make about fluxes near the ends of phase space, it should be a useful guide for estimating yields at NAL and ISR. As data become available, our hypotheses should provide a useful framework within which to assess significance of results.

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Separation of N and Δ Exchanges in πN Scattering and Deduction of Amplitude Zeros*

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Linear combinations of πN differential cross sections and polarizations are used to isolate the $I_u = \frac{1}{2}$ and $I_u = \frac{3}{2}$ exchange amplitudes directly from data. For $I_u = \frac{1}{2}$ exchange the cross section has a zero at $u = -0.15$ (GeV/c)^2 and the polarization becomes large and negative for $|u| > 0.25$ (GeV/c)^2. Our phenomenological analysis indicates that in the dip region the $I_u = \frac{1}{2}$ amplitude is inconsistent with appreciable secondary trajectories or absorptive corrections. For $I_u = \frac{3}{2}$ exchange we infer that the imaginary s-channel helicity-flip amplitude has a zero near $u = -0.15$ (GeV/c)^2, consistent with peripherality for Δ exchange. We also derive a sum rule relating the πN differential cross sections at 180° to the Regge-trajectory difference $\alpha_\Delta - \alpha_N$.

I. INTRODUCTION

The experimental structure of high-energy πN differential cross sections[1] and polarizations[2] near the backward direction has resulted in a puzzling phenomenological situation.[3] When the fixed-u dip was observed at $u = -0.15$ (GeV/c)^2 in $\pi^+ p$ backward scattering, it was first presumed to be due to an amplitude zero of N_α exchange at the $\alpha = -\frac{1}{2}$ wrong-signature–nonsense point. Subsequent measurements of $\pi^- p \to \pi^0 n$ exhibited a fixed-u dip at $u = -0.25$ (GeV/c)^2. This called into question the dominance of the N_α trajectory, and models involving an N_γ contribution were suggested. Following these developments absorptive and fixed-cut models were suggested as alternative explanations of the differential cross-section data. In spite of this proliferation of suggested possibilities none of the existing Regge-pole or -cut fits[4] came close to describing recent measurements of polarization in $\pi^\pm p$ backward scattering.[2] It is therefore imperative to determine the features of u-channel exchange amplitudes needed for a successful description of the scattering data in order to make progress in understanding the nature of the dynamics.

In this paper we show that a surprising amount of information about the u-channel exchange amplitudes can be deduced directly from the differential cross-section and polarization data without the need of specific model assumptions. The differential cross-section contribution due to $I_u=\frac{1}{2}$ exchange can be isolated from elastic and charge-exchange cross sections. The polarization due to interference of $I_u=\frac{1}{2}$ exchange with itself can be bounded from triangular isospin inequalities. Combinations of polarized and unpolarized differential cross sections can be used to study interference between $I_u=\frac{1}{2}$ and $I_u=\frac{3}{2}$ exchanges. Following this model-independent approach we reach several general conclusions about the baryon exchange amplitudes.

Our analysis is presented in terms of s-channel helicity amplitudes G_{++} and G_{+-}. We introduce a vector $\vec{G}$ in helicity space,

$$\vec{G} \equiv G_{++}\hat{l} + G_{+-}\hat{m}\,, \tag{1}$$

where $\hat{l}$ and $\hat{m}$ are orthogonal unit vectors. The differential cross-section and polarization expressions are given by

$$\sigma \equiv \frac{d\sigma}{du} = \vec{G}^* \cdot \vec{G}\,, \tag{2}$$

$$P\sigma = i(\vec{G}^* \times \vec{G}) \cdot \hat{n}\,, \tag{3}$$

where

$$\hat{n} = \hat{l} \times \hat{m}\,.$$

The behavior of the helicity amplitudes near $\cos\theta = -1$ is

$$G_{++} \sim (u_0 - u)^{1/2}\,, \qquad G_{+-} \sim 1\,, \tag{4}$$

where

$$u_0 \equiv (M^2 - \mu^2)^2/s\,.$$

The πN amplitudes are decomposed according to u-channel isospin as

$$(\pi^- p \to \pi^- p)\,, \quad \vec{G}_- = \vec{\Delta}\,,$$
$$(\pi^- p \to \pi^0 n)\,, \quad \vec{G}_0 = \tfrac{1}{3}\sqrt{2}\,(\vec{N} - \vec{\Delta})\,, \tag{5}$$
$$(\pi^+ p \to \pi^+ p)\,, \quad \vec{G}_+ = \tfrac{1}{3}(2\vec{N} + \vec{\Delta})\,.$$

