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Are There Fixed Singularities in T_1?

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Nonforward scaling and the light-cone commutator are shown to require the existence of a Kronecker δ term in T_1. The present data may be consistent with the possibility that this Kronecker δ term vanishes at $t=0$ and $q^2=-\infty$. Analyticity implies a connection between scaling behavior and Regge behavior.

Recently, Gross[1] argued on the basis of unitarity and the assumed absence of oscillatory behavior that the absorptive parts of the nonforward Compton amplitude also scale according to Bjorken's suggestion. Unfortunately, the prospect of verifying this conclusion in the foreseeable future is dim. In this note we show that nonforward scaling, when supplemented by a number of currently acceptable assumptions, implies the existence of a Kronecker δ term δ_{J0} in the analytic J-plane continuation of T_1.

Our assumptions are the following:

(a) The Bjorken-Johnson-Low limit[2] $\lim_{\rm BJL} {Q_0}^2 T_{ij}$ exists.

(b) The behavior of the current commutator near the light cone is relevant for the deep-inelastic region.[3]

Assumption (b) actually implies nonforward scaling.

We now demonstrate our assertion. The target spin-averaged nonforward Compton amplitude may be decomposed as[4]

$$T_{\mu\nu}(q,q',p,p') = -g_{\mu\nu}T_1(\nu,Q^2,t,\delta) + \frac{P_\mu P_\nu}{m^2}T_2(\nu,Q^2,t,\delta)+\cdots, \quad (1)$$

where

$$P=\tfrac{1}{2}(p+p'),\quad Q=\tfrac{1}{2}(q+q'),\quad \Delta=p-p'=q'-q,$$
$$\nu=P\cdot Q,\quad \delta=Q\cdot\Delta=q'^2-q^2,\quad t=\Delta^2,$$

and

$$\omega=-Q^2/2\nu.$$

Suppose that there is *no* fixed singularity in the J plane. Then T_1 presumably satisfies an unsubtracted dispersion relation for $t<t_0$ for some $t_0<0$, viz.,

$$T_1(\nu,Q^2,t,\delta)=\int_{{\nu_0}^2}^\infty \frac{d\nu'^2}{\nu'^2-\nu^2}W_1(\nu',Q^2,t,\delta) \quad (2a)$$

$$=2\omega^2\int\frac{d\omega'}{\omega'}\,\frac{W_1(\omega',Q^2,t,\delta)}{\omega^2-\omega'^2}. \quad (2b)$$

We now let $Q_0\to i\infty$ keeping t and δ fixed:

$$T_1(\nu,Q^2,t,\delta)\underset{\rm BJL}{\sim}2\int_0^1\frac{d\omega'}{\omega'}\lim_{Q^2\to-\infty}W_1(\omega',Q^2,t,\delta). \quad (3)$$

Assumption (a) then implies that[5]

$$\lim_{Q^2\to-\infty}Q^2W_1(\omega,Q^2,t,\delta)=-\bar{F}_1(\omega,t,\delta). \quad (4)$$

[We exclude the possibility that $\lim W_1(\omega,Q^2,t,\delta)=F_1(\omega,t)$ and that $\int_0^1 d\omega\,\omega^{-1}F_1(\omega,t)=0$ for all $t<t_0$.] Translated into the language of light-cone dominance this means that the operator of lowest twist[6] contributing has twist four. For $t>t_0$, however, T_1 satisfies a once-subtracted dispersion relation. As is well known, assumption (a) leads to[7]

$$\lim_{Q^2\to-\infty}W_1(\omega,Q^2,t,\delta)=F_1(\omega,t,\delta)\quad(\text{for } t>t_0) \quad (5)$$

and the statement that the operator of lowest twist contributing has twist two. [Actually, $F_1(\omega,t,\delta)$ is independent of δ as a consequence of locality.[1]] Now the matrix element of the local operator of twist two $\langle p'|O_{\mu_1\cdots\mu_J}(0)|p\rangle$ involves form factors which have standard analyticity properties in $t=(p-p')$.[2] They cannot be nonzero for $t>t_0$ and identically zero for $t<t_0$. Hence the dispersion representation for T_1 cannot be unsubtracted and our assertion is proved. We conclude with a number of remarks.

