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Search for Direct Processes in $K^{\pm} \to \pi^{\pm} \pi^0 \gamma$ Decays*

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We have found 33 events of the type $K^+ \to \pi^+\pi^0\gamma$ and 31 of the type $K^- \to \pi^-\pi^0\gamma$ in an experiment using spark chambers. The angles of the observed decay products were used to determine the angle between the charged pion and the γ ray in the K rest frame. The distribution of this angle showed no indication of a direct radiative process but instead was consistent with internal bremsstrahlung. In order to be quantitative concerning the amount of direct interaction, we selected 24 events with the charged-pion kinetic energy T_π restricted to $58 \le T_\pi \le 90$ MeV and carried out maximum-likelihood fits for various assumptions regarding the radiative amplitudes, final-state interaction, and amount of *CP* violation. Assuming no magnetic dipole, the largest value found by the maximum-likelihood fit for the direct (electric dipole) amplitude is $A_E = -0.12^{+0.21}_{-0.25}$ relative to the internal-bremsstrahlung amplitude. If the direct process occurs, and if *CP* is appreciably violated in the $K^\pm \to \pi^\pm\pi^0\gamma$ decay, there would be a difference between the rates for $K^+ \to \pi^+\pi^0\gamma$ and $K^- \to \pi^-\pi^0\gamma$. Our result is $\Gamma(K^- \to \pi^-\pi^0\gamma)/\Gamma(K^+ \to \pi^+\pi^0\gamma) - 1 = 0 \pm 0.24$. Thus a large asymmetry suggested by some theories is ruled out by this observation.

I. INTRODUCTION

The decay $K^\pm \to \pi^\pm\pi^0\gamma$ can occur through either the internal-bremsstrahlung (IB) mechanism or a so-called direct process. In the IB process, one of the charged particles which appears in the laboratory ($K^\pm$ or $\pi^\pm$) emits the photon, and the weak interaction $K^\pm \to \pi^\pm\pi^0$ therefore proceeds with that particle off the mass shell. In the direct process, it is impossible to separate the radiative vertex from the weak decay interaction. The two types of processes are represented in simplified form by the diagrams of Fig. 1.

The decay $K^\pm \to \pi^\pm\pi^0$ violates the $\Delta I = \frac{1}{2}$ rule because the pions must be in an $I=2$ final state. It is suppressed relative to the decay $K^0 \to \pi\pi$ by a factor of about 600; therefore the direct decay, $K^\pm \to \pi^\pm\pi^0\gamma^0$, can compete more favorably with the IB rate, which is on the order of 1% of the $K^+ \to \pi^+\pi^0$ rate. If a direct amplitude is observable, then it is possible to test *CP* invariance in the electromagnetic and/or weak interaction by comparing the partial decay rates and Dalitz-plot densities for the reactions $K^+ \to \pi^+\pi^0\gamma$ and $K^- \to \pi^-\pi^0\gamma$.[1] According to some estimates,[2] the ratio $\Gamma(K^- \to \pi^-\pi^0\gamma)/\Gamma(K^+ \to \pi^+\pi^0\gamma)$ of these partial rates might be as large as 3.

Three experiments have been reported[3-5] on the decay $K^+ \to \pi^+\pi^0\gamma$ to date. None of these experiments shows evidence of an appreciable amplitude for the direct process.

In this paper we report the observation of 33 events of the type $K^+ \to \pi^+\pi^0\gamma$ and 31 of the type $K^- \to \pi^-\pi^0\gamma$. Section II presents a description of the experimental procedure including the event reconstruction, and Sec. III contains a description of the analysis procedure. In Sec. IV we present the conclusions.

II. EXPERIMENTAL PROCEDURE

A. Apparatus and Technique

The $K^\pm \to \pi^\pm\pi^0\gamma$ events were obtained during a measurement of the relative partial decay rates for $K^\pm \to \pi^\pm\pi^0\pi^0$.[6] The apparatus is described in detail in Ref. 6, so we present only a summary of

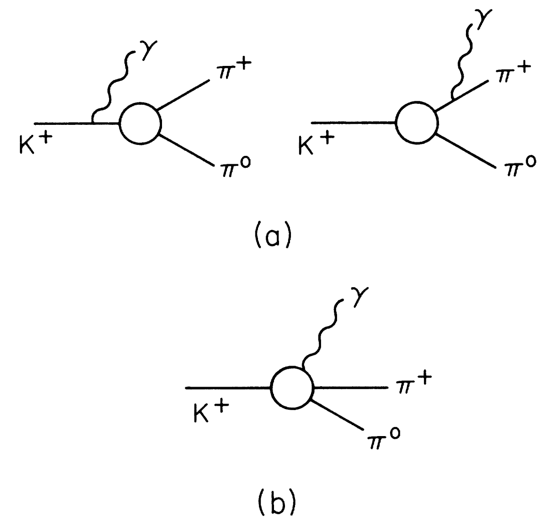

FIG. 1. Graphs for the two types of processes considered in this experiment: (a) charged K decaying to two pions with IB (b) direct radiative decay.

the significant aspects here.

