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[17]The validity of the quark charge sum rule has since become less plausible. Modifying this constraint these authors obtain

$$F_2(\omega) \to 0.17 + 0.339\omega^{1/2} + 3.42\omega^{3/2}$$

instead of Eq. (17). (F. Close, private communication.) With this fit we obtain $R(Q^2=\infty,\ t=0) \simeq 0.152$, still a rather small number. This fit and the one in Eq. (8) are favored by a recent analysis based on complex scaling [N. Khuri, Phys. Rev. D 5, 462 (1972)].

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Inelastic Scattering from Deuterium in the Impulse Approximation*

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We evaluate in detail the differential cross sections for arbitrary processes $ad \to bpp$ and $ad \to b'pn$ in the impulse approximation, with spin and isospin effects and the deuteron D state properly included. Approximations valid when a single term of the free-nucleon amplitude dominates are given.

I. INTRODUCTION

The only way to study inelastic scattering from neutrons is by observing the corresponding inelastic processes on deuterium targets. A method of relating the scattering from a nucleon bound in the deuteron and that from a free nucleon is provided by the impulse and closure approximations. The form of the result depends crucially upon proper evaluation of matrix elements between appropriate states symmetrized in spin and isospin. This is a formidable task, which heretofore has been carried through only in a few special cases.[1] In this paper we shall evaluate the impulse approximation for arbitrary deuteron breakup reactions $ad \to bpp$ and $ad \to b'pn$ with spin and isospin properly included.

II. CALCULATION OF DIFFERENTIAL CROSS SECTION

We write the final-state wave function in the laboratory system as the direct product of the scattered particle state and a two-nucleon state decomposed into spatial and spin-isospin parts,

$$|\psi_f\rangle = |b\rangle |\varphi_f\rangle |\chi_f\rangle . \tag{1}$$

Neglecting final-state interactions,[2] we shall take the spatial state as simple plane waves,

$$|\varphi_{f\pm}\rangle = 2^{-1/2}(|\vec{p}_1\rangle|\vec{p}_2\rangle \pm |\vec{p}_2\rangle|\vec{p}_1\rangle) \tag{2}$$

describing the symmetrized combinations of states with three-momenta $\vec{p}_1$ and $\vec{p}_2$. The spin-isospin wave function $|\chi_f\rangle$ is correspondingly symmetrized so that $|\varphi_f\rangle|\chi_f\rangle$ is antisymmetric. The initial state is

$$\begin{aligned} |\psi_i\rangle &= |a\rangle|\varphi_d\rangle|\chi_d\rangle , \\ |\varphi_d\rangle &= 2^{-1/2}\int d^3p\, \varphi(\vec{p})|\vec{p}\rangle|-\vec{p}\rangle , \end{aligned} \tag{3}$$

where $\varphi(\vec{p})$ is the momentum-space deuteron wave function and $|\chi_d\rangle$ is the $S=1$, $I=0$ SU(4) wave function.[3]

The impulse approximation relates the transition matrix T for the deuteron process to free-nucleon scattering via

$$\langle\psi_f|T|\psi_i\rangle = \langle\psi_f|t_1+t_2|\psi_i\rangle , \tag{4}$$

where t_i describes the corresponding free process taking place on the ith nucleon. A similar equality holds between the scattering amplitudes F and f obtained from T and t. Carrying out the spatial integration, we find that the amplitude for scattering to the final state involving (2) is

$$F_\pm(\vec{p}_1, \vec{p}_2) = \tfrac{1}{2}[\varphi(\vec{p}_1) \pm \varphi(\vec{p}_2)] f_\pm(\vec{p}_1 + \vec{p}_2)\,, \qquad (5)$$

where

$$f_\pm(\vec{\Delta}) = \langle \chi_f | f_1(\vec{\Delta}) \pm f_2(\vec{\Delta}) | \chi_i \rangle\,, \qquad (6)$$

$f_i(\vec{\Delta})$ being the scattering amplitude for the corresponding free process with momentum transfer $\vec{\Delta}$, considered as an operator in the nucleon spin-isospin space. The closure approximation can be applied to (5) to obtain the net differential cross section, summed over all final two-nucleon states, as

$$\frac{d\sigma}{d\Omega} = \tfrac{1}{2}[1 + S(\vec{\Delta})] \sum_{i,f} |f_+|^2 + \tfrac{1}{2}[1 - S(\vec{\Delta})] \sum_{i,f} |f_-|^2. \qquad (7)$$

In (7) the summations indicate averaging over initial spin states and summing over final spin states, and $S(\vec{\Delta})$ is the deuteron form factor.

