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Produced Particle Fluxes at Very High Energies*

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A conjecture, motivated by dual-resonance models and scaling laws, is presented which simply relates the production rates for various particles *in the central region,* at limiting high energies. At zero c.m. longitudinal momentum ($x = 0$) and zero transverse momentum, we predict the kaon/pion ratio to be of order 0.02. An operational definition is proposed for the term "central region"; it is used subsequently to determine the energy region of applicability of our predictions. The results given here may be useful for estimating secondary particle yields at the CERN Intersecting Storage Rings and the National Accelerator Laboratory.

Recently developed models of inclusive reactions at high energies based on dual-resonance models[1] or semiclassical radiation concepts[2] have been successful in explaining qualitative features of particle production at high energies. Sharp damping in transverse momentum, scaling laws in longitudinal momentum, logarithmic growth of multiplicities, and inseparability of the invariant cross section into transverse and longitudinal variables are present to the degree suggested by data.[3]

However, one important aspect of production at high energies has not yet been provided by these models; this is the realistic prediction of relative production rates for particles with various masses. To relate K-meson to π-meson production, one could simply include internal-symmetry arguments, for example; but SU(3) Clebsch-Gordan coefficients provide factors of only 2 to 3 in relative production rates, whereas it is known that factors of 20 to 30 are typically observed favoring π-meson to K-meson production. The only models which provide reasonable estimates of these ratios have been phase-space or statistical-thermodynamic models[4] which include Boltzmann (or Planck) distributions characteristic of a limiting temperature of order 140 MeV. The connection between such models and the previously mentioned lines of investigation (or any approach based on analyticity and unitarity) is absent, and it is clearly desirable to advance alternative hypotheses in any case, since the predictions of the thermodynamic model have often been the only serious proposals for estimating beam fluxes at high energies. Thus we are led to make a conjecture, based on an abstraction from dual models, which allows predictions of relative yields.

In calculations of inclusive spectra based on dual-resonance models,[1,5] it is observed that in the central region[6] (where $x \to 0$) the invariant cross section $E\, d^3\sigma/dp_L dp_T^2$ depends explicitly only on the variable

$$\kappa = p_T^2 + m^2 .$$

Here, m and p_T^2 are, respectively, the mass and the square of the transverse momentum of the produced particle. There is no dependence on m other than that which enters through κ. As an example of the form of dependence on κ, we reproduce in Fig. 1 results obtained by DeTar *et al.*[1] from their dual-model calculation.

The variable κ mentioned above is related to the Toller angle of the produced particle vertex, in the O(2, 1) phenomenological formalism introduced by Mueller[7] and developed further by Abarbanel.[8] The variable enters as a limiting ratio of subenergies in any theory based on Lorentz invariants, and appears to be the characteristic variable to describe transverse-momentum distributions, at least in models with finite range order in rapidity. Therefore, the fact that the invariant cross section depends only on κ, for values of x near zero, is doubtlessly much more general than dual models. Moreover, the general shape of the distribution given in Fig. 1 is also more general. Whereas the precise shape and numerical value of the curve do depend on details of the model, both the rapid fall of $d\sigma/d\kappa$ as κ is increased from zero and a more gradually decreasing behavior at large κ may be expected from general arguments[9] concerning the analytic structure of the central two-Reggeon two-particle vertex of the graph given in Fig. 1 (inset).

Following these observations, we find it natural to abstract the following conjecture from explicit dual-model calculations:

Within small ratios (e.g., provided perhaps by internal-symmetry Clebsch-Gordan coefficients), the inclusive transverse momentum spectrum $E\, d^2\sigma/dp_L dp_T^2$ of any produced particle, *including*

