

Light-Cone Analysis of Spin-Dependent Deep-Inelastic Electron Scattering*

A. J. G. Hey† and J. E. Mandula

California Institute of Technology, Pasadena, California 91109

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The light-cone analysis of deep-inelastic electron-proton scattering is extended to non-spin-averaged scattering. Scaling is deduced for the spin-dependent structure functions $M^2\nu G_1$ and $M\nu^2 G_2$. The connection between moments of the scaling functions and matrix elements of operators in the operator-product expansion is described in detail and leads to two sum rules for the scaling functions when the quark light-cone algebra of Fritzsche and Gell-Mann is assumed. Predictions of other models of scaling are briefly compared.

I. INTRODUCTION

The inelastic electron scattering experiments at SLAC¹ appear to give results which are consistent with the scaling behavior predicted by Bjorken.² Several theories have subsequently been advanced to account for the SLAC results besides the "parton model" of Feynman³ and of Bjorken and Paschos.⁴ Prominent among these are the Pom-eranchuk-exchange theory of Abarbanel, Gold-berger, and Treiman⁵ and Harari,⁶ the cutoff field theory of Drell, Levy, and Yan,⁷ and the light-cone analysis of Brandt and Preparata,⁸ Frishman,⁹ and of Fritzsche and Gell-Mann.¹⁰ The purpose of this paper is to apply the light-cone analysis to deep-inelastic scattering of polarized electrons from polarized nucleons, and to compare the results of that analysis with the predictions of other models. As might be expected, our results are contained in the predictions of parton models and cutoff field theory.

The total inelastic scattering of polarized electrons from polarized nucleons is described by four structure functions. In addition to the structure functions W_1 and W_2 of unpolarized scattering, there are two structure functions,¹¹ which we denote by G_1 and G_2 , which contribute to the anti-symmetric piece of the absorptive part of forward virtual Compton scattering:

$$W_{\mu\nu}^{[A]} = \frac{1}{M} \epsilon_{\mu\nu\lambda\sigma} q^\lambda [M^2 s^\sigma G_1(p \cdot q, q^2) + (p \cdot q s^\sigma - s \cdot q p^\sigma) G_2(p \cdot q, q^2)], \quad (1)$$

where p , M , s , and q are the target 4-momentum, mass, covariant polarization, and the virtual-photon 4-momentum, respectively. The functions G_1 and G_2 contribute only to the asymmetries of polarized electron scattering from polarized targets, and can be measured only in experiments in which both the lepton and target nucleon are polarized.

Both structure functions are functions only of the invariants q^2 and $p \cdot q/M = \nu$. With the usual assumptions about the light-cone singularities of current commutators, they exhibit the following scaling behavior in the deep-inelastic region ν , $-q^2 \rightarrow \infty$ with $\xi = -q^2/2M\nu$ fixed:

$$M^2 \nu G_1(\nu, q^2) \rightarrow g_1(\xi), \quad (2)$$

$$M\nu^2 G_2(\nu, q^2) \rightarrow g_2(\xi). \quad (3)$$

If the currents obey the light-cone algebra abstracted from the quark model there result the following sum rules for $g_1(\xi)$ and $g_2(\xi)$:

$$\int_0^1 d\xi [g_1^p(\xi) - g_1^n(\xi)] = \frac{1}{6} \frac{G_A}{G_V}. \quad (4)$$

The superscripts denote a neutron or proton target. This is essentially a restatement of a current-algebra sum rule due to Bjorken¹²:

$$\int_0^1 d\xi g_2(\xi) = 0. \quad (5)$$

This sum rule for g_2 is a scaling version of the superconvergent sum rule of Burkhardt and Cottingham.¹³ These sum rules have also recently been derived by methods similar to ours by Dicus, Jackiw, and Teplitz.¹⁴

If the currents are constructed as in the algebra of fields there is a different sum rule for g_1 :

$$\int_0^1 d\xi g_1(\xi) = 0. \quad (6)$$

These sum rules are the analogs in polarized scattering of the Callan-Gross sum rules,¹⁵ but here there is a technical difference. The Callan-Gross sum rule sets the integral of a positive definite quantity to zero, and so permits the conclusion that the integrand vanishes. The functions g_1 and g_2 need not be positive, however, and so no such conclusion obtains.

