virtual photoabsorption do not scale. Our result on the Drell-Yan-West relation is a more stringent application of unitarity and analyticity. We conclude that the proton's Dirac form factor $F_1(Q^2) = F(m^2, Q^2)$ is bounded by $(\ln Q^2)^c Q^{-(p+1)}$ if $F(s, Q^2)$ is a smooth but sufficiently varying function of s, near $s = m^2$, as $Q^2 \to \infty$. This is true whether or not the $J^\pi = \frac{1}{2}^+$ contributions scale.

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[1]A. M. Bincer, Phys. Rev. 118, 855 (1960).

[2]F. Cooper and H. Pagels, Phys. Rev. D 2, 228 (1970).

[3]G. B. West, Phys. Rev. Letters 27, 762 (1971).

[4]E. D. Bloom and F. J. Gilman, Phys. Rev. D 4, 2901 (1971), and G. Miller, Stanford thesis and Report No. SLAC-129, 1970 (unpublished).

[5]S. D. Drell and T.-M. Yan, Ann. Phys. (N.Y.) 66, 578 (1971), and references contained therein.

[6]S. D. Drell and T.-M. Yan, Phys. Rev. Letters 24, 181 (1970).

[7]G. B. West, Phys. Rev. Letters 24, 1206 (1970). Footnote 8 of this reference suggests the relevance of sideways dispersion relations.

[8]E. D. Bloom and F. J. Gilman, Phys. Rev. Letters 25, 1140 (1970).

[9]S. D. Drell and T. D. Lee, Phys. Rev. D 5, 1738 (1972).

[10]R. Chanda and C. Ryan, Nucl. Phys. B40, 573 (1972).

[11]These numerical improvements may be broken down as follows. Our inequalities (13) and (14) are better by factors of 2 and 4 than those of West (Ref. 3) and Cooper and Pagels (Ref. 2), respectively, probably because of our inclusion of the spin sum in the Schwarz inequality and the resultant frame independence. By bounding the combination of positive and negative cuts we gain a further factor of 4. Finally, Cooper and Pagels appear to have lost a factor of 2 between their Eqs. (2.18) and (2.19). This completes the tally.

[12]D. J. Gross, Phys. Rev. D 4, 1130 (1971).

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Comparison of the Parton and Light-Cone Analyses of Highly Inelastic Leptonic Processes*

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We study the relationship between the parton model and the analysis of light-cone singularities for highly inelastic leptonic processes. For deep-inelastic lepton scattering the parton model is found to be a momentum-space representation of any model in which free-field singularities on the light cone are dominant. Scaling laws and sum rules derived in one approach are shown to obtain in the other with equivalent assumptions. For massive-muon-pair production the two approaches are found to differ fundamentally. In the parton model the leading singularity is dominated in the high-mass limit by the nonsingular annihilation diagram. The scaling law which is obtained in the parton model is not obtained from a light-cone analysis without additional, seemingly arbitrary assumptions. Massive-muon-pair production therefore tests the parton model in a region where it is not equivalent to the light-cone approach. Several other processes are studied including one-particle inclusive e^+e^- annihilation and photoproduction of muon pairs.

I. INTRODUCTION

The theoretical effort to understand the highly inelastic interactions of leptons with hadrons has been extensive in the past few years. Much of this work comes in response to the SLAC-MIT inelastic electron scattering experiments[1] and, in particular, to the observation of scale independence at large energy and momentum transfer. Our object is to explore the relationship between two of the multitude[2] of theoretical models in which this process has been studied: the parton model and the analysis of light-cone singularities.

Since originally being applied to highly inelastic electroproduction, both of these approaches have been developed extensively and applied to other physical processes. Despite the similarity of their predictions for inelastic lepton scattering, the parton model and light-cone analysis (augmented by additional dynamical assumptions) lead to strikingly different predictions for several other processes. We compare the two approaches in order to understand how this comes about and in hopes of gaining some insight into various problems which arise in each model.

The parton model and the analysis of light-cone singularities approach the problem of highly inelastic lepton scattering from different directions. On the one hand, the parton model, as developed by Feynman[3] and by Bjorken and Paschos,[4,5] assumes the existence of pointlike constituents or partons in the target nucleon. A simple intuitive picture of highly inelastic electron scattering then arises when the process is observed in an infinite-momentum frame. There, it is argued, the partons are scattered elastically and incoherently by the incident electron. The large inelastic scattering cross section observed at SLAC derives from the assumed pointlike nature of the partons: there are no form factors to diminish the process at large momentum transfer, while the scaling behavior arises from the kinematic restriction that the partons be scattered elastically by the electromagnetic current. In the parton picture, the explicit shape of the highly inelastic scattering cross section measures the as yet unspecified longitudinal-momentum distribution of the target's partons in an infinite-momentum frame.

In other applications, it is useful to distinguish between "conservative" parton models in which the longitudinal distribution of partons is left unspecified and which are applied only where the assumptions of elasticity and incoherence can be motivated by appeal to some type of cutoff field theory, and more speculative parton applications[5,6] in which specific distributions are assumed or in which elasticity and incoherence are not directly supported by recourse to field theory. Among "conservative" applications of parton ideas are the extensive calculations of Drell, Levy, and Yan[7] and those calculations of Landshoff, Polkinghorne, and their collaborators which do not involve duality constraints on the parton distributions.[8] Except where specifically noted, we adopt the spirit of "conservative" models.

The light-cone analysis,[9] on the other hand, proceeds primarily in coordinate space without reference to any decomposition of the target into constituent states characteristic of parton models. First the inelastic scattering cross section is related to the imaginary part of the forward virtual Compton-scattering amplitude. It is then argued that, barring pathologies, the leading contribution to highly inelastic electron scattering arises when the two currents are separated by a nearly lightlike distance. The current product is then expanded in a series of terms with differing singularities on the light cone and the inelastic scattering cross section appropriate to each term is calculated. It is found that the observed scale independence obtains only if the dominant light-cone singularities are those appropriate to free-field theory, i.e., calculated as if the local electromagnetic currents were built up out of some noninteracting fields. Once again, the explicit shape of the highly inelastic scattering cross section is unspecified: Here it is linked to the variation along the light cone of the leading singularity. As with the parton model, we will distinguish between the "conservative" approach outlined above and one in which additional assumptions are required (e.g., Regge bounds on multiparticle matrix elements or specific forms for matrix elements near the light cone).

Despite the superficial differences outlined above, we find that there are intimate connections between the parton and light-cone approaches. It is often noted that both analyses achieve scaling predictions for inelastic electron scattering by assuming certain free-field behavior: for the scattering amplitude in particular regions of momentum space in the one instance, and for current products near the light cone in the other. This complementary relationship between coordinate and momentum space is borne out in our analysis. In fact, *as far as inelastic leptonic scattering is concerned*, the parton model is simply a momentum-space realization of a light-cone operator-product expansion with free-field singularities dominant. When reformulated in coordinate space a conservative parton model displays dominant free-field singularities on the light cone and enough smoothness off the light cone to admit the usual arguments for light-cone dominance.

This connection provides considerable insight into the formal manipulations of the light-cone analysis. In particular, we are led to identify the free-field singularity on the light cone with the propagator of an elastically scattered parton. Also, the variation of the leading singularity along the light cone is linked with the parton longitudinal-momentum distribution in a straightforward way. These correspondences will be elaborated upon in the following sections.

The identification of the parton and light-cone analyses of electroproduction is perhaps not unexpected. More surprising, however, is our conclusion that *the identification cannot be extended to other important highly inelastic leptonic processes*. In particular, we study in detail the attempts to apply the two approaches to the production of massive muon-antimuon pairs in high-energy hadron-hadron collisions ($P+P\to\mu^+ +\mu^- +$"anything"). This process has been analyzed in parton models by Drell and Yan[10] and Landshoff and Polkinghorne,[11] and in a light-cone analysis augmented with assumptions of Regge behavior by Altarelli, Brandt, and Preparata[12] with very different results. We find that the parton-model cross section for this process is not light-cone-dominated in the conventional sense, nor does the dominant piece possess the free-field singularities assumed on Ref. 12. This dissimilarity arises in as natural a way from the parton model as does the similarity for electroproduction.

This situation enhances the experimental importance of $P+P\to\mu^+ +\mu^- +$"anything" and related processes. At the same time, it provides a convenient postponement of an old problem for parton theorists. The problem is the existence of real physical partons. It is hard to envision a parton model without at least occasionally producing a parton in the final state (see Ref. 13 for a discussion of this point and Refs. 14 and 15 for possible ways of avoiding the prediction). Currently popular models, such as the quark parton model, are thereby trapped with the embarrassing prediction of production of quarks or some other unusual particles at SLAC energies. This problem is avoided by viewing the partons, not as the quanta of some underlying field theory, but merely as a convenient and intuitive representation of the underlying light-cone singularity structure, circumventing the whole question of physical partons. If, however, it could be shown that the free-field singularities on the light cone necessarily imply the existence of corresponding physical eigenstates, the problem would reappear.[16] As yet, no one has been able to establish this.

The importance of muon-pair production is now evident: unlike highly inelastic electron scattering, *the parton-model scaling law for this process cannot be directly attributed to light-cone behavior alone*. If it is verified experimentally, the notion of pointlike constitutents in the nucleon will be much more compelling.

The above results are derived in the following sections. Our procedure is as follows: For a given process, we first display explicitly the coordinate-space structure in the perturbation-theoretic parton model of Drell, Levy, and Yan (DLY).[7] With this intuitive picture in mind, we then rederive the coordinate-space structure directly from the scaling laws which obtain in a wider class of parton models. Finally, we examine this result and compare and contrast it with light-cone-singularity analyses. In Sec. II we consider highly inelastic electron scattering, in Sec. III the muon-pair production process. Section IV combines several less extensive analyses, including $e^+ + e^- \to P+$anything and $\gamma+P\to\mu^+ +\mu^- +$anything. Appendices A, B, and C treat some peripheral aspects of the electron scattering problem. A brief summary of this work may be found in Ref. 17.

II. HIGHLY INELASTIC ELECTRON SCATTERING

A. Kinematics

To begin, we briefly review the kinematics and our conventions for highly inelastic electron scattering. Assuming single-photon exchange as shown in Fig. 1(a), all hadronic information is summarized in the usual tensor $W_{\mu\nu}$,[18]

$$W_{\mu\nu}\equiv 4\pi^2\frac{E_P}{M}\sum_n \langle P|J_\mu(0)|n\rangle\langle n|J_\nu(0)|P\rangle \times(2\pi)^4\delta^4(q+P-P_n), \qquad (2.1)$$

where single-particle states are normalized to $\langle P'|P\rangle=\delta^3(\vec{P}'-\vec{P})$. Translating and performing the sum over states,

$$W_{\mu\nu}\equiv 4\pi^2\frac{E_P}{M}\int d^4y\, e^{iq\cdot y}\langle P|J_\mu(y)J_\nu(0)|P\rangle. \qquad (2.2)$$

For positive-frequency photons, the current product may be converted to a commutator:

$$W_{\mu\nu}= 4\pi^2\frac{E_P}{M}\int d^4y\, e^{iq\cdot y}\langle P|[J_\mu(y), J_\nu(0)]|P\rangle. \qquad (2.3)$$

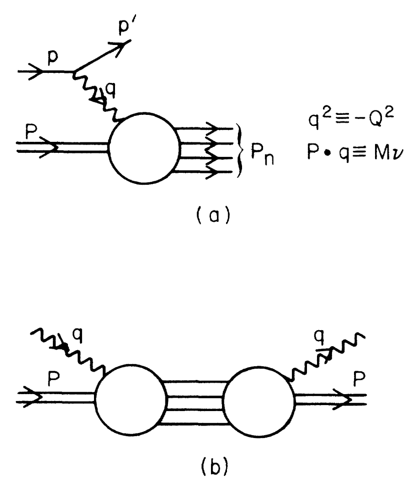

FIG. 1. (a) Kinematics of inelastic electroproduction. (b) Discontinuity in forward virtual Compton scattering for spacelike q^2.