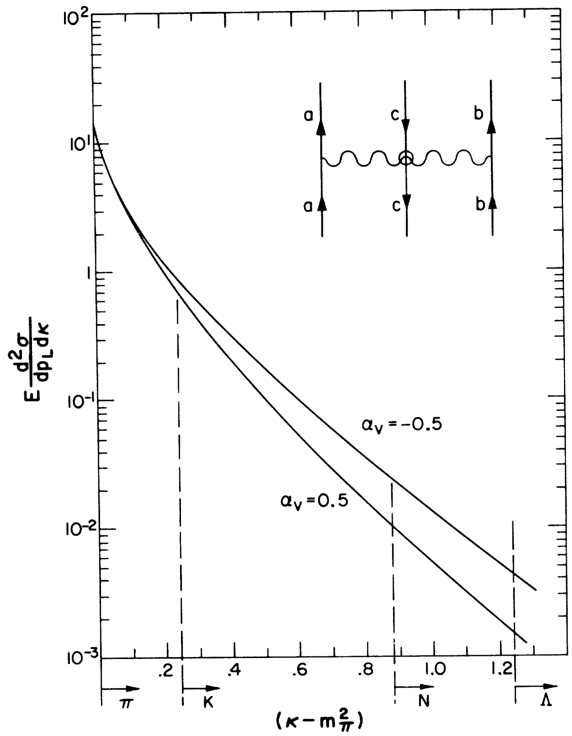

FIG. 1. Plot of the universal, invariant inclusive cross section versus $(\kappa - m_\pi{}^2)$ for fixed $p_L = 0$. Solid curves were obtained by DeTar *et al.* (Ref. 1); as noted, they correspond to two different choices of intercept α_V of the vacuum trajectory. Absolute normalization is arbitrary; only relative normalization along the curves is meaningful. Marked along the κ axis are beginnings of the physical region for π, K, N, and Λ production. *Inset:* Mueller diagram appropriate in the central region for inclusive reaction $a + b \rightarrow c +$ anything. Wavy lines are exchanged vacuum trajectories.

normalizations, is a universal function of κ, for small x. At a given value of x near zero, this universal function can be obtained from the (experimental) spectrum of the lightest produced particle (pion) by substituting (through $p_T{}^2 = \kappa - m_\pi{}^2$) the variable κ defined above. For a heavier particle of mass M, the physical region (starting, of course, at $\kappa = M^2$) begins at $p_{T,\pi}{}^2 = M^2 - m_\pi{}^2$.

Imagine that the experimental spectrum for pions at $x = 0$ were given by one of the solid lines in Fig. 1. Then the *normalized* $p_T{}^2$ spectrum for K's, $\overline{N}$'s, Λ's, etc., could be read off from this curve. We would predict that the kaon/pion ratio should be approximately 0.02 *at (both)* $p_T{}^2 = 0$. Likewise, the antiproton/pion ratio is of order 10^{-3} for antiprotons and pions with $p_T{}^2 = 0$. Alternatively, we could integrate under the curves to obtain *total* fluxes at $x = 0$; these values are $K/\pi = 0.13$ to 0.16, $\overline{p}/\pi \simeq 3\times 10^{-3}$, and $\Lambda/\pi \simeq 10^{-3}$.

Although our conjecture is motivated in the simplest dual model *only at* $x = 0$, we propose that the universal κ distribution holds also for small finite x, e.g., $|x| < 0.2$. Moreover, in spite of the fact that dual models have not been very successful for quantitative fits to data, we are encouraged to take our conjecture seriously because other qualitative suggestions abstracted from dual models have led to useful insight.[10]

For what range of energies do we expect our conjecture to be valid? The energy must be sufficiently high that there is a clearly defined *central region*. In the literature, existence of a central region is almost synonymous with presence of a flat plateau in a rapidity plot.[11] This is difficult to deal with operationally. We therefore propose the following more restrictive criterion: For a specific inclusive process, initiated by given particles, a distinct central region exists (in the neighborhood of $x = 0$) only if, in that region, one finds equal numbers of π^+ and π^-, of K^+ and K^-, of p and $\overline{p}$, and so forth. A simple glance at the Mueller diagram with two Pomeranchukons, shown in Fig. 1 (inset), will confirm that particle-antiparticle equality will occur in the kinematic region in which that (central region) diagram dominates. Moreover, our operational definition (flat rapidity spectrum and equal particle-antiparticle cross sections) is in accord also with the substance of other models. For example, in the multiperipheral (MP) model the central region is, by definition, many correlation lengths (in rapidity) from the incident particle and target ends of the MP chain.[12] Therefore, in the central region, particles and antiparticles should be produced in equal numbers, regardless of the quantum numbers of particles initiating the interaction.