The data consist of spark-chamber photographs (90° stereoscopic) of decays in flight of 3.5 ± 0.06-GeV/c $K^{\pm}$ mesons. The unseparated charged beam was produced at an angle of 2° relative to the first external proton beam of the Argonne Zero Gradient Synchrotron. Particle identification was achieved electronically with a differential Čerenkov counter and a threshold Čerenkov counter. The positive beam contained about 1.4% K^+ mesons, and the negative beam contained 1.0% K^- mesons. The Čerenkov selection reduced the pion and proton contributions to less than 5×10^{-4} of the kaon intensity. The total intensity for each polarity was limited to 2×10^5 particles/sec to avoid systematic errors arising from intensity-dependent effects such as random coincidences.

The detection apparatus (Fig. 2) consisted of a vacuum tank in which most of the decays occurred, spark chambers upstream and downstream from the vacuum tank to determine the positions of the incident particle and decay products, respectively, and appropriate scintillation counter hodoscopes to trigger the spark chambers. One large thin-foil spark chamber with four gaps was placed immediately downstream from the decay volume to detect the charged pion and to verify that no other charged particles emerged from the decay volume. Behind this chamber were four (36 in.×40 in.) spark chambers each containing 22 plates of $\frac{1}{8}$-in. aluminum. Each of these chambers was 0.80 radiation length thick. The three heavy plate chambers farthest upstream had 6-in.-diameter holes to allow the beam to pass through without interacting. The fourth heavy plate chamber had no hole.

The information contained in the photographs is the position of the incident K at the beam chamber and the trajectories of the $\pi^{\pm}$ and photon conversion products. Assuming the direction of the K to be within the divergence of the beam, one has sufficient information to provide a one-constraint fit to the $K^{\pm} \to \pi^{\pm}\pi^0\gamma$ hypothesis.

B. Scanning and Measuring

As mentioned previously, this is a subset of the data from a measurement of the relative partial

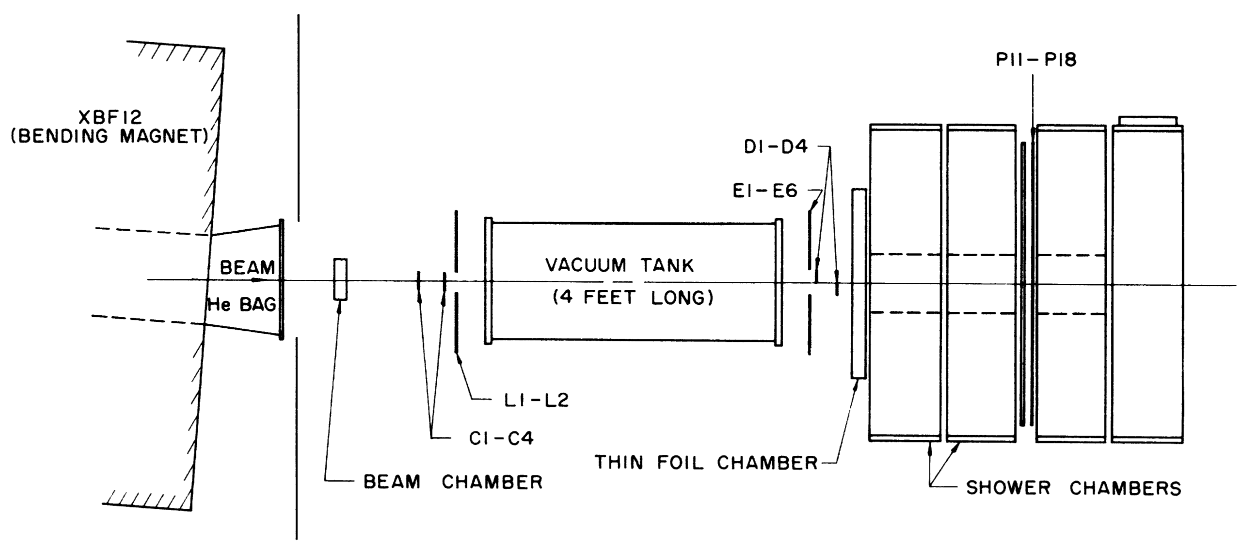

FIG. 2. Plan view of the decay observation apparatus: Counter arrays $C1$–$C4$ and $L1$–$L2$ specify exactly one incident charged particle; $D1$–$D4$, $E1$–$E6$, and $P11$–$P18$ select events with one outgoing charged particle and three or more showers.

