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Low-Energy $\Sigma^- d$ Scattering*

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We use a Faddeev formalism and two-body, S-wave, central separable potentials to calculate $\Sigma^- d$ quartet elastic and reaction cross sections for Σ^- incident lab momentum in the range 30–90 MeV/c. We use a three-channel isospin-$\frac{1}{2}$ hyperon-nucleon potential with a Λn resonance below the $\Sigma^0 n$ threshold, but in the three-body calculation the Λn channel is only included implicitly. We find that $\Sigma^- d$ cross sections are sensitive to the size of the $\Sigma^- p$ scattering length, but insensitive to the exact position and width of the Λn resonance.

I. INTRODUCTION

The purpose of this work is to construct a model of $\Sigma^- d$ scattering below the threshold for deuteron breakup and to use it to investigate the sensitivity of the elastic and reaction cross sections to some of the physical parameters of the two-body zero-charge Σ-nucleon (ΣN) interactions.

Our philosophy is that this work is a first crude step in the theoretical investigation of the attributes of the ΣN interaction (e.g., its off-shell behavior) that may not be directly accessible from a study of free $\Sigma^- p$ scattering. Ultimately we should like to be able to test whether two different meson-theoretic potentials which give the same results for on-shell scattering parameters for free $\Sigma^- p$ scattering can be distinguished by analysis of $\Sigma^- d$ scattering. At this stage of the game when very little $\Sigma^- p$ scattering data is available,[1] we feel it makes more sense to work with simple phenomenological potentials rather than full-blown meson-theoretic potentials. For the same reason we shall use only a very crude model for the three-body system. We emphasize that we do not expect the absolute size of the cross sections we calculate to be of any great significance, but rather it is the variation (or the lack of variation) of these cross sections as the two-body interactions are changed, upon which we shall focus.

The only previous calculations of low-energy $\Sigma^- d$ cross sections that we know of are those of Day, Snow, and Sucher,[2] Chen,[3] and Neville,[4] all of whom calculated the ratio of the cross section for Σ^0 production to the total reaction cross section at zero incident Σ^- energy. All three of these works were basically impulse approximation calculations, although Chen did include the $^1S_0$ neutron-neutron final-state interaction. Because of the low Σ^- energy this is not a valid way to proceed with our problem.

We investigate $\Sigma^- d$ scattering in the quartet spin state so that the nucleon-nucleon and hyperon-nucleon interactions are all spin-triplet interactions. We use central, S-wave, two-body potentials to represent these interactions and we do not include the Coulomb interaction. Our two-body potentials are all $^3S_1$ potentials. They are the np potential, the $\Sigma^- n$ potential, and the three-channel ($\Sigma^- p$, $\Sigma^0 n$, Λn) potential which we shall refer to hereafter as the YN potential.[5] Each of the first two potentials and each matrix element of the YN potential we represent by a nonlocal separable (NLS) potential. We concentrate on Σ^-d scattering below the threshold for deuteron breakup because this region should be most sensitive to the resonance structure of the YN interaction that lies just below the $\Sigma^0 n$ threshold and because our use of S-wave potentials for the two-body interactions is more justifiable here than at higher energies. We use a Faddeev type of formalism[6] to calculate the amplitude for elastic Σ^-d scattering with incident Σ^- laboratory momentum in the range 30–90 MeV/c, the threshold for deuteron breakup being 93.4 MeV/c. From this amplitude we calculate directly the elastic scattering cross section and, with the use of the optical theorem, the total cross section. The reaction cross section (i.e., the cross section for $\Sigma^-d \to \Sigma^0 nn$ plus the cross section for $\Sigma^-d \to \Lambda nn$) we obtain by subtracting the first of these results from the second.