At present accelerator energies, there is no such equality, even for pions. As shown in Fig. 2, for 12.5-GeV/c pp interactions,[13] at $p_L = 0.6$ GeV/c, the π^+/π^- ratio is between 2 and 3 over the full range of $p_T{}^2$ values. Similarly, the K^+/K^- ratio is about 10. Although $p_L = 0.6$ GeV/c, corresponding to $x = 0.25$, may be deemed too large for the central region, a similar π^+/π^- ratio is observed also in the same experiment[13] at the lowest measured value of p_L, viz., 0.11 GeV/c ($x \simeq 0.05$). Likewise, in pp interactions at 19.2 GeV/c,[14] for $x \simeq 0.2$, $\pi^+/\pi^- \simeq 2.2$, and $K^+/K^- \simeq 3.5$, indicative of slow, if any, energy dependence of the π^+/π^- ratio.

It is clear therefore that our conjecture, or any other hypothesis based on the existence of a *central region*, is not relevant at present conventional accelerator energies ($p_{\rm lab} \lesssim 70$ GeV/c). On the other hand, by $p_{\rm lab} \gtrsim 300$ GeV/c, one may expect that the increased size of longitudinal phase space will allow separation of events into a central region and two fragmentation classes. Thus we suggest that our hypothesis be used *now* to estimate expected secondary fluxes at the National Accelerator Laboratory (NAL) and the CERN Intersecting Storage Rings (ISR). Once data near $x=0$ are available from these accelerators, our operational definition can be used to ascertain whether a central region exists. Then, tests of our conjecture of a universal $d\sigma/d\kappa$ dependence should provide important insight into mass dependence of vertices such as the two-particle two-Reggeon vertex shown in Fig. 1 (inset). Fortunately, in the one case in which the energy is sufficiently high and a clear identification of particles has been made,[15] it does seem that the K^+/π^+ ratio, at least, is not incompatible with our expectations. At both 1100 and 1500 GeV/c, for $x \simeq 0.15$ and $p_T^2 = 0.16$ (GeV/c)^2, the upper limit[15] for the K^+/π^+ ratio is about 10%, to be compared with the value 13% obtained from Fig. 1 [for $\kappa_\pi = 0.18$ (GeV/c)^2, $\kappa_K = 0.40$ (GeV/c)^2]. Investigations over a more extended range of x and p_T^2 are clearly crucial.

In the paragraphs above, we have concentrated on only one consequence of our conjecture, namely, its prediction for the *absolute value* of K/π, $\bar{p}/\pi$, Λ/π, etc., ratios. We turn now to a brief examination of other related aspects of data.

(1) Shapes of spectra. At low energies, in all inclusive processes,[13,14] a universal behavior of the type $\exp(-3p_T^2)$ is observed at large values of p_T^2 [$\gtrsim 0.2$ (GeV/c)^2]. An example of this is shown in Fig. 2; note that the *shapes* of the spectra for $\pi^\pm$ and K^- are nearly identical, in spite of the fact that $K^-/\pi^- < 0.1$ at a given value of p_T^2. Our conjecture suggests that this universality of *shape* at large p_T^2 should persist to asymptotic energies. This is a weaker statement than our full conjecture, which specifies also relative *normalization*.

(2) Sharp forward peak. At low energy, as shown in Fig. 2, a sharp peak near $p_T^2 = 0$ is present in the p_T^2 distribution for pions. As explained by Yen and Berger,[16] this forward peak is built up by pions which are decay products of N^* resonances with small Q value. With such a model, it is difficult to predict the energy dependence of the forward peak since, as energy is increased, one must deal with more N^*'s, of higher and higher mass and with complicated decay chains. As shown in Fig. 1, the dual model suggests that the steep forward peak for π's persists to asymptotic energies and, moreover, is present in the central region. This latter point is interesting because the Yen-Berger explanation is linked to fragmentation concepts (N^* production and decay), not ordinarily thought to be applicable at $x=0$ for very large energy. On the other hand, the dual spectrum suggests no pronounced forward peak for K's or $\bar{p}$'s, in some qualitative disagreement with low-energy results.[13,17] More detailed experimental investigations are needed of π and K spectra, at small x and very small p_T^2, over a wide range of energies ($p_{lab} = 5$ to 500 GeV/c).