The spin dependence of deep-inelastic scattering

has been considered within other theories which account for the spin-averaged scattering. The spin dependence is quite different in these models, and so can serve to distinguish between them.

The organization of the paper is as follows. In Sec. II we review the kinematics of polarized electron-nucleon scattering and define the structure functions G_1 and G_2 . In Sec. III we apply the light-cone analysis of current commutators and derive the scaling predictions for νG_1 and $\nu^2 G_2$, along with the sum rules for the scaling functions. In Sec. IV we compare the light-cone results with the expectations of other models.

II. KINEMATICS

The purpose of this section is to define the structure functions for forward off-shell Compton scattering and to relate these structure functions to asymmetries in the cross section for polarized inelastic electron-nucleon scattering. We define the structure functions through the Fourier transform of the one-nucleon matrix elements of the commutator of two electromagnetic currents:

$$\begin{aligned} W_{\mu\nu}(p, q, s) &= \frac{1}{2\pi} \int d^4x e^{iq \cdot x} \langle ps | [J_\mu(x), J_\nu(0)] | ps \rangle \\ &= W_{\mu\nu}^{[S]} + iW_{\mu\nu}^{[A]}, \end{aligned} \quad (7)$$

where

$$\begin{aligned} W_{\mu\nu}^{[S]} &= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(p \cdot q, q^2) \\ &\quad + \frac{1}{M^2} \left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right) \left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) W_2(p \cdot q, q^2) \end{aligned} \quad (8)$$

and

$$\begin{aligned} W_{\mu\nu}^{[A]} &= \frac{1}{M} \epsilon_{\mu\nu\lambda\sigma} q^\lambda [M^2 s^\sigma G_1(p \cdot q, q^2) \\ &\quad + (p \cdot q s^\sigma - s \cdot q p^\sigma) G_2(p \cdot q, q^2)]. \end{aligned}$$

Note that the contributions of W_1 and W_2 to $W_{\mu\nu}$ are independent of the nucleon polarization, while those of G_1 and G_2 change sign under reversal of the nucleon polarization. Thus W_1 and W_2 describe

the imaginary part of the spin-averaged virtual Compton amplitude while G_1 and G_2 describe the polarization-dependent asymmetries.

The cross section for the scattering of a polarized electron from a polarized nucleon, summed over final electron polarization and over all final hadronic states, to lowest order in electromagnetism, is proportional to $l_{\mu\nu} W_{\mu\nu}$, where the leptonic tensor is

$$\begin{aligned} l_{\mu\nu}^{(\circ)} &= \sum_{\sigma'} \bar{u}_{k\sigma} \gamma_\mu u_{k'\sigma'} \bar{u}_{k'\sigma'} \gamma_\nu u_{k\sigma} \\ &= l_{\mu\nu}^{(\circ)[S]} + i l_{\mu\nu}^{(\circ)[A]}, \end{aligned} \quad (9)$$

$$l_{\mu\nu}^{(\circ)[S]} = 2(k_\mu k'_\nu + k_\nu k'_\mu + \frac{1}{2} q^2 g_{\mu\nu}),$$

and

$$l_{\mu\nu}^{(\circ)[A]} = 2m_e \epsilon_{\mu\nu\lambda\rho} \sigma^\lambda q^\rho,$$

