end{aligned} \tag{A2}$$

In terms of the invariants,

$$Q_x^2+Q_z^2+Q_t^2-\frac{2P}{E}Q_zQ_t=2\frac{P'\cdot Q\,P\cdot Q}{P\cdot P'}-Q^2, \tag{A3}$$

so we can write

$$F(Q)=\frac{MM'}{P\cdot P'}\exp\left[-\frac{3}{4\alpha}\left(2\frac{P'\cdot Q\,P\cdot Q}{P\cdot P'}-Q^2\right)\right]. \tag{A4}$$

In the case of the η integration we see from I, I^μ that $Q=(\frac{2}{3})^{1/2}(P'-P)$, whereas in the ξ integration, $Q=0$, so that every term in I, I^μ will contain the factor

$$G(q^2)\equiv F((\tfrac{2}{3})^{1/2}(P'-P))F(0)=\left(\frac{2MM'}{M^2+M'^2-q^2}\right)^2\exp\left(-\frac{(M^2-M'^2)^2-q^2(M^2+M'^2)}{2\alpha(M^2+M'^2-q^2)}\right). \tag{A5}$$

In the case of the elastic form factors, $M=M'$, and $G(q^2)$ reduces to the expression in Eq. (14a)

$$G(q^2)=\left(1-\frac{q^2}{2M^2}\right)^{-2}\exp\left[\frac{1}{2\alpha}q^2\left(1-\frac{q^2}{2M^2}\right)^{-1}\right]. \tag{A6}$$

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$$\begin{aligned}I_{n,m}(\nu,\mu)\equiv&\left(\frac{\alpha}{\pi}\right)^2\int d^4n\, H_n\left(\sqrt{\alpha}\left(n_\nu-\frac{P\cdot n}{M^2}P_\nu\right)\right)\\&\times H_m\left(\sqrt{\alpha}\left(n_\mu-\frac{P'\cdot n}{M'^2}P'_\mu\right)\right)\\&\times\exp\left\{\alpha\left[n^2-\left(\frac{P'\cdot n}{M'}\right)^2-\left(\frac{P\cdot n}{M}\right)^2\right]\right\}.\end{aligned}$$

If we evaluate in the rest frame of the initial particle and choose ν,μ to be along the direction of motion of the final state (which we define as the z axis), then

$$I_{n,m}(z,z)=\begin{cases}\dfrac{M'}{E'}2^n\left(\dfrac{P'}{M'}\right)^{m-n}\dfrac{m!}{[\frac{1}{2}(m-n)]!} & \text{if } m\geq n \text{ and } (-1)^{m+n}=+1\\ 0 & \text{if } m<n \text{ or } (-1)^{m+n}=-1,\end{cases}$$

where P', E' are the final-state momentum, energy, respectively, and the Hermite polynomials satisfy

$$\int_{-\infty}^{\infty} H_n(x)\, H_m(x)\, e^{-x^2}\, dx = \sqrt{\pi}\, 2^n n!\, \delta_{mn}\,.$$

Notice that when $P' = 0$ the usual orthogonality condition is recovered. The point is that the overlap integral is not a Lorentz scalar, so its value is frame-dependent and cannot always be zero for arbitrary relative motion when $m \neq n$.

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