For spacelike virtual photons, Eq. (2.1) is the imaginary part of the forward virtual Compton-scattering amplitude shown in Fig. 1(b).

The well-known structure functions W_1 and W_2 are defined by the invariant decomposition of $W_{\mu\nu}$:

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2, \nu) + \frac{1}{M^2}\left(P_\mu - \frac{P\cdot q}{q^2} q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2} q_\nu\right) W_2(q^2, \nu), \tag{2.4}$$

where $P\cdot q \equiv M\nu$ and $q^2 \equiv -Q^2 \leq 0$. With these definitions, Bjorken's scaling hypothesis[19] takes the form

$$\lim_{\text{Bj}} MW_1(q^2, \nu) = F_1(x),$$
$$\lim_{\text{Bj}} \nu W_2(q^2, \nu) = F_2(x), \tag{2.5}$$

where the Bjorken limit ($\lim_{\text{Bj}}$) is $\nu \to \infty$, $Q^2 \to \infty$, with $x \equiv Q^2/2M\nu$ fixed. Equation (2.3) has the crossing properties of the physical Compton amplitude,

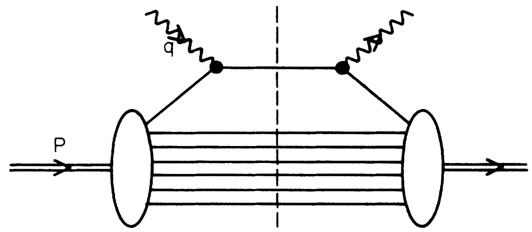

FIG. 2. Parton-model diagram for deep-inelastic electroproduction (in an infinite-momentum frame of P).

$$W_{\mu\nu}(q^2, -\nu) = -W_{\mu\nu}(q^2, \nu),$$

which imply

$$W_i(q^2, -\nu) = -W_i(q^2, \nu).$$

The structure functions $F_i(x)$ are defined for $0 < x \leq 1$ if $W_{\mu\nu}$ is defined by Eq. (2.2) or for $-1 \leq x \leq 1$ if $W_{\mu\nu}$ is defined by Eq. (2.3).

B. Coordinate-Space Structure in a Perturbative Parton Model

The assumptions of elasticity and incoherence at the heart of the parton model for highly inelastic electron scattering are summarized in a single formula from the work of DLY:

$$\lim_{\text{Bj}} W_{\mu\nu} = 4\pi^2 \frac{E_P}{M} \sum_n |a_n|^2 \int d^4y\, e^{iq\cdot y} \langle n | j_\mu(y) j_\nu(0) | n \rangle \tag{2.6}$$

valid in the Bjorken limit for $x \neq 0$.

Elasticity and incoherence allow the replacement of the fully interacting Heisenberg current operator, $J_\mu(y)$, by the "bare" current operator, $j_\mu(y)$, constructed from free fields. Interactions appear only in the coefficients $|a_n|^2$ which weight the importance of various parton configurations $|n\rangle$. The appearance of $|a_n|^2$ rather than off-diagonal terms $a_m^* a_n$ with $m \neq n$ is also a consequence of incoherence.

Equation (2.6) is derived only in an infinite-momentum frame of the target. This restriction reveals a basic limitation of perturbative parton models and their associated physical picture (see Fig. 2) of the incoming hadron developing into some constituent state $|n\rangle$ with amplitude a_n, followed by the elastic, incoherent scattering of individual partons by the bare current $j_\mu(0)$.

Justification of Eq. (2.6) is a substantial task. DLY accomplish it order-by-order in a γ_5 perturbation theory of pions and nucleons with a transverse-momentum cutoff. For a more thorough discussion of their work, we refer the reader to Ref. 20.

To display the coordinate-space structure of Eq. (2.6), we write out explicitly the current matrix element for a constituent state $|n\rangle$ containing spin-0 partons with charges λ_i and spin-$\frac{1}{2}$ partons with charges λ_j,[21]

$$\langle n | j_\mu(y) j_\nu(0) | n \rangle = \frac{1}{(2\pi)^6}\left[\sum_i \lambda_i^2 \int \frac{d^3P'}{2E'2E_i} e^{i(P_i - P')\cdot y}(P_i + P')_\mu (P_i + P')_\nu + \sum_j \lambda_j^2 \int \frac{d^3P'}{2E'2E_j} e^{i(P_j - P')\cdot y}\, 2(P_{j\mu}P'_\nu + P_{j\nu}P'_\mu - g_{\mu\nu} P_j \cdot P')\right].$$

P'_μ is the momentum of the scattered parton. Because the currents j_μ are devoid of strong interactions, the sum over a complete set of final states reduces to the phase-space integral for the elastically scattered parton. Some elementary algebra yields

$$\langle n | j_\mu(y) j_\nu(0) | n \rangle = \frac{1}{(2\pi)^3}\left\{\sum_i \lambda_i^2 \frac{1}{2E_i}(2P_i + i\partial)_\mu (2P_i + i\partial)_\nu \Delta_+(y, m^2) e^{iP_i\cdot y}\right.$$

$$+\sum_j \lambda_j^2 \frac{1}{2E_j}[4P_{j\mu}P_{j\nu}+2i(P_{j\mu}\partial_\nu+P_{j\nu}\partial_\mu)-g_{\mu\nu}\Box]\Delta_+(y,m^2)e^{iP_j\cdot y}\Big\},$$

where Δ_+ is the singular function defined by

$$\Delta_\pm(y,m^2)\equiv\frac{1}{(2\pi)^3}\int d^4k\,\theta(k_0)\delta(k^2-m^2)e^{\mp ik\cdot y}$$

and m is the parton mass which for convenience is taken to be the same for all partons. Since all partons are assumed to have a positive fraction of the target's infinite momentum and limited transverse momenta, we may replace P_i^μ by $\eta_i P^\mu$ up to terms of order $1/P$ where $0<\eta_i\leqslant 1$. If $\eta_i\lesssim M/P$, this is not a valid substitution. The problem of "wee" ($\eta\approx 0$) partons is discussed in Appendix B, where it is shown that errors incurred by replacing P_i^μ by $\eta_i P^\mu$ are limited to the region $x\lesssim\sqrt{M/P}$ where Eq. (2.6) is not valid in the first place. Making this replacement in Eq. (2.6),

$$\lim_{\mathrm{Bj}} W_{\mu\nu}=\frac{1}{4\pi M}\sum_n |a_n|^2\int d^4y\,e^{iq\cdot y}\Big\{\sum_i \lambda_i^2[4\eta_i^2P_\mu P_\nu+2i\eta_i(P_\mu\partial_\nu+P_\nu\partial_\mu)-\partial_\mu\partial_\nu]\frac{\Delta_+(y,m^2)e^{iP_i\cdot y}}{\eta_i}$$
$$+\sum_j\lambda_j^2[4\eta_j^2P_\mu P_\nu+2i\eta_j(P_\mu\partial_\nu+P_\nu\partial_\mu)-g_{\mu\nu}\Box]\frac{\Delta_+(y,m^2)e^{iP_j\cdot y}}{\eta_j}\Big\}.$$

This equation may be simplified considerably by separating the sum over longitudinal momentum from the rest of the constituent sum, i.e.,

$$\sum_i \to \int_0^1 d\eta\,\delta(\eta-\eta_i)\sum_i .$$

We may now use the parton-model expressions for the scaling functions F_1 and F_2 which may be read off as the coefficients of $P_\mu P_\nu$ and $g_{\mu\nu}$ in the previous equation:

$$F_2(\eta)=\eta\sum_n|a_n|^2\Big[\sum_i\lambda_i^2\delta(\eta-\eta_i)+\sum_j\lambda_j^2\delta(\eta-\eta_j)\Big],$$

$$F_1(\eta)=\tfrac{1}{2}\sum_n|a_n|^2\sum_j\lambda_j^2\delta(\eta-\eta_j).$$

As a result, we obtain

$$\lim_{\mathrm{Bj}} W_{\mu\nu}=\frac{1}{4\pi M}\int_0^1 d\eta\int d^4y\,e^{iq\cdot y}\Big\{2(-g_{\mu\nu}\Box+\partial_\mu\partial_\nu)\Delta_+(y,m^2)\frac{F_1(\eta)}{\eta}e^{iP_\eta\cdot y}$$
$$+[4\eta^2P_\mu P_\nu+2\eta i(\partial_\mu P_\nu+\partial_\nu P_\mu)-\partial_\mu\partial_\nu]\Delta_+(y,m^2)\frac{F_2(\eta)}{\eta^2}e^{iP_\eta\cdot y}\Big\}. \quad (2.7)$$

Finally, we compare Eqs. (2.7) and (2.2) and conclude that up to terms whose *Fourier transforms vanish in the scaling limit*, the matrix element of the current product is given by

$$\frac{4\pi^2E_P}{M}\langle P|J_\mu(y)J_\nu(0)|P\rangle\doteq\frac{1}{4\pi M}\Big\{(-g_{\mu\nu}\Box+\partial_\mu\partial_\nu)\Big[\Delta_+(y,m^2)\int_0^1\frac{d\eta}{\eta}2F_1(\eta)e^{iP_\eta\cdot y}\Big]$$
$$+\int_0^1 d\eta\Big[4P_\mu P_\nu+\frac{2i}{\eta}(\partial_\mu P_\nu+\partial_\nu P_\mu)-\frac{1}{\eta^2}\partial_\mu\partial_\nu\Big][\Delta_+(y,m^2)F_2(\eta)e^{iP_\eta\cdot y}]\Big\}. \quad (2.8)$$

We have introduced the notation $\doteq$ to indicate equality up to terms whose Fourier transforms vanish in the scaling limit. Even in the parton model, a complete expression for the matrix element on the left of Eq. (2.8) contains other terms. For example, Eq. (2.8) cannot adequately describe virtual Compton scattering for $q^2>0$ since its Fourier transform vanishes in that region (the mass-shell δ function in Δ_+ requires $2M\nu\eta=-q^2$ with $0\leqslant\eta\leqslant 1$, while the spectral condition requires $|2M\nu|\geqslant q^2$ or $\eta\geqslant 1$). Also, there are presumably other terms (e.g., terms everywhere smooth in coordinate space) whose Fourier transform vanishes as the Bjorken limit is approached. Equation (2.8) *does* contain those pieces of the current product matrix element which dominate the Bjorken limit.