and k, σ and k', σ' denote the initial and final electron momenta and polarizations. Note that $l_{\mu\nu}^{(\circ)[S]}$ is independent of the electron polarization, and contributes to the initial-lepton-spin-averaged cross section, while $l_{\mu\nu}^{(\circ)[A]}$ depends on the polarization and makes no contribution to the spin-averaged scattering. Because all nucleon polarization asymmetries are contained in the antisymmetric part of $W_{\mu\nu}$, to experimentally observe these it is necessary to use polarized leptons. At high lepton energies E , the antisymmetric part of the lepton tensor for transversely polarized leptons is suppressed by a factor m_e/E with respect to that for longitudinally polarized leptons. We will consider only leptons polarized along their direction of motion, for which

$$l_{\mu\nu}^{[A]} = -2\epsilon_{\mu\nu\lambda\rho} q^\lambda k'^\rho, \quad (11)$$

with the neglect of the lepton mass.

The difference between the cross sections for nucleons polarized at an angle ϕ and at an angle $\phi + \pi$ to the incident longitudinally polarized lepton beam, in the lepton scattering plane, summed over final lepton polarizations and final hadron states, is

$$\begin{aligned} \frac{d\sigma^{\uparrow(\phi)}}{dq^2 dE'} - \frac{d\sigma^{\uparrow(\phi+\pi)}}{dq^2 dE'} &= -\frac{2\pi\alpha^2}{E^2 q^4} l_{\mu\nu}^{[A]} W_{\mu\nu}^{[A]} \\ &= \frac{4\pi\alpha^2}{E^2 q^2} \{ [E \cos\phi + E' \cos(\theta - \phi)] M G_1(p \cdot q, q^2) - 2EE' [\cos\phi - \cos(\theta - \phi)] G_2(p \cdot q, q^2) \}, \end{aligned} \quad (12)$$

where $E' = k'_0$ is the final lepton energy and θ is the lepton scattering angle. For the two cases of longitudinal nucleon polarization asymmetries ($\phi = 0$) and transverse nucleon polarization asymmetries ($\phi = \frac{1}{2}\pi$), this reduces to

$$\frac{d\sigma^{\uparrow\uparrow}}{dq^2 dE'} - \frac{d\sigma^{\uparrow\downarrow}}{dq^2 dE'} = \frac{4\pi\alpha^2}{E^2 q^2} [(E + E' \cos\theta) M G_1 + q^2 G_2]$$

$$= \frac{4\pi\alpha^2}{E^2q^2} [(E+E')MG_1 - E'(1-\cos\theta)(MG_1+2EG_2)] \quad (13)$$

and

$$\frac{d\sigma^{1\rightarrow}}{dq^2dE'} - \frac{d\sigma^{1\leftarrow}}{dq^2dE'} = \frac{4\pi\alpha^2E'}{E^2q^2} (\sin\theta)(MG_1+2EG_2). \quad (14)$$

So in order to extract the two structure functions one may either measure the longitudinal asymmetry at small and large lepton scattering angles, or measure the longitudinal asymmetry at small angles and the transverse asymmetry at moderate angles.

III. LIGHT-CONE ANALYSIS

In this section we derive the expected scaling behavior of the structure functions G_1 and G_2 from the small- x^2 operator light-cone expansion and then specialize the result to the case of the quark-model light-cone algebra. We make the assumption that the diagonal matrix elements of the commutator of two electromagnetic currents $[J_\mu(x), J_\nu(0)]$ can be expanded near the light cone as

$$\begin{aligned} [J_\mu(x), J_\nu(0)] \cong & -(g_{\mu\nu}\square^2 - \partial_\mu\partial_\nu)O_1(x, 0) - (g_{\mu\lambda}g_{\nu\sigma}\square^2 - g_{\mu\lambda}\partial_\nu\partial_\sigma - g_{\nu\sigma}\partial_\mu\partial_\lambda + g_{\mu\nu}\partial_\lambda\partial_\sigma)O_2^{\lambda\sigma}(x, 0) \\ & - i\epsilon_{\mu\nu\lambda\sigma}\partial^\lambda Q_1^\sigma(x, 0) - i(\partial_\nu\epsilon_{\mu\rho\lambda\sigma}\partial^\rho - \partial_\mu\epsilon_{\nu\rho\lambda\sigma}\partial^\rho - \square^2\epsilon_{\mu\nu\lambda\sigma})Q_2^{\lambda\sigma}(x, 0), \end{aligned} \quad (15)$$

where $O_2^{\lambda\sigma}$ is symmetric in its Lorentz indices while $Q_2^{\lambda\sigma}$ is antisymmetric.