We shall now evaluate the summations of $|f_\pm|^2$ appearing in (7) and compare them with the corresponding sums for free-nucleon scattering. The f_i, as spin-isospin operators, can be written in the general form

$$f_i = A + \vec{B} \cdot \vec{\sigma}_i + \vec{C} \cdot \vec{\tau}_i + D_{\alpha\lambda} \sigma_{i\alpha} \tau_{i\lambda}\,, \qquad (8)$$

where $\vec{\sigma}_i$ and $\vec{\tau}_i$ are the spin and isospin operators of nucleon i. The coefficients A, $\vec{B}$, $\vec{C}$, and $D_{\alpha\lambda}$ must be constructed appropriately from the available momenta and the operators describing $|a\rangle$ and $|b\rangle$. The differential cross sections resulting from (8) for free-nucleon processes are

$$\frac{d\sigma}{d\Omega}(an \to bp) = \tfrac{1}{4} |C_-|^2 + \tfrac{1}{16} |\vec{D}_-|^2\,, \qquad (9)$$

where $C_- = C_1 - iC_2$ and $D_{\alpha -} = D_{\alpha 1} - iD_{\alpha 2}$, and $|\vec{D}_-|^2 = D^*_{\alpha -} D_{\alpha -}$; and

$$\frac{d\sigma}{d\Omega}(ap \to b'p) + \frac{d\sigma}{d\Omega}(an \to b'n) = 2|A|^2 + \tfrac{1}{2}|\vec{B}|^2 + \tfrac{1}{2}|C_3|^2 + \tfrac{1}{8}|\vec{D}_3|^2. \qquad (10)$$

To obtain the deuteron cross sections one must study the matrix elements of f_+ and f_-. The former involves a sum of spin and isospin operators which was studied intensively for pion scattering by Dean and Friar,[4] and the results of that work can be generalized immediately. It follows that

$$\sum_{i=1,2} f_i = 2A + \vec{B} \cdot \vec{S} + \vec{C} \cdot \vec{T} + \tfrac{1}{2} D_{\alpha\lambda} \mathcal{S}_\alpha \mathcal{T}_\lambda\,, \qquad (11)$$

where $\vec{S}$ and $\vec{T}$ are the spin and isospin operators of the two-nucleon state and $\mathcal{S}_\alpha \mathcal{T}_\lambda$ denotes a "multiplet-changing operator" within the SU(4) supermultiplet. (For further details, see Ref. 4.) Only the final term of (11) contributes to the two-proton final state, yielding

$$\sum_{i,f} |f_+|^2 = \tfrac{1}{24} |\vec{D}_-|^2 \qquad (12)$$

for the pn final state, the analogous result is

$$\sum_{i,f} |f_+|^2 = \tfrac{1}{3}(12|A|^2 + 2|\vec{B}|^2 + \tfrac{1}{4}|\vec{D}_3|^2)\,. \qquad (13)$$

For the antisymmetric states we must study correspondingly

$$f_- = \vec{B} \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) + \vec{C} \cdot (\vec{\tau}_1 - \vec{\tau}_2) + D_{\alpha\lambda}(\sigma_{1\alpha}\tau_{1\lambda} - \sigma_{2\alpha}\tau_{2\lambda}),$$

which is a more complicated operator; it connects the deuteron to the symmetric 10 representation of SU(4). The result, obtained after some labor, is

$$\sum_{i,f} |f_-|^2 = \tfrac{1}{2} |C_-|^2 + \tfrac{1}{12} |\vec{D}_-|^2\,, \qquad (14)$$

for pp final states, and

$$\sum_{i,f} |f_-|^2 = \tfrac{1}{3} |\vec{B}|^2 + |C_3|^2 + \tfrac{1}{6} |\vec{D}_3|^2\,, \qquad (15)$$

for pn final states.