In order to reduce the number of coupled integral equations that must be solved to obtain the Σ^-d elastic scattering amplitude we introduce the approximation of treating the Λn channel of the three-channel hyperon-nucleon potential *implicitly*. That is, we solve the two-body three-channel (Σ^-p, $\Sigma^0 n$, Λn) problem exactly, but then we use only the (Σ^-p, $\Sigma^0 n$) two-channel part of our result in our Σ^-d calculations. In the three-body problem, the Λn interaction appears only as an energy-dependent, absorptive part of the $\Sigma^-p \to \Sigma^-p$, $\Sigma^0 n$ and $\Sigma^0 n \to \Sigma^0 n$, Σ^-p amplitudes. The basis of this approximation is that the Λn threshold is $\simeq 80$ MeV below the Σ^-p threshold. Once produced the relatively high energy Λ is assumed to leave the interaction volume so quickly that it does not interact further with the two neutrons. If there is a Λn resonance just below the $\Sigma^0 n$ threshold – as indeed we shall assume below – so that the above assumption is not completely correct, not only will the cross section for Λ production be significantly larger than we are accounting for, but the Σ^0 production cross section will also be large, so that the error in our calculation of the combined reaction cross section should again be small. Preliminary results of an exact calculation indicate that in treating the Λ implicitly our error in the elastic scattering cross section is at most 10% while our error in the reaction cross section is 20% or less.

In Sec. II we describe in detail the separable potentials used to represent the two-body interactions. The input parameters for the np potential are the triplet scattering length and the deuteron binding energy. For the Σ^-n potential we choose the intrinsic range and adjust the well-depth parameter to its maximum value consistent with some early Σ^+p elastic scattering data.[7] For the three-channel potential we initially have six strength parameters and three range parameters to be determined. On the basis of the small Σ^0-Σ^- and n-p mass differences we reduce these to three strength parameters, a ΣN range parameter $1/\beta_\Sigma$, and a Λn range parameter $1/\beta_\Lambda$. The input parameters are the Λp scattering length a_Λ and effective range $r_{0\Lambda}$ and the position E_0 and width Γ of the Λ channel resonance which is assumed to exist somewhat below the Σ^0 channel threshold. Alternatively we also use as input the real and imaginary parts of the Σ^-p scattering length A_- instead of the resonance parameters. We determine all of the potential parameters by assuming a given value for $\beta_\Lambda/\beta_\Sigma$.

In Sec. III we present and discuss the results of our calculations of the Σ^-d elastic and reaction cross sections. For fixed a_Λ, $r_{0\Lambda}$, E_0, and Γ we find that these cross sections are quite sensitive to the value of $\beta_\Sigma/\beta_\Lambda$, i.e., to the value of A_-. On the other hand for fixed a_Λ, $r_{0\Lambda}$, and A_- the Σ^-d cross sections are not sensitive to changes of the order of 3–5 MeV in the position or width of the Λ channel resonance.

II. TWO-BODY POTENTIALS

For the np potential we use the form of NLS potential first given by Yamaguchi.[8] In a relative momentum-space representation we have for the matrix element of the np potential-energy operator[9]

$$\langle \vec{k}' | V_{np} | \vec{k} \rangle = \lambda_N v_N(k') v_N(k) , \quad (1)$$

with

$$v_N(k) = 1/(k^2 + \beta_N^2) . \quad (2)$$

We use the values $M_n = 939.5527$ MeV and $M_p = 938.256$ MeV, respectively, for the neutron and proton masses. We determine the strength parameter λ_N and range parameter $1/\beta_N$ by the requirements that our potential yield the deuteron binding energy $\epsilon = 2.226$ MeV and the triplet scattering length $a_N = 5.39$ F. By standard methods we obtain the values

$$\lambda_N = -84.3966 \times (2\pi/10)^3 \text{ F}^{-2} , \quad (3)$$

$$1/\beta_N = 0.698\,339 \text{ F}. \quad (4)$$

For the Σ^-n potential we choose the same basic form as we use for the np potential, i.e.,

$$\langle \vec{k}' | V_{-n} | \vec{k} \rangle = \lambda_- v_-(k') v_-(k) , \quad (5)$$