rates for $K^+ \to \pi^+\pi^0\pi^0$ and $K^- \to \pi^-\pi^0\pi^0$ (called τ') decays. About 10 000 events of this type of each sign were identified by observing the charged pion and three or four photon showers from the π^0 decays. On the basis of the previously measured branching ratio[7] and our Monte Carlo detection efficiencies, we expect that the three-shower events include at least 60 events of each of the decay modes $K^\pm \to \pi^\pm\pi^0\gamma$. Independent of the type of interaction, at least one-third of these events should have the laboratory angle of the charged pion relative to the beam direction larger than the kinematic maximum angle for the τ' decays.

The criteria used in scanning for τ' events were the following:

(1) At least one nonshowering track must be seen entering the spark-chamber array through the thin-foil spark chamber.

(2) Three or four independent showers must originate in the first three thick plate chambers.

(3) One extra track entering the array through the thin-foil chamber was tolerated to avoid possible asymmetries due to slightly different compositions of the positive and negative beams.

In addition to τ' events, the above criteria accept $K^\pm \to \pi^\pm\pi^0\gamma$ events and a relatively small number of other background events which will be discussed later.

The definition of a shower originating within the thick plate chambers is of particular relevance to culling $K^\pm \to \pi^\pm\pi^0\gamma$ events from the scanning results. A curving or showering track containing six sparks or more, which is clearly independent from all other tracks in the chambers, is considered a γ-ray shower. This includes showers from γ rays with possibly as little as 15 MeV. Therefore no reasonable candidate was rejected in the scanning.

As a first step in selecting those events for which the angle of the charged track was too large to be a τ' decay (0.136 rad relative to the nominal beam direction), we eliminated all events in which the angle was less than 4° (0.070 rad) in both 90° stereo views by using a template on the scanning table. This reduced the number of events under consideration from 20 000 to 2400. The four-shower events were carried through the analysis to determine the τ' background and were ruled out.

Most of the 2400 events from the wide-angle scan were measured on a Scamp image-plane digitizer. A few were measured on Prevost scanning tables.

The position and direction of the charged track (the track seen in the foil chamber) were measured by laying a straight-line template over the continuing track in that thick plate chamber which gave the best directional information. (For some events, the charged-particle trajectory is through the hole in the first thick plate chamber, and the track does not appear in this chamber.) Any event in which the charged track interacted before the first half of the first chamber was rejected as unmeasurable (less than 6 in. of visible track). The projected angular error was ±6.0 mrad. The largest part of the error is a result of multiple scattering. The error in the position of the charged track was ±0.02 in.

The positions and directions of the showers were determined by laying a straight-line template over the tracks. The error in the angle measurement was estimated by inspection of the angular divergence of the early part of the shower. The average projected angular error assigned was ±24 mrad.

The position of the $K^\pm$ was determined by the beam chamber within an error of ±0.02 in. Because the track length in the beam chamber was inadequate for measuring individual $K^\pm$ directions, the $K^\pm$ direction was taken to be the nominal beam direction. The error assumed for the $K^\pm$ direction was the beam divergence, which was determined from photographs of beam tracks penetrating all the chambers. This projected beam divergence was determined to be ±4.6 mrad.

Of the 2400 events measured, about 1000 involved $\pi^\pm$ angles larger than 0.136 rad. About 500 of these were rejected if

(1) more than one charged track entered the spark-chamber array, which could not be clearly dissociated from the decay event, or

(2) the charged particle track initiated a shower typical of an electron, or

(3) a two-shower event ($K^\pm \to \pi^\pm\pi^0$) was misinterpreted as a three-shower event.

The remaining 487 events were fitted to determine the vertex location and the laboratory angles of all the particles. All events with normalized χ^2 values less than 2.0 were accepted.

C. Kinematic Fitting

Once the vertex was located, the directions of the four outgoing decay products relative to the beam direction could be determined. These eight angles together with the incident K momentum gave a one-constraint fit for $K \to \pi\pi\gamma$ kinematics.