The full results are therefore

$$\frac{d\sigma}{d\Omega}(ad \to bpp) = \tfrac{1}{4}[1 - S(\vec{\Delta})] |C_-|^2 + \tfrac{1}{16}[1 - \tfrac{1}{3}S(\vec{\Delta})] |\vec{D}_-|^2\,,$$
$$\frac{d\sigma}{d\Omega}(ad \to b'pn) = 2[1 + S(\vec{\Delta})] |A|^2 + \tfrac{1}{2}[1 + \tfrac{1}{3}S(\vec{\Delta})] + \tfrac{1}{2}[1 - S(\vec{\Delta})] |C_3|^2 + \tfrac{1}{8}[1 - \tfrac{1}{3}S(\vec{\Delta})] |\vec{D}_3|^2. \qquad (16)$$

These expressions can be compared with (9) and (10) to relate the deuteron process to the free process. The differential cross sections are identical when $S(\vec{\Delta}) = 0$, as we would expect since this corresponds to a "non-overlapping" deuteron, i.e., essentially two free nucleons. Thus we write

$$\frac{d\sigma}{d\Omega}(ad \to bpp) = \frac{d\sigma}{d\Omega}(an \to bp) - S(\vec{\Delta}) R(an \to bp)\,,$$
$$\frac{d\sigma}{d\Omega}(ad \to b'pn) = \frac{d\sigma}{d\Omega}(ap \to b'p) + \frac{d\sigma}{d\Omega}(an \to b'n) - S(\Delta) R(ap \to b'p)\,,$$

to isolate the correction terms

$$R(an \to bp) = \tfrac{1}{4} |C_-|^2 + \tfrac{1}{48} |\vec{D}_-|^2\,,$$
$$R(ap \to b'p) = -2|A|^2 - \tfrac{1}{6} |\vec{B}|^2 + \tfrac{1}{2} |C_3|^2 + \tfrac{1}{24} |\vec{D}_3|^2. \qquad (17)$$

For the charge-exchange process it is possible to identify the C_- and $D_{\alpha -}$ terms as spin-nonflip and spin-flip, respectively (although for the general case, it must be realized that D_{3-} need not vanish, so the latter term is not necessarily zero in the forward direction). Then one obtains

$$\frac{d\sigma}{d\Omega}(ad \to bpp) = [1 - S(\vec{\Delta})] \frac{d\sigma_{nf}}{d\Omega} + [1 - \tfrac{1}{3}S(\vec{\Delta})] \frac{d\sigma_f}{d\Omega}\,, \qquad (18)$$

which is simply a generalization of the result found originally for $K^+d \to K^0pp$ by Lee.[1] For the non-charge-exchange reaction, however, no such simple result follows.

It should be noted, finally, that the ratio of $|f_+|^2$ and $|f_-|^2$ terms determines both the relative magnitude of the deuteron corrections and the spin state of the final two-nucleon system. For example, if the spin-nonflip term C_- dominates the charge-exchange reaction, one has

$$\frac{d\sigma}{d\Omega}(ad \to bpp) \approx [1 - S(\vec{\Delta})]\frac{d\sigma}{d\Omega}(an \to bp),$$

and also finds the two protons predominantly in a triplet spin state. If the spin-flip amplitude D dominates instead, one finds

$$\frac{d\sigma}{d\Omega}(ad \to bpp) \approx [1 - \tfrac{1}{3}S(\vec{\Delta})]\frac{d\sigma}{d\Omega}(an \to bp),$$

and the triplet spin state has probability $\frac{2}{3}$, the singlet $\frac{1}{3}$. The importance of the spin-state ratio is that it determines the nature of interference effects in the two-proton distribution; in a separate paper,[5] we have shown how experimental measurement of these effects can determine which of the above equations is appropriate in a given reaction.

