[8] See, for example, G. Mack, Nucl. Phys. B35, 592 (1971); Y. Frishman, Phys. Rev. Letters 25, 966 (1970), and Ann. Phys. (N.Y.) 66, 373 (1971); Mandula *et al.*, Ref. 6; B. W. Lee and J. E. Mandula, Phys. Rev. D 4, 3475 (1971).

[9] We are using units in which the mass of the nucleon is 1.

[10] I. M. Gel'fand and G. E. Shilov, *Generalized Functions* (Academic, New York, 1964), Vol. 1.

[11] See, for example, R. Jackiw, R. Van Royen, and G. B. West, Phys. Rev. D 2, 2473 (1970).

[12] Equation (3.4) may be proven from the observed scaling in electroproduction and the MCA.

[13] γ was called α in Ref. 2.

[14] M. Gell-Mann, in *Proceedings of the Third Hawaii Topical Conference on Particle Physics,* edited by S. F. Tuan (Western Periodicals, North Hollywood, Calif., 1969).

[15] K. G. Wilson, Phys. Rev. D 3, 1818 (1971).

[16] This can be regarded as an application of the "totalitarian principle": "Everything not forbidden is compulsory." See T. H. White, *The Once and Future King* (Putnam's Sons, New York, 1958), p. 121; O.-M. Bilaniuk and E. C. G. Sudarshan, Phys. Today 22, No. 5, 43 (1969); G. L. Trigg, *ibid.* 23, No. 10, 79 (1970).

[17] See P. Nath, R. Arnowitt, and M. H. Friedman, Northeastern University Report No. NUB 2091, 1971 (unpublished), and references contained therein.

[18] E. D. Bloom *et al.*, MIT-SLAC Report No. SLAC-PUB-796, 1970 (unpublished), presented at the Fifteenth International Conference on High-Energy Physics, Kiev, U.S.S.R., 1970.

[19] Mack, Ref. 8.

[20] Aside from the invalidity of (8.2) when $\Delta_1 < 2$.

[21] C. G. Callan and D. J. Gross, Phys. Rev. Letters 22, 156 (1969).

PHYSICAL REVIEW D VOLUME 5, NUMBER 10 15 MAY 1972

Partons, Electromagnetic Mass Shifts, and the Approach to Scaling

William I. Weisberger*
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11790
(Received 24 January 1972)

In parton models the most singular part of the electromagnetic level shift of a physical state is predicted to be a weighted sum of the level shifts of the partons. This leads to a tadpole model for masses squared. Requiring consistency of this result with Cottingham's formula leads to a prediction of the cross section for absorption of virtual scalar as well as transverse photons in the Bjorken scaling limit. If these cross sections are in a particular ratio to each other, mass shifts or differences will be finite in conventional electrodynamics.

I. INTRODUCTION

The apparent scaling behavior of the structure functions measured in deep-inelastic electron-nucleon scattering[1] have been accounted for by two basic theoretical approaches. The more abstract and formal approach assumes a simple form for the commutator of the hadronic electromagnetic current on the light cone.[2] Kinematic analysis shows this to be the space-time region ultimately proved in deep-inelastic scattering.[3] The MIT-SLAC results have also been realized more concretely in parton models in which the virtual photons scatter incoherently from the nucleonic constituents (partons) which can be considered instantaneously as free, bare particles.[4] These two approaches are really complementary, for the parton models can be regarded as a particular realization of a light-cone algebra isomorphic to that generated by the electromagnetic currents of the free parton fields.

In this paper, we look at parton pictures for the electromagnetic mass shifts of hadrons. This is, in a sense, a contraction of the problem studied in the inelastic scattering experiments since the most singular parts of the electromagnetic mass shifts are governed by the behavior of products of currents near the tip of the light cone. The parton models give a realization of the original operator expansion suggested by Wilson,[5] and are consistent with the general formula for leading singularities proposed by Bjorken.[6] In another sense, however, the mass differences study more than the inelastic structure functions since the latter are proportional to the absorptive part of the forward Compton amplitudes for virtual photons while the mass differences depend upon the real parts as well. There have been previous attempts[7] to relate the scaling behavior of structure functions to divergences in calculations of electromagnetic mass differences from Cottingham's formula,[8,9] but our model being more specific leads to more definite and complete results.

