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Finite-Energy Sum Rules, Regge Cuts, and Forward πN Charge-Exchange Scattering

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Finite-energy sum rules (FESR) with logarithmic terms are formulated in a way amenable to simple and accurate approximation, thus allowing their convenient use in data fitting with Regge cuts. It is observed that a typical cut FESR contribution is similar to that of a pole lying perhaps half a unit lower in the J plane. Therefore $\rho+\rho'$ fits to $\pi^- p \to \pi^0 n$ seem to imply decoupling of the $\rho \otimes P$ Regge cut from the nonflip amplitude at $t=0$. This is investigated in more detail, and its implications are pointed out.

I. INTRODUCTION

Finite-energy sum rules (FESR) are usually written for amplitudes with assumed power (Regge pole) asymptotic behavior.[1] However, now it is believed that at high energy there are also significant Regge-cut (i.e., logarithmic) contributions,[2] and so to help in the construction of realistic phenomenological models it is desirable to generalize the sum rules to include such terms.

Unfortunately this is impossible in closed form[3-5] (except in special models[6]), and the repeated numerical evaluations necessary in typical parameter-search calculations are often prohibitively time consuming.

In this paper we first examine the structure of FESR's with logarithmic terms and put them into a form where a simple approximation can be made to overcome the time problem, so giving a practical way to include FESR constraints in data fitting with general types of Regge cuts.

We then observe that in a FESR the simplest kind of Regge cut behaves like a pole with intercept lower in the J plane by about $\frac{1}{4}$ to $\frac{1}{2}$ a unit. Consequently, for example, some $\rho+\rho'$ models of $\pi^- p \to \pi^0 n$ may be good approximations to the physics of a ρ pole plus a $\rho \otimes P$ cut and imply that the cut essentially decouples at $t=0$. (We use the notation $A \otimes B$ to denote the cut resulting from the simultaneous exchange of Regge poles A and B in the t channel.)

The situation is explored in more detail, and quantitative results are given. We point out the major implications of a small cut amplitude at $t=0$, regarding especially the crossover mechanism and the difference of total cross sections at high energy, and note the similarity of the situation in charged-pion photoproduction.

II. SUM RULES

A standard FESR derivation[3] deals with an amplitude $F(\nu)$ $[\nu=(s-u)/4m]$ at fixed t or u (suppressed) with the usual analytic properties and an assumed asymptotic ($|\nu| \geq N$) model form. Cauchy's theorem $[\int_C F(\nu)d\nu=0]$ is used, where the contour C lies along the real axis above the physical cuts from $-N$ to N and closes with a semicircle $|\nu|=N$ in the upper half-plane. Thus the low-energy amplitude is integrated from $-N$ to N, and the asymptotic model round the semicircle (not to threshold, nor to ∞). For an amplitude of definite crossing symmetry $[F(-\nu)=\pm F^*(\nu)]$ the left- and right-hand parts of the low-energy integral can be combined.

A crossing-odd amplitude with Regge-pole (power) behavior,

$$F(\nu)=i\gamma(-i\nu)^{\alpha}$$
$$=i\gamma e^{-i\pi\alpha/2}\nu^{\alpha}, \quad |\nu| \geq N \tag{1}$$

is then found to obey the FESR

$$\frac{1}{N}\int^{N} \mathrm{Im}F(\nu)d\nu=\gamma\frac{N^{\alpha}}{\alpha+1}\cos\tfrac{1}{2}\pi\alpha\,. \tag{2}$$

A Regge-cut amplitude contains powers of $\ln\nu$, and a simple crossing-odd term of the type suggested by current models[3-8] is

$$F(\nu)=i\gamma(-i\nu)^{\alpha_c}[c+\ln(-i\nu)]^{\beta}$$
$$=i\gamma e^{-i\pi\bar{\alpha}/2}q(\nu)\nu^{\alpha_c}, \quad |\nu|\geq N \tag{3}$$

where

$$q(\nu)=[(c+\ln\nu)^2+\tfrac{1}{4}\pi^2]^{\beta/2},$$
$$\bar{\alpha}(\nu)=\alpha_c+\frac{2\beta}{\pi}\arctan\left[\frac{\pi}{2(c+\ln\nu)}\right]. \tag{4}$$