III. INCLUSION OF D-STATE EFFECTS

The importance of the deuteron's D-state admixture in considering the detailed structure of deuteron reactions is well established.[6] We shall therefore now extend the calculations of Sec. II to include those effects for the case of charge-exchange reactions, which are of the most interest experimentally. To do this, we must replace the simple deuteron wave function in (3) by

$$|\varphi_d\rangle|\chi_d\rangle = 2^{-1/2}\int d^3p\,[\varphi_S(\vec{p}) + (\tfrac{1}{8})^{1/2}\varphi_D(\vec{p})S_{12}(\vec{p})] \times |\vec{p}\rangle|-\vec{p}\rangle|\chi_d\rangle, \tag{19}$$

where

$$S_{12}(\vec{p}) = 2[3(\hat{p}\cdot\vec{S})^2 - 2]. \tag{20}$$

Then (5) is replaced by

$$F_\pm(\vec{p}_1,\vec{p}_2) = F_\pm^S(\vec{p}_1,\vec{p}_2) + (\tfrac{1}{8})^{1/2}F_\pm^D(\vec{p}_1,\vec{p}_2), \tag{21}$$

with

$$F_\pm^S(\vec{p}_1,\vec{p}_2) = \tfrac{1}{2}[\varphi_S(\vec{p}_1) \pm \varphi_S(\vec{p}_2)]f_\pm(\vec{p}_1+\vec{p}_2), \tag{22}$$

as in (6), and

$$F_\pm^D(\vec{p}_1,\vec{p}_2) = \tfrac{1}{2}\langle\chi_f|(f_1 \pm f_2)[\varphi_D(\vec{p}_1)S_{12}(\vec{p}_1) \pm \varphi_D(\vec{p}_2)S_{12}(\vec{p}_2)]|\chi_d\rangle. \tag{23}$$

We consider first the symmetric term F_+. Since the only nonvanishing contributions here arise when $|\chi_f\rangle$ is a spin singlet, the commutation relation $[\mathcal{S}_\alpha, S_\beta] = i\epsilon_{\alpha\beta\gamma}\mathcal{S}_\gamma$ can be used to show that

$$\langle\chi_f|D_{\alpha-}\mathcal{S}_\alpha\mathcal{T}_+(\hat{p}\cdot\vec{S})^2|\chi_d\rangle = [\delta_{\alpha\beta} - \hat{p}_\alpha\hat{p}_\beta]D_{\beta-}\langle\chi_f|\mathcal{S}_\alpha\mathcal{T}_+|\chi_d\rangle.$$

This result allows us to make a simple redefinition of $D_{\alpha-}$, and it follows by comparison with (12) that

$$\sum_{i,f}|F_+|^2 = \tfrac{1}{24}\{(|\varphi_{S+}|^2 + |\varphi_{D+}|^2)|\vec{D}_-|^2 + \sqrt{2}\,\mathrm{Re}[\varphi_D^*(\vec{p}_1)(\varphi_{S+} - 8^{-1/2}\varphi_{D+})](|\vec{D}_-|^2 - 3|\hat{p}_1\cdot\vec{D}_-|^2)$$
$$+\sqrt{2}\,\mathrm{Re}[\varphi_D^*(\vec{p}_2)(\varphi_{S+} - 8^{-1/2}\varphi_{D+})](|\vec{D}_-|^2 - 3|\hat{p}_2\cdot\vec{D}_-|^2) + \Lambda_+\}, \tag{24}$$

where $\varphi_{S+} = \varphi_S(\vec{p}_1) + \varphi_S(\vec{p}_2)$, $\varphi_{D+} = \varphi_D(\vec{p}_1) + \varphi_D(\vec{p}_2)$, $\vec{p}\cdot\vec{D}_- = p_\alpha D_{\alpha-}$, and Λ_+ contains terms which are of second order in the D-state wave function *and* vanish when $\Delta \to 0$.

As before, the antisymmetric terms are considerably more laborious, and we only give the final result

$$\sum_{i,f}|F_-|^2 = (|\varphi_{S-}|^2 + |\varphi_{D-}|^2)(\tfrac{1}{2}|C_-|^2 + \tfrac{1}{12}|\vec{D}_-|^2)$$
$$-\tfrac{1}{6}(8^{-1/2})\,\mathrm{Re}[\varphi_D(\vec{p}_1)(\varphi_{S-} - 8^{-1/2}\varphi_{D-})](|\vec{D}_-|^2 - 3|\hat{p}_1\cdot\vec{D}_-|^2)$$
$$+\tfrac{1}{6}(8^{-1/2})\,\mathrm{Re}[\varphi_D(\vec{p}_2)(\varphi_{S-} - 8^{-1/2}\varphi_{D-})](|\vec{D}_-|^2 - 3|\hat{p}_2\cdot\vec{D}_-|^2) + \Lambda_-, \tag{25}$$

with notations similar to (24).