In the parton model we picture hadrons as a collection of bare particles which are instantaneously free. This suggests that the contribution to the level shift from photons of large virtual momenta should be a sum over the parton distribution functions of the level shifts of the partons. This turns out to be the case, where it is the *energy shifts* in the infinite-momentum frame which are directly related by the summation. This leads to a corresponding formula for the shifts of the squared masses of the physical particles as a sum of shifts of the squared masses of the partons.

If the charged partons have spin 0 or spin $\frac{1}{2}$, the calculated shift diverges quadratically or logarithmically, respectively. The divergent term takes the form of an infinite c number multiplying the expectation value of a regular operator (more precisely, it is a sum of such contributions). We can replace the infinite c number by a numerical parameter to be determined from experiment either in the spirit of renormalization theory or from a belief that some as yet undiscovered mechanism will eventually cut off the q^2 integrations and give us a finite calculation of the second-order electromagnetic mass differences. The resulting terms have the form of tadpole contributions to the shifts or differences of the squared masses.[10]

If the partons are isosinglets and isodoublets only, as in quark or other triplet models, these singular terms contribute only to the $I=0$ and $I=1$ pieces of the mass shifts. Mass differences transforming as $I=2$ should, therefore, be finite when calculated from Cottingham's formula.

The coefficients of the divergent terms can be compared with those obtained by using the scaling hypothesis for the absorptive parts of the Compton amplitude in the dispersion formulation of Cottingham. For spin-$\frac{1}{2}$ partons, consistency requires a certain sum of the structure functions to approach the scaling limit as $1/q^2$.[11] The specific form of the coefficient is determined from the parton distribution function. This result predicts the form of σ_s, the total cross section for the absorption of virtual scalar photons in the scaling region.

If the charged partons have spin $\frac{1}{2}$ and zero rest mass, the mass shifts are finite. This implies a simple proportionality between σ_s and σ_t in the scaling regions (σ_t is the total cross section for transverse photons). Assuming this relation to hold everywhere, we can estimate the continuum contribution to the neutron-proton mass difference from recent MIT-SLAC results.[1] This continuum term has the correct sign but is two orders of magnitude too small to account for the observed mass difference.[12]

The necessity for subtractions in the dispersion relations of the Cottingham formula[13] corresponds to the presence of contributions to mass shifts or differences from "wee" partons whose momentum distribution cannot be expressed as a function of the scaling variable. The apparent degree of divergence of the final q^2 integrations is unaffected by the subtractions.

These ideas may be extendable to the strong mass differences among SU(3) multiplets and to the weak parity-conserving spurions which, according to the current-algebra predictions of Suzuki and Sugawara, are supposed to determine nonleptonic s-wave decays of hyperons.[14]

In Sec. II we review the parton model and derive our basic result, Eq. (13). More explicit calculations in Sec. III using the Bjorken limit and a perturbation calculation in field theory confirm the deduction. In Sec. IV, we calculate the mass shifts from Cottingham's formula and investigate the implications of consistency between this formalism and our earlier formulation.

II. DERIVATION OF THE MASS-SHIFT FORMULA IN PARTON MODELS

The derivation of our results is carried out conveniently in the formalism of Drell, Levy, and Yan,[15] which we summarize very briefly. To establish normalization conventions we note that our one-particle plane-wave states are normalized as

$$\langle P' | P \rangle = (2\pi)^3 \delta^{(3)}(\vec{P}' - \vec{P}) . \tag{1}$$

Introducing the U matrix,

$$T\left(\exp\left(-i\int_{-\infty}^{0} H_s(t)dt\right)\right),$$

where H_s is the Hamiltonian for strong interactions, we can expand the physical state as a superposition of plane-wave states of the free particles ("partons") of the underlying field theory:

$$\langle P' | P \rangle = \sum_n \langle UP' | p_1, p_2, \ldots, p_n \rangle \times \prod_{i=1}^{n} \left(\frac{d^3p_i}{(2\pi)^3}\right) \langle p_1, p_2, \ldots, p_n | UP \rangle . \tag{2}$$

Since the U matrix conserves three-momentum, we define

$$\langle p_1, p_2, \ldots, p_n | UP \rangle = \psi^n(p_1, p_2, \ldots, p_n)(2\pi)^3 \delta^{(3)}\Big(\vec{P} - \sum_i \vec{p}_i\Big). \tag{3}$$