The first two terms in the light-cone expansion contribute to $W_{\mu\nu}^{[S]}$, which is measured in the unpolarized inelastic electron-proton scattering experiments, while the second two terms, with which we are concerned here, contribute to $W_{\mu\nu}^{[A]}$ and can be measured only in polarized scattering experiments. Note that the bilocal operator Q_1 contributes to both structure functions G_1 and G_2 , while Q_2 contributes only to G_2 .

The essential ingredient of the light-cone approach is that the bilocal operators may be written as the product of c -number functions, singular at $x^2=0$, and nonsingular bilocal operators:

$$\begin{aligned} Q_1^\sigma(x, 0) &= E(x; c_1)\mathcal{Q}_1^\sigma(x, 0), \\ Q_2^{\lambda\sigma}(x, 0) &= E(x; c_2)\mathcal{Q}_2^{\lambda\sigma}(x, 0), \end{aligned} \quad (16)$$

where the nonsingular operators \mathcal{Q}_i have Taylor-series expansions with local operators as coefficients:

$$\begin{aligned} \mathcal{Q}_1^\sigma(x, 0) &= \sum_n x_{\mu_1} \cdots x_{\mu_n} R_1^{\sigma\mu_1 \cdots \mu_n}(0), \\ \mathcal{Q}_2^{\lambda\sigma}(x, 0) &= \sum_n x_{\mu_1} \cdots x_{\mu_n} R_2^{\lambda\sigma\mu_1 \cdots \mu_n}(0) \end{aligned} \quad (17)$$

and the singular functions are¹⁶

$$\begin{aligned} E(x; c) &= \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot x} \left[\left(\frac{1}{-k^2 - i\epsilon k_0} \right)^{2-c/2} - \left(\frac{1}{-k^2 + i\epsilon k_0} \right)^{2-c/2} \right] \\ &= \frac{-i}{2^{4-c}\pi^2} \frac{\Gamma(\frac{1}{2}c)}{\Gamma(2-\frac{1}{2}c)} [(-x^2 - i\epsilon x_0)^{-c/2} - (-x^2 + i\epsilon x_0)^{-c/2}]. \end{aligned} \quad (18)$$

The matrix elements of the local fields R between identical non-spin-averaged proton states are expressible in terms of numerical parameters a, b, d :

$$\begin{aligned} \langle p_S | R_1^{\sigma\mu_1 \cdots \mu_n}(0) | p_S \rangle \\ = a_n s^\sigma p^{\mu_1} \cdots p^{\mu_n} + b_n p^\sigma \{s^{\mu_1} p^{\mu_2} \cdots p^{\mu_n}\}, \end{aligned} \quad (19)$$

$$\langle p_S | R_2^{\lambda\sigma\mu_1 \cdots \mu_n}(0) | p_S \rangle = d_n \frac{1}{2} (p^\lambda s^\sigma - p^\sigma s^\lambda) p^{\mu_1} \cdots p^{\mu_n}. \quad (20)$$

The symbol $\{ \}$ in Eq. (19) denotes complete sym-

metrization of Lorentz indices. Note that b_0 cannot be defined.

Translational invariance applied to $\langle p_S | [J_\mu(x), J_\nu(0)] | p_S \rangle$ gives the symmetries of the bilocal operators

$$\begin{aligned} \langle p_S | \mathcal{Q}_1^\sigma(x, 0) | p_S \rangle &= \langle p_S | \mathcal{Q}_1^\sigma(-x, 0) | p_S \rangle, \\ \langle p_S | \mathcal{Q}_2^{\lambda\sigma}(x, 0) | p_S \rangle &= -\langle p_S | \mathcal{Q}_2^{\lambda\sigma}(-x, 0) | p_S \rangle, \end{aligned} \quad (21)$$

which is equivalent to the Taylor expansion of \mathcal{Q}_1^σ having only even powers of x and that of $\mathcal{Q}_2^{\lambda\sigma}$ having only odd powers.