Rather than directly proceed to a least-squares fitting with the possibility of spurious local minima, we took advantage of the transformation properties of photons to eliminate many false solutions. Since the incident kaon four-momentum is known, the six photon angles can be transformed unambiguously to the K rest frame. Then pairs of photons with opening angles smaller than the minimum for $\pi^0 \to \gamma\gamma$ in this frame can be ruled out immediately.

The pion angles cannot be transformed uniquely, as can those of the photons, but if the angle of the charged pion in the K rest frame is specified, there are enough constraints to permit two independent determinations of both the momentum and the angle of the π^0. For each choice of the odd photon, the K-frame angle of the charged pion is stepped through the interval of values consistent with the corresponding laboratory direction; the value which minimizes the vector difference between the two determinations of the π^0 momentum is chosen.

With this approximate determination of the charged pion momentum in the K frame, a least-squares fit to all the measurements was carried out. The center-of-mass quantities were varied, transformed to the laboratory system, and compared with the measurements. The errors in particle directions after vertex fitting were ±8 mrad for the projected angle of the charged pion and ±16 mrad for the projected angle of each photon.

Of the 72 events which survived all experimental cuts, 14 resulted in unambiguous determination of the odd photon after the above fitting procedure. In order to estimate the fraction of events which should be ambiguous, a Monte Carlo program was written. In this program, each wrong choice of the odd photon had a 50% probability of giving a good fit. Thus 75% of all fitted Monte Carlo events had ambiguities in the choice of the odd photon. Since the number of successfully fitted events from our experiment was reduced to 64 by elimination of wrong energy showers as described below, the ratio 14/64 of unambiguous events was in satisfactory agreement with the Monte Carlo results.

The relation between γ-ray energy and shower appearance could be calibrated very well by fitting the many two-shower events to the $K^{\pm} \to \pi^{\pm}\pi^{0}$ decay process. The eight events removed from the final sample were those for which the appearance of one or more of the showers was grossly inconsistent with the energy determined by the kinematic fit.

TABLE I. Estimates of backgrounds.

Process	Number of events (total + and − events)
$K^{\pm} \to e^{\pm}\nu\pi^{0}\gamma$	0
$K^{\pm} \to \mu^{\pm}\nu\pi^{0}\gamma$	~2
$K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$	~5
$K^{\pm} \to \pi^{\pm}\pi^{0}$ + extraneous γ	<0.1
$K^{\pm}$ interaction	0
	7 events

III. ANALYSIS

A. Background

There are three possible sources of contamination of the sample of fitted $K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$ events: (i) other K decay modes, (ii) $K^{\pm} \to \pi^{\pm}\pi^{0}$ decays in which another γ ray unrelated to the event created a shower in the heavy plate spark chamber, and (iii) interactions of beam kaons with material in the decay region. Estimates of these backgrounds are discussed in Ref. 6 and summarized in Table I. $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ events contributed to the background under the following circumstances:

(1) The incident K angle was large compared with the nominal beam divergence;

(2) the charged pion had large multiple scattering in the shower chamber;

(3) measuring error.

In order to minimize this contamination, the cutoff on the charged pion laboratory angle was chosen to be 0.144 rad. The contribution of interactions of beam kaons was eliminated by choosing the fiducial volume for decays to exclude the region of the upstream counters and the downstream Mylar window. As can be seen in Table I, the sum of all of these backgrounds is estimated to be seven events, or about 10% of the sample.

B. Fit to the Data

In the analysis of the $\pi\pi\gamma$ sample, the variable of most interest is $\hat{p}\cdot\hat{q}$, where $\hat{p}$ is the unit vector

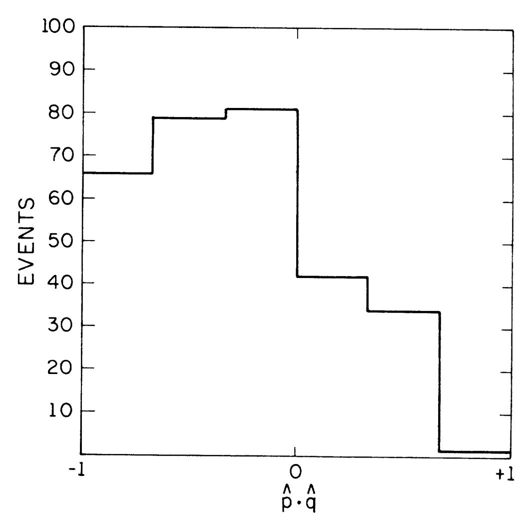

FIG. 3. Distribution of $\hat{p}\cdot\hat{q}$ for Monte-Carlo-generated direct decays, including the effect of ambiguities in choice of the odd γ.

in the direction of the charged pion and $\hat{q}$ is the unit vector in the direction of the odd γ (both in the K rest frame). The angle between the charged pion and the odd γ is interesting because it is usually less than 90° for internal bremsstrahlung and is usually greater than 90° for an $E1$ or $M1$ direct process (see Appendix, Fig. 7). Also, the distributions in this angle are fairly independent of the γ energy and therefore of the detection efficiency for low-energy γ's.

Fourteen events in the final sample of 64 have a good fit in which there is no ambiguity in the choice of the odd γ. Ten of these events have the angle between the $\pi^{\pm}$ and γ less than 90°.

A Monte Carlo program was used to study expected distributions and the effects of the ambiguity in the choice of the odd γ ray. The Monte Carlo program showed that typically internal bremsstrahlung gives one choice which leads to a large value of $\hat{p}\cdot\hat{q}$ and another with $\hat{p}\cdot\hat{q}$ less than zero. It also showed that for direct decays 74% of the values for $\hat{p}\cdot\hat{q}$ are less than zero including all ambiguities.

Figure 3 shows the distribution in $\hat{p}\cdot\hat{q}$ for Monte Carlo-generated direct-process decays in which the highest value of $\hat{p}\cdot\hat{q}$ was chosen for events with ambiguous solutions. Figure 4 shows the same distribution for Monte Carlo-generated IB events with the same choice in the case of ambiguities.

For events with ambiguous solutions, we therefore chose the solution with the highest value of $\hat{p}\cdot\hat{q}$. The effect of this choice is to move internal bremsstrahlung events away from $\hat{p}\cdot\hat{q}=-1$ preferentially compared to direct process events. We then generated Monte Carlo events from a pure IB amplitude and again chose the highest value of $\hat{p}\cdot\hat{q}$ in the case of ambiguous solutions. The presence of direct processes will be indicated by an excess of events in the bins of low $\hat{p}\cdot\hat{q}$ in the data relative to the number of events in the same bins in the Monte Carlo set.

The experimental distribution in $\hat{p}\cdot\hat{q}$ with the above choice for ambiguous solutions is shown in Fig. 5. Also shown in the figure is the distribution of Monte Carlo events generated from pure IB and normalized such that the number of events in the highest bin is the same as for the data.[8] A comparison of the two distributions shows no indication of direct-process events in the data.

In order to enhance the effect of any direct interaction, we restricted the charged pion kinetic energy to the range $58 \leq T_\pi \leq 90$ MeV. This cut reduces the sample which we analyze from 64 events to 24 events. The distribution in $\hat{p}\cdot\hat{q}$ of the 24 events in this sample is shown in Fig. 6.

A maximum-likelihood fit to the angular distribution was performed on the sample of 24 events with $58 \leq T_\pi \leq 90$ MeV in order to test for the presence of direct processes. In principle, the maximum-

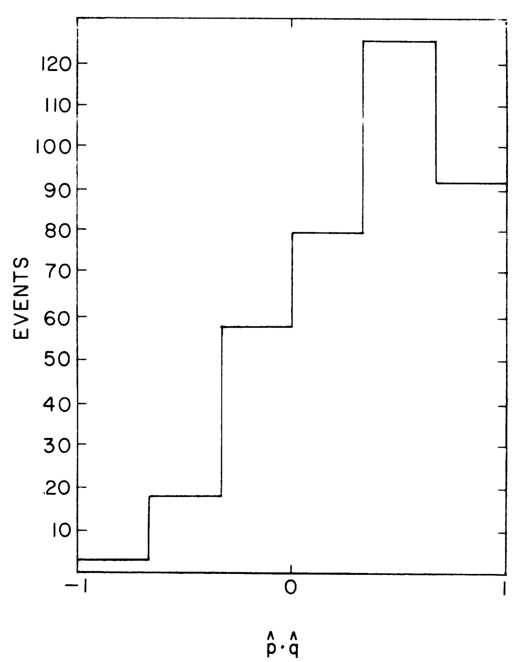

FIG. 4. Distribution of $\hat{p}\cdot\hat{q}$ for Monte Carlo-generated IB events, including the effect of ambiguities.