where

$$v_-(k) = 1/(k^2 + \beta_-^2) , \quad (6)$$

and $\vec{k}$ ($\vec{k}'$) is the Σ^-n relative wave vector with magnitude k (k'). We determine the parameters λ_- and $1/\beta_-$ by following the treatment of Tang and Herndon.[10] We first fix the intrinsic range b_- of our potential at 1.5 F, one of the values used in Ref. 10. Thus[8] $1/\beta_- = \frac{1}{3}b_- = 0.5$ F. We next assume that the $\Sigma^- n$ scattering cross section is the same as the $\Sigma^+ p$ cross section. To maximize the size of the $\Sigma^- n$ $^3S_1$ interaction (so that its presence is sure to show up in the $\Sigma^- d$ problem) we assume that the experimental value of the $\Sigma^+ p$ cross section is due purely to $^3S_1$ scattering. We then choose the value of the well-depth parameter s_- of our potential so that at a Σ^- momentum of 135 MeV/c with a Σ^- mass of 1197.32 MeV we obtain a cross section in agreement with the $\Sigma^+ p$ experimental value of 185 ± 55 mb.[11] With $s_- = 0.6$ we obtain a cross section of 185.7 mb so we stick with this value of s_-. With these values of β_- and s_-, we then obtain by standard methods

$$\lambda_- = -91.1485 \times (2\pi/10)^3 \text{ F}^{-2}. \tag{7}$$

The three-channel YN potentail-energy operator we write as a 3×3 matrix whose ij element in a relative momentum-space representation is

$$\langle \vec{k}_i' \,|\, V_{ij} \,|\, \vec{k}_j \rangle = \lambda_{ij} v_i(k_i') v_j(k_j), \tag{8}$$

where

$$v_j(k_j) = 1/(k_j^2 + \beta_j^2), \tag{9}$$

and $\vec{k}_i$ is the hyperon-nucleon relative wave vector in the i^{th} channel with magnitude k_i. We choose channels 1, 2, and 3 to be, respectively, the $\Sigma^- p$, $\Sigma^0 n$, and Λn channels. To reduce the number of free parameters we first set the range parameters in the first two channels equal:

$$\beta_1 \equiv \beta_2 \equiv \beta_\Sigma. \tag{10}$$

We then choose the strength parameters λ_{ij} to be related to each other in the same way as they would be if there were no n-p and Σ^--Σ^0 mass differences and if our YN potential were a pure isospin-$\frac{1}{2}$ interaction. If these conditions were in fact true our YN potential could be reduced to the two-channel potential

$$V = \begin{pmatrix} \lambda_\Lambda v_\Lambda v_\Lambda & \lambda_x v_\Lambda v_\Sigma \\ \lambda_x v_\Sigma v_\Lambda & \lambda_\Sigma v_\Sigma v_\Sigma \end{pmatrix}, \tag{11}$$

where $v_\Sigma \equiv v_1 = v_2$, $v_\Lambda \equiv v_3$. The isospin-$\frac{1}{2}$ state of a Σ and a nucleon with third component of isospin-$\frac{1}{2}$ would be related to the two charge-zero states of a Σ and a nucleon by

$$|\Sigma N\rangle = (\tfrac{2}{3})^{1/2} |\Sigma^- p\rangle - (1/\sqrt{3})\, |\Sigma^0 n\rangle. \tag{12}$$

We calculate the $\Sigma N \to \Sigma N$ scattering amplitude using the potential of Eq. (11) and the state of Eq. (12) in a two-channel Lippmann-Schwinger equation and compare the results with a similar calculation using the potential of Eqs. (8), (9), and (10) in a three-channel Lippmann-Schwinger equation without the nucleon and Σ mass splittings. We obtain

$$\lambda_{11} = 2\lambda_{22} = -\sqrt{2}\,\lambda_{12} = \tfrac{2}{3}\lambda_\Sigma, \tag{13a}$$

$$\lambda_{13} = -\sqrt{2}\,\lambda_{23} = (\tfrac{2}{3})^{1/2}\lambda_x, \tag{13b}$$

$$\lambda_{33} = \lambda_\Lambda. \tag{13c}$$

We use Eq. (13) in our three-channel YN potential so that knowledge of five parameters – λ_Λ, λ_Σ, λ_x, β_Λ, and β_Σ – completely determines our YN interaction.