From Eqs. (2) and (5) we can write

$$\sigma_- = |\vec{\Delta}|^2 = \sigma_\Delta\,, \tag{6}$$

$$\sigma_0 = \tfrac{2}{9}(\sigma_N - 2D) + \tfrac{2}{9}\sigma_-\,, \tag{7}$$

$$\sigma_+ = \tfrac{4}{9}(\sigma_N + D) + \tfrac{1}{9}\sigma_-\,, \tag{8}$$

where

$$\sigma_N = |\vec{N}|^2\,, \tag{9}$$

$$D = \tfrac{1}{2}(\vec{N}^* \cdot \vec{\Delta} + \vec{\Delta}^* \cdot \vec{N})\,. \tag{10}$$

For polarizations we can similarly write

$$P_-\sigma_- = i\vec{\Delta}^* \times \vec{\Delta} \cdot \hat{n}\,, \tag{11}$$

$$P_+\sigma_+ = \tfrac{1}{9}P_-\sigma_- + \tfrac{4}{9}X\,, \tag{12}$$

where

$$X = \tfrac{1}{2}i(\vec{\Delta}^* \times \vec{N} + \vec{N}^* \times \vec{\Delta}) \cdot \hat{n} + P_N\sigma_N\,, \tag{13}$$

$$P_N\sigma_N = i\vec{N}^* \times \vec{N} \cdot \hat{n}\,. \tag{14}$$

The defined quantities σ_N, D, and X can be determined from experimental data by the inverse relations

$$\sigma_N = \tfrac{1}{2}[3(\sigma_+ + \sigma_0) - \sigma_-]\,, \tag{15}$$

$$D = \tfrac{3}{4}(\sigma_+ - 2\sigma_0 + \tfrac{1}{3}\sigma)\,, \tag{16}$$

$$X = \tfrac{1}{4}(9P_+\sigma_+ - P_-\sigma_-)\,. \tag{17}$$

Equations (6)–(17) form the basis for our interpretation of the data.

II. N-EXCHANGE CROSS SECTION

The $I_u=\frac{1}{2}$ contribution to the differential cross sections at 6 GeV/c obtained from Eq. (15) is shown in Fig. 1. The data indicate that a nearly complete zero $|\vec{N}|^2$ occurs at $u=-0.15$ (GeV/c)^2. This means that the real and imaginary parts of both helicity amplitudes vanish at $u=-0.15$ (GeV/c)^2. A wrong-signature-nonsense zero of the $\vec{N}$ amplitude is expected at this location if an N_α Regge pole is the dominant exchange. A total zero in $|\vec{N}|^2$ would not be anticipated in strong-Regge-cut absorption models, fixed-cut models, or geometric peripheral models. The extent to which the zero appears in σ_N also places a severe limit on any contributions from N_γ exchange.[5]

The order of the zero in σ_N can be inferred from simple fits to the data in the vicinity of $u=-0.15$ in units of (GeV/c)^2 with a functional dependence $\sigma_N = (u+0.15)^k f(u)$. A quadratic zero ($k=2$) agrees well with the structure in σ_N whereas a quartic zero ($k=4$) is not compatible with the data.

Results for σ_N at 10 GeV/c look very similar to the 6-GeV/c results in Fig. 1, showing the same zero structure at $u = -0.15$ (GeV/c)^2. An effective Regge trajectory for $|\vec{N}|^2$ can be calculated from the data at these two energies. We find an effective trajectory

$$\alpha_N = -0.4 + u \tag{18}$$

$$\frac{1}{4}\left[3\left(\frac{\sigma_+}{\sigma_N}(1+P_+)\right)^{1/2} - \left(\frac{\sigma_-}{\sigma_N}(1+P_-)\right)^{1/2}\right]^2 - 1 \le P_N \le 1 - \frac{1}{4}\left[3\left(\frac{\sigma_+}{\sigma_N}(1-P_+)\right)^{1/2} - \left(\frac{\sigma_-}{\sigma_N}(1-P_-)\right)^{1/2}\right]^2 . \tag{19}$$

For momentum transfers outside the vicinity of the $u \simeq -0.15$ (GeV/c)^2 dip, $\sigma_- \ll \sigma_N$ and $\sigma_+ \simeq \frac{4}{9}\sigma_N$ and we obtain the result