(1) It has been known[8] for some time now that general principles allow, but do *not* require, the presence of a Kronecker δ term, δ_{J0}, in T_1. ($J=0$ is a sense point for T_1, the amplitude proportional to the t-channel helicity amplitude F^t_{00}. F^t_{00} is linked via conspiracy relations to F^t_{+-} for which $J=0$ is a nonsense point. Hence F^t_{+-} *may* have a fixed pole at $J=0$, *possibly* leading to a δ_{J0} term in F^t_{00}. See Ref. 8.) The precise nature of the fixed singularity does not concern us here. Current algebra does *require* that the crossing-odd $I_t=1$ amplitude has a fixed pole.[9] No such

argument exists previously for the Compton amplitudes.

(2) Our conclusion may be avoided if (i) the Pomeranchukon is a fixed singularity, (ii) $\alpha_P(t)$ is never negative, *or* (iii) the cuts generated by the Pomeranchukon pile up in such a way that $\alpha_P^{\text{effective}}(t)$ is never negative. We assume that these cases do not occur. This caveat need not be posted if one considers the difference between the Compton amplitudes for proton and for neutron. T_1^{proton} and T_1^{neutron} cannot be both free from fixed singularities.

(3) Indeed, there exists no twist-two and isospin $I=2$ operator in the quark-gluon model. This fact is consistent with the expectation that the Compton amplitudes with $I=2$ in the t channel satisfy an unsubtracted dispersion representation. Conversely, if one is able to construct twist-two operators in any given model which satisfies our assumption, then the corresponding Compton amplitude cannot satisfy an unsubtracted dispersion representation. It is usually, and correctly, stated that what happens near the light cone is irrelevant for the Regge region. Here, assumption (a) and analyticity link these two regions together. Otherwise one may always add a real function constant in ν to T_1 without affecting the light-cone commutator.

Also, it has been stated erroneously in the literature that the scaling of W_1 and νW_2 follows from dimensional arguments alone. The present discussion shows this to be false.

(4) Our observation implies that the deep-inelastic region does not contribute to $\Delta I=1$ and $\Delta I=2$ mass differences in the same manner. In particular T_1 ($I_t=2$) satisfies an unsubtracted dispersion representation and hence

$$\lim_{q^2\to -\infty} + q^2 W_1(\omega, q^2) = -\bar{F}_1(\omega)\,. \tag{6}$$

We obtain the contribution of the deep-inelastic region

$$\delta m_\infty^{\Delta I=2} = \tfrac{3}{4}\pi e^2 \int_{|q_0|^2}^{\infty} \frac{dq_E^2}{q_E^2}\left[-2\int_0^1 \frac{d\omega}{\omega}\bar{F}_1(\omega) + \int_0^1 d\omega F_2(\omega)\right]. \tag{7}$$

This is to be contrasted with the more familiar expression[10]

$$\delta m_\infty^{\Delta I=1} = \tfrac{3}{4}\pi e^2 \int_{|q_0|^2}^{\infty} \frac{dq_E^2}{q_E^2} \times\left(-q_E^2 T_1(-q_E^2) + \int_0^1 d\omega[F_2(\omega) + 2\omega F_1(\omega) m^2]\right). \tag{8}$$

In particular, there is a possibility that the logarithmic divergences in $\delta m_\infty^{\Delta I=2}$ cancel,[11] which is desirable in view of the success of theoretical calculations of δm_π.

(5) The correct dispersive representation of T_1 for $t<t_0$ reads

$$T_1(\nu, Q^2, t, \delta) = R(Q^2, t, \delta) + \int_{\nu_0^2}^{\infty} \frac{d\nu'^2}{\nu'^2-\nu^2} W_1(\nu', Q^2, t, \delta)\,, \tag{9}$$

rather than Eq. (2a). $R(Q^2, t, \delta)$ is the term coming from the Kronecker δ. Assumption (a) then leads to Eq. (5) and

$$\lim_{Q^2\to\infty} R(Q^2, t, \delta) = -2\int_0^1 \frac{d\omega}{\omega} F_1(\omega, t)\,. \tag{10}$$

We note that as a result of locality $\lim_{Q^2\to\infty} R(Q^2, t, \delta)$ is independent of δ. There have been some speculations[12] on whether $R(Q^2, t, \delta)$ is in fact a polynomial in Q^2. It follows from (10) that if $R(Q^2, t, \delta)$ is a polynomial in Q^2 it is a constant.