Several other remarks should be made at this point. First, Eq. (2.8) has been obtained only in the infinite-momentum frame of $|P\rangle$. This restriction is peculiar to the perturbative parton model and will be removed in the following section. Second, the transverse momentum in the vector $P_{i\mu}$ was ignored in the steps preceding Eq. (2.7). Since transverse momenta are bounded in the parton model, this does not matter to leading order. And third, the corresponding expression for the *current commutator* may be obtained in a similar manner:

$$\frac{4\pi^2 E_P}{M}\langle P|[J_\mu(y), J_\nu(0)]|P\rangle \doteq \frac{1}{2\pi i M}\left\{(-g_{\mu\nu}\Box + \partial_\mu\partial_\nu)\Delta(y, m^2)\int_0^1 \frac{d\eta}{\eta}2F_1(\eta)\cos P_\eta\cdot y + \int_0^1 d\eta\left[4P_\mu P_\nu + \frac{2i}{\eta}(P_\mu\partial_\nu + P_\nu\partial_\mu) - \frac{1}{\eta^2}\partial_\mu\partial_\nu\right]F_2(\eta)\Delta(y, m^2)\cos P_\eta\cdot y\right\}, \tag{2.9}$$

where $\Delta(y, m^2) \equiv i[\Delta_+(y, m^2) - \Delta_-(y, m^2)]$. The changes reflect the locality $[\Delta(y, m^2) = 0$ for $y^2 < 0]$ and crossing symmetry $[\cos P_\eta\cdot y = \cos P_\eta\cdot(-y)]$ of the commutator.

The last step is to write Eq. (2.9) in a form in which current conservation is manifest. The mathematical details of this step are contained in Appendix A. The result is as follows:

$$\frac{4\pi^2 E_P}{M}\langle P|[J_\mu(y), J_\nu(0)]|P\rangle \doteq \frac{1}{2\pi M i}\left\{(g_{\mu\nu}\Box - \partial_\mu\partial_\nu)\left[\Delta(y, m^2)\int_0^1 \frac{d\eta}{\eta^2}[F_2(\eta) - 2\eta F_1(\eta)]\cos P_\eta\cdot y\right] + 4[P_\mu P_\nu\Box - (P\cdot\partial)(P_\mu\partial_\nu + P_\nu\partial_\mu) + g_{\mu\nu}(P\cdot\partial)^2] \times\left[\Delta'(y, m^2)\int_0^1 \frac{d\eta}{P_\eta\cdot y}\sin P_\eta\cdot y\, F_2(\eta)\right]\right\}, \tag{2.10}$$

where $\Delta'(y, m^2) \equiv -d\Delta(y, m^2)/dm^2$.

This equation contains the results of the light-cone analyses of Jackiw, Van Royen, and West.[22] We defer, however, a detailed discussion of the light-cone behavior of Eq. (2.10) until we have rederived it from a more general standpoint.

C. More General Parton Models

In this section Eq. (2.10) is rederived from the following two assumptions:

I. MW_1 and νW_2 scale in the Bjorken limit.

II. The singularities of the current commutator, if any, are on the light cone rather than somewhere else in coordinate space.

Assumption I is, of course, the motivation for constructing a parton model in the first place. Assumption II is valid in all parton models which are known to us. In the previous section we saw it to be true in the perturbative model of DLY. In the nonperturbative model of Landshoff, Polkinghorne, and their coworkers[8,11] the singularities are associated with the propagation of a parton between current vertices and therefore lie on the light cone. Many other models (and perhaps the real world) may satisfy these assumptions: The analysis of this section applies equally well to them.

Inserting Assumption I into Eq. (2.4) and rearranging terms,

$$\lim_{\mathrm{Bj}} M^2 W_{\mu\nu} = -(g_{\mu\nu}q^2 - q_\mu q_\nu)\left(-\frac{F_1(x)}{2\nu x} + \frac{F_2(x)}{4x^2\nu}\right) + \left(P_\mu P_\nu + \frac{1}{2x}(P_\mu q_\nu + P_\nu q_\mu) + \frac{q^2}{4x^2}g_{\mu\nu}\right)\left(\frac{F_2(x)}{\nu}\right), \tag{2.11}$$

where $W_{\mu\nu}$ is defined in terms of the commutator [Eq. (2.3)] for $-1 \leq x \leq 1$. The combinations of structure functions in parentheses are odd under crossing ($\nu \leftrightarrow -\nu$, q^2 fixed), which implies

$$\mathfrak{F}_i(x, q^2) = \int_0^1 d\eta\, \mathfrak{F}_i(\eta, q^2)[\delta(\eta - x) - \delta(-\eta - x)] \tag{2.12}$$

for any of the combinations of structure functions. Now observe the following identities:

$$\int e^{iq\cdot y}\Box[\Delta(y, m^2)\cos P_\eta\cdot y]d^4y = \pi i\eta[\delta(\eta - x) - \delta(-\eta - x)],$$

$$\int e^{iq\cdot y}\partial_\mu\partial_\nu[\Delta(y, m^2)\cos P_\eta\cdot y]d^4y = -\frac{q_\mu q_\nu}{|2M\nu|}i\pi[\delta(\eta - x) - \delta(-\eta - x)],$$

$$\int e^{iq\cdot y}\Delta(y, m^2)\cos P_\eta\cdot y\, d^4y = \frac{1}{|2M\nu|}i\pi[\delta(\eta - x) - \delta(-\eta - x)], \tag{2.13}$$

$$\int e^{iq\cdot y}\partial_\mu[\Delta(y, m^2)\cos P_\eta\cdot y]d^4y = \frac{-iq_\mu}{|2M\nu|}i\pi[\delta(\eta - x) - \delta(-\eta - x)],$$

for $-1 \leq x \leq 1$ and $\eta \geq 0$. These are identities only if $P_\eta \cdot q = \eta M\nu$ and $P_\eta{}^2 = m^2$, where m^2 is an arbitrary but fixed parameter. It is not possible to find a P_η which satisfies both restrictions for arbitrary m^2. However, our representation for the current commutator need only be valid up to terms whose Fourier transform vanishes in the scaling limit, so we may choose P_η to satisfy the restriction to leading order in the scaling limit. As an example, consider the infinite-momentum frame where

$$q=\left(\frac{2M\nu - Q^2}{4P},\, 0,\, \sqrt{-q^2}\,,\, -\frac{2M\nu+Q^2}{4P}\right)$$

and choose $P_\eta = ((\eta^2P^2+m^2)^{1/2}, 0, 0, \eta P)$. For finite η, $P_\eta \cdot q = \eta M\nu + O(2M\nu m^2/\eta P^2)$ and $P_\eta{}^2 = m^2$, so in this case corrections to Eq. (2.13) are controlled by the large parameter P. The restriction to finite η has been noted before. It is discussed further in Appendix B where several explicit representations for P_η (not necessarily in an infinite-momentum frame) are given.

With these identities, the derivation is easily completed: substitution of Eqs. (2.12) and (2.13) into Eq. (2.11) yields Eq. (2.9) of the previous section. This derivation avoids two of the difficulties which arose in the previous section. First, it is not limited to an infinite-momentum frame. Second, it was not necessary to assume $P_\eta^\mu \sim \eta P^\mu$ to leading order. These limitations are replaced by the single problem of choosing P_η^μ to satisfy $P_\eta \cdot q = \eta M\nu$ and $P_\eta{}^2 = m^2$, which receives attention in Appendix B.

Since W_1 and W_2 are known only over a limited region of momentum space (in particular for $Q^2 > 0$), it is not, in general, possible to invert the Fourier transform and (uniquely) display them in coordinate space. However, by means of Eq. (2.13), the scaling functions are "mapped" onto a particular set of singular functions in coordinate space. This particular choice of singular functions was made on the basis of Assumption II. Assumption II fixes uniquely the coordinate-space behavior from which the dominant term arises because scaling cannot be obtained from a current matrix element which is smooth everywhere in coordinate space. This assertion is proved in Appendix C. There must be singularities and Assumption II places them on the light cone where a singularity of a given order is linked uniquely to scaling with a particular power of ν.[9] This, therefore, determines the particular choice of identities in Eq. (2.13). Given Assumptions I and II, the current commutator must be given by Eq. (2.10) (up to terms whose Fourier transform vanishes in the scaling limit).

Having derived Eq. (2.10) under more general assumptions, we proceed to investigate its light-cone structure.

D. Light-Cone Dominance and the Parton Model

In the Bjorken limit, the Fourier transform of the parton-model current commutator [cf. Eq. (2.10)] is light-cone dominated. Light-cone dominance may be interpreted to mean that the leading term in $W_{\mu\nu}$ must come from the leading light-cone singularity in the current product matrix element. On the other hand, it may be interpreted to require that $W_{\mu\nu}$ receive contributions primarily from a region of coordinate space restricted to $y^2 \lesssim 1/Q^2$. Whether or not these are identical criteria need not concern us here in this process, since both are satisfied by the matrix element of Eq. (2.10).

Considering the first criterion, note that the singular functions Δ and Δ' may be written[23]

$$\begin{aligned}\Delta(y, m^2) &= \frac{1}{2\pi}\delta(y^2)\epsilon(y_0) - \frac{m^2}{4\pi}\epsilon(y_0)\theta(y^2)\frac{J_1(m\sqrt{y^2})}{m\sqrt{y^2}}\,,\\ \Delta'(y, m^2) &= \frac{1}{8\pi}\theta(y^2)\epsilon(y_0) - \frac{m^2y^2}{8\pi}\epsilon(y_0)\theta(y^2)\left[\frac{1-J_0(m\sqrt{y^2})}{m^2y^2}\right],\end{aligned} \tag{2.14}$$

where the term in brackets goes to $\frac{1}{4}$ as $y^2 \to 0$. When substituted in Eq. (2.10) and Fourier transformed, the first terms in Eqs. (2.14) give the usual scaling law for $W_{\mu\nu}$ [see, e.g., Eq. (2.11)] while the less singular second terms vanish in the scaling limit as m^2/Q^2 or $m^2/M\nu$. This satisfies the first criterion for light-cone dominance.

Concerning the second criterion, it is obviously fulfilled for terms in $W_{\mu\nu}$ arising from $\Delta(y, m^2)$. These are dominated by the singularity $\delta(y^2)$ in the Bjorken limit so only the light cone itself contributes. Terms in $W_{\mu\nu}$ proportional to $\Delta'(y, m^2)$ are given by integrals over $\theta(y^2)$ in the Bjorken limit. It is not obvious that these integrals receive contributions only from $y^2 \lesssim 1/Q^2$. That this is so may be proved in the manner of Appendix C.

Since only the leading light-cone singularity contributes in the Bjorken limit, Eq. (2.10) may be rewritten

$$\begin{aligned}\frac{4\pi^2E_P}{M}\langle P|[J_\mu(y), J_\nu(0)]|P\rangle = \frac{1}{2\pi^2 iM}\Bigg\{ &-(g_{\mu\nu}\Box - \partial_\mu\partial_\nu)\left[\delta(y^2)\epsilon(y_0)\int_0^1\frac{d\eta}{\eta}\left(F_1(\eta)-\frac{1}{2\eta}F_2(\eta)\right)\cos P_\eta\cdot y\right]\\ &+\tfrac{1}{2}[P_\mu P_\nu\Box - (P\cdot\partial)(P_\mu\partial_\nu + P_\nu\partial_\mu) + g_{\mu\nu}(P\cdot\partial)^2]\left[\theta(y^2)\epsilon(y_0)\int_0^1 d\eta\, F_2(\eta)\frac{\sin P_\eta\cdot y}{P_\eta\cdot y}\right]\Bigg\}\,.\end{aligned} \tag{2.15}$$

This differs from the results of conventional light-cone analyses (cf. Ref. 22) only in having P_η everywhere instead of ηP. Up to this point, we have not assumed $P_\eta^\mu \approx \eta P^\mu$ (see previous section). In an infinite-momentum frame we could replace P_η^μ by ηP^μ to leading order but we do not want to be limited to an infinite-momentum frame. In Appendix B it is shown that regardless of whether $P_\eta^\mu \approx \eta P^\mu$, Eq. (2.15) gives the same expression for $W_{\mu\nu}$ when either is used. With this, we obtain agreement with the results of Ref. 22.