To determine the five YN potential parameters we proceed as follows. First we solve in a straightforward manner the three-channel YN coupled Lippmann-Schwinger equation for the YN t matrix using the YN NLS potential described above. From the results we obtain for δ_Λ, δ_-, and δ_0 the phase shifts for $\Lambda n \to \Lambda n$, $\Sigma^- p \to \Sigma^- p$, and $\Sigma^0 n \to \Sigma^0 n$ scattering, respectively, the expressions

$$k_3 \cot\delta_\Lambda = -\frac{2\pi(-g_3^p + 1/\gamma_3)}{\mu_3 v_3^2(k_\Lambda)}, \tag{14}$$

$$k_1 \cot\delta_- = -\frac{2\pi(-g_1^p - 0.5 g_2 + 1.5/\gamma_\Sigma)}{\mu_1 v_1^2(k_1)}, \tag{15}$$

$$k_2 \cot\delta_0 = -\frac{2\pi(-g_2^p - 2g_1 + 3/\gamma_\Sigma)}{\mu_1 v_1^2(k_1)}. \tag{16}$$

Here, for $j = 1, 2, 3$, μ_j is the hyperon-nucleon reduced mass in channel j, $\gamma_\Sigma = \lambda_\Sigma + \lambda_x^2 g_3/(1 - \lambda_\Lambda g_3)$,

$$\gamma_3 = \lambda_\Lambda + \lambda_x^2 g_\Sigma/(1 - \lambda_\Sigma g_\Sigma), \quad g_\Sigma = (2g_1 + g_2)/3,$$
$$g_j = -\frac{\mu_j}{\pi^2}\int_0^\infty \frac{q^2 v_j^2(q)\,dq}{q^2 - k_j^2 - i\eta}, \quad \eta \to 0^+ \tag{17}$$

and g_j^p is the principal-value part of g_j. Note that below the channel j threshold $k_j \to ik_j$, $k_j > 0$, so that the $i\eta$ in Eq. (17) may be dropped. For $j = 1, 2$, the k_j's are related to k_3 by

$$k_j^2 = (\mu_j/\mu_3)(k_3^2 - k_{j0}^2), \tag{18a}$$

$$k_{j0} = [2\mu_3(M_j - M_3)]^{1/2}, \tag{18b}$$

and M_j is the total mass in channel j for all values of j. In all our YN potentials in addition to the masses given above we used for the values of the Σ^0 and Λ masses, respectively, $M_0 = 1192.46$ MeV and $M_\Lambda = 1115.6$ MeV, so that $M_1 - M_3 = 80.42$ MeV and $M_2 - M_3 = 76.86$ MeV.

Next we assume that the Λn scattering length a_Λ and effective range $r_{0\Lambda}$ are known. We used the values for these parameters given by Satoh and Nogami[12] for the Λp interaction:

$$a_\Lambda = -1.6523 \text{ F}, \quad r_{0\Lambda} = 3.1717 \text{ F}. \tag{19}$$

TABLE I. Λn resonance energy and width, range ratio, $\Sigma^- p$ scattering length and $\Sigma^0 n$ scattering length for 12 different YN potentials. For all potentials $a_\Lambda = -1.6523$ F and $r_{0\Lambda} = 3.1717$ F.

Potential No.	E_0 (MeV)	Γ (MeV)	$\beta_\Sigma/\beta_\Lambda$	A_- (F)	A_0 (F)
1	70.85	10.00	0.5	3.0069−i1.7324	3.0821−i0.8215
2	70.85	10.00	1.0	1.9344−i1.0528	1.9852−i0.7118
3	70.85	10.00	1.5	1.6547−i0.8714	1.6543−i0.6641
4	70.85	5.00	1.0	2.0573−i0.9160	2.1633−i0.3988
5	73.85	10.00	0.5	2.8188−i2.1500	3.0337−i1.6215
6	73.85	10.00	1.0	1.8463−i1.4046	1.9086−i1.4038
7	73.85	10.00	1.5	1.6015−i1.2064	1.5693−i1.3082
8	73.85	5.00	1.0	2.1707−i1.3150	2.7264−i1.1464
9	73.69	4.54	1.5	1.9345−i1.0528	2.3995−i0.9412
10	73.72	2.12	1.5	2.0573−i0.9160	2.6244−i0.4687
11	75.10	5.52	1.5	1.8464−i1.4045	2.1426−i1.9545
12	75.32	2.58	1.5	2.1707−i1.3150	3.4449−i1.6882

We emphasize that for our purposes these numbers need only have the right approximate size. Any of the other sets of values for low-energy ΛN scattering parameters appearing in the recent literature[13] could have been used with equal effect.[14] We expand the right-hand side of Eq. (14) in a power series in k_3^2 and compare the result to

$$k_3 \cot\delta_\Lambda = -\frac{1}{a_\Lambda} + \tfrac{1}{2} r_{0\Lambda} k_3^2 + \cdots , \tag{20}$$

to obtain two relations among the YN potential parameters.