(6) The expression (10) for $R(Q^2=\infty, t)$ is valid for $t<t_0$ only. As $\omega\to 0$,

$$F_1(\omega, t) \to F_1^{\text{Regge}}(\omega, t) \equiv \sum_i \gamma_i(t)\omega^{-\alpha_i(t)}\,, \tag{11}$$

where the sum runs over $i=P$ and R with $\alpha_P(0)=1$ and $\alpha_R(0)\simeq\frac{1}{2}$. (It may be emphasized that this behavior is proven in Ref. 1 from the locality of the light-cone operators.) Hence the integral representation (10) for $R(Q^2=\infty, t, \delta)$ must be analytically continued in t past t_0 to $t=0$, the point of interest. We note that $A(J, t) \equiv \int_0^1 d\omega\, \omega^{J-1}F_1(\omega, t)$ is (a) an analytic function in J for Re $J>1$ and (b) proportional to the single-particle matrix element of a spin-J local operator appearing in the light-cone commutator if J is equal to an integer. $A(J, t)$ is supposed to be analytic in t except for a cut starting at $t=4m_\pi^2$, as expressed by the representation

$$A(J, t) = \sum_i \frac{\gamma_i(t)}{J-\alpha_i(t)} + \int_0^1 d\omega\, \omega^{J-1}[F_1(\omega, t) - F_1^{\text{Regge}}(\omega, t)]\,. \tag{12}$$

Continuation of $A(J, t)$ to $J=0$ gives the Kronecker δ term at $Q^2=\infty$ and $t=0$,

$$+\tfrac{1}{2}R(Q^2=\infty, t=0) = +\sum_i \frac{\gamma_i(0)}{\alpha_i(0)} - \int_0^1 \frac{d\omega}{\omega}[F_1(\omega) - F_1^{\text{Regge}}(\omega)]\,. \tag{13}$$

If there exists a $J=0$ local operator such that its single particle matrix element coincides with the analytic continuation of $A(J, t)$ to $J=0$, then one may not have to continue in t past t_0. The reason is that $\gamma(t_0)$ may vanish in order to avoid the presence of a ghost pole in $A(J=0, t)$. This question hinges on the inclusion of effects of higher order in α (see Ref. 13).

It is interesting to note that the fixed pole in T_2 is simply given by

$$R_2(q^2=\infty, t=0)=\int_0^1 \frac{d\omega}{\omega^2}[F_2(\omega)-F_2^{\text{Regge}}(\omega)]. \quad (14)$$

[$R_2(q^2, t=0)$ is defined by

$$q^2 R_2(q^2, t=0)=\lim_{\nu\to\infty} \nu^2 (T_2 - T_2^{\text{Regge}}). \quad (15)$$

T_2^{Regge} contains Regge poles with intercept >0 and

$$F_2^{\text{Regge}}(\omega)=(1/\pi)\lim_{\text{scaling}} \nu \operatorname{Im} T_2^{\text{Regge}}.]$$

This may be traced to the fact that helicity flip suppresses Regge behavior by two powers of ν so that $T_2 \to \nu^{\alpha-2}$. Thus an unsubtracted dispersion representation exists for $\nu(T_2 - T_2^{\text{Regge}})$ and Eq. (14) follows. In contrast, $T_1 \to \nu^{\alpha}$ in the Regge region and thus the extra term $\sum \gamma_i \alpha_i^{-1}$ appears in Eq. (13).

(7) A reliable evaluation of Eq. (13) is not possible with the SLAC-MIT data at present. The evaluation depends sensitively on the small-ω region where data tend to be poor and q^2 tends to be small. What follows is to be understood as a preliminary attempt to evaluate $R(Q^2=\infty, t=0)$, an attempt presumably to be modified when more accurate data become available.

We exploit the fact that $F_2(\omega)=2\omega m^2 F_1(\omega)$ is a good approximation in the region explored in the SLAC-MIT experiment. There exist reasonable evidences[14] that

$$R_2(q^2=0, t=0)\simeq 1. \quad (16)$$

Close and Gunion[15] in a constrained fit obtained

$$F_2(\omega)\to 0.12+0.462\omega^{+1/2}+4.02\omega^{3/2} \quad (17)$$

as $\omega \to 0$. They *imposed* the constraint[16]

$$R_2(q^2=\infty, t=0)=1, \quad (18)$$

in addition to other constraints, such as various quark charge sum rules involving the neutron data.[17] Using these figures we *tentatively* obtain

$$m^2 R(Q^2=\infty, t=0)\simeq 0.12+0.924-1\simeq 0. \quad (19)$$

Thus, it is possible that the Kronecker δ term vanishes for $t=0$ *and* $q^2=\infty$.

Our result that a Kronecker δ term is present in T_1 is hardly surprising. In fact, it appears to be difficult[18] to construct a model with no fixed J-plane singularity in T_1 and T_2. What is surprising is that it may turn out that no Kronecker δ is present in T_1 at $t=0$.