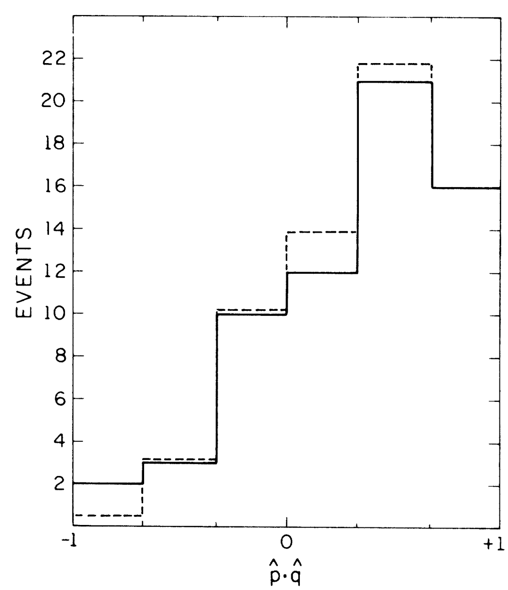

FIG. 5. Distribution of $\hat{p}\cdot\hat{q}$ for the 64 events found in this experiment (solid line) compared with the Monte Carlo-generated IB events (dotted line). The two distributions are normalized to the same value for the bin of greatest $\hat{p}\cdot\hat{q}$.

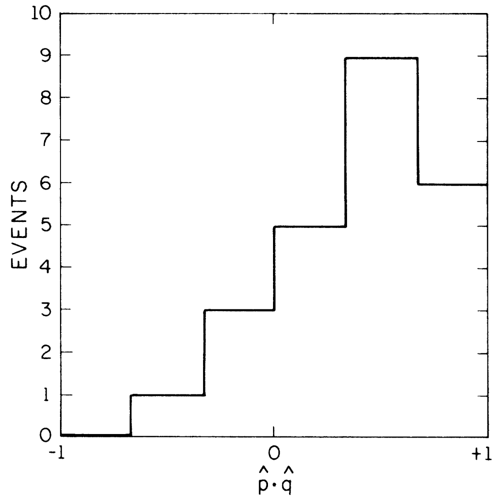

FIG. 6. Distribution of $\hat{p}\cdot\hat{q}$ for the 24 observed events with $58 \leq T_\pi \leq 90$ MeV.

likelihood calculation requires four parameters (see Appendix): A_E and A_M, the amplitudes for electric and magnetic dipole radiation relative to IB; $\bar{\delta}$, the average final-state π-π interaction phase; and ϕ_E, the CP-violating phase of the electric dipole amplitude relative to IB.

Because the sample did not warrant a four-parameter fit, various hypotheses were made about one or more parameters and the remaining parameters were fitted. The results are summarized in Table II. For details of the treatment, see Ref. 9.

From Table II we see that to 1 standard deviation, the amplitude for direct processes is between -0.37 and $+0.26$ relative to IB. If we assume $A_E=0$, the maximum contribution to the decay rate is 7% to 1 standard deviation. On the other hand, if we assume $A_M=0$, the amplitude A_E (the electric dipole direct-process amplitude) is model-dependent. From Table II we see that the maximum-likelihood fit gives $A_E=-0.12$ with standard deviations that depend on the phases. The largest standard deviations are $+0.21$ and -0.25.

We have also obtained the branching ratio for $K^{\pm}\to\pi^{\pm}\pi^0\gamma$ relative to all decay modes for $58 \leq T_\pi \leq 90$ MeV. Comparing the number of $\pi\pi\gamma$ events with the number of τ' events found yields

$$\frac{\Gamma(K^{\pm}\to\pi^{\pm}\pi^0\gamma)}{\Gamma(K\to \text{all})}=\frac{\Gamma(K^{\pm}\to\pi^{\pm}\pi^0\pi^0)}{\Gamma(K\to \text{all})}\frac{N(\pi^{\pm}\pi^0\gamma)}{N(\tau')}\times\frac{\text{efficiency } \tau'}{\text{efficiency } \pi\pi\gamma},$$

where $N(\pi^{\pm}\pi^0\gamma)$ is the number of $\pi\pi\gamma$ decays and $N(\tau')$ is the number of τ' decays found. The efficiencies were calculated by a Monte Carlo program which included the detection efficiency as well as the probability of surviving the cuts involving vertex position, vertex χ^2, and kinematic χ^2 values. If we subtract 10% background from the $\pi\pi\gamma$ rates, we obtain

$$\frac{\Gamma(K^{\pm}\to\pi^{\pm}\pi^0\gamma)}{\Gamma(K^{\pm}\to \text{all})}=(2.4\pm0.8)\times10^{-4} \quad (58 \leq T_\pi \leq 90 \text{ MeV}).$$

This result agrees with the IB calculation of 2.4×10^{-4} for this energy interval.