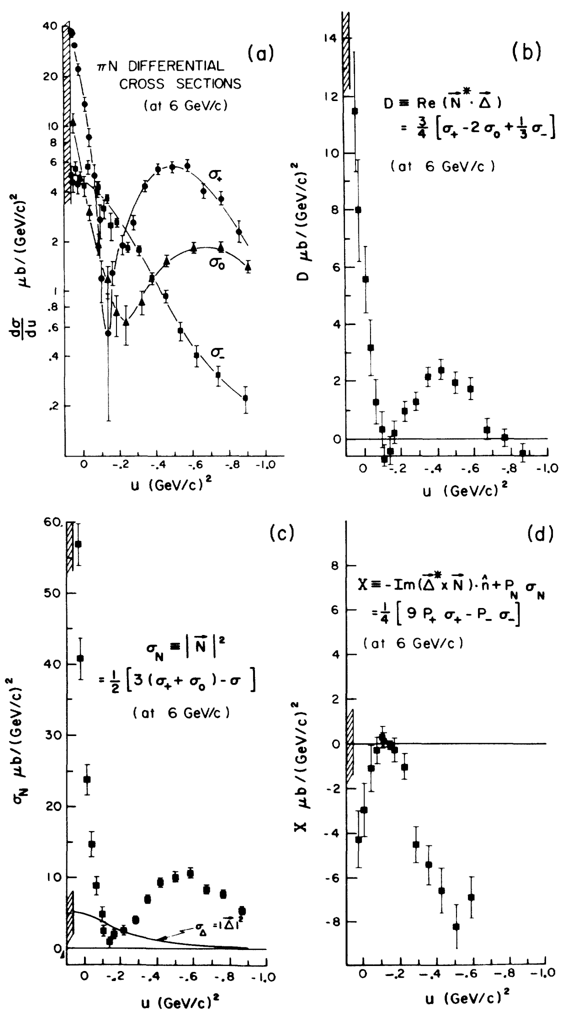

FIG. 1. Experimental data on πN backward scattering at 6 GeV/c. (a) Measured differential cross sections $\sigma_+, \sigma_0, \sigma_-$. (b) $I_u = \frac{1}{2}$ differential cross sections σ_N. (c) Interference term D [cf. Eqs. (10) and (16) of text]. (d) Interference term X [cf. Eqs. (13) and (17) of text].

which agrees well with the usual N_α trajectory.

III. N-EXCHANGE POLARIZATION

The polarization P_N in Eq. (14) due to interference of the $I_u = \frac{1}{2}$ exchange amplitude with itself can be bounded from an isospin inequality written for the combination $N_{++} \pm iN_{+-}$.[6] We find the following restrictive bounds:

$$P_N \simeq P_+ \tag{20}$$

from Eq. (19). Since the measured $\pi^+ p$ polarization[2] is large and negative for $|u| > 0.3$ (GeV/c)^2 (e.g., at $u = -0.48$, $P_+ = -0.65 \pm 0.09$), the polarization from $I_u = \frac{1}{2}$ exchange is also large and negative. In Fig. 2, experimental bounds on P_N from the isospin inequality of Eq. (19) are shown.

A plausible interpretation of the polarization from the $I_u = \frac{1}{2}$ exchange is the existence of a $\sqrt{u}$ term in the N_α Regge trajectory

$$\alpha = a + bu + c\sqrt{u} . \tag{21}$$

A $\sqrt{u}$ term in the trajectory does not alter the effective energy dependence of the differential cross section.[7,8] For a small coefficient c in Eq. (21) the wrong-signature–nonsense dip in σ_N at $u = -0.15$ (GeV/c)^2 will not be appreciably filled in. The polarization for P_N from Eq. (21) is[7,8]

$$P_N = -\left(\frac{u - u_0}{u}\right)^{1/2} \tanh(c\pi\sqrt{-u}) . \tag{22}$$

In order to shift the odd parity partners of the N_α trajectory to higher mass values, the coefficient c must be positive. The corresponding polarization P_N is then negative. For $c = 0.25$ the predicted polarization falls within the allowed corridor for P_N, as illustrated in Fig. 2.

The polarization P_N enters in the quantity X of Eqs. (13) and (17). We can write X as

$$X = -\mathrm{Im}(\vec{\Delta}^* \times \vec{N}) \cdot \hat{n} + P_N \sigma_N . \tag{23}$$

The structure of X in Fig. 1 is similar to σ_N, with opposite sign. This reinforces our supposition that P_N falls monotonically with increasing $|u|$ to a rather large negative value. The $\mathrm{Im}(\vec{\Delta}^* \times \vec{N})$ term in Eq. (23) is presumably a minor perturbation on the contribution from $P_N \sigma_N$. Assuming this is the case, we expect $P_0 \simeq P_+$.