Notice that the integrals in Eq. (2.15) are divergent at $\eta = 0$ unless the structure functions were to vanish sufficiently rapidly there. Experimentally they do not. Problems near $\eta = 0$ have occurred before in our analysis, in choosing the vector P_η and in making the substitution $P_\eta \to \eta P$. In Appendix B it is shown that all these problems are related and only affect the $x \to 0$ limit of the scaling region. We conclude that Eq. (2.15), with the integrals regulated near $\eta = 0$, is valid up to terms which vanish in the scaling limit for $x \neq 0$.

This establishes the correspondence between light-cone dominance and free-field singularities on the one hand and incoherent and elastic scattering of partons on the other. Had we begun with the light-cone analysis [as summarized, for example, in Eq. (2.15)] and attempted to derive the parton model, it would be necessary at some point to *assume* that partons exist, i.e., that it makes sense in an infinite-momentum frame to write $|P\rangle = \sum_n a_n |n\rangle$. For this reason the parton model is not formally equivalent to the light-cone analysis but is instead a particular realization of the light-cone analysis in momentum space.

To complete this discussion, it is necessary to investigate how symmetry properties and other constraints in one approach transform to the other. Our discussion parallels that of Fritzsch and Gell-Mann.[16] If various properties are attributed to the current operators or their matrix elements on the light cone [e.g., SU(3)×SU(3) symmetry, spin-$\frac{1}{2}$ transformation laws], sum rules and other relations emerge among the structure functions for highly inelastic electroproduction and the corresponding weak processes.[24,16] Likewise, if symmetry properties are ascribed to the partons or if assumptions are made about the momentum distribution of partons, another set of relations among structure functions emerges. The equivalence is straightforward: An algebra of operators on the light cone corresponds to a symmetry among partons. To see this, assume some symmetry among partons; rederive Eq. (2.15); the symmetry emerges as the corresponding algebra of operators. Notice that in neither approach are assumptions made about the symmetries of physical states. Assumptions about the *distribution* or *types* of partons constituting a physical particle transform into statements about the coordinate

TABLE I. Transformation between parton-model and light-cone-analysis assumptions and their physical consequences.

Parton-model assumption		Light-cone-analysis assumption	Physical consequence [a]
Elasticity, incoherence	⟷	Light-cone dominance, free-field singularities	Scaling in Bjorken limit
Properties of partons (e.g., symmetries, spins)	⟷	Operator properties on the light cone (e.g., algebra, spin structure)	Sum rules: e.g., Adler sum rule: $\int_0^1 \frac{dx}{x}(F_2^{\nu n} - F_2^{\nu p}) = 2$ Gross–Llewellyn Smith sum rule: $\int_0^1 dx\,(F_3^{\nu p} + F_3^{\nu n}) = -6$ Callan-Gross relation: $F_1(x) = \frac{1}{2x} F_2(x)$
Parton composition of physical states	⟷	Matrix elements of operators on the light cone	Various relations, e.g., Bjorken-Paschos sum rule: $\int_0^1 dx\,F_2(x) = \left\langle \frac{Q^2}{N} \right\rangle$ Dual-quark-model sum rule: $\int_0^1 \frac{dx}{x}(F_2^{ep} - F_2^{en}) = \frac{1}{3}$

[a] See Refs. 2 and 24 for references.

space or algebraic structure of the *matrix elements* of operators near the light cone. Any relation derived in one approach from such an assumption may be derived from the other approach with an equivalent (though perhaps less intuitively compelling) assumption. Table I summarizes the transformation from partons to the light cone and vice versa.

In deriving Eq. (2.15), some physical understanding of the light-cone singularities has been obtained. First, the free-field singularity on the light cone arises from the phase-space integral for the elastically scattered parton, i.e., *from its free propagation into the final state*. Pictorially, we associate $\Delta_+(y, m^2)$ with the scattered parton in Fig. 2. This association will help us to interpret the results of the analysis of muon-pair production in the next section. Second, the $P\cdot y$ dependence is given by

$$F(P\cdot y) \equiv \sum_n |a_n|^2 \left(\sum_i \lambda_i^2 \frac{e^{iP_i\cdot y}}{2E_i} + \sum_j \lambda_j^2 \frac{e^{iP_j\cdot y}}{2E_j} \right).$$

Noticing that $\psi_i(y) = e^{-iP_i\cdot y}/\sqrt{2E_i}$ is a parton's "wave function," we may rewrite the above equation as

$$F(P\cdot y) = \langle P|\psi^\dagger(y)\Lambda^2\psi(0)|P\rangle, \tag{2.16}$$

where Λ is a charge operator. $F(P\cdot y)$ measures the average correlation of a parton at y with one at zero weighted by the squared charge. The association of the $P\cdot y$ dependence of the singularity with longitudinal coherence, as discussed by Ioffe and others,[9,25] is borne out in the parton model.

III. PRODUCTION OF MASSIVE MUON PAIRS IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES

A. Kinematics

Figure 3 shows the reaction which concerns us in this section. We define $Q^2 \equiv (p_+ + p_-)^2$ to be the squared mass of the muon pair and $s \equiv (P+P')^2$. We restrict ourselves to the limit $s, Q^2 \to \infty$ with $Q^2/s \equiv \tau$ fixed ($Q^2 < s$). Only the total cross section for a given squared mass Q^2 will be discussed here. Other experimentally useful cross sections have been discussed elsewhere.[10,26] When Q^2 and s are much greater than the squared masses of the muon and the incident particles, the cross section is

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{3Q^2 s} W(Q^2, s),$$

where

$$W(Q^2,s) = -16\pi^2 EE' \int_R d^4q\, \delta(q^2 - Q^2) \sum_n (2\pi)^4 \delta^4(P+P'-q-P_n) \langle PP'^{\text{in}}|J_\mu(0)|n\rangle\langle n|J^\mu(0)|PP'^{\text{in}}\rangle. \tag{3.1}$$

R is the phase-space region defined by

$$\sqrt{Q^2} \leq q_0 < \frac{s+Q^2}{2\sqrt{s}}. \tag{3.2}$$

$W(Q^2, s)$ may be rewritten as

$$W(Q^2,s) = -4EE'(2\pi)^5 \int d^4y\, \Delta_+^R(y, Q^2) \langle PP'^{\text{in}}|J_\mu(y) J^\mu(0)|PP'^{\text{in}}\rangle, \tag{3.3}$$

where Δ_+^R is the singular function defined earlier with the momentum constrained by Eq. (3.2). As Q^2 and s become infinite Δ_+^R reduces to Δ_+.[12]

B. Coordinate-Space Structure in a Perturbative Parton Model

Application of parton-model ideas to massive-muon-pair production rests on the observation that parton-antiparton annihilation is a more efficient way of producing a massive photon than is an exchange process. Drell and Yan[10] have used this observation to obtain a scaling law for this process. Their analysis may be summarized as follows. As s becomes large, the center of mass approaches an infinite-momentum frame. The colliding hadrons may be viewed as collections of colliding partons which can produce a massive photon either by annihilation [shown in Fig. 4(a)] or by bremsstrahlung [shown in Fig. 4(b)]. To conserve energy and momentum, the bremsstrahlung of Fig. 4(b) must be accompanied by an exchange of momentum. In order to produce a system of mass greater than Q from an incoming particle of mass M_N and energy $s/2M_N$, a squared momentum transfer of $|t| \geq M_N^2(Q^4/s^2) = M_N^2\tau^2$ is required. In the limit of interest $t_{\min}$ remains finite and of the order of 0.5 GeV^2. From high-energy scattering experiments, we know that hadronic cross sections are dominated by Pomeranchukon exchange which decreases exponentially with momentum transfer. Even for fairly small values of τ the bremsstrahlung diagram will be reduced by a substantial factor. Since the annihilation diagram of Fig. 4(a) experiences no such damping, it will dominate at large s and Q^2. We refer the reader to the work of Drell and Yan[10] for a more detailed derivation of this result. In their work a transverse-momentum cutoff replaces the Regge asymptotics as the origin of the damping.

The parton-model prediction for $W(Q^2, s)$ may be obtained from inspection of Fig. 4(a). The cross section is proportional to the probability of finding a parton of type b moving to the right, convoluted with the probability of finding an antiparton $\bar{b}$ moving to the left, times the total annihilation cross section for point particles summed over all *types* of partons. The annihilating partons' momenta are constrained to form a photon of mass Q^2:

$$\frac{d\sigma}{dQ^2} = \int_0^1 d\eta_1 \int_0^1 d\eta_2 \sum_b \mathcal{P}_b(\eta_1)\mathcal{P}_{\bar{b}}(\eta_2) \times \delta((P_{\eta_1} + P'_{\eta_2})^2 - Q^2)[\sigma(Q^2)]_{\text{point}},$$

where $P_{\eta_1} = ([(\eta_1 P)^2 + m^2]^{1/2}, \vec{0}, \eta_1 P)$ and $P'_{\eta_2} = ([(\eta_2 P)^2 + m^2]^{1/2}, \vec{0}, -\eta_2 P)$, $s \cong 4P^2$, ignoring transverse momenta which are assumed to be limited. The point cross section is $\sigma(Q^2) = (4\pi\alpha^2/3Q^2)\lambda_b^2$ and the probabilities are found in Sec. II:

$$\mathcal{P}_b(\eta) = F_{2b}(\eta)/\eta\lambda_b^2,$$

where F_{2b} is the contribution to the highly inelastic structure function from partons of type b. Using $(P_{\eta_1} + P'_{\eta_2})^2 \cong \eta_1\eta_2 s$, we have finally

$$\lim_{s, Q^2 \to \infty;\ \tau \text{ fixed}} W(Q^2, s) \equiv W(\tau) = \int_0^1 \frac{d\eta_1}{\eta_1} \int_0^1 \frac{d\eta_2}{\eta_2} \delta(\eta_1\eta_2 - \tau) \sum_b F_{2b}(\eta_1) F'_{2\bar{b}}(\eta_2) \frac{1}{\lambda_b^2}. \tag{3.4}$$

The second structure function is allowed to differ from the first in case the incident particles are not identical.

The coordinate-space version of this model may be extracted directly from Eq. (3.4) by substituting the identity (to leading order as $s, Q^2 \to \infty$)

$$\int d^4y\, \Delta_+(y, Q^2) \exp[i(P_{\eta_1} + P'_{\eta_2}) \cdot y] = \frac{2\pi}{s} \delta(\eta_1\eta_2 - \tau).$$

Some elementary algebra yields

$$W(\tau) = \frac{s}{2\pi} \int d^4y\, \Delta_+(y, Q^2) \sum_b \frac{1}{\lambda_b^2} \int_0^1 \frac{d\eta_1}{\eta_1} \int_0^1 \frac{d\eta_2}{\eta_2} \exp[i(P_{\eta_1} + P'_{\eta_2}) \cdot y] F_{2b}(\eta_1) F'_{2\bar{b}}(\eta_2).$$

Comparing this with the definition of W in Eq. (3.3),

$$\langle PP'^{\text{in}} | J_\mu(y) J^\mu(0) | PP'^{\text{in}} \rangle \doteq -\frac{1}{(2\pi)^6} \sum_b \frac{1}{\lambda_b^2} \int_0^1 \frac{d\eta_1}{\eta_1} \exp(iP_{\eta_1} \cdot y) F_{2b}(\eta_1) \int_0^1 \frac{d\eta_2}{\eta_2} \exp(iP'_{\eta_2} \cdot y) F_{2\bar{b}}(\eta_2). \tag{3.5}$$

We emphasize that there are terms in the current product other than the one displayed in Eq. (3.5) [for example, the bremsstrahlung diagram of Fig. 4(b)]. *These terms, when integrated against $\Delta_+(y, Q^2)$ (as we have argued for the bremsstrahlung diagram), are lower order in the $s, Q^2 \to \infty$ limit.* Equation (3.5) displays the coordinate-space dependence of the DLY parton model. We postpone a discussion of its light-cone behavior until we note how specific our result is to the model of Drell and Yan.

C. More General Parton Models

A result similar to Eq. (3.5) may be obtained under somewhat more general assumptions. In deriving Eq. (3.5), it was assumed that the distribution of partons in each incident hadron is unaffected by the presence of the other. Drell and Yan note that this assumption need not be made, in which case one still obtains scaling [$d\sigma/dQ^2 \propto 1/Q^4 f(\tau)$] but $f(\tau)$ is no longer explicitly given in terms of the electroproduction structure functions $F_2(\eta)$. Likewise Landshoff and Polkinghorne obtain scaling but not factorization in their nonperturbative model.[11] Scaling alone is enough to obtain a form similar to Eq. (3.5). To see this, write

$$\lim_{Q^2, s \to \infty;\ Q^2/s \text{ fixed}} W(Q^2, s) = W(\tau) = \int_0^1 d\alpha\, W(\alpha) \delta(\alpha - \tau)$$

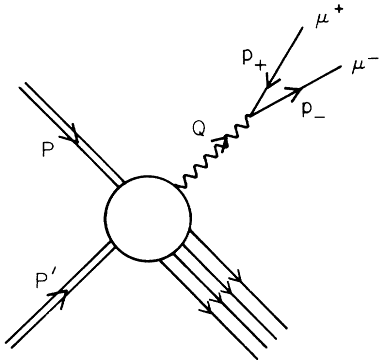

FIG. 3. Kinematics of massive muon-pair production in hadron-hadron collisions, $s=(P+P')^2$.

$$=\frac{s}{2\pi}\int d^4y\,\Delta_+(y,Q^2)\int_0^1 d\alpha\,W(\alpha)e^{i\tilde{P}_\alpha\cdot y},$$

where $\tilde{P}_\alpha{}^2=\alpha s$ to leading order as $s,\,Q^2\to\infty$. Then up to terms whose integrals against $\Delta_+(y,Q^2)$ vanish in the limit,

$$\langle PP'^{\text{in}}|J_\mu(y)J^\mu(0)|PP'^{\text{in}}\rangle \doteq -\frac{1}{(2\pi)^6}\int_0^1 d\alpha\,W(\alpha)e^{i\tilde{P}_\alpha\cdot y}. \tag{3.6}$$

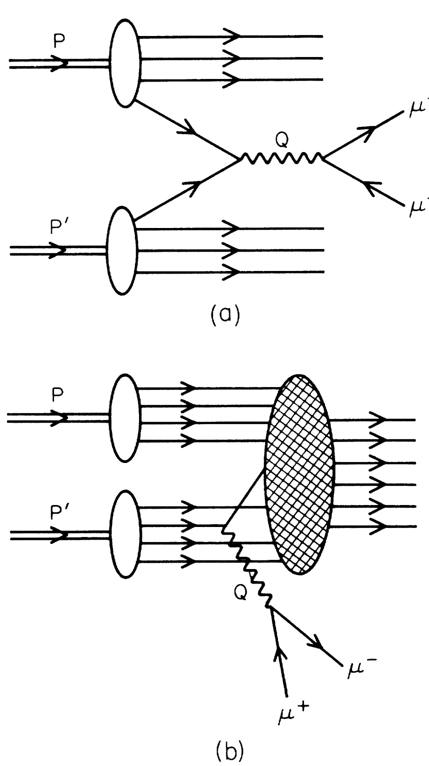

FIG. 4. Parton-model diagrams for (a) parton-pair annihilation into a massive muon pair; (b) parton bremsstrahlung of a massive muon pair.

In this expression, just as in Eq. (3.5), scaling is linked with high-frequency oscillations. Of course, there is no experimental evidence as yet that $W(Q^2,s)$ scales so Eq. (3.6) is at best a form shared by models which predict scaling (cf. Refs. 10, 11, 27) and may or may not describe the data.

A correspondence between scaling and high oscillations is not unique. Any attempt to extract the matrix element uniquely by inverting the Fourier transform of Eq. (3.3) fails because of the restriction to positive-energy photons. For electroproduction, assuming the singularities (if any) to be on the light cone (Assumption II) was sufficient, together with scaling, to determine the leading term in the current product uniquely. For muon-pair production scaling can arise from a matrix element smooth everywhere in coordinate space [see Eq. (3.6)] so a restriction analogous to Assumption II is not sufficient to "map" scaling behavior onto the light cone.

If scaling is not uniquely connected to high oscillations, it should be possible to find other forms for the current product matrix element which scale. One such form, singular on the light cone and not highly oscillatory away from it, is discussed in the next section [cf. Eqs. (3.7) and (3.8)]. The problem with this example and others like it is their arbitrariness: There is as yet no convincing theoretical reason to choose such a form. If scaling is observed experimentally, it is explained simply and unambiguously in the parton model. It could be accounted for by a matrix element singular on the light cone and not highly oscillating away from it, but not without additional assumptions to single out the behavior desired.

Before discussing the light-cone behavior of the parton model, it is necessary to relate $W(\alpha)$ appearing in Eq. (3.6) to the various structure functions appearing in Eq. (3.5). As Drell and Yan note,[10] factoring obtains only if one rules out parton annihilation *accompanied* by exchange of wee partons. Wee-parton exchanges are called upon to build up high-energy diffractive scattering (Pomeranchukon) in Feynman's original parton work.[3]

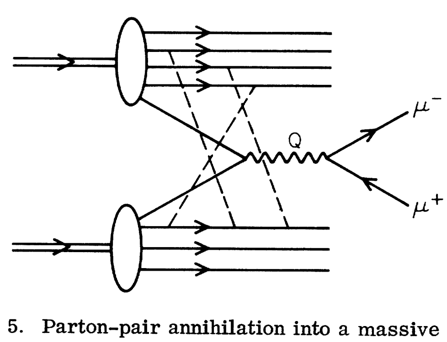

FIG. 5. Parton-pair annihilation into a massive muon pair accompanied by the exchange of wee partons.

If, as shown in Fig. 5, wee-parton exchanges take place, the parton distribution in one hadron is modified by the presence of the other. This effect also prevents factoring in the nonperturbative model of Landshoff and Polkinghorne.[11]

Nevertheless wee-parton (Pomeranchukon) exchanges should not modify substantially the original parton distributions of the incident particles. Wee-parton exchanges carry asymptotically small longitudinal momentum (of order $1/\sqrt{s}$) and are independent of s as $s\to\infty$. Because the exchanges are s-independent, the scaling law is not affected. Because they carry vanishingly small momentum, we expect that $W(\alpha)$ is approximately what it is in the Drell-Yan model, i.e., that Eq. (3.4) is approximately correct.

D. Light-Cone Dominance and the Parton Model

The role of the light cone in massive-muon-pair production is more complex than in electroproduction. We show, on the one hand, that with the parton-model matrix element of Eq. (3.6), $W(Q^2,s)$ is dominated by the region of $y^2\lesssim n/Q^2$ for some dimensionless constant n. On the other hand, we show that terms in the matrix element *are not ordered in relative importance as* $Q^2,s\to\infty$ *by the strength of their singularities alone* (as was the case in electroproduction). Rather, their importance is determined by the singularity and other dynamical information such as the s dependence of the matrix element and the frequency of its oscillations away from the light cone.

In the parton model itself, we find that the leading light-cone singularity (bremsstrahlung diagram) is dominated in the $s,Q^2\to\infty$ limit by the nonsingular annihilation diagram, just the opposite of what would be expected on the basis of light-cone considerations alone.

The primary advantage of the light-cone analysis in electroproduction has been to relieve the theorist of having to study the whole two-current matrix element in favor of looking only at its leading light-cone singularity. This advantage is lost in muon-pair production where terms less singular on the light cone are not *a priori* less important than the leading singularity. In light of this, it is somewhat academic what region of coordinate space contributes to $W(Q^2,s)$ in the parton model. Nevertheless, let us show that $W(Q^2,s)$ receives important contributions only when $y^2\lesssim n/Q^2$, when the matrix element is given by Eq. (3.6).

To do this, rewrite the definition of $W(Q^2,s)$ in terms of the dimensionless variable $\xi^\mu\equiv\sqrt{s}\,y^\mu$ in the rest system of $P+P'$:

$$W(Q^2,s)=\frac{1}{2\pi}\int d^4\xi\,\Delta_+(\xi,\tau)\int_0^1 d\alpha\,W(\alpha)e^{i\xi_0\sqrt{\alpha}}\,.$$

This expression involves no dimensional parameters and remains unchanged as $Q^2,s\to\infty$ with fixed τ. Moreover, since the integral over ξ converges, there must be some N such that the region $\xi^2>N$ gives as small a contribution to the integral as desired. The dominant contribution comes, therefore, from $\xi^2\lesssim N$ or $y^2\lesssim N\tau/Q^2$.

We now turn to the central question: whether terms more singular than Eq. (3.6) necessarily give larger contributions to $W(Q^2,s)$ as $Q^2,s\to\infty$. This is not the case in the parton model. Note, first, that Eq. (3.6) *has no light-cone singularity at all*.[28] The $\Delta_+(y,m^2)$ found in electroproduction is not present. As was noted, such singularities arise from the free propagation of elastically scattered partons. In the annihilation diagram for muon-pair production there is no scattered parton, therefore no singularity. Notice, however, that there *is* an elastically scattered parton and an associated free-field singularity in the bremsstrahlung diagram of Fig. 4(b). We have argued that this mechanism is damped relative to parton annihilation as $Q^2,s\to\infty$. The diagram *more singular* on the light cone is *less important* at large Q^2 and s.

To understand better how this behavior might arise, consider a specific example chosen to simulate the bremsstrahlung diagram:

$$\langle PP'^{\text{in}}|J_\mu(y)J^\mu(0)|PP'^{\text{in}}\rangle=-\frac{1}{(2\pi)^4}\Delta_+(y,m^2)\frac{a^2}{s\{a^2+[(P+P')\cdot y]^2\}}\,. \tag{3.7}$$

The power of s is dictated by the dimension of the matrix element [$\langle PP'^{\text{in}}|J_\mu(y)J^\mu(0)|PP'^{\text{in}}\rangle$ is dimensionless and $\Delta_+\sim 1/(\text{length})^2$].

Multiplying Eq. (3.7) by $\Delta_+^R(y,Q^2)$ and integrating over all space, we recover $W(Q^2,s)$:

$$W(Q^2,s)=\int_{\sqrt{\tau}}^1 d\alpha\,f(\alpha)\frac{|\vec{q}|}{2\alpha\sqrt{s}}\,, \tag{3.8}$$

where $f(\alpha)$ is the Fourier transform of $a^2/\{a^2+[(P+P')\cdot y]^2\}$, $f(\alpha)=\frac{1}{2}ae^{-\alpha a}$, and $|\vec{q}|=(\alpha^2 s-Q^2)/2\alpha\sqrt{s}$. This final integral is bounded as follows:

$$|W(Q^2,s)|\leq\frac{(1+\tau)}{\sqrt{\tau}}(e^{-a\sqrt{\tau}}-e^{-a})\,.$$

By choosing the dimensionless parameter a as reasonably large, this contribution to $W(Q^2,s)$ may be made as small as desired. This term, more singular on the light cone than the parton annihilation contribution, is less important in the scaling region by a fixed (for fixed τ) exponential factor.