IV. CONCLUSIONS

We have seen no evidence (Figs. 5 and 6) for direct amplitudes in the reactions $K^{\pm}\to\pi^{\pm}\pi^0\gamma$. The limits on the direct amplitudes are model dependent and are summarized in Table II. The branching ratio for $K^{\pm}\to\pi^{\pm}\pi^0\gamma$ in the interval $58 \leq T_\pi \leq 90$

TABLE II. Results of maximum-likelihood calculations for various assumptions.

Assumptions	Values of other parameters	Result
Direct part is pure $M1$, magnetic dipole	$A_E=0$	$A_M=0.0\pm0.26$
Direct part is pure $E1$ and has maximum CP violation	$A_M=0$, $\phi_E=\pi/2$, $\bar{\delta}=23°$	$A_E=0.0\pm0.26$ $\frac{\text{Maximum interference}}{\text{IB}}=\pm0.12$ $\frac{\Gamma(K^-\to\pi^-\pi^0\gamma)}{\Gamma(K^+\to\pi^+\pi^0\gamma)}-1<0.24$
Direct part pure $E1$ and no CP violation and maximum interference	$A_M=0$, $\phi_E=0$, $\bar{\delta}=0$	$A_E=-0.12\pm0.19$
Direct part pure $E1$, no CP violation, current value for phase difference	$A_M=0$, $\phi_E=0$, $\bar{\delta}=23°$	$A_E=-0.12^{+0.21}_{-0.25}$

MeV is $(2.4\pm0.8)\times10^{-4}$, consistent with a pure IB calculation. From the discussion of the material in Table II, one sees that if there is no magnetic dipole transition, the largest value found by the maximum-likelihood method for the direct (electric dipole) amplitude is $A_E = -0.12^{+0.21}_{-0.25}$ relative to the IB amplitude.

We are also able to rule out a large asymmetry in the relative partial rates which could be present if there were a CP-violating direct amplitude. If we double the figure ±0.12 given in Table II for the ratio of maximum interference to internal bremsstrahlung for the maximum CP violation, we obtain

$$\frac{\Gamma(K^-\to\pi^-\pi^0\gamma)}{\Gamma(K^+\to\pi^+\pi^0\gamma)}-1=0\pm0.24\,.$$

This result agrees with the straightforward comparison of the 31 events $K^-\to\pi^-\pi^0\gamma$ to the 33 events $K^+\to\pi^+\pi^0\gamma$, which yields the value -0.06 ± 0.24 for the above expression.

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APPENDIX

In order to parametrize the data, we use a model developed by Good[10] and elaborated by Costa and Kabir[11] and by Christ.[1] In this model the direct process is approximated by the lowest terms (electric and magnetic dipole) in a multipole expansion. The total amplitude is the sum of three terms $M_{\rm IB}$, M_E, and M_M corresponding to IB and $E1$ and $M1$ direct processes:

$$M=M_{\rm IB}+M_E+M_M\,. \tag{1}$$

If T invariance is assumed, the Fermi-Watson theorem specifies the phase of the amplitude of a definite isospin and angular momentum for the $\pi\pi$ system to be $\delta_{I,J}(\text{mod}\pi)$, where $\delta_{I,J}$ is the $\pi\pi$ scattering phase shift. If T invariance is not assumed, the requirement of CPT in this model is

$$M^+_{I,J,\sigma}=(M^-_{I,J,\sigma})^*\exp[2i\delta_{I,J}]\,, \tag{2}$$

where the plus and minus refer to K^+ and K^- amplitudes, and σ refers to the photon polarization.

These amplitudes therefore can be written

$$\begin{aligned} M^+_{I,J,\sigma}&=|M_{I,J,\sigma}|\exp(i\delta_{I,J}+i\phi)\,,\\ M^-_{I,J,\sigma}&=|M_{I,J,\sigma}|\exp(i\delta_{I,J}-i\phi)(\text{mod}\pi)\,, \end{aligned} \tag{3}$$

where ϕ is the T-violating part of the phase.