These assumptions then lead to scaling of W_1 and

W_2 with the power of scaling directly related to the strength of the c -number singularity. This power may be obtained from the assumed conservation of scale dimension of the light-cone expansion if one knows the dimensions of the bilocal operators. The observed scaling of νW_2 and W_1 is consistent with the spin- J operators having scale dimensions $J+2$. We will henceforth keep this connection between the spins and scale dimensions of the operators in the light-cone expansion. With the assumption that dimension is conserved on the light cone, it gives the singularities of the c -number functions associated with Q_1 and Q_2 as

$$c_1=2, \quad c_2=0. \quad (22)$$

The implied scaling of the structure functions G_1 and G_2 is

$$\lim_{\nu, -q^2 \rightarrow \infty; q^2/\nu \text{ fixed}} M^2 \nu G_1(\nu, q^2) \rightarrow g_1(\xi), \quad (23a)$$

$$\lim_{\nu, -q^2 \rightarrow \infty; q^2/\nu \text{ fixed}} M \nu^2 G_2(\nu, q^2) \rightarrow g_2(\xi), \quad (23b)$$

where

$$\xi = -q^2/2p \cdot q = -q^2/2M\nu. \quad (24)$$

Moments of the scaling functions $g_1(\xi)$ are related to the parameters a_n , b_n , and d_n which characterize the matrix elements of the local operators in the Taylor expansion of the bilocal operators. Explicitly, the relation is (for n even)

$$\int_{-1}^1 \xi^n g_1(\xi) d\xi = -\frac{1}{2} (i)^n n! [a_n + b_n], \quad (25)$$

$$\int_{-1}^1 \xi^n g_2(\xi) d\xi = \frac{1}{2} (i)^n n! [b_n + \frac{1}{2} d_{n-1}].$$

The quark-field-light-cone algebra gives the light-cone expansion

$$[J_{i\mu}(x), J_{j\nu}(0)]^{[A]} \doteq \frac{1}{4\pi} \partial_\rho [\epsilon(x_0) \delta(x^2)] \{ f_{ijk} \epsilon_{\mu\nu\rho\sigma} [\mathfrak{F}_{k\sigma}^{(5)}(x, 0) - \mathfrak{F}_{k\sigma}^{(5)}(0, x)] - i d_{ijk} \epsilon_{\mu\nu\rho\sigma} [\mathfrak{F}_{k\sigma}^{(5)}(0, x) + \mathfrak{F}_{k\sigma}^{(5)}(x, 0)] \}. \quad (26)$$

When $J_{i\mu}$ and $J_{j\nu}$ are both electromagnetic currents only the term involving d -type coupling contributes, and since

$$\epsilon(x_0) \delta(x^2) = 2\pi E(x; 2) \quad (27)$$

we identify

$$\mathfrak{Q}_1^a(x, 0) = [d_{33k} + (2/\sqrt{3})d_{38k} + \frac{1}{3}d_{88k}] \times \frac{1}{2} [\mathfrak{F}_k^{(5)\sigma}(x, 0) + \mathfrak{F}_k^{(5)\sigma}(0, x)], \quad (28)$$

and the second term $Q_2^{\lambda\sigma}(x, 0)$ of the more general expansion (15) is absent. So in the quark model all the coefficients d_n in Eq. (20) are zero. The coefficient a_0 is proportional to matrix elements of $J_{3\sigma}^{(5)}(0)$, the $SU(3) \times SU(3)$ axial-vector currents. The difference of a_0 for protons and neutrons is proportional to matrix elements of $J_{3\sigma}^{(5)}(0)$, which is an isotopic partner of the weak-interaction $\Delta S = 0$ axial-vector current, and can be expressed in terms of (G_A/G_V) . Using Eq. (25) to relate a_0 to the scaling function $g_1(\xi)$, and recalling that b_0 is absent, one obtains the sum rule