We next obtain two more relations among the YN potential parameters in two different ways. In the first we use the position E_0 and width Γ of the resonance in the Λn channel. From Eqs. (14) and (18) we obtain the resonance energy $E_0 = k_3^2/2\mu_3$ by finding the value of k_3 for which

$$1 - g_3^p \gamma_3 = 0 . \tag{21}$$

At the resonance we let $k_3 \equiv k_0$. Following Ref. 12 we obtain the width of the resonance from

$$\Gamma = -(k_0/\mu_3) \Big/ \frac{d}{dk_3^2}(k_3 \cot\delta_\Lambda)\Big|_{k_3=k_0} . \tag{22}$$

In the second way, we take the $\Sigma^- p$ scattering length A_-, which is of course complex – as known – and use Eq. (15) at $k_1 = 0$. In both cases we then fix the ratio $\beta_\Sigma/\beta_\Lambda$ to give us five relations among the five YN potential parameters.

In Tables I and II we give the results of our determination of the YN potential for 12 different cases. For the first three cases of Table I we chose the value of E_0 to be that used in case A of Ref. 12,[15] we chose the value of Γ to be the maximum consistent with that value of E_0, and we varied the ratio $\beta_\Sigma/\beta_\Lambda$ over the values 0.5, 1.0, and 1.5. The value 0.5 is consistent with a one-pion exchange mechanism in the ΣN interaction and a two-pion exchange mechanism in the Λn interaction,

TABLE II. YN potential parameters for the 12 potentials of Table I.

Potential No.	β_Λ (F^{-1})	$\beta_\Sigma/\beta_\Lambda$	$-\lambda_\Lambda/(\frac{1}{5}\pi)^3$ (F^2)	$-\lambda_\Sigma/(\frac{1}{5}\pi)^3$ (F^2)	$-\lambda_x/(\frac{1}{5}\pi)^3$ (F^2)
1	1.32835	0.5	14.6861	16.3412	19.5271
2	1.32200	1.0	13.2566	81.1913	34.3604
3	1.31945	1.5	12.5227	226.842	48.9749
4	1.39360	1.0	22.6187	91.9795	26.7714
5	1.31515	0.5	12.5164	13.4593	21.3853
6	1.30690	1.0	10.6786	70.9973	37.9374
7	1.30365	1.5	9.7401	203.638	54.2967
8	1.38625	1.0	21.1065	81.5307	29.9332
9	1.39300	1.5	21.8380	242.055	41.0782
10	1.43185	1.5	27.9608	260.329	29.5899
11	1.36615	1.5	17.5553	219.463	48.3158
12	1.41790	1.5	25.4782	240.267	36.1839

the value 1.0 is the simplest value from a phenomenological point of view, and the value 1.5 gives a third value for this parameter which eventually yields a value of $1/\beta_\Sigma$ that is not unreasonably small. Potential 4 of Table I was chosen to have the same input as potential 2 but with a resonance width half as wide. For these four potentials the resonance is 6 MeV below the $\Sigma^0 n$ threshold. Potentials 5 through 8 were chosen in the same way as 1–4, but with $E_0 = 73.85$ MeV, i.e., with the resonance 3 MeV below the $\Sigma^0 n$ threshold. After determining the potential parameters we calculated the values of A_- for each of these potentials. Potentials 9, 10, 11, and 12 were chosen to have $\beta_\Sigma/\beta_\Lambda = 1.5$ and the same values of A_- as potentials 2, 4, 6, and 8, respectively. (We were unable to obtain a similar match to the $\beta_\Sigma/\beta_\Lambda = 1.00$ values of A_- when we used 0.5 for the ratio of the β's.) After determining the potential parameters for potentials 9–12 we calculated the values of E_0 and Γ for these potentials. For completeness we also list in Table I the values we calculated for the $\Sigma^0 n$ scattering length A_0. In Table II we give the values of the potential strength and range parameters for each of the 12 potentials listed in Table II.

III. $\Sigma^- d$ CALCULATIONS

The application of the Faddeev formalism used here parallels that used by Hetherington and Schick[16] in their treatment of $K^- d$ elastic scattering, the main difference being that instead of a two-channel $\bar{K}N$ interaction we have a three-channel YN interaction. However, since we treat the Λn channel implicitly we too finally reduce the problem to solving a set of four coupled one-dimensional integral equations for each partial-wave amplitude. Details of such a calculation have appeared many times in the literature.[17] We see no need to go into further detail here, other than to say we included the first four partial waves in the single-scattering terms and the first two partial waves in all the multiple-scattering terms, we used the contour-rotation method[17] to smooth the integrals, and we used a mesh of 96×96 points in our numerical integration.[18] For a given set of two-body input parameters we obtained a value for $f_{-d}(\theta)$ the $\Sigma^- d$ elastic scattering amplitude as a function of the c.m. scattering angle θ. We then found the elastic cross section σ_{el} from

$$\sigma_{el} = \int d\Omega |f_{-d}(\theta)|^2 ,$$

where the angular integration was done numerically. We found the total cross section σ_{tot} from

$$\sigma_{tot} = (4\pi/k)\,\mathrm{Im}[f_{-d}(0)] ,$$

where k is the $\Sigma^- d$ relative momentum. We found the reaction cross section from

$$\sigma_{re} = \sigma_{tot} - \sigma_{el} .$$

We confined our work to energies below the deuteron breakup threshold so that only the two processes $\Sigma^- d \to \Sigma^0 nn$ and $\Sigma^- d \to \Lambda nn$ are included in σ_{re}. In particular we have calculated σ_{el} and σ_{re} for incident Σ^- lab momenta of 30, 50, 70, and 90 MeV/c for which the relative $\Sigma^- d$ momenta are 18.31, 30.52, 42.73, and 54.93 MeV/c, respectively, and the energies available in the $\Sigma^- d$ and $\Sigma^0 nn$ c.m. systems are −2.00 and 1.56 MeV, −1.59 and 1.97 MeV, −0.98 and 2.58 MeV, and −0.16 and 3.40 MeV, respectively. We would have liked to go to even lower momenta but our numerical work becomes unreliable at very low momenta. The results of our $\Sigma^- d$ calculations are given in Tables III, IV, and V.

The first question we asked was whether the low energy $\Sigma^- d$ cross sections are sensitive to the $\Sigma^- p$ scattering lengths. We asked this question be

TABLE III. $\Sigma^- d$ elastic and reaction cross sections with $\Sigma^- d$ lab momentum of 30–90 MeV/c for YN potentials 1–4.

Potential No.	Type of cross section	Cross section in mb at 30 MeV/c	50 MeV/c	70 MeV/c	90 MeV/c
1	Elastic	1584	1343	1108	899
	Reaction	933	706	616	553
2	Elastic	950	842	731	625
	Reaction	666	466	390	344
3	Elastic	758	678	594	514
	Reaction	619	416	337	293
4	Elastic	990	894	787	682
	Reaction	443	354	326	305

TABLE IV. $\Sigma^- d$ elastic and reaction cross sections with Σ^- lab momentum of 30–90 MeV/c for YN potentials 5–8.

Potential No.	Type of cross section	Cross section in mb at 30 MeV/c	50 MeV/c	70 MeV/c	90 MeV/c
5	Elastic	1631	1337	1071	848
	Reaction	1255	870	714	611
6	Elastic	1037	890	749	625
	Reaction	966	622	488	412
7	Elastic	854	740	630	532
	Reaction	906	567	434	361
8	Elastic	1156	1012	864	728
	Reaction	685	508	441	395

cause it is possible that the Λn resonance, being just below the $\Sigma^0 n$ threshold and hence in the middle of the integration region of the internal-momentum variable in the $\Sigma^- d$ problem, might contribute the dominant part of σ_{el} and σ_{re}. Further, we concentrated on the sensitivity of σ_{el} and σ_{re} to A_- rather than A_0 since the former is much more likely to become available through two-body scattering experiments.

To answer the above question we used potentials 1, 2, and 3 to obtain the values of σ_{el} and σ_{re} shown in Table III. We emphasize that potentials 1, 2, 3 not only all have the same values of a_Λ, $r_{0\Lambda}$, E_Λ, Γ – so that the Λn on-shell amplitude between the Λn and $\Sigma^0 n$ thresholds is for all practical purposes the same for all three potentials – but the values of β_Λ are so close for these potentials that between the Λn and $\Sigma^0 n$ thresholds they all have the same Λn half-off-shell amplitude. On the other hand A_- for potential 1 is about twice the size of A_- for potential 3, which is reflected in the values for σ_{el} and σ_{re} listed in Table III. The ratio of σ_{el} for potential 1 to σ_{el} for potential 3 lies within 1.8–2.1 for the energy range shown, while the analogous ratio for σ_{re} lies within 1.5–1.6.

To make sure these results were not particular to the value of E_0 used in potentials 1, 2, and 3 we repeated the above calculation for potentials 5, 6, and 7. As shown in Table IV we obtained similar results. These results, along with those obtained above, indicate that the $\Sigma^- p$ scattering length must be fairly well known – at least to within (say) a 50% error – from, for example, $\Sigma^- p$ scattering experiments, before we can hope to obtain from low-energy $\Sigma^- d$ scattering further information on the YN interaction.

The next question we asked was, given that we knew A_- approximately, say to within 25%, were σ_{el} and σ_{re} sensitive to the position of the Λn resonance? We may look at Tables III and IV for the answer. We see from comparing σ_{el} for potentials 1, 2, 3 (Table III) with σ_{el} for potentials 5, 6, 7 (Table IV), respectively, that for the momentum range shown there is little difference ($\lesssim 12\%$) in having the resonance 6 MeV or 3 MeV below the $\Sigma^0 n$ threshold. This difference is larger for the

TABLE V. $\Sigma^- d$ elastic and reaction cross sections with Σ^- lab momentum of 30–90 MeV/c for YN potentials 9–12.

Potential No.	Type of cross section	Cross section in mb at 30 MeV/c	50 MeV/c	70 MeV/c	90 MeV/c
9	Elastic	950	844	732	627
	Reaction	569	418	362	324
10	Elastic	982	885	777	671
	Reaction	391	335	319	300
11	Elastic	1023	884	748	627
	Reaction	821	541	433	370
12	Elastic	1114	980	840	709
	Reaction	581	445	394	356

lower momentum values since there the Σ^-d c.m. energy is, relative to its distance from the lower value of the resonance energy, much closer to the higher value of the resonance energy. For the same reason the increase of σ_{el} with decreasing energy is more marked in Table IV than it is in Table III. The reaction cross section appears to be more strongly affected by the change in position of E_0. From Table I, however, this effect can be attributed in large measure to the fact that for potentials 5, 6, and 7 A_- is much more absorptive than it is for potentials 1, 2, and 3, respectively.

Again assuming a minor variation in A_- is not ruled out, we asked about the variation of σ_{el} and σ_{re} with Γ given that E_0 is fixed. To answer this question we compare the results for potentials 2 and 4 (whose A_-'s differ by <10%) in Table III and we also compare the results for potentials 6 and 8 (whose A_-'s differ by $\lesssim$16%) in Table IV. We see that the values obtained for σ_{el} are insensitive to the change in Γ throughout the momentum range covered. The values obtained for σ_{re} are very strongly Γ dependent ($\simeq$50% variation in σ_{re}) at the lowest momentum and almost Γ independent ($\lesssim$10% variation in σ_{re}) at the higher momenta. This is just the sort of behavior we would expect in that as the Σ^-d energy moves away from the Λn resonance energy – and hence outside the width of the resonance – the width becomes unimportant, whereas close to the resonance the width (which of course determines how close is close) becomes very important, with the larger value Γ producing the larger values of σ_{re}.