Here α_c is the branch-point trajectory, β is related to the nature of the J-plane cut discontinuity at its tip [see Eq. (14) below], and c is included to account for a possible difference in the scale of the logarithmic and power energy-dependences (as the absorptive/eikonal prescriptions would suggest[3-8]). Note that $\bar{\alpha}(\nu)$ controls the phase of the cut, and we emphasize that there is no assumption about t dependence.

Values of these parameters may either be inserted into (3) according to theoretical knowledge (or prejudice), or else left as phenomenological parameters to be determined by data (just like pole parameters).

Following the derivation outlined above, the Regge-cut FESR can be put into the form

$$\frac{1}{N}\int^{N}\mathrm{Im}F(\nu)d\nu=\gamma\frac{N^{\alpha_c}}{\alpha_c+1}q(N)\,\mathrm{Re}\{e^{-i\pi\bar{\alpha}(N)/2}g(\beta,x)\}, \tag{5}$$

where

$$x=(\alpha_c+1)[c+\ln(-iN)].$$

This equation is similar in structure to the pole sum rule (2) except for the appearance of the "phase function" $g(\beta,x)$, which comes essentially from the semicircle integral and which cannot be expressed in simple closed form. Its numerical evaluation is the key to the practical use of (5).

The continuous-moment generalizations of (2) and (5) have $F(\nu)$ replaced by $(\nu_0^2-\nu^2)^{(-\epsilon-1)/2}F(\nu)$ on the left-hand side (ν_0=threshold), and the substitution $\alpha\to\alpha-\epsilon-1$ or $\alpha_c\to\alpha_c-\epsilon-1$ as appropriate on the right-hand side. It is easy to prove that the error committed is less than $(\nu_0/N)^2$, which is usually negligible, except perhaps for large positive ϵ, which emphasizes the threshold region ("negative moment" sum rules). In any case negative-moment FESR's with logarithmic terms tend to be unreliable because the amplitude itself does not have logarithmic singularities, either at $\nu=0$ or $\nu=\nu_0$.

III. PHASE FUNCTION

The phase function $g(\beta,x)$ is related to the incomplete gamma function. It is defined by

$$g(\beta,x)=e^{-x}x^{-\beta}\int^{x}(x')^{\beta}e^{x'}dx', \tag{6}$$

and is subject to the boundary conditions appropriate to the pole limit

$$\beta=0:\; g(0,x)=1, \tag{7a}$$
$$N\to\infty:\; g(\beta,\infty)=1. \tag{7b}$$

The lower integration limit in (6) is independent of β and x, and is otherwise arbitrary because its contribution cancels from (5) when the real part is taken. Note that the denominator factor $\alpha+1$ is compensated at $\alpha=-1$ by the vanishing of the whole Re{ } term.

a. Exact Evaluation. Exact evaluation of g involves summing the convergent series obtained by expanding the inner exponential in (6) and termwise integration. (For $\beta=-1,-2,\ldots,$etc. this picks up logarithmic terms). Numerical experience shows that $g(\beta,x)$ is accurate to within one percent if more than about $2|x|$ terms are retained, subject to a minimum of 4 or 5.

With typical values $\alpha=0.5$ (e.g., $\rho\otimes P$ trajectory), $c=4$ (absorption model, Refs. 3–8), N=2 GeV (Barger and Phillips, Ref. 9), we have $|x|\simeq 7.5$. Therefore for reasonable accuracy during a parameter search it is necessary to use perhaps 10–15 terms in the expansion of g, which can be excessively time consuming. A simpler, accurate,

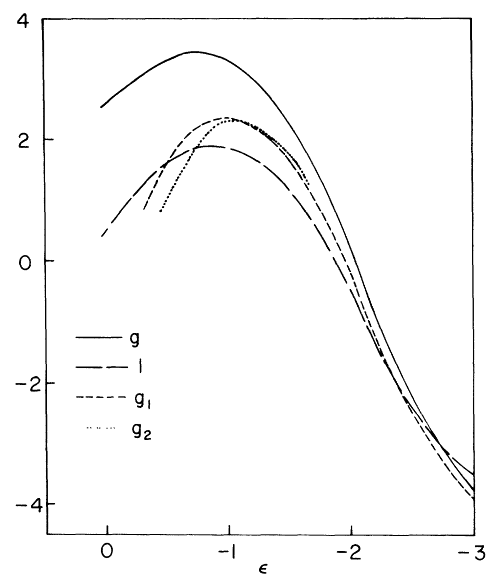

FIG. 1. Comparison of exact (full line, g) and several approximate cut contributions to a continuous-moment FESR, plotted as a function of moment parameter ϵ (Ref. 9). The long-dash line is $g\simeq 1$ [Eq. (8)], the short-dash and dotted lines are $g\simeq g_1$ [Eq. (9)] and $g\simeq g_2$ [Eq. (10)], respectively. The FESR is cut off at N =2 GeV (Ref. 9) and the cut parameters are α_c=0.55, $\beta=-1$, c=1.25 [see Eq. (3)]. Note that the example with $g\simeq g_2$ is indistinguishable from that with $g\simeq g_1$ for $\epsilon\leq-1$.

approximation to g is useful for the main part of such a calculation.

b. Approximations. The simplest approximation is

$$g(\beta, x) = 1\,. \tag{8}$$

This obeys (7), but on the right-hand side of (5) the Re{ } no longer vanishes to cancel the denominator zero.

The approximation (8) is tested in Fig. 1, where approximate and exact cut contributions to a typical πN continuous-moment FESR are compared. Away from the pole at $\epsilon = \alpha$ Eq. (8) is quite satisfactory, and it is especially accurate for higher moments, as expected from the asymptotic condition (7b).

Also included in Fig. 1 are the results of the simplest of a class of rational approximations to g:

$$g(\beta, x) = x/(x+\beta) \quad (\equiv g_1) \tag{9}$$

and

$$g(\beta, x) = x(x+\beta)/[\beta + (x+\beta)^2] \quad (\equiv g_2)\,. \tag{10}$$

These too satisfy (7), and have the advantage of effecting the cancellation required in (5) when $\alpha + 1 = 0$.

They are derived as Padé approximants to the asymptotic series for g, which is obtained from (6) by the usual repeated integrations by parts. [The finite-order Padé approximants to the convergent expansion do not obey condition (7a)].

Figure 1 shows that both g_1 and g_2 are reasonably accurate, again especially for higher moments, as expected. For $\epsilon \leq 0$ in this example there is no clear reason for preferring them to the simplest expression (8).

IV. POLES AND CUTS

It is well known in at least one specific example[5] that FESR's alone are not sufficient to distinguish between possible alternative models. They cannot determine unaided whether (for example) a model of pole plus cut is better physics than one of pole plus pole; they only help to fix the parameters of the model chosen.

Figure 2 compares some pole and cut FESR contributions, and confirms that this is likely to be a quite general result. The sum rules cannot discriminate on the basis of phase between a typical cut and an effective pole. The ϵ dependence of the cut term is always to a good approximation the same as that of a pole, displaced in the J plane.

In the examples shown, the position $\alpha_{\rm eff}$ of the "phase-equivalent pole" is about $\frac{1}{4}$ to $\frac{1}{2}$ unit lower than the actual branch-point trajectory, α_c.

Comparing (2) and (5) the reason is clear: To the extent that $g(\beta, x) \simeq 1$, we have

$$\alpha_{\rm eff} \simeq \bar{\alpha}(N)\,, \tag{11}$$

where $\bar{\alpha}(\nu)$ is given by (4). That is, if (8) holds, we have an explicit expression for the position of the phase-equivalent pole:

$$\alpha_c - \alpha_{\rm eff} = -\frac{2\beta}{\pi} \arctan\left[\frac{\pi}{2(c + \ln N)}\right] \quad (\equiv \delta)\,. \tag{12}$$

From (12) it follows that the typically negative sign of β ($\beta \leq -1$ usually; $\beta = -1$ in Figs. 1 and 2) determines $\alpha_{\rm eff} < \alpha_c$, and the size of the difference δ is roughly inversely proportional to the size of the scale constant c.

The energy dependence of the cut amplitude (3) is different from ν^{α_c} by the logarithmic factor $q(\nu)$ [see (4)], which means that also from this viewpoint it appears to be effectively a lower-lying pole (if $\beta < 0$).

Thus for data fitting over a restricted finite range of ν and t, *including FESR constraints*, it may be a reasonable approximation to make the following replacement in (3):

$$e^{-i\pi\bar{\alpha}(\nu)/2}\, q(\nu)\, \nu^{\alpha_c} \to e^{-i\pi\alpha_{\rm eff}/2}\, \nu^{\alpha_{\rm eff}}\,, \tag{13}$$

where $\alpha_{\rm eff} \simeq \alpha_c - \delta$ and $\delta = \frac{1}{4}$ to $\frac{1}{2}$.

In this context it is worth examining the J-plane

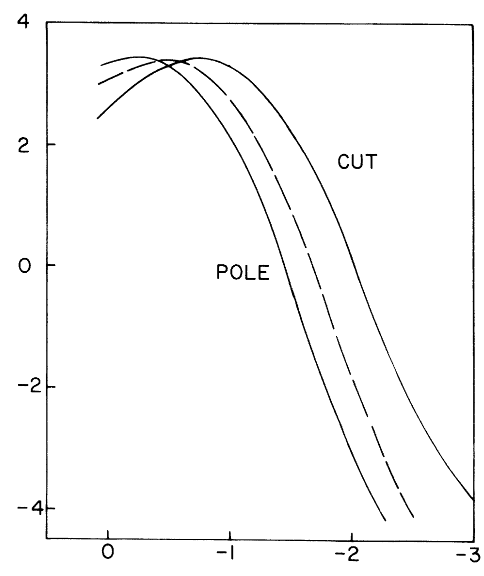

FIG. 2. Comparison of pole and cut FESR contributions for coincident trajectory values, and the same sign of over-all coupling. The cut contribution (CUT) is as in Fig. 1, and the pole contribution (POLE) has $\alpha = 0.55$ ($= \alpha_c$). Shown as a dashed line is a cut contribution with $c = 4$, a typical absorption-model value (Ref. 3–8). Note that the cuts look like poles lying lower in the J plane by about $\frac{1}{2}$ unit ($c = 1.25$) or $\frac{1}{4}$ unit ($c = 4$).

discontinuity function $\Delta(\alpha)$ of the cut amplitude. The interest at this stage is theoretical rather than directly phenomenological, for the models most usually used to generate Regge cuts (absorptive, eikonal[3-8]) do not operate in such terms.

The cut amplitude (3) can alternatively be written

$$i\int^{\alpha_c}\Delta(\alpha)(-i\nu)^{\alpha}d\alpha\,, \tag{3$'$}$$

and correspondingly the right-hand side of (5) becomes

$$\int^{\alpha_c}\Delta(\alpha)\frac{N^{\alpha}}{\alpha+1}\cos(\tfrac{1}{2}\pi\alpha)d\alpha\,, \tag{5$'$}$$

where α_c is the position of the branch point.

Part of the connection between (3), (5) and (3$'$), (5$'$) is

$$\Delta(\alpha)\underset{\alpha\to\alpha_c}{\sim}\text{const}(\alpha-\alpha_c)^{-1-\beta}, \tag{14}$$

and evidently from the preceding discussion of an equivalent pole the main contribution of $\Delta(\alpha)$ is bunched approximately at $\alpha=\alpha_c-\delta$. That is, the form of energy dependence assumed in (3), which is perhaps the simplest, and certainly typical of current models, is essentially a very simple assumption about $\Delta(\alpha)$, the cut J-plane discontinuity. Possibly, future models will be able to give more detailed structure to $\Delta(\alpha)$ and lead to an energy variation which is a superposition of terms like (3).

V. AN IMPLICATION

The discussion in Sec. IV has one (at least) very interesting phenomenological implication.

The process $\pi^-p\to\pi^0n$ has been extensively investigated with various $\rho+\rho'$ Regge-pole models, and some of the fits have included FESR constraints.[9-11] The authors' conclusions concur: If the ρ'-pole intercept is about $\frac{1}{2}$ unit lower than that of the ρ, then to a good approximation the secondary pole decouples entirely from the nonflip amplitude at $t=0$.[9-11]

We have shown that the $\rho\otimes P$ Regge cut can be regarded effectively as such a ρ' pole, and therefore this result seems to have the implication that the cut vanishes in the forward direction.

We have investigated this remarkable situation in more detail, using continuous-moment FESR evaluations for the forward nonflip $\pi^-p\to\pi^0n$ amplitude made by Olsson[11] ($N=5$ GeV, $0.6\geqslant\epsilon\geqslant-2$) and by Barger and Phillips[9] ($N\simeq 2$ GeV, $0\geqslant\epsilon\geqslant-3$). The latter were checked against new evaluations[12] using the latest CERN phase shifts[13]; the agreement is satisfactory.

We assumed the following form of $\rho+\rho\otimes P$ model:

$$T(E)=i(-iE)^{\alpha}\left(\gamma+\frac{\lambda}{c+\ln(-iE)}\right), \tag{15}$$

where E is the pion lab energy and $T(E)$ is the forward nonflip amplitude normalized as in Ref. 11. The procedure was to seek a least-squares fit to the two sets of FESR's with the different cutoff energies by adjusting the four parameters α, γ, λ, and c, and check that the resulting amplitude gives total cross-section differences $\Delta\sigma_T$ and forward differential cross sections in agreement with high-energy measurements.

Note that the use of FESR's with different cutoffs provides an extra lever on the model, being in principle equivalent to a simultaneous fit to $\Delta\sigma_T$ and $(d\sigma/dt)_{t=0}$ at separated energies, which should be sufficient to determine four parameters.

The results are summarized in Fig. 3, where the predicted ratios of cut to pole contributions to the real and imaginary parts of $T(E)$ are plotted as functions of E.

Reasonable fits are possible in fact with both $\alpha=0.483$ (Lovelace-Veneziano, Ref. 14) and $\alpha=0.55$ (from an effective-pole fit to the energy dependence of $d\sigma/dt$, which is dominated by the presumably pure Regge-pole spin-flip amplitude[2]), but if α is allowed to vary, the value $\alpha=0.53$ is strongly preferred.

In the fits shown in Fig. 3 which are for the two bracketing α values, the parameters γ, λ, and c were varied from a large number of starting values.

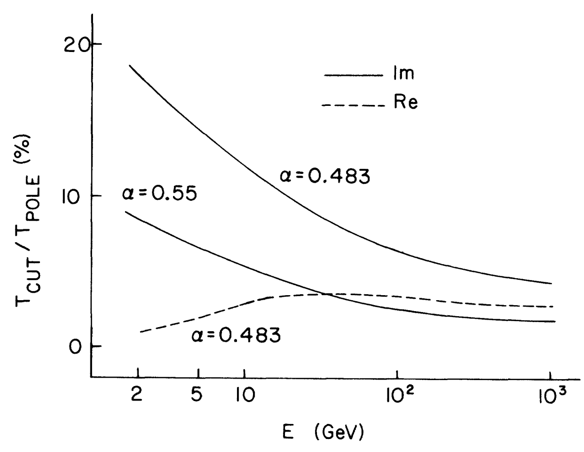

FIG. 3. Predicted ratios of cut to pole contributions to the imaginary (full lines) and real (dashed line) parts of $T(E)$ as functions of E, for $\alpha=0.483$ and $\alpha=0.55$. The ReT ratio for $\alpha=0.55$ is omitted – it is very much smaller than for $\alpha=0.483$. The parameters of the fits as described in the text are

(i) $\alpha=0.483$, $\gamma=2.04$ GeV^{-2}, $\lambda=-0.57$ GeV^{-2}, $c=0.43$.

(ii) $\alpha=0.55$, $\gamma=1.8$ GeV^{-2}, $\lambda=-0.19$ GeV^{-2}, $c=0.00$.

Fixing c at 4 or 5 (typical absorptive/eikonal model values[3-8]) gives a distinctly poor fit to the FESR's. Values of c between 0 and 1 are preferred, corresponding to a phase-effective ρ' pole with intercept around zero (i.e., $|\mathrm{Re}\,T_{\mathrm{cut}}| \ll |\mathrm{Im}\,T_{\mathrm{cut}}|$).

In all cases the cut amplitude turns out to be destructive in sign (i.e., $\gamma > 0$ and $\lambda < 0$) as predicted by the physical picture of absorption,[2-8] but small in magnitude compared to the pole.

Attempted fits using larger constrained cut amplitudes give very inferior results, and alternatively replacing $[c+\ln(-iE)]^{-1}$ in (15) by $[c+\ln(-iE)]^{\beta}$ and varying β gives preferred values $|\beta|<0.01$. The sum rules insist on almost pure Regge-pole amplitudes.

The results presented in Fig. 3 can be converted to an upper limit at (say) 6 GeV of

$$|\mathrm{Im}\,T_{\mathrm{cut}}| < 0.15\,|\mathrm{Im}\,T_{\mathrm{pole}}| \tag{16}$$

corresponding to $\alpha \geq 0.483$, and a much smaller limit if α is larger. Note that the smaller α is, the larger the destructive cut contribution required to maintain the observed energy dependence, which gives for a single pole $\alpha = 0.57$.[11]

Accepting therefore that the strength of the destructive (absorptive) $\rho\otimes P$ cut is rather small at $t=0$, we are necessarily forced to the conclusion that in order to achieve cancellation of the ρ pole and the production of a crossover zero near $t = -0.2$ (GeV/c)^2 the cut amplitude must increase considerably in strength at first as $-t$ increases. This must be true regardless of dip mechanism, although of course the effect is far more dramatic in a purely geometric[8] (as opposed to nonsense-wrong-signature zero[7]) picture.

A cut almost decoupling at $t=0$ and rising sharply for $t<0$ is completely different from the large contribution ($|\mathrm{Im}\,T_{\mathrm{cut}}| \sim 0.8\,|\mathrm{Im}\,T_{\mathrm{pole}}|$) at $t=0$, followed by a rather featureless exponential falloff in t, predicted by the popular models,[3-8] and this situation is currently under further investigation.

It is worth remarking that this cut decoupling at $t=0$ contradicts the natural expectation[2] that strong absorptive cut contributions are responsible for the apparently anomalous energy dependence of $\sigma_T(\pi^- p) - \sigma_T(\pi^+ p)$ in the Serpukhov energy range.[15] In fact, we are inclined to dismiss the "anomaly" because the energy dependence of $\sigma_T(\pi^+ n) - \sigma_T(\pi^+ p)$ at similar energies[15] is in no way anomalous, and disagrees with the other data by a large margin. The amplitudes resulting from the sum-rule analysis agree very well with all the nonanomalous $\Delta\sigma_T$ measurements, and the forward $d\sigma/dt$, as they should.

It is also worth pointing out a similar result reached by Worden[4] in a Regge-pole and -cut analysis of photoproduction. The conclusion is that in charged-pion photoproduction ($\gamma p \to \pi^+ n$, $\gamma n \to \pi^- p$) the $\rho\otimes P$ cut must be small at $t=0$ and stronger for $t<0$.

The main point of the argument is simple. The ratio of differential cross sections (π^-/π^+) differs from unity at $t=0$ only to the extent that the interference between the $\rho\otimes P$ and $\pi\otimes P$ Regge cuts is nonzero. Experimentally the π^-/π^+ ratio is consistent with unity at $t=0$. Since the amplitudes are not orthogonal and the $\pi\otimes P$ cut (responsible for the forward spike) is certainly not zero, the $\rho\otimes P$ cut amplitude must be small. The evidence of FESR's agrees with this reasoning.