$$\int_0^1 [g_1^p(\xi) - g_1^n(\xi)] d\xi = \frac{1}{6} (G_A/G_V), \quad (29)$$

which was originally derived by Bjorken from the $U(6) \times U(6)$ algebra.¹² Also, since both b_0 and d_{-1} do not exist, one obtains

$$\int_0^1 g_2(\xi) d\xi = 0, \quad (30)$$

whether or not the $Q_2^{\lambda\sigma}(x, 0)$ is present.

The first sum rule is peculiar to the quark-model algebra, but the second sum rule is true in many models including field algebra and ϕ^4 -theory algebra. In field algebra the right-hand side of the sum rule (29) for $g_1(\xi)$ is replaced by zero.

The second sum rule for $g_2(\xi)$ is analogous to the Callan-Gross sum rule,¹⁵

$$\int_0^1 \sigma_L(\xi) d\xi = 0,$$

but because $g_2(\xi)$ is not a positive-definite quantity (it is related to differences of cross sections) one cannot infer from it the vanishing of $g_2(\xi)$.¹⁷

IV. CONCLUSIONS

From the quark-light-cone algebra, predictions were obtained for the scaling behaviors of G_1 and G_2 , and Bjorken's remarkable sum rule for G_1 was rederived. The scaling behavior of the spin-dependent structure functions may be used to distinguish between the many models which attempt to account for the spin-averaged scaling. It is not surprising that our results are contained among those of various "parton" calculations. The scaling of $M^2 \nu G_1$ and $M \nu^2 G_2$ has been obtained in the Drell-Levy-Yan model,^{7,18} in the nonperturbative parton model,¹⁹ and by a more straightforward application of parton ideas.^{20,21} In these latter parton calculations, by ignoring the possibility of angular

momenta between the partons, one obtains the result

$$M\nu^2G_2 = 0.$$

With additional assumptions about the parton distribution, a sum rule relating an integral over the one remaining spin-dependent scaling function to an integral over the spin-independent scaling function was obtained.²⁰ Such results are seen, in the light of the light-cone approach, to involve specific assumptions about matrix elements of bilocal operators, and are not obtainable from the more general idea of a light-cone algebra abstracted from a pure quark model.

In the direct-channel-resonance model of Domokos *et al.*,²² besides the scaling of νG_1 and $\nu^2 G_2$, the result

$$M\nu G_1 \cong \nu^2 G_2$$

is obtained. Such a relation is not implied by the quark-light-cone algebra.

Note also that the spin-dependent scattering provides a very direct test of the suggestion that the Pomernanchuk exchange is responsible for the scaling of deep-inelastic electron scattering^{5,6} which, besides predicting the equality of neutron and proton spin-averaged scattering cross sections, also requires zero asymmetry. Subsidiary Regge ex-

changes are compatible with scaling and nonzero asymmetries, and have been considered by Galfi *et al.*^{18,20}

Similar quark-light-cone-algebra predictions exist for neutrino scattering from polarized targets,²³ but such measurements seem far beyond present experimental capabilities.

In conclusion, we have seen that the quark-light-cone algebra makes very specific predictions for the spin-dependent scaling functions. Note, however, that these experiments do not test the bilocal density algebra but only (if the algebra is true) measure the non-spin-averaged matrix elements of the bilocal density.

After the completion of this work, the authors learned that the question of what may be gleaned from the measurement of polarization asymmetries in deep-inelastic electron scattering has also been considered by Carlson and Tung.²⁴ These authors have especially emphasized that measuring both longitudinal and transverse polarization asymmetries is the best way of separating the independent structure functions G_1 and G_2 .

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†Harkness Fellow.

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