Of course, one can write down expressions similar to Eq. (3.7) which *do* dominate the annihilation diagram [just replace $1/s$ by $1/M_N^2$ in Eq. (3.7)]. The point is that *the light cone does not dominate a priori,* additional assumptions are needed.

Altarelli, Brandt, and Preparata[12] achieve light-cone dominance by bounding terms in their operator-product expansion with Regge asymptotics. It is, therefore, not possible to build up large-phase oscillations, so a term analogous to parton annihilation is excluded from their analysis. Their conclusions rest heavily on the assumption that Regge asymptotics can be applied to the creation of a particle of asymptotically infinite mass and are presumably valid in the limit $s \gg Q^2$ where appeal to Regge theory is well supported. Our analysis relies heavily on taking $\tau = Q^2/s$ finite. If τ is very small the parton bremsstrahlung diagram discussed in Sec. III B is no longer negligible and may in fact dominate the scaling, annihilation diagram. It is not surprising that the results quoted in Ref. 12 disagree with the parton model since the analyses apply to different kinematic regions.

Massive-muon-pair production distinguishes clearly between the light-cone approach and the parton model. The parton-model scaling law arises from a piece of the two-current matrix element which is nonsingular on the light cone and highly oscillatory away from it. As noted earlier, it is possible to find some model for the matrix element which both is light-cone dominated and produces scaling, but only by making what appear to be arbitrary assumptions about the nonsingular part of the matrix element. If the scaling law is verified, it will strongly enhance the attractiveness of the parton model for highly inelastic processes.

IV. OTHER INELASTIC LEPTONIC PROCESSES

A. $e^+ + e^- \to P +$ anything

Drell, Levy, and Yan[7] have studied the annihilation of an electron-positron pair to an arbitrary hadronic state from which a single hadron is detected. The squared amplitude is shown in Fig. 6. All hadronic information is contained in the tensor $\overline{W}_{\mu\nu}$:

$$\overline{W}_{\mu\nu} = \int d^4y\, e^{iq\cdot y} \frac{4\pi^2 E_P}{M} \sum_n \langle 0|J_\mu(y)|Pn\rangle\langle Pn|J_\nu(0)|0\rangle\,, \tag{4.1}$$

with invariant decomposition analogous to Eq. (2.4). ($\nu, q^2 \to \infty$; $x \equiv q^2/2M\nu$ fixed; $1 \le x < \infty$.) In the Bjorken limit and in their cutoff field theory of pions and nucleons, DLY find that the structure functions $(-)M\overline{W}_1$ and $\nu\overline{W}_2$ scale as functions of x and, moreover, that the resulting $\overline{F}_1$ and $\overline{F}_2$ are continuations of the electroproduction structure functions $F_1(x)$ and $F_2(x)$ to the region $x>1$:

$$\overline{F}_i(x) = F_i(x)\,. \tag{4.2}$$

Repeating the analysis of Sec. II B, we obtain (for simplicity we consider the trace $\overline{W}^\mu_\mu$):

$$\frac{4\pi^2 E_P}{M} \sum_n \langle 0|J_\mu(y)|Pn\rangle\langle Pn|J^\mu(0)|0\rangle = \frac{1}{2\pi M}\,\Box\left[\Delta_+(y,m^2)\int_1^\infty \frac{d\eta}{\eta}\left(-3\overline{F}_1(\eta)+\frac{1}{2\eta}\overline{F}_2(\eta)\right)e^{-iP\eta\cdot y}\right], \tag{4.3}$$

which should be compared with the trace of Eq. (2.8):

$$\frac{4\pi^2 E_P}{M}\langle P|J_\mu(y)J^\mu(0)|P\rangle = \frac{1}{2\pi M}\Box\left[\Delta_+(y,m^2)\int_0^1 \frac{d\eta}{\eta}\left(-3F_1(\eta)+\frac{1}{2\eta}F_2(\eta)\right)e^{iP\eta\cdot y}\right].$$

As in electroproduction, it is easy to show that only the leading singularity of Δ_+ [i.e., $i/4\pi^2(y^2+i\epsilon y_0)$] contributes in the Bjorken limit, so that the process is light-cone-dominated.

Nevertheless Eq. (3.3) cannot be obtained from a light-cone analysis without further assumptions. Assuming free-field singularities, it is straightforward to find the piece of the current product which contributes to the imaginary part of forward Compton scattering for $q^2>0$:

$$\frac{4\pi^2 E_P}{M}\langle P|J_\mu(y)J^\mu(0)|P\rangle = \frac{i}{8\pi^3 M}\Box\left[\frac{1}{y^2+i\epsilon y_0}\int_1^\infty \frac{d\eta}{\eta}\left(-3\tilde{F}_1(\eta)+\frac{1}{2\eta}\tilde{F}_2(\eta)\right)e^{iP\eta\cdot y}\right],$$

where $\tilde{F}_i$ are the (scaling) structure functions derived from the discontinuity in the forward virtual Compton amplitude $T_{\mu\nu}$. There are four distinct pieces in this discontinuity for timelike photons (see Fig. 7) one of which [Fig. 7(a)] is related (by crossing) to the left-hand side of Eq. (4.3). Although the object on the left of Eq. (4.3) is related to one piece of the current product in the above equation, and although the $\overline{F}_i(x)$ contribute to $\tilde{F}_i(x)$, we know of no way of retrieving Eq. (4.3) from the light-cone analysis without additional assumptions. Ellis[29] has derived the parton-model scaling laws for this process from a light-cone analysis. We refer the reader to his work for a discussion of the additional assumptions necessary to obtain this result.

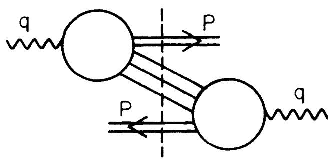

FIG. 6. Squared amplitude for positron-electron annihilation into a hadron, P, plus anything.

B. $\gamma+P\rightarrow\mu^++\mu^-+$anything

In an earlier paper[26] we showed that the high-energy photoproduction of massive muon pairs may be described in terms of parton annihilation much like the process $P+P\rightarrow\mu^++\mu^-+$anything. Separating the parton contribution from the large background of Bethe-Heitler pairs (see Fig. 8) necessitates measuring a cross section symmetric in the produced muons and differential in their longitudinal momentum,

$$\left(\frac{d\sigma}{dQ^2dQ_3}\right)_{\text{symmetric}}=\frac{\alpha^2}{Q_0{}^2Q^4}\sum_b\frac{f_b}{\lambda_b{}^2}F_{2b}(\eta_1)G_{2\bar{b}}(\eta_2)\,, \tag{4.4}$$

where $f_a=2$ for spin-0 partons and $f_a=1$ for spin-$\frac{1}{2}$ partons. G_{2b} is the structure function for partons of type b in the photon. η_1 and η_2 are the fractional momenta of the partons in the center-of-mass system, constrained by

$$\eta_1\eta_2=Q^2/s \quad\text{and}\quad \eta_1-\eta_2=2\frac{Q_3^{\text{c.m.}}}{\sqrt{s}}\,.$$

As in proton-proton production of muon pairs, this cross section is proportional to the two-particle matrix element of a current product,

$$\left(\frac{d\sigma}{dQ^2dQ_3}\right)_{\text{symmetric}}$$

$$\propto\int d^4y\, e^{iq\cdot y}\langle Pk^{\text{in}}|J_\mu(y)J_\nu(0)|Pk^{\text{in}}\rangle l^{\mu\nu}\,,$$

where the muon currents combine to form the tensor $l^{\mu\nu}$. When integrated over all muon momenta for fixed Q^2, $l^{\mu\nu}$ becomes proportional to $g^{\mu\nu}$. In forming the symmetric cross section differential in Q_3 additional components would in general enter into $l^{\mu\nu}$. If, however, Q^2 and s are very large and the transverse momenta of the annihilating partons are limited, then to leading order $l^{\mu\nu}$ is again proportional to $g^{\mu\nu}$. Since the same matrix element enters $d\sigma/dQ^2$ and $(d\sigma/dQ^2dQ_3)_{\text{symmetric}}$ it is not necessary to transform the cross section of Eq. (4.4) to coordinate space. Instead we consider $d\sigma/dQ^2$ and write down the relevant matrix element by analogy to the process $P+P\rightarrow\mu^++\mu^-+$anything:

$$\langle Pk^{\text{in}}|J_\mu(y)J^\mu(0)|Pk^{\text{in}}\rangle$$

$$=-\frac{1}{(2\pi)^6}\sum_b\frac{1}{\lambda_b{}^2}\int_0^1\frac{d\eta_1}{\eta_1}\exp(iP_{\eta_1}\cdot y)F_{2b}(\eta_1)$$

$$\times\int_0^1\frac{d\eta_2}{\eta_2}\exp(iP'_{\eta_2}\cdot y)G_{2\bar{b}}(\eta_2)\,.$$

This process has the same light-cone behavior as hadronic production of muon pairs. Photoproduction of muon pairs also provides an experimental arena in which to distinguish the parton from the light-cone approach to highly inelastic

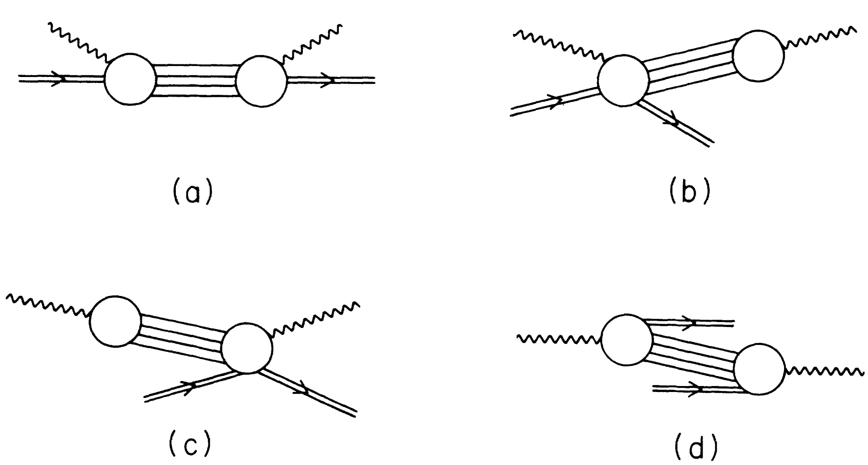

FIG. 7. Discontinuities in forward virtual Compton scattering for timelike q^2.

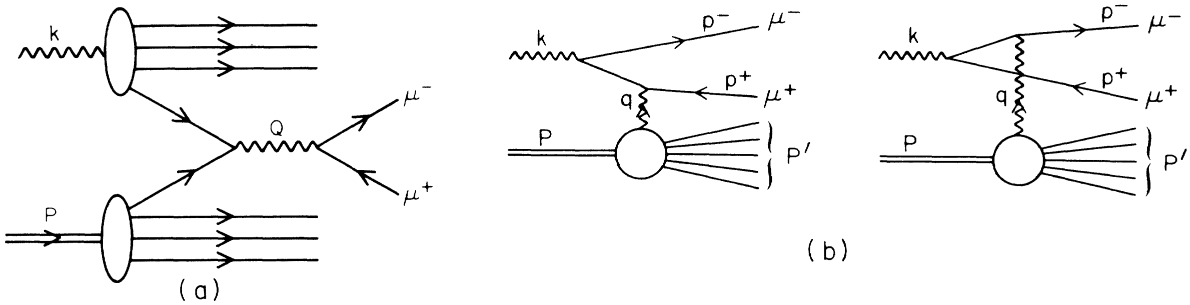

FIG. 8. Contributions to $\gamma + P \rightarrow \mu^+\mu^-$ +anything from (a) parton-pair annihilation; (b) Bethe-Heitler process.

electromagnetic processes.

C. Other Processes

We have translated several other parton-model analyses into the language of the light cone. Among these are the highly inelastic scattering of neutrinos and of polarized electrons from hadronic targets, and one-particle inclusive electroproduction ($e + P \rightarrow e + h$ +anything). In these cases nothing unexpected is encountered. Perturbative parton-model analyses of these processes may be found in Refs. 7 and 30. In the Bjorken limit, the parton model predicts scaling laws analogous to electroproduction. In coordinate space, the scaling laws translate into free-field singularities and light-cone dominance. The light-cone structure derived in this manner is well known from other light-cone analyses.[16,31,29] Since there are no surprises or additional insights, the details are not presented here.

V. SUMMARY

Massive-muon-pair production at high energy emerges from our analysis as the proper experiment in which to study the notion of "pointlike constituents in the nucleon." The scaling law of Drell and Yan [Eq. (4.4)] is obtained naturally in the parton model but not in the more general analysis of light-cone singularities. Experiments to date have been performed at only one value of s,[32] so the scaling law is as yet untested. In the near future it will be possible to test this prediction. If verified, it provides strong support for the parton viewpoint.

On the other hand, it appears to be impossible to differentiate the parton model from the analysis of light-cone singularities in highly inelastic electroproduction and related experiments. Any effect understood in terms of the elastic, incoherent scattering of partons is equally well attributed to free-field singularities on the light cone. Neither scaling nor any other aspect of inelastic leptonic scattering necessitates the existence of "pointlike constituents." While they need not actually exist, we have seen how useful partons are in developing an intuition for the analysis of light-cone singularities. We may identify the free-field singularities with the free propagation of an elastically scattered parton, and the "longitudinal" variation of the singularities along the light cone with the correlation of the points at which the virtual photon is absorbed and reemitted in forward Compton scattering.

In electroproduction it is a matter of taste how much reality one ascribes to this momentum-space realization of canonical light-cone singularities. In muon-pair production there is not so much freedom. Experiment will make it clear how seriously the idea of "pointlike constituents in the nucleon" must be taken.

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APPENDIX A

To obtain a manifestly current-conserving form of Eq. (2.9) we first separate out the longitudinal structure function $F_L(\eta) = F_2(\eta) - 2\eta F_1(\eta)$:

$$\frac{4\pi^2 E_P}{M}\langle P|[J_\mu(y),J_\nu(0)]|P\rangle \doteq \frac{1}{2\pi i M}\Big\{(g_{\mu\nu}\Box-\partial_\mu\partial_\nu)\Big[\Delta(y,m^2)\int_0^1\frac{d\eta}{\eta^2}F_L(\eta)\cos P_\eta\cdot y\Big] + \int_0^1 d\eta\Big(4P_\mu P_\nu+2\frac{i}{\eta}(P_\mu\partial_\nu+P_\nu\partial_\mu)-\frac{g_{\mu\nu}}{\eta^2}\Box\Big)F_2(\eta)\Delta(y,m^2)\cos P_\eta\cdot y\Big\}. \quad \text{(A1)}$$

Equation (2.9) was derived by identifying two quantities under a Fourier transform and is therefore valid up to terms whose Fourier transform vanishes in the Bjorken limit. The same is true for Eq. (A1). We may manipulate Eq. (A1) disregarding terms which vanish when Fourier transformed in the scaling limit. This allows us to make use of the following equivalences (proved below):

$$\Delta(y,m^2)\cos P_\eta\cdot y \doteq \Box\Big[\frac{\Delta'(y,m^2)\sin P_\eta\cdot y}{P_\eta\cdot y}\Big], \quad \text{(A2)}$$

$$\Box[\Delta(y,m^2)\cos P_\eta\cdot y] \doteq -4(P_\eta\cdot\partial)^2\Big[\frac{\Delta'(y,m^2)\sin P_\eta\cdot y}{P_\eta\cdot y}\Big], \quad \text{(A3)}$$

$$\Delta(y,m^2)\cos P_\eta\cdot y \doteq 2i(P_\eta\cdot\partial)\Big[\frac{\Delta'(y,m^2)\sin P_\eta\cdot y}{P_\eta\cdot y}\Big], \quad \text{(A4)}$$

where $\Delta'(y,m^2)=-d\Delta(y,m^2)/dm^2$. The equality is up to terms which vanish when Fourier transformed in the scaling limit. Using these substitutions in the second term of Eq. (A1),

$$\frac{4\pi^2 E_P}{M}\langle P|[J_\mu(y),J_\nu(0)]|P\rangle = \frac{1}{2\pi i M}\Big\{(g_{\mu\nu}\Box-\partial_\mu\partial_\nu)\Big[\Delta(y,m^2)\int_0^1\frac{d\eta}{\eta^2}F_L(\eta)\cos P_\eta\cdot y\Big] + 4[P_\mu P_\nu\Box-(P\cdot\partial)(P_\mu\partial_\nu+P_\nu\partial_\mu)+g_{\mu\nu}(P\cdot\partial)^2]\Big[\Delta'(y,m^2)\int_0^1\frac{d\eta}{\eta}F_2(\eta)\frac{\sin P_\eta\cdot y}{P_\eta\cdot y}\Big]\Big\}.$$

This is, of course, the manifestly current-conserving form desired.

It remains to derive Eqs. (A2)–(A4). Since the equivalence need only obtain under the Fourier transform, we will establish that the Fourier transforms of both sides of each equation are equal in the Bjorken limit. Beginning with Eq. (A2),

$$I \equiv \int e^{iq\cdot y}\Box\Big[\frac{\Delta'(y,m^2)\sin P_\eta\cdot y}{P_\eta\cdot y}\Big]d^4y$$

$$= -\frac{1}{2m^2}\frac{d}{d\beta}\int_{-1}^{1}d\alpha\int e^{iq\cdot y}\Box[\Delta(y,m^2\beta)e^{i\alpha P_\eta\cdot y}]d^4y\Big|_{\beta=1}.$$

To leading order in the scaling limit,

$$I=-\frac{Q^2}{2m^2}\frac{d}{d\beta}\int_{-1}^{1}d\alpha\, 2\pi i\delta(2\alpha\eta M\nu-Q^2+m^2\alpha-m^2\beta)\epsilon(\alpha\eta M\nu+m^2)\Big|_{\beta=1}.$$

Finally,

$$I=\frac{\pi i}{2M\nu}[\delta(\eta-x)-\delta(-\eta-x)].$$

An analogous and easier calculation of the left-hand side of Eq. (A2) yields the same result. The other identities may be derived from Eq. (A2) by partial integration of the Fourier transform.

APPENDIX B

Here we discuss three problems which arose in our analysis of highly inelastic electroproduction:

(1) Finding a four-vector P_η to satisfy $P_\eta\cdot q=\eta M\nu$ and $P_\eta{}^2=m^2$ to leading order.

(2) Replacing P_η^μ by ηP^μ whether or not $P_\eta^\mu\approx\eta P^\mu$.

(3) Divergences in the η integrals of Eq. (2.15) (for example) near $\eta=0$.

Problems (1) and (3) are solved by deleting the point $x=0$ from the region of our analysis and removing $\eta=0$ from the integrals. Problem (2) is resolved by direct calculation.

First we write down some representations for P_η and see what problems arise:

(I) In an infinite-momentum frame

$$q\equiv\Big(\frac{2M\nu-Q^2}{4P},0,\sqrt{-q^2},-\frac{2M\nu+Q^2}{4P}\Big),$$

$$P_\eta=((\eta^2P^2+m^2)^{1/2},0,0,\eta P).$$

Then

$$P_\eta \cdot q = \eta M\nu + O\left(\frac{M\nu m^2}{\eta P^2}\right),$$

$$P_\eta{}^2 = m^2.$$

(II) In the rest frame of P

$$q \equiv (\nu, 0, 0, (\nu^2 + Q^2)^{1/2}),$$

$$P_\eta = \tfrac{1}{2}\left(\eta M + \frac{m^2}{\eta M}, 0, 0, -\eta M + \frac{m^2}{\eta M}\right),$$

$$P_\eta \cdot q = \eta M\nu + O(m^2 x/\eta),$$

$$P_\eta{}^2 = m^2.$$

(III) In the rest frame of P with the parameter $m^2 = 0$

$$P_\eta = (\eta M, \eta M, 0, 0),$$

$$P_\eta \cdot q = \eta M\nu,$$

$$P_\eta{}^2 = 0.$$

Although (III) satisfies the restrictions exactly, it is of less interest to us since the intuitive correspondence to the parton model is lost when m^2 (identified with the parton mass) is set to zero. Representations (I) and (II) break down for small η when the second term in $P_\eta \cdot q$ becomes comparable to the first: $\eta \sim m/P$ for (I) and $\eta \sim (m^2 x/M\nu)^{1/2}$ for (II). If the η integrals converged near $\eta = 0$, this discrepancy would be inconsequential since it is important only over an infinitesimal region. This region of the integrals must be avoided.

To do this, define the region $\mathfrak{L}$ as the Bjorken-limit region minus the point $x=0$, i.e., $Q^2, M\nu \to \infty$, $\epsilon < |x| \le 1$ for fixed ϵ chosen as small as desired. Now repeat the steps of Eqs. (2.12) and (2.13) [consider $W^\mu_\mu(x) \equiv -3F_1(x) + (1/2x)F_2(x)$ for simplicity]:

$$W^\mu_\mu(x) = \int_\epsilon^1 d\eta[\delta(\eta - x) - \delta(-\eta - x)]W^\mu_\mu(\eta), \qquad \epsilon < |x| \le 1.$$

The analogs of Eq. (2.13) are

$$\int d^4y\, e^{iq\cdot y}\Box[\Delta(y, m^2)\cos P_\eta \cdot y] = \pi i\eta[\delta(\eta - x) - \delta(-\eta - x)]$$

and so on, for $\eta > \epsilon$, $\epsilon < |x| \le 1$.

Finally,

$$\frac{4\pi^2 E_P}{M}\langle P|[J_\mu(y), J^\mu(0)]|P\rangle \doteq \frac{1}{2\pi M i}\Box\left\{\Delta(y, m^2)\int_\epsilon^1 \frac{d\eta}{\eta^2}[F_2(\eta) - 6\eta F_1(\eta)] \times \cos P_\eta \cdot y\right\}. \quad \text{(B1)}$$

This is accurate up to terms whose Fourier transforms vanish in the region $\mathfrak{L}$, but by construction it *does not* contain the leading contribution in the region $|x| < \epsilon$.

The cutoff at ϵ removes the divergences in Eq. (2.15). It also removes the difficulties in choosing P_η. Consider, for example, Representation (I) and choose $\epsilon = (M/P)^{1/2}$. If (I) is an acceptable representation of P_η, the Fourier transform of Eq. (B1) should yield $W^\mu_\mu(x)$ with corrections which vanish as $P \to \infty$. Let T be that Fourier transform:

$$T = \frac{Q^2}{2}\int_\epsilon^1 \frac{d\eta}{\eta^2}[F_2(\eta) - 6F_1(\eta)]\,\delta(2P_\eta \cdot q - Q^2) \quad \text{for} \quad q_0 > 0$$

$$= Q^2\int_\epsilon^1 \frac{d\eta}{\eta} W^\mu_\mu(\eta)\delta\left(2\eta M\nu + \frac{m^2}{4\eta P^2}(2M\nu - Q^2) - Q^2\right),$$

where we have kept the leading correction to $P_\eta \cdot q$:

$$T = x\left[\frac{W^\mu_\mu(\eta)}{\eta[1 - (m^2/4P^2\eta^2)(1 - x)]}\right]_{\eta = x - (m^2/4P^2)(1-x)/x},$$

$$T = W^\mu_\mu\left(x - \frac{m^2}{4P^2 x}(1 - x)\right)\left(1 + \frac{m^2}{2P^2 x^2}(1 - x)\right).$$

These corrections are largest at $x = \epsilon$ but with $\epsilon = (m/P)^{1/2}$ the corrections vanish like m/P or faster as $P \to \infty$. A similar procedure may be used for Representation (II) if ϵ is chosen to be $(m^2/M\nu)^{1/4}$.