Combining these two results and using the closure approximation, we find that (neglecting the small $\Lambda_\pm$ terms) the differential cross section of (16) is replaced by

$$\frac{d\sigma}{d\Omega}(ad \to bpp) = \tfrac{1}{4}[1 - S_0(\vec{\Delta})]|C_-|^2 + \tfrac{1}{16}\{1 - \tfrac{1}{3}[S_0(\vec{\Delta}) + 2S_2(\vec{\Delta})]\}|\vec{D}_-|^2 + \tfrac{1}{8}S_2(\vec{\Delta})|\hat{\Delta}\cdot\vec{D}_-|^2, \tag{26}$$

where we have introduced the usual spherical and quadrupole form factors of the deuteron,

$$S_0(\vec{\Delta}) = \int d^3p\,[\varphi_S^*(\vec{p})\varphi_S(\vec{\Delta}-\vec{p}) + \varphi_D^*(\vec{p})\varphi_D(\vec{\Delta}-\vec{p})]$$

$$= \int_0^\infty dr\,[u^2(r) + w^2(r)]\,j_0(\Delta r)$$

$$S_2(\vec{\Delta}) = 2^{1/2}\int_0^\infty dr\,w(r)[u(r) - 8^{1/2}w(r)]\,j_2(\Delta r)\,.$$

It may be noted that the differential cross section (26) contains, via the quadrupole form factor, a dependence on the direction of the momentum transfer $\hat{\Delta}$ relative to the vector $\vec{D}_-$, i.e., relative to whatever vectors describing the particles a, b are used to construct the amplitude. (For example, if b is a vector meson, $\vec{D}_-$ will be proportional to its polarization vector.) If the states of a and b are summed over, this dependence will be replaced by an appropriately incoherent sum.

Multiple scattering corrections to the amplitudes C_- and $\vec{D}_-$ can be calculated via the Glauber theory, although one may question whether that method properly includes all inelastic intermediate states for breakup scattering. If it is assumed that the only important corrections arise from (a) elastic scattering of a before the breakup, and (b) elastic scattering of b after the breakup, the results will be essentially those obtained in an earlier paper[7] neglecting spin, isospin, and symmetrization effects.

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Application of the New Interference Model to the Meson-Baryon Hypercharge-Exchange Reactions

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The hypercharge-exchange reactions $K^-n \to \pi^-\Lambda^0$, $K^-p \to \pi^0\Lambda^0$, $\pi^-p \to K^0\Lambda^0$, $K^-n \to \pi^-\Sigma^0$, $K^-p \to \pi^-\Sigma^+$, $\pi^-p \to K^0\Sigma^0$, and $\pi^+p \to K^+\Sigma^+$ are studied within the framework of the new interference model. It is found that the differential cross section and polarization can be predicted in reasonable agreement with experiments in the intermediate momentum range.

I. INTRODUCTION

Several attempts[1-3] have been made to explain the observed differential cross-section and polarization data for the various hypercharge-exchange $0^-\frac{1}{2}^+ \to 0^-\frac{1}{2}^+$ reactions. In the Regge-model approach[1] the possible t-channel Regge poles $K^*(890)$ $J^P = 1^-$ and $K^{**}(1420)$ $J^P = 2^+$ are taken as nondegenerate; trajectory functions are modified, and a cross-over term is introduced to obtain reasonable success, with eight parameters in the cross-section formula. Absorptive peripheral models[2,3] have been tried with and without exchange degeneracy. By using trajectory parameters which are different from those determined from a Chew-Frautschi plot acceptable fits have been obtained.

In the intermediate momentum region, it has been shown[4,5] that the new interference model of Coulter *et al.*,[6] which is free from double-counting errors, gives a satisfactory explanation both for angular distribution and polarization. In this paper the calculations have been extended to the following hypercharge-exchange reactions in the intermediate momentum region: