(1967); 179, 1330 (1969).
[8]M. Goldberger and K. Watson, *Collision Theory* (Wiley, New York, 1964), p. 231.
[9]On this point we disagree with a statement in Bawin (Ref. 6).
[10]D. H. Herndon, A. Barbaro-Galtieri, and A. H. Rosenfeld, LRL Report No. UCRL-20030πN, 1970 (unpublished).
[11]W. Rarita, R. J. Riddell, Jr., C. B. Chiu, and R. J. N. Phillips, Phys. Rev. 165, 1615 (1968).
[12]S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Harper and Row, New York, 1962), p. 372,
[13]W. M. Layson, Nuovo Cimento 20, 1207 (1961).
[14]R. H. Landau, S. Phatak, and F. Tabakin (unpublished).
[15]J. P. Dedonder, Nucl. Phys. A174, 251 (1971); T. E. O. Ericson and J. Hüfner, Phys. Letters 33B, 601 (1970); R. H. Landau, Nucl. Phys. B35, 390 (1971).
[16]L. L. Foldy and J. D. Walecka, Ann. Phys. (N.Y.) 54, 447 (1969).

PHYSICAL REVIEW D VOLUME 5, NUMBER 11 1 JUNE 1972

K^+p and K^-p Elastic Scattering and Total Cross Sections*

R. Henzi,† A. Mackay, B. Margolis,† and W. A. Ross
Department of Physics, McGill University, Montreal 110, P. Q., Canada
(Received 27 December 1971)

We present the results of a fit to K^+p and K^-p total cross sections and elastic differential cross sections and polarizations, as well as a calculation of the ratios of the real to the imaginary part of the forward elastic scattering amplitudes using the Frautschi-Margolis Regge-eikonal model. The results of the fits, especially in large-angle K^-p scattering, present evidence for the validity of a multiple-scattering picture.

Some time ago Frautschi and Margolis[1] developed the proposition that a moving-pole Pomeranchukon plus lower trajectories be eikonalized. This provides a model of moving Regge poles and cuts with no parameters other than those for the poles. The model as has been pointed out[1] predicts the following properties of high-energy scattering:

(1) The total cross sections for hadron-hadron interactions will eventually rise to their asymptotic finite values from below.

(2) The ratios of the real to the imaginary part of forward elastic scattering amplitudes will all be positive (although very small in magnitude) at high enough energies.

(3) The Pomeranchukon contribution to the elastic differential cross sections will continue falling as energy increases at all but the smallest values of momentum transfer.

(4) Diffraction dips due to multiple Pomeranchukon exchange will be present at high enough energy.

(5) Mild violations of factorization ($\approx 20\%$) of amplitudes exist.

Further, the model can account for the following features:

(6) At intermediate energies ($\lesssim 20$ GeV/c) there will be strong shrinkage in exotic channels in contrast to nonexotic channels, where there will be less or no shrinkage or even antishrinkage.

(7) Crossover points which are at too large $|t|$ values in Regge-pole theory (if they are assumed to be associated with nonsense-wrong-signature points) are moved towards their correct values when eikonal-model cuts are introduced.

(8) Turning for a moment from elastic scattering, the extension of eikonalization to nonelastic processes converts forward dips into spikes.[1,2]

The specific purpose of this paper is to examine in what detail the Frautschi-Margolis Regge-eikonal model is capable of describing high-energy elastic scattering. We have undertaken to do the analysis to be described because of new data from Serpukhov[3] for K^+p and K^-p total cross sections as well as new data from other sources for elastic differential cross sections[4] and polarization.[5]

Data on the line-reversed reactions[6] $K^-p \to \bar{K}^0 n$ and $K^+n \to K^0 p$ at around 5 GeV/c and above indicate approximate exchange degeneracy for the K-nucleon system between trajectories of opposite signature. Assuming exchange degeneracy and dominance of the trajectories ρ, ω, P', and A_2 in addition to the Pomeranchukon, one can describe high-energy scattering in terms of relatively few parameters.[7,8]

Preliminary calculations indicate that we will be able to describe $\pi^\pm p$ as well as pp and $\bar{p}p$ elastic scattering at high energies equally well. We leave these cases for later discussion.

In the exotic K^+p channel we write the Regge-pole amplitude for helicity nonflip and helicity flip, respectively,

$$A_N = iC_P(e^{\gamma_1 t} + C_1 e^{\gamma_2 t}) - C_K(1+bt)e^{\alpha t}\nu^{-0.57}, \quad (1a)$$

$$A_F = iC_{PF}\sqrt{-t}\,(e^{\gamma_1 t} + C_1 e^{\gamma_2 t}) - C_{KF}\sqrt{-t}\; e^{\alpha t}\nu^{-0.57}, \quad (1b)$$

where

$$\frac{d\sigma}{dt} = \pi(|A_N|^2 + |A_F|^2)$$

and

$$\nu = \tfrac{1}{2}(s-u),$$

$$\gamma_j = a_j + \alpha_P'(\ln\nu - i\pi/2), \quad j = 1, 2$$

$$\alpha = a_R + 0.95\ln\nu.$$

The formulas above have energy in units of GeV. We have taken the exchange-degenerate Regge trajectory to be given by

$$\alpha_R(t) = 0.43 + 0.95t.$$

It follows that for the K^-p channel, using crossing symmetry, we have nonflip and flip amplitudes $\overline{A}_N$ and $\overline{A}_F$ through the replacement of α by $\overline{\alpha} = a_R + 0.95(\ln\nu - i\pi)$ and (C_K, C_{KF}) by $(C_K e^{-0.43i\pi}, C_{KF}e^{-0.43i\pi})$.

The two exponentials in the Pomeranchukon residue allow for a t-dependent slope[9,10] of differential elastic scattering at small momentum transfers which will decrease with increasing $|t|$. The linear term in t in the spin-nonflip non-Pomeranchukon Regge residue provides some freedom to arrange the details of the surface peaking of the interactions.[11]

We eikonalize the amplitudes above following Frautschi and Margolis[1] and Frautschi, Kofoed-Hansen, and Margolis.[12] We replace[8] ν by s in the calculations.

Results for the total cross sections, differential cross sections, and polarizations, fitted to experiment, are shown in Figs. 1–3. The predicted ratios of forward real to imaginary part of the scattering amplitudes are also shown in Fig. 1. Values for the fit parameters are the following: $\alpha_P' = 0.51$, $a_1 = 2.47$, $a_2 = 0.20$, $a_R = 0.28$, $b = 0.63$ or 1.02, all in $(\mathrm{GeV}/c)^{-2}$; $C_P = 2.00$, $C_K = 7.64$, both in $[\mathrm{mb}/(\mathrm{GeV}/c)^2]^{1/2}$; $C_{PF} = 0.64$, $C_{KF} = -4.16$, both in $[\mathrm{mb}/(\mathrm{GeV}/c)^4]^{1/2}$; $C_1 = 1.08$. One of the two fits has a value of $b = 1.02\ (\mathrm{GeV}/c)^{-2}$ which provides a factor $1 + bt \propto 2\alpha_R(t) + 1$ as part of the nonflip Regge residue.

One sees in Fig. 1 that there is reasonable agreement with total cross-section data at and above 10 GeV/c. The K^+p total cross section rises slowly with increasing energy. Roughly speaking, the rise is described mathematically by the form[1]

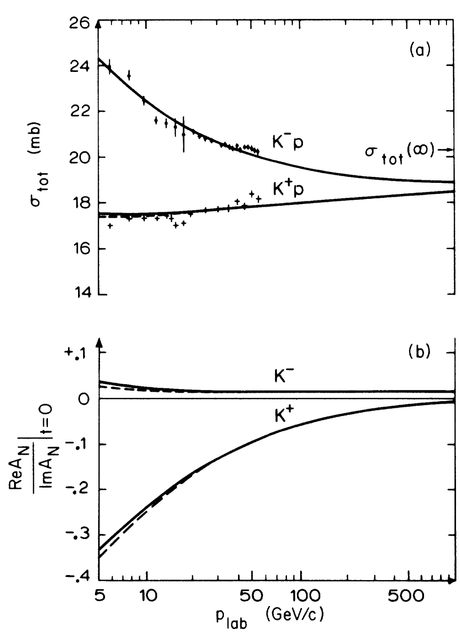

FIG. 1. (a) K^+p and K^-p total cross sections as function of p_{lab}. Data from Ref. 3. (b) Ratios of forward real part to the imaginary part of K^+p and K^-p elastic scattering amplitudes as function of p_{lab}. Solid line: $b = 0.63\ (\mathrm{GeV}/c)^{-2}$. Dashed line: $b = 1.02\ (\mathrm{GeV}/c)^{-2}$.

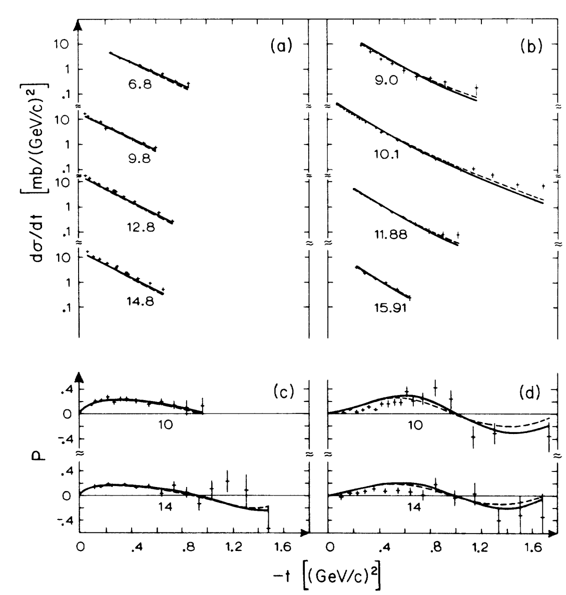

FIG. 2. (a) K^+p elastic differential cross section at $p_{lab} = 6.8$–14.8 GeV/c. Data from Ref. 4. (b) K^-p elastic differential cross section at $p_{lab} = 9.0$–15.91 GeV/c. Data from Ref. 4. (c) K^+p elastic polarization. Data from Ref. 5. (d) K^-p elastic polarization. Data from Ref. 5. Solid line: $b = 0.63\ (\mathrm{GeV}/c)^{-2}$. Dashed line: $b = 1.02\ (\mathrm{GeV}/c)^{-2}$.

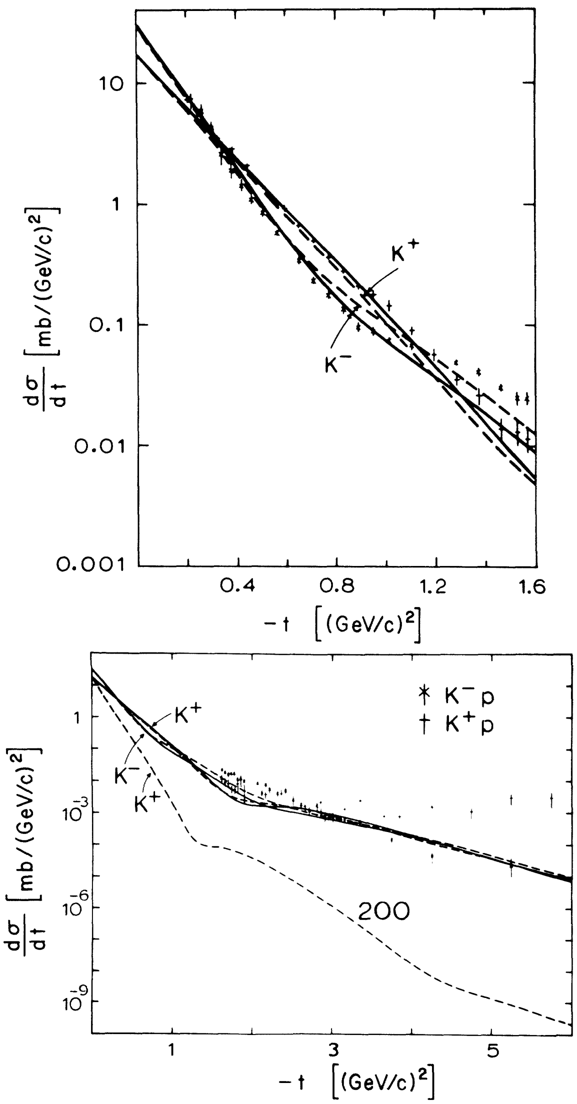

FIG. 3. K^+p and K^-p elastic differential cross sections at 5 GeV/c and K^+p scattering prediction at 200 GeV/c. Data from Ref. 4. Solid line: $b=0.63$ (GeV/c)$^{-2}$. Dashed line: $b=1.02$ (GeV/c)$^{-2}$.

$$\sigma_{\text{tot}}(s) \cong \sigma_{\text{tot}}(\infty)\left[1-\frac{A}{\ln(s/s_0)}\right].$$

There is agreement with experiment in the position of the crossover point between K^-p and K^+p scattering which occurs near $t=-0.3$ (GeV/c)^2 at $p_{\text{lab}}=5$ GeV/c. For $\pi^{\pm}p$ as well as pp and $\bar{p}p$ one can expect the crossover to occur at smaller $|t|$ because the Regge trajectories which produce it apparently couple more strongly when compared to the Pomeranchukon coupling.[8] We note that the form of the Regge residue used is of some importance in determining the exact position of the crossover.

We remark further that this model yields through a series of small momentum-transfer scatterings (multiple scattering) an amplitude at large momentum transfer which is in good agreement with K^-p data. It is to be noted that there are no nonexotic baryon exchanges which will permit backward K^-p scattering, unlike K^+p scattering. The possible importance of baryon exchange cuts at intermediate angles has been discussed elsewhere for channels which can have nonexotic backward exchanges.[13]

There is some disagreement, mainly at 5 GeV/c, between the fits and experiment in the differential cross sections in the region of momentum transfer $t=-1.0$ to -2.5 (GeV/c)^2. We expect that this could be corrected by consideration of different residue functions for the pole amplitudes and possibly modest deviations from exchange degeneracy. Lower trajectories could of course contribute non-negligibly as well.

Returning to total cross sections, in Fig. 1 the K^+p total cross-section fit is perhaps slightly more constant as a function of energy than the data. Within the framework of this model a faster rise with energy (and more flatness in the K^-p case) can be obtained with a larger Pomeranchukon slope than that found here, $\alpha_P'=0.51$ (GeV/c)$^{-2}$. The Pomeranchukon slope obtained in these fits is not bigger, partly because of the constraint of fitting polarization data as well as $d\sigma/dt$ and σ_{tot}. An important test of the model will be measurements of total cross sections at National Accelerator Laboratory energies.

It has been pointed out above that the good fit to the large-angle K^-p scattering data at 5 GeV/c provides evidence for the dominance of multiple-scattering effects at these large momentum transfers. A further consequence of multiple scattering is a repeated change in sign of the polarization for both K^+p and K^-p scattering. The first sign change in the fits shown here is near $t=-1.0$ (GeV/c)^2 for both $K^{\pm}p$. There is evidence in the data for this feature and further measurements would be of interest. Fits of Tuan *et al.*[14] with only single and double exchanges (scatterings) show no sign change near $t=-1.0$ (GeV/c)^2 for K^+p scattering, and a sign change near $t=-1.5$ (GeV/c)^2 for K^-p scattering.

*Work supported in part by the National Research Council of Canada and the Government of Quebec.

†Part of this work was done at CERN.

[1]S. Frautschi and B. Margolis, Nuovo Cimento 56A, 1155 (1968). See also R. C. Arnold, Phys. Rev. 153, 1523 (1967). For a compilation of much other work see the review talk of R. J. N. Phillips, in Proceedings of the Amsterdam International Conference on Elementary Particles, 1971 (unpublished).

[2]E. M. Henley and I. J. Muzinich, Phys. Rev. 136, B1783 (1964); G. A. Ringland and R. J. N. Phillips, Phys. Letters 12, 62 (1964).

[3]S. P. Denisov *et al.*, Phys. Letters 36B, 415 (1971); 36B, 528 (1971); W. Galbraith *et al.*, Phys. Rev. 138, B913 (1965).

[4]C. Baglin *et al.*, CERN report, 1971 (unpublished). (Also see Ref. 6); ABCLV collaboration, Nucl. Phys. B29, 398 (1971); G. C. Fox and C. Quigg, LRL Report No. UCRL-20001, 1970 (unpublished).

[5]M. Borghini *et al.*, Phys. Letters 36B, 493 (1971).

[6]M. Jacob, CERN Report No. CERN-TH-1328, 1971 (unpublished).

[7]S. Frautschi, C. J. Hamer, and F. Ravndal, Phys. Rev. D 2, 2681 (1970).

[8]C. J. Hamer and F. Ravndal, Phys. Rev. D 2, 2687 (1970).

[9]R. A. Carrigan, Jr., Phys. Rev. Letters 24, 168 (1970).

[10]M. Holder *et al.*, Phys. Letters 36B, 400 (1971); U. Amaldi *et al.*, *ibid.* 36B, 504 (1971).

[11]R. Henzi, A. Kotański, D. Morgan, and L. van Hove, Nucl. Phys. B16, 1 (1970); R. Henzi, Nucl. Phys. B35, 434 (1971); H. Harari, Ann. Phys. (N.Y.) 63, 432 (1971).

[12]S. Frautschi, O. Kofoed-Hansen, and B. Margolis, Nuovo Cimento 61A, 41 (1969).

[13]P. R. Auvil, F. Halzen, and B. Margolis, Phys. Letters 32B, 769 (1970); F. Halzen, in High Energy Phenomenology, Proceedings of the Second Session of the Sixth Rencontre de Moriond, edited by J. Tran Thanh Van (Orsay, 1971), p. 65 (unpublished).

[14]R. Hong Tuan, J. M. Kaplan, and G. Sanguinetti, Nucl. Phys. B32, 655 (1971).

PHYSICAL REVIEW D VOLUME 5, NUMBER 11 1 JUNE 1972

High-Energy Bounds from Low-Energy Data*

Richard Blankenbecler and Robert Savit
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 14 February 1972)

A method for deriving rigorous upper bounds on asymptotic values of total cross sections is discussed. The size of the bound is determined by low-energy data. A functional form for the total cross section at high energies is assumed and bounds on the parameters introduced are obtained. The method is applied to π-π scattering, and numerical estimates, using experimental data, are given for different parametrizations. For an asymptotically constant $\sigma_T(s)$, we find $\sigma_T(\infty) \lesssim 40$ mb. Better data would improve the accuracy of this bound and could lower it significantly.

One problem often faced in physics is to estimate, or at least to limit, an experimental quantity in regions where it has not been measured. In this note the Froissart-Gribov representation and a knowledge of finite-energy scattering will be used to limit the value of the total cross section at asymptotic energies, assuming a functional form for σ_T in this region.

Suppose that the experimental situation is as follows: The total cross section is known for x, the square of the center-of-mass momentum, between zero and c. Further, a partial-wave analysis of elastic scattering has been performed for $0 \leq x \leq b \leq c$, so that a finite number of partial-wave elastic cross sections are known. For values of x above c, the functional form of σ_T will be assumed. For example, one could take the cross section to be given by

$$\sigma_T(x) = \sigma_T(\infty) + [\sigma_T(c) - \sigma_T(\infty)](c/x)^{1/2}, \quad x \geq c \qquad (1)$$

as suggested by Regge theory, and derive an upper bound on $\sigma_T(\infty)$.[1]

The Froissart-Gribov formula for the D-wave scattering length d for spinless particles of unit mass (isospin will be neglected for the moment) is

$$d = \int_0^\infty dx\, K(x) \sum_l (2l+1) a_l(x) P_l(w), \qquad (2)$$

where

$$K(x) = [30\pi(1+x)^{5/2} x^{1/2}]^{-1},$$

$$w = 1 + 2/x,$$

and d is defined by

$$d = \lim_{x\to 0} \frac{(x+1)^{1/2}}{x^{5/2}} e^{i\delta_2(x)} \sin\delta_2(x). \qquad (3)$$

The a_l's are the partial-wave amplitudes for the absorptive part in the crossed channel.