(3) Mean transverse momentum. The only difference between p_T^2 spectra for π, K, $\bar{p}$, Λ, etc., occurs at very small p_T^2. The large-p_T^2 behavior is universal. Our model and data (cf. Figs. 1 and 2) show a pion spectrum which drops much more sharply at small p_T^2 than at large p_T^2; i.e., the curve is concave upwards on a log plot vs p_T^2. Consequently, as remarked before,[16] the sharp forward peak in the π spectrum implies that the mean p_T^2 for pions should be less than for kaons.

In summary, we have extracted from dual models a simple conjecture relating production rates for various particles at very high energies. Although our conjecture is applicable only in the central region, and we have no simple statement to

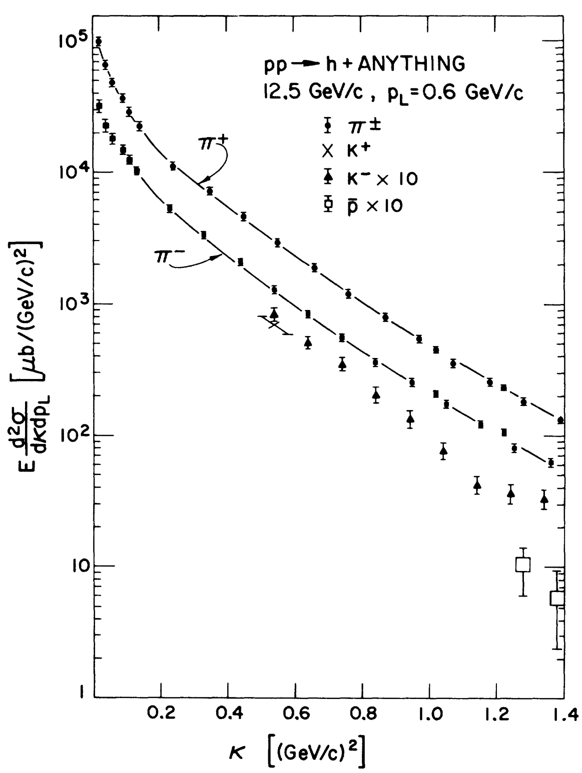

FIG. 2. Experimental data of Akerlof *et al.* (Ref. 13) replotted as invariant inclusive cross section versus $\kappa = p_T^2 + m^2$, where m and p_T^2 are, respectively, the mass and the square of the transverse momentum of hadron h from $pp \to h +$ anything. Center-of-mass longitudinal momentum p_L is fixed at 0.6 GeV/c. As noted, K^- and $\bar{p}$ cross sections are multiplied by 10 in this figure. Dashed lines joining sets of $\pi^\pm$ points are drawn to guide the eye.

make about fluxes near the ends of phase space, it should be a useful guide for estimating yields at NAL and ISR. As data become available, our hypotheses should provide a useful framework within which to assess significance of results.

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Separation of N and Δ Exchanges in πN Scattering and Deduction of Amplitude Zeros*

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Linear combinations of πN differential cross sections and polarizations are used to isolate the $I_u = \frac{1}{2}$ and $I_u = \frac{3}{2}$ exchange amplitudes directly from data. For $I_u = \frac{1}{2}$ exchange the cross section has a zero at $u = -0.15$ $(\text{GeV}/c)^2$ and the polarization becomes large and negative for $|u| > 0.25$ $(\text{GeV}/c)^2$. Our phenomenological analysis indicates that in the dip region the $I_u = \frac{1}{2}$ amplitude is inconsistent with appreciable secondary trajectories or absorptive corrections. For $I_u = \frac{3}{2}$ exchange we infer that the imaginary s-channel helicity-flip amplitude has a zero near $u = -0.15$ $(\text{GeV}/c)^2$, consistent with peripherality for Δ exchange. We also derive a sum rule relating the πN differential cross sections at 180° to the Regge-trajectory difference $\alpha_\Delta - \alpha_N$.

I. INTRODUCTION

The experimental structure of high-energy πN differential cross sections[1] and polarizations[2] near the backward direction has resulted in a puzzling phenomenological situation.[3] When the fixed-u dip was observed at $u = -0.15$ $(\text{GeV}/c)^2$ in $\pi^+ p$ backward scattering, it was first presumed to be due to an amplitude zero of N_α exchange at the $\alpha = -\frac{1}{2}$ wrong-signature–nonsense point. Subsequent measurements of $\pi^- p \to \pi^0 n$ exhibited a fixed-u dip at $u = -0.25$ $(\text{GeV}/c)^2$. This called into question the