Finally we asked, given that A_- is known exactly – say on the basis of experimental results plus a meson-theoretic model – what is the sensitivity of σ_{el} and σ_{re} to the values of E_0 and Γ? For the answer to this question we compare the results of Table V for potentials 9 through 12 with the results obtained for potentials 2, 4, 6, and 8, respectively, given in Tables III and IV. For example, from Table I we see that potential 9 has a width about half that of potential 2 and its resonance is about 3 MeV closer to the $\Sigma^0 n$ threshold (i.e., for the Λn channel parameters potential 9 looks much like potential 8). However, potential 9 has the same A_- as potential 2. From Tables II and V we see the differences in the Σ^-d cross sections obtained using potential 9 instead of potential 2 is in all cases <16% and in most cases the difference is 10% or less. Similar close results hold for comparisons of the Σ^-d cross sections from potentials 4 and 10, 6 and 11, and 8 and 12. We note that the variation of σ_{re} with changes in E_0 and Γ together is much smaller than it was with a change in either of these parameters alone. This is not unexpected since in going from potential 2 to potential 4 (i.e., decreasing Γ with E_0 fixed) σ_{el} decreases – as the effect of the resonance is spread over a narrower energy range – while in going from potential 2 to potential 6 (i.e., increasing E_0 with Γ fixed) σ_{el} increases – as the resonance is closer to the Σ^-d physical scattering region. The variations in σ_{re} are still a good deal larger than the variations in σ_{el} for corresponding cases. This could merely be a reflection of the fact that the error we make in treating the Λn channel implicitly is not only relatively larger for σ_{re} but it also depends on the values of E_0 and Γ. Whether this sort of error accounts for most of this difference, or whether this difference is real, is now under study.

In summation then, we calculated Σ^-d quartet cross sections below the threshold for deuteron breakup. We used a three-channel (Σ^-p, $\Sigma^0 n$, Λn) S-wave central potential in this calculation, but treated the Λn channel implicitly. We fixed the Λn scattering length and effective range and investigated the variation of the Σ^-d cross sections as the Λn resonance energy E_0 and width Γ were varied as well as when the Σ^-p scattering length A_- was varied. We found that for fixed E_0 and Γ varying A_- by a factor of two caused about a factor 2 variation in the Σ^-d elastic cross section and about a factor 1.5 variation in the Σ^-d reaction cross section. We found that for fixed A_-, within the errors introduced by our implicit Λ channel approximation, the variation of the Σ^-d cross sections was negligibly small. These results indicate that measurement of low-energy Σ^-d cross sections may help in determining the Σ^-p scattering length, but not in determining the position and width of a Λn resonance that lies 3–6 MeV below the $\Sigma^0 n$ threshold.

Two further questions are now under study within the same general framework used here. First, how badly has the implicit Λn channel approximation used here distorted the three-body scattering results? Second, are the cross sections calculated here sensitive to the parameters of the Σ^-n potential?

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Properties of Hadronic Amplitudes in an Absorptive Model*

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Hadronic two-body amplitudes involve two components. The imaginary part of the nondiffractive component R is dominated by the most peripheral impact parameters ($b \sim r$). The imaginary part of the diffractive component P has substantial contributions from all impact parameters $b \lesssim r$. We study the energy dependence of the b representations of both components as well as various possible forms for the corresponding real parts. We show that the following three assumptions are mutually inconsistent: (i) $\mathrm{Im}R(s,t)$ is always dominated by $b \sim r$ terms; (ii) $\mathrm{Im}R(s,t)$ shrinks indefinitely as $s \to \infty$; (iii) r approaches a constant as $s \to \infty$. We define three classes of models obtained by abandoning, one at a time, these three assumptions. We discuss the complex-J-plane structure as well as the asymptotic phase of the R amplitude for each of these classes and propose various experimental ways of distinguishing between the models. A detailed analysis of $\mathrm{Re}R$ indicates that, while in certain cases it reaches its asymptotic phase at relatively low energies, in other cases the asymptotic phase is approached very slowly and it has no resemblance to the observed phase at present energies.

I. INTRODUCTION

The phenomenological description of hadronic scattering amplitudes for two-particle final states involves two components.[1] The first component, $R(s,t)$, contributes to both elastic and inelastic processes. According to the usual duality ideas[2] it can be viewed either as a sum of s-channel resonances or as a combination of "ordinary" t-channel exchanges (poles and cuts). The second component, $P(s,t)$, is the diffractive "Pomeranchukon-exchange" part and it contributes only to elastic (or quasielastic) processes.

Both the t-channel and the s-channel points of view seem to be crucial for the description of various systematic features of elastic and inelastic amplitudes. In general, the t-channel picture has been more successful in explaining the s dependence of hadronic amplitudes while the s-channel picture has been very useful in understanding the