The wave function ψ^n is normalized to

$$1 = \sum_n \int \prod_i \left(\frac{d^3p_i}{(2\pi)^3}\right) |\psi^n(p_1, \ldots, p_n)|^2 (2\pi)^3 \times \delta^{(3)}\Big(\vec{P} - \sum_j \vec{p}_j\Big). \tag{4}$$

A one-particle distribution function is defined by

$$f^n(p)=\int\prod_{i=2}^{n}\left(\frac{d^3p_i}{(2\pi)^3}\right)|\psi^n(p,p_2,\ldots,p_n)|^2 \times\delta^{(3)}\Big(\vec{P}-\vec{p}-\sum_{j=2}\vec{p}_j\Big) \tag{5}$$

and normalized to

$$1=\sum_n\int d^3p f^n(p). \tag{6}$$

Extending our notation to include different types of partons, we label the states by the set of occupation numbers of the identical particles $\{N_j\}$, and define in analogy to (5) a one-particle distribution function for a particle of type i, $f_i^{\{N_j\}}(\vec{p})$.

The normalization is

$$1=\sum_{\{N_j\}}\sum_i\frac{N_i}{N}\int f_i^{\{N_j\}}(\vec{p})d^3p. \tag{7a}$$

In an infinite-momentum frame, $P=P_z\to\infty$, the longitudinal momentum of a parton is a fraction, $x=p_{iz}/P$, of the momentum of the physical particle; the transverse momenta have a finite cutoff. The energy dependence of the wave functions scales so that we can write the momentum distribution as functions of x and $\vec{p}_t$, $f_i^{\{N_j\}}(x,\vec{p}_t)$, normalized to

$$1=\sum_{\{N_j\}}\sum_i\frac{N_i}{N}\int_0^1dx\int d^2p_tf_i^{\{N_j\}}(x,\vec{p}_t). \tag{7b}$$

If $A(0)$ is any dressed local operator, we can write

$$\lim_{P\to\infty}\langle P|A(0)|P\rangle=\lim_{P\to\infty}\sum_{\substack{\{N_j\}\\\{N_k\}}}\langle UP|\{N_k\}\rangle\langle\{N_k\}|UA(0)U^{-1}|\{N_j\}\rangle\langle\{N_j\}|UP\rangle. \tag{8}$$

$a(0)=UA(0)U^{-1}$ is the undressed operator. If $a(0)$ is a one-particle operator, bilinear in the bare fields, we get

$$\lim_{P\to\infty}\langle P|A(0)|P\rangle=\sum_{\{N_j\}}\sum_iN_i\int f_i^{\{N_j\}}(x,\vec{p}_t)\Big[\lim_{P\to\infty}\langle xP,\vec{p}_t|a(0)|xP,\vec{p}_t\rangle\Big]dx\,d^2p_t. \tag{9}$$

In particular, let $A=J_0(0)$, the electric charge density. Then the total electric charge is given by

$$Q=\langle P|J_0(0)|P\rangle =\sum_{\{N_j\}}\sum_iN_iQ_i\int f_j^{\{N_j\}}(x,\vec{p}_t)dx\,d^2p_t. \tag{10}$$

In general, this is an additional normalization constraint on possible distribution functions. If, however, $\psi^{\{N_j\}}$ is symmetric in the momenta of all the constituents, so that $f_i^{\{N_j\}}$ is independent of i, we can use conservation of charge,

$$Q=\sum_iN_iQ_i\,|_{\{N_j\}\,\text{fixed}},$$

to reduce (10) to our original normalization condition.

The well-known scaling limits of the inelastic structure functions in the parton model are recorded for reference[16]:

$$W_{\mu\nu}=2E\sum_n\langle P|J_\mu(0)|n\rangle\langle n|J_\nu(0)|P\rangle(2\pi)^4\delta^{(4)}(P+q-P_n) =\left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)W_1(q^2,\nu)+\frac{1}{M^2}\left(P_\mu-\frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu-\frac{P\cdot q}{q^2}q_\nu\right)W_2(q^2,\nu). \tag{11}$$

The scaling limit ($\lim_{\text{Bj}}$) is defined by

$$P\cdot q\equiv M\nu\to\infty,\quad q^2\to-\infty,\quad M\nu/q^2\ \text{fixed},$$

$$\lim_{\text{Bj}}W_1(q^2,\nu)=F_1(x=-q^2/2M\nu) =(2\pi)\sum_{\{N_j\}}\sum_i\delta_{si}N_iQ_i^2f_i^{\{N_j\}}(x),$$

$$\lim_{\text{Bj}}2M\nu W_2(q^2,\nu)=F_2(x) =8\pi M^2\sum_{\{N_j\}}\sum_iN_iQ_i^2f_i(x)x,$$

where $f(x)$ is the one-particle distribution integrated over p_t. $\delta_{si}=0, 1$ if the parton has spin 0 or

$\frac{1}{2}$, respectively.