(8) As $q^2 \to 0$ the kinematical constraint $\nu^2 T_2 + q^2 m^2 T_1 \to 0$ holds. (For $t \neq 0$ the constraint holds as $qq' \to 0$.) This implies the relation

$$R_2(q^2=0, t=0)=m^2 R(q^2=0, t=0). \quad (20)$$

If both R and R_2 are constants[12] in q^2 then

$$R_2(q^2=\infty, t=0)=m^2 R(q^2=\infty, t=0), \quad (21)$$

which is contradicted by our preliminary analysis. Naturally, when better data become available the question of whether $R_2(q^2, t=0)$ is a constant in q^2 may be settled by directly analyzing the data, rather than by this convoluted reasoning. We note that Eq. (21) may be rewritten as

$$\sum_i \frac{\gamma_i(0)}{\alpha_i(0)}=\frac{1}{m^2}\int_0^1 \frac{d\omega}{\omega^2}[F_2(\omega)-F_2^{\text{Regge}}(\omega)]. \quad (22)$$

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[17]The validity of the quark charge sum rule has since become less plausible. Modifying this constraint these authors obtain

$$F_2(\omega) \to 0.17 + 0.339\omega^{1/2} + 3.42\omega^{3/2}$$

instead of Eq. (17). (F. Close, private communication.) With this fit we obtain $R(Q^2 = \infty,\ t = 0) \simeq 0.152$, still a rather small number. This fit and the one in Eq. (8) are favored by a recent analysis based on complex scaling [N. Khuri, Phys. Rev. D 5, 462 (1972)].

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Inelastic Scattering from Deuterium in the Impulse Approximation*

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We evaluate in detail the differential cross sections for arbitrary processes $ad \to bpp$ and $ad \to b'pn$ in the impulse approximation, with spin and isospin effects and the deuteron D state properly included. Approximations valid when a single term of the free-nucleon amplitude dominates are given.

I. INTRODUCTION

The only way to study inelastic scattering from neutrons is by observing the corresponding inelastic processes on deuterium targets. A method of relating the scattering from a nucleon bound in the deuteron and that from a free nucleon is provided by the impulse and closure approximations. The form of the result depends crucially upon proper evaluation of matrix elements between appropriate states symmetrized in spin and isospin. This is a formidable task, which heretofore has been carried through only in a few special cases.[1] In this paper we shall evaluate the impulse approximation for arbitrary deuteron breakup reactions $ad \to bpp$ and $ad \to b'pn$ with spin and isospin properly included.

II. CALCULATION OF DIFFERENTIAL CROSS SECTION

We write the final-state wave function in the laboratory system as the direct product of the scattered particle state and a two-nucleon state decomposed into spatial and spin-isospin parts,

$$|\psi_f\rangle = |b\rangle |\varphi_f\rangle |\chi_f\rangle . \tag{1}$$

Neglecting final-state interactions,[2] we shall take the spatial state as simple plane waves,

$$|\varphi_{f\pm}\rangle = 2^{-1/2}(|\vec{p}_1\rangle|\vec{p}_2\rangle \pm |\vec{p}_2\rangle|\vec{p}_1\rangle) \tag{2}$$

describing the symmetrized combinations of states with three-momenta $\vec{p}_1$ and $\vec{p}_2$. The spin-isospin wave function $|\chi_f\rangle$ is correspondingly symmetrized so that $|\varphi_f\rangle|\chi_f\rangle$ is antisymmetric. The initial state is

$$\begin{aligned} |\psi_i\rangle &= |a\rangle |\varphi_d\rangle |\chi_d\rangle , \\ |\varphi_d\rangle &= 2^{-1/2}\int d^3p\, \varphi(\vec{p})|\vec{p}\rangle|-\vec{p}\rangle , \end{aligned} \tag{3}$$

where $\varphi(\vec{p})$ is the momentum-space deuteron wave function and $|\chi_d\rangle$ is the $S=1$, $I=0$ SU(4) wave function.[3]

The impulse approximation relates the transition matrix T for the deuteron process to free-nucleon scattering via

$$\langle\psi_f|T|\psi_i\rangle = \langle\psi_f|t_1 + t_2|\psi_i\rangle , \tag{4}$$

where t_i describes the corresponding free process taking place on the ith nucleon. A similar equality holds between the scattering amplitudes F and f obtained from T and t. Carrying out the spatial integration, we find that the amplitude for scatter-