We conclude that all of the representations for the current commutator given in Sec. II are valid in the $\mathfrak{L}$ region: the Bjorken region minus the point $x = 0$. The η integrals in Sec. II must be understood to range from ϵ to 1 for arbitrarily small ϵ.

Lastly we verify that P^μ_η may be replaced by ηP^μ to leading order in the Bjorken limit. Consider the identities of Eq. (2.13) with P^μ_η replaced by ηP^μ:

$$\int e^{iq\cdot y}\Box[\Delta(y, m^2)\cos\eta P \cdot y]\,d^4y = \pi i\eta\left[\delta\left(\eta - x + \frac{(\eta^2 - 1)m^2}{2M\nu}\right) - \delta\left(-\eta - x + \frac{(\eta^2 - 1)m^2}{2M\nu}\right)\right]$$

and so on. Since $\epsilon < \eta \le 1$, the additional term $(\eta^2 - 1)m^2/2M\nu$ contributes only to lower order as $\nu \to \infty$. This allows us freely to replace P^μ_η by ηP^μ although the four-vectors themselves need not be equal.

It may seem artificial to use P_η in Sec. II when ηP could have been used all along. We chose this approach because it preserves the connection with the intuitive parton-model picture of P_η as the four-momentum of the parton which is elastically scattered. In the conventional light-cone analysis,

η appears as the Fourier conjugate variable to $P\cdot y$ without any particular physical correspondence.

APPENDIX C

Here we show that scaling in electroproduction cannot be obtained from a matrix element which is smooth everywhere in coordinate space. For definiteness we consider $\nu W_2(Q^2,\nu)$; the proof for $MW_1(Q^2,\nu)$ is similar. Isolating the contribution to νW_2 from the definition of $W_{\mu\nu}$ we obtain

$$\nu W_2(Q^2,\nu)=Q^2M\nu\int d^4y\, e^{iq\cdot y}C_2(y^2,y\cdot P)\,, \qquad \text{(C1)}$$

where

$$C_2(y^2,y\cdot P)=-C_2(y^2,-y\cdot P)$$

and

$$C_2(y^2,y\cdot P)=0 \quad \text{for} \quad y^2<0\,. \qquad \text{(C2)}$$

We prove that $\lim_{\text{Bj}}\nu W_2(Q^2,\nu)=0$ if $C_2(y^2,y\cdot P)$ is continuous in y^2 [which requires that $\lim_{y^2\to 0}C_2(y^2,y\cdot P)$ exists and equals zero]. First use Eq. (C2) to write

$$C_2(y^2,y\cdot P)=\epsilon(y\cdot P)\int_0^\infty da^2\int_{-\infty}^{\infty}d\alpha\,\delta(y^2-a^2)\times e^{i\alpha P\cdot y}C_2(a^2,\alpha)\,.$$

Equation (C1) may be rewritten

$$\nu W_2(Q^2,\nu)=Q^2M\nu\int_0^\infty da^2\int_{-\infty}^{\infty}d\alpha\, C_2(a^2,\alpha)\times(2\pi)^3\Delta(q+\alpha P,a^2)\,, \qquad \text{(C3)}$$

where Δ is the causal propagator defined in Sec. II.

Take the Bjorken limit of νW_2 by considering $\nu W_2(\beta Q^2,\beta\nu)$ and letting β approach infinity:

$$\lim_{\beta\to\infty}\nu W_2(\beta Q^2,\beta\nu)\propto\lim_{\beta\to\infty}\beta^2Q^2M\nu\int_0^\infty da^2\int_{-\infty}^{\infty}d\alpha\, C_2(a^2,\alpha)\Delta(2aM\nu\beta-Q^2\beta+\alpha^2M^2,a^2)\,.$$

Observe that $\Delta(\beta R^2,a^2)=\beta^{-1}\Delta(R^2,\beta a^2)$ and define $\beta a^2\equiv u^2$:

$$\lim_{\text{Bj}}\nu W_2(Q^2,\nu)\propto\lim_{\beta\to\infty}Q^2M\nu\int_0^\infty du^2\int_{-\infty}^{\infty}d\alpha\, C_2\!\left(\frac{u^2}{\beta},\alpha\right)\Delta\!\left(2M\nu\alpha-Q^2+\frac{\alpha^2M^2}{\beta},u^2\right). \qquad \text{(C4)}$$

We may take the limit under the integral provided the limit of the integrand and the integral of the limit exist. The continuity of $C_2(y^2,y\cdot P)$ guarantees the existence of the integrand. It remains to show that the following integral exists:

$$I\equiv\int_0^\infty du^2\int_{-\infty}^{\infty}d\alpha\, C_2(0,\alpha)\Delta(2M\nu\alpha-Q^2,u^2)\,.$$

The techniques of Frishman[9] may be used to perform this integral obtaining

$$I\propto\frac{1}{(2M\nu)^2}\frac{d}{d\alpha}C_2(0,\alpha)\,. \qquad \text{(C5)}$$

Since we have required $C_2(y^2,y\cdot P)$ to be continuous across the light cone, $C_2(0,\alpha)=0$ and $\lim_{\text{Bj}}\nu W_2(Q^2,\nu)=0$, completing the proof. Ways in which additional singularities in $C_2(y^2,y\cdot P)$ can invalidate this result are discussed elsewhere.[33]

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[1]J. I. Friedman *et al.*, SLAC Report No. SLAC-PUB-907 (unpublished); G. Miller *et al.*, SLAC Report No. SLAC-PUB-815 (unpublished); G. Miller, SLAC Report No. SLAC-REPORT- 129 (unpublished).

[2]For a brief review of some of this work, see C. H. Llewellyn Smith, CERN Report No. CERN-TH-1188, 1970 (unpublished).

[3]R. P. Feynman (unpublished); Phys. Rev. Letters 23, 1415 (1969); in *High-Energy Collisions*, Third International Conference held at State University of New York, Stony Brook, 1969, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969).

[4]J. D. Bjorken, in *Particle Physics*, edited by J. Steinberger (Academic, New York, 1968).

[5]J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969); Phys. Rev. D 1, 1450 (1970).

[6]J. Kuti and V. Weisskopf, Phys. Rev. D 4, 3418 (1971).

[7]S. D. Drell, D. J. Levy, and T.-M. Yan, Phys. Rev.

Letters 22, 744 (1969); Phys. Rev. 187, 2159 (1969); Phys. Rev. D 1, 1035 (1970); 1, 1617 (1970); S. D. Drell and T.-M. Yan, *ibid.* 1, 2402 (1970). See especially the third reference.

[8]P. V. Landshoff, J. C. Polkinghorne, and R. D. Short, Nucl. Phys. B28, 222 (1971); P. V. Landshoff and J. C. Polkinghorne, *ibid.* B28, 240 (1971); Phys. Letters 34B, 621 (1971); J. C. Polkinghorne, Nuovo Cimento 8A, 592 (1972).

[9]R. A. Brandt, Phys. Rev. Letters 23, 1260 (1969); R. A. Brandt and G. Preparata, Nucl. Phys. B27, 541 (1971); L. S. Brown, in *Lectures in Theoretical Physics*, edited by W. E. Brittin, B. W. Downs, and J. Downs (Interscience, New York, 1969); Y. Frishman, Phys. Rev. Letters 25, 966 (1970); Ann. Phys. (N.Y.) 66, 373 (1971); B. L. Ioffe, Phys. Letters 30B, 123 (1969).

[10]S. D. Drell and T.-M. Yan, Phys. Rev. Letters 25, 316 (1970); Ann. Phys. (N.Y.) 66, 578 (1971).

[11]P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. 33B, 221 (1971); and Erratum to University of Cambridge Report No. DAMTP 71/22 (unpublished).

[12]G. Altarelli, R. Brandt, and G. Preparata, Phys. Rev. Letters 26, 42 (1971); G. Preparata, Brookhaven National Laboratory Report No. BNL 16173 (unpublished); R. Brandt and G. Preparata, Brookhaven National Laboratory Report No. BNL 16183 (unpublished).

[13]C. H. Llewellyn Smith, Lectures at the International Summer Institute in High Energy Physics, Hamburg, 1971 (unpublished).

[14]S. M. Berman, J. D. Bjorken, and J. B. Kogut, Phys. Rev. D 4, 3388 (1971).

[15]See the discussion of the first section of the second paper of Ref. 8.

[16]H. Fritzsch and M. Gell-Mann, in *Broken Scale Invariance and the Light Cone*, Proceedings of the 1971 Coral Gables Conference on Fundamental Interactions at High Energy, Vol. 2, edited by M. Dal Cin, G. J. Iverson, and A. Perlmutter (Gordon and Breach, New York, 1971).

[17]R. L. Jaffe, Phys. Letters 37B, 517 (1971).

[18]We use the metric $a\cdot b \equiv a^0 b^0 - \vec{a}\cdot\vec{b}$ and other conventions of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964) throughout.

[19]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[20]S. D. Drell, in *Subnuclear Phenomena*, edited by A. Zichichi (Academic, New York, 1970).

[21]Generalization to partons of higher spin is straightforward although Eq. (2.6) cannot be derived by the perturbation-theory techniques of DLY in this case.

[22]R. Jackiw, R. Van Royen, and G. West, Phys. Rev. D 2, 2473 (1970).

[23]See, for example, N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Interscience, New York, 1959).

[24]C. H. Llewellyn Smith, Phys. Reports (to be published).

[25]V. N. Gribov, B. L. Ioffe, and I. Ya. Pomeranchuk, Yad. Fiz. 2, 768 (1965) [Sov. J. Nucl. Phys. 2, 549 1966)]. J. Pestieau, P. Roy, and H. Terazawa, Phys. Rev. Letters 25, 402 (1970).

[26]R. L. Jaffe, Phys. Rev. D 4, 1507 (1971).

[27]K. Subbarao, Princeton University report (unpublished). Subbarao assumes a point coupling to the massive photon thereby reducing the multiperipheral model to a type of parton model.

[28]The apparently logarithmically divergent piece of the η integrals of Eq. (3.5) does not contribute to $W(Q^2, s)$ in the limit $s, Q^2 \to \infty$, τ fixed. The proof is similar to that discussed in Appendix B for electroproduction.

[29]J. Ellis, Phys. Letters 35B, 537 (1971).

[30]S. D. Drell and T.-M. Yan, Phys. Rev. Letters 24, 855 (1970).

[31]D. Dicus, R. Jackiw, and V. Teplitz, Phys. Rev. D 4, 1733 (1971).

[32]J. H. Christenson, G. S. Hicks, L. M. Lederman, P. J. Limon, B. G. Pope, and E. Zavattin, Phys. Rev. Letters 25, 316 (1970).

[33]R. L. Jaffe, SLAC Report No. SLAC-PUB-1023 (unpublished).