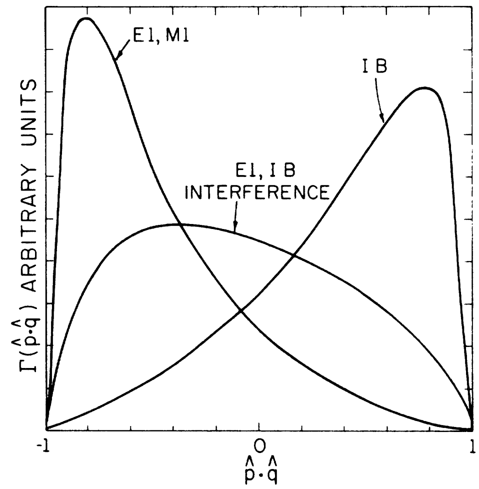

FIG. 7. Theoretical distributions of $\hat{p}\cdot\hat{q}$ for the three contributions to the decay rate: direct radiative processes ($E1,M1$), IB and interference. Normalizations are arbitrary.

Upon squaring the amplitude M and summing over photon polarization, one finds that the interference terms containing the $M1$ amplitude drop out. The T-violating effects therefore involve only the relative phases of the $E1$ and IB amplitudes.

We write Eq. (22) of Christ[1] in terms of the charged-pion kinetic energy T and the cosine of the angle between the charged pion and photon in the K rest frame $\mu\equiv\hat{p}\cdot\hat{q}$:

differential decay rate

$$=b(T,\mu)+i(T,\mu)\cos(\delta\pm\phi_E)+d(T,\mu)\,. \tag{4}$$

Here b, i, and d represent the contributions of IB, IB-$E1$ interference, and the $E1$ and $M1$ direct processes, respectively. The one-dimensional distributions in $\mu=\hat{p}\cdot\hat{q}$ for the IB, interference, and direct-process contributions are shown in Fig. 7. The relative normalizations of the distributions are arbitrary.

The CP-violating $E1$ phase is ϕ_E, and $\delta=\delta_1-\delta_2$, where δ_1 and δ_2 are the $I=1$, p-wave and $I=2$, s-wave $\pi\pi$ phase shifts. The strengths of the direct processes are defined relative to the strength of the internal bremsstrahlung process[12]:

$$A_E{}^2+A_M{}^2=\frac{\int_{-1}^{1}d\mu\int_{T_1}^{T_2}dT\,d(T,\mu)}{\int_{-1}^{1}d\mu\int_{T_1}^{T_2}dT\,b(T,\mu)}\,, \tag{5}$$

$$A_E\cos(\bar{\delta}\pm\phi_E)=\frac{\int_{-1}^{1}d\mu\int_{T_1}^{T_2}dT\,i(T,\mu)\cos(\delta\pm\phi_E)}{\int_{-1}^{1}d\mu\int_{T_1}^{T_2}dT\,b(T,\mu)}\,, \tag{6}$$

where $\bar{\delta}$ is the average of the $\pi\pi$ phase shifts over the available phase space,

$T_1 = 58$ MeV, and $T_2 = 90$ MeV.

The decay rate in terms of these quantities is

$$(\text{constant}) \times [1 + 1.23 A_E \cos(\bar{\delta} \pm \phi_E) + A_E{}^2 + A_M{}^2]. \tag{7}$$

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Search for a Weakly Decaying $S = +2$ Exotic Meson*

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A search for a strangeness-2 exotic meson (X^+) with mass less than two kaon masses has resulted in a limit on the cross section for the production of this particle in the reaction $K^+ + n \to \Lambda^0 + X^+$. For 1.9-GeV/$c$ incident K^+'s, this cross section is less than 3.3 μb with 90% confidence. The known cross section for the analogous reaction $\pi^+ + n \to \Lambda^0 + K^+$ is about 170 μb at this energy, or more than 50 times larger.

The remarkable success of SU(3) in explaining the grouping of elementary particles and predicting certain new particles is well known. At the same time, particles not consistent with the simplest SU(3) classification (exotic particles) are notably absent. Searches have been made for two particular types of exotic particles: (I) $S = +1$, $B = +1$, and (II) $S = 2$, $B = 0$. Particles of type I have been searched for in K^+p total cross sections and phase-shift analyses,[1] and some evidence exists for baryons produced with a cross section of ~4 mb at 1.91 GeV/c^2, for example.

Strangeness-2 mesons (type II) have been looked for in reactions of the type $K^+p \to K^+K^+\Lambda$, where both final-state kaons are observed, and the invariant mass of the kaons ($X \to KK$ strongly) was investigated for evidence of structure. For a time, there was evidence for enhancements[2] at 1280 MeV/c for K^+ incident at 3–5 GeV/c. Later experiments[3] showed, with increased statistics, no evidence for the effect.

We report here a search for an exotic boson,