IV. REGGE PHASE RELATION AT 180°

The quantity D in Eqs. (10) and (16) is a measure of the projection of the $\vec{\Delta}$ amplitude on the $\vec{N}$ am-

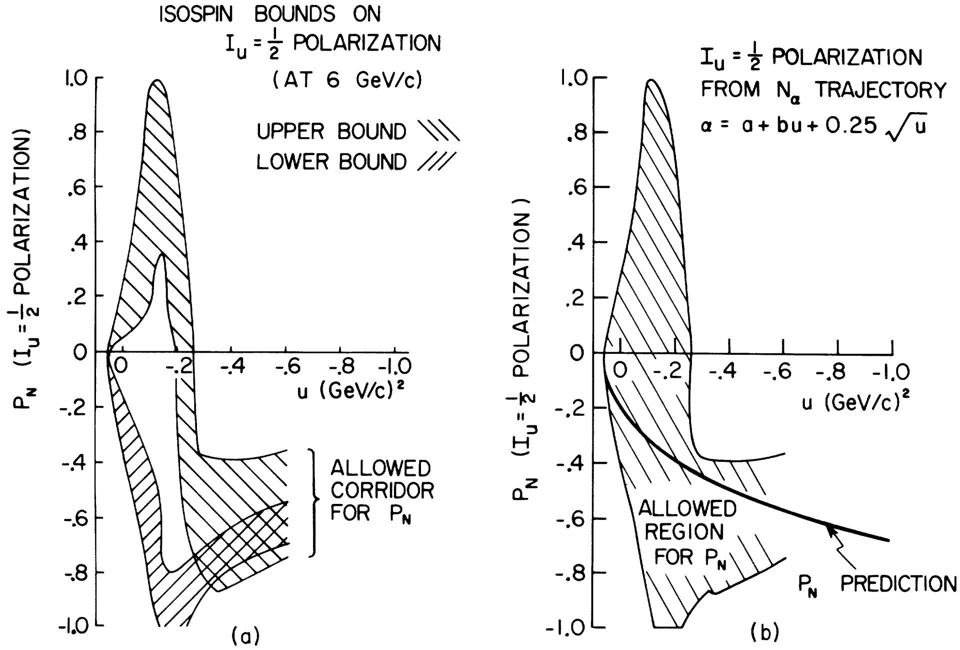

FIG. 2. πN polarization for backward angles at 6 GeV/c. (a) Isospin bounds on the $I_u = \frac{1}{2}$ polarization P_N. (b) Predicted $I_u = \frac{1}{2}$ polarization with an N_α trajectory of the form $\alpha = a + bu + 0.25\sqrt{u}$.

plitude in helicity space. At the backward direction the helicity-nonflip amplitude vanishes according to Eq. (4) and D is given by

$$D(u_0) = \mathrm{Re}(N^*_{+-}\Delta_{+-}) . \qquad (24)$$

With a Regge phase representation

$$N_{+-} = -|N_{+-}|\, e^{-(i\pi/2)(\alpha_N - 1/2)},$$
$$\Delta_{+-} = i\,|\Delta_{+-}|\, e^{-(i\pi/2)(\alpha_\Delta - 1/2)} \qquad (25)$$

the result in Eq. (24) can be expressed as

$$D(u_0) = (\sigma_N\sigma_-)^{1/2} \sin\tfrac{1}{2}\pi(\alpha_\Delta - \alpha_N) . \qquad (26)$$

Combining Eqs. (16) and (26) we obtain the Regge phase sum rule

$$\sin\tfrac{1}{2}\pi(\alpha_\Delta - \alpha_N) = \frac{\frac{3}{4}(\sigma_+ - 2\sigma_0 + \frac{1}{3}\sigma_-)}{(\sigma_N\sigma_-)^{1/2}} \qquad (27)$$

at $u = u_0$. In either Regge-pole or Regge-cut models we expect the phases in Eq. (27) to be correctly specified at $u = u_0$ by the Δ_δ and N_α trajectories, for which

$$\alpha_\Delta - \alpha_N \simeq \tfrac{1}{2},$$
$$\sin\tfrac{1}{2}\pi(\alpha_\Delta - \alpha_N) \simeq 0.7 . \qquad (28)$$

Calculations of the right-hand side of Eq. (27) from the differential cross-section data at 180° give

$$0.70 \pm 0.2 \text{ at } 6\ \mathrm{GeV}/c,$$
$$0.78 \pm 0.3 \text{ at } 10\ \mathrm{GeV}/c, \qquad (29)$$

in good agreement with the value of the left-hand side of Eq. (27) in Eq. (28).