VI. CONCLUSIONS

Our results are as follows:

(i) FESR's can be fitted with Regge-cut amplitudes as conveniently and routinely as with the usual poles. For the main part of the parameter search use an approximation to the phase function [e.g., Eqs. (8)–(10)]; use the accurate function only for the final approach to the χ^2 minimum.

(ii) FESR's cannot be expected to distinguish poles from cuts just on the basis of phase. A typical cut with energy dependence of the sort generated by absorptive models looks like a pole lower in the J plane by perhaps $\frac{1}{4}$ to $\frac{1}{2}$ a unit (its J-plane discontinuity is approximately bunched).

(iii) The $\rho\otimes P$ cut appears almost to decouple from nonflip $\pi^- p \to \pi^0 n$ at $t=0$. This is remarkable, contradicting completely expectation based on models for the angular dependence. It is extremely interesting, however, to note that the $\rho\otimes P$ cut also seems to decouple at $t=0$ in charged-pion photoproduction, and the questions arise concerning other cuts, other processes.

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Low-Energy $\Sigma^- d$ Scattering*

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We use a Faddeev formalism and two-body, S-wave, central separable potentials to calculate $\Sigma^- d$ quartet elastic and reaction cross sections for Σ^- incident lab momentum in the range 30–90 MeV/c. We use a three-channel isospin-$\frac{1}{2}$ hyperon-nucleon potential with a Λn resonance below the $\Sigma^0 n$ threshold, but in the three-body calculation the Λn channel is only included implicitly. We find that $\Sigma^- d$ cross sections are sensitive to the size of the $\Sigma^- p$ scattering length, but insensitive to the exact position and width of the Λn resonance.

I. INTRODUCTION

The purpose of this work is to construct a model of $\Sigma^- d$ scattering below the threshold for deuteron breakup and to use it to investigate the sensitivity of the elastic and reaction cross sections to some of the physical parameters of the two-body zero-charge Σ-nucleon (ΣN) interactions.

Our philosophy is that this work is a first crude step in the theoretical investigation of the attributes of the ΣN interaction (e.g., its off-shell behavior) that may not be directly accessible from a study of free $\Sigma^- p$ scattering. Ultimately we should like to be able to test whether two different meson-theoretic potentials which give the same results for on-shell scattering parameters for free $\Sigma^- p$ scattering can be distinguished by analysis of $\Sigma^- d$ scattering. At this stage of the game when very little $\Sigma^- p$ scattering data is available,[1] we feel it makes more sense to work with simple phenomenological potentials rather than full-blown meson-theoretic potentials. For the same reason we shall use only a very crude model for the three-body system. We emphasize that we do not expect the absolute size of the cross sections we calculate to be of any great significance, but rather it is the variation (or the lack of variation) of these cross sections as the two-body interactions are changed, upon which we shall focus.

The only previous calculations of low-energy $\Sigma^- d$ cross sections that we know of are those of Day, Snow, and Sucher,[2] Chen,[3] and Neville,[4] all of whom calculated the ratio of the cross section for Σ^0 production to the total reaction cross section at zero incident Σ^- energy. All three of these works were basically impulse approximation calculations, although Chen did include the $^1S_0$ neutron-neutron final-state interaction. Because of the low Σ^- energy this is not a valid way to proceed with our problem.

We investigate $\Sigma^- d$ scattering in the quartet spin state so that the nucleon-nucleon and hyperon-nucleon interactions are all spin-triplet interactions. We use central, S-wave, two-body potentials to represent these interactions and we do not include the Coulomb interaction. Our two-body potentials are all $^3S_1$ potentials. They are the np potential, the $\Sigma^- n$ potential, and the three-channel ($\Sigma^- p$, $\Sigma^0 n$, Λn) potential which we shall refer to hereafter as the YN potential.[5] Each of the