V. N-Δ INTERFERENCE

The experimental form of D obtained from Eq. (16) is shown in Fig. 1. At both 6 and 10 GeV/c this quantity also exhibits a quadratic zero at $u \simeq -0.15$ (GeV/c)^2 similar to σ_N. The dip location is sufficiently near to the backward direction that helicity-flip amplitudes should dominate there. Hence we can still approximate D by Eq. (24) in the vicinity of the dip,[9]

$$D \simeq \mathrm{Re}N_{+-}\mathrm{Re}\Delta_{+-} + \mathrm{Im}N_{+-}\mathrm{Im}\,\Delta_{+-} . \qquad (30)$$

Given that σ_N is explained by a N_α Regge-pole amplitude, the structure of D provides information on the behavior of the Δ helicity-flip contribution. For N_α exchange, $\mathrm{Im}N_{+-}$ has a linear zero and $\mathrm{Re}N_{+-}$ a quadratic zero at the dip. To produce the quadratic zero in D, $\mathrm{Im}\,\Delta_{+-}$ must then have a linear zero. This structure in $\mathrm{Im}\,\Delta_{+-}$ is similar to a Bessel function $J_0(R(u_0 - u)^{1/2})$ with $R \simeq 1$ F. Such peripheral behavior of the imaginary part of the Δ_{+-} exchange amplitude is a natural consequence of duality, due to the fact that the leading direct-channel resonances are peripheral.[10]

The displacement of the dip location from $u = -0.15$ (GeV/c)^2 in σ_+ to $u = -0.25$ (GeV/c)^2 in σ_0 is easily explained in terms of the similarity in structure of D and σ_N using Eqs. (7) and (8). Since D has the same sign and shape as σ_N, the structure due to $(\sigma_N - 2D)$ in σ_0 is suppressed relative to that arising from $4(\sigma_N + D)$ in σ_+. Consequently the addition of the smoothly falling σ_- terms causes the "effective" dip location to be shifted farther out in $|u|$ for σ_0 than for σ_+. The failure of conventional N_α and Δ_δ Regge-pole fits to correctly explain the separation of the σ_+ and σ_0 dips can be traced to the linear zero in D at $u = -0.15$ (GeV/c)^2 in those parametrizations (corresponding to the absence of a zero in Im Δ_{+-}).

VI. Δ-EXCHANGE POLARIZATION

The $\pi^- p$ polarized differential cross section[2] remains positive in the vicinity of $u = -0.15$ (GeV/c)^2. Since Im Δ_{+-} vanishes at $u = -0.15$ (GeV/c)^2, ReΔ_{+-} must be nonzero there. The observed zero in P_- at $u \simeq -0.4$ (GeV/c)^2 can arise from either (i) a 180° phase difference of Δ_{++} and Δ_{+-} or (ii) a zero of Δ_{++} or Δ_{+-} in both real and imaginary parts. The zero of D at $u \simeq -0.7$ (GeV/c)^2 seen in Fig. 1 is probably of the same origin as the zero in P_- at $u \simeq -0.4$ (GeV/c)^2.

VII. SUMMARY

From a direct analysis of the πN backward scattering data we find the following conclusions about the baryon exchange amplitudes[11]:

(i) σ_N has a quadratic zero at $u \simeq -0.15$ (GeV/c)^2, consistent with the wrong-signature–nonsense zero[5] of the N_α Regge pole at $\alpha = -\frac{1}{2}$.

(ii) The effective trajectory from the energy dependence of σ_N agrees well with the N_α trajectory.

(iii) P_N is large and negative for $|u| > 0.25$ (GeV/c)^2. The data are suggestive of the predicted polarization from a small $\sqrt{u}$ term in the N_α trajectory.

(iv) A relation among the σ_+, σ_0, and σ_- cross sections at 180° follows from Δ_δ and N_α Regge phases for the helicity-flip amplitude. For $\alpha_\Delta - \alpha_N = \frac{1}{2}$ the relation agrees with the data.

(v) ImΔ_{+-} has a linear zero at $u = -0.15$ (GeV/c)^2, consistent with a peripheral character. Re$\Delta_{+-} \neq 0$ near this u value.

(vi) The separation of the fixed-u dip locations in σ_+ and σ_0 is easily understood in terms of the linear zeros of ImN_{+-} and ImΔ_{+-} at $u = -0.15$ (GeV/c)^2 and the smoothly falling σ_- differential cross section.

(vii) The failure of previous N_α and Δ_δ Regge-pole fits to the πN backward cross sections can be attributed to an incorrect ImΔ_{+-} parametrization.

A quantitative study of the exchange amplitudes following the approach of this paper is in progress.

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