With these preliminaries, we can write down the leading behavior of the electromagnetic mass shift quite easily. To the first order in the fine-structure constant, the mass shift is determined from an effective operator

$$S(0)=\int d^4r\, D^{\mu\nu}(r)\, T^*(J_\mu(r)J_\nu(0)), \tag{12}$$

where $D^{\mu\nu}$ is the free photon propagator and $J_\mu(x)$ is the dressed electric current density of the hadrons. The energy level shift is given by $\delta E =\langle P|S(0)|P\rangle$, and is related to the mass shift by $\delta E=\delta M^2/2E$.

The most singular contribution to δE comes from the region near $r_\mu=0$. At this point $S(0)$ approaches a local operator. If we evaluate the energy shift in an infinite momentum frame, in the limit $P\to\infty$, the most singular term is given by

$$\delta E^{(s)}=\sum_{\{N_j\}}\sum_i N_i\int_0^1 dx f_i^{\{N_j\}}(x)\delta E_i^{(s)}. \tag{13}$$

The energy shift of the particle is the sum of the energy shifts of the partons weighted by the probability for a given parton to be present.

For the parton energy shift, we can write

$$\delta E_i^{(s)}=\frac{\delta(M_i^2)^{(s)}}{2E_i}=\frac{M_i\delta M_i^{(s)}}{E_i}.$$

For all except "wee partons" $E_i=xE$, so that

$$\delta(M^2)^{(s)}=\sum_{\{N_j\}}\sum_i N_i\delta(M_i^2)^{(s)}\int_0^1\frac{dxf_i^{\{N_j\}}(x)}{x}. \tag{14}$$

These formulas will be rederived by more explicit calculations in Sec. III. At this point, however, we can discuss certain consequences of our result.

(1) If the charged partons have spin 0 or $\frac{1}{2}$, the expressions for δM_i^2 are, respectively, quadratically or logarithmically divergent.

(2) If the charged partons all have spin $\frac{1}{2}$ and they belong to isodoublet or isosinglet charge multiplets, then the formally logarithmically divergent expression has pieces which transform as $I_3=0$, $I=0$ or 1 isotopic tensors. Mass differences which transform as $I=2$ are finite in such a model and can be computed from Cottingham's formula.

(3) The parton mass shifts δM_i^2 can be written as Q_i^2C where Q_i^2 is the squared charge of a parton, and C is a divergent number which is formally equal for each member of a parton isomultiplet. If, in the spirit of renormalization theory, we regard the C's as c numbers to be fitted to experimental mass differences, our singular contribution to the mass shift is equivalent to a tadpole model. For models restricted to partons with quantum numbers as specified in comment (2) above, we have obviously a tadpole model for $I=1$ electromagnetic mass differences. Actually, we have tadpoles for masses squared, but for purely electromagnetic differences, this hardly matters.

(4) In addition to the ultraviolet singularity, expression (14) appears to have an additional divergence in the integration over x if $f(x)$ does not go to zero sufficiently rapidly as $x\to 0$. This effect is spurious, however. The troublesome factor of $1/x$ is actually $\lim_{p\to\infty}(E/E_i)$, which is not equal to $1/x$ very close to $x=0$. If the distribution functions do not vanish fast enough for the integral to converge, then it should be cut off at some $x_0\ll 1$ and the contribution from wee partons $x<x_0$ must be treated separately. The impulse approximation we have used to get our result breaks down for this part of the parton spectrum.

If we assume a power behavior, $f(x)\sim x^a$, near $x=0$, then convergence of the integral requires $a>0$. If $a\le 0$, the contributions from wee partons must be handled more carefully and the singular contribution to the mass shifts cannot be expressed in terms of the scaling limit of the parton distribution function.[17]

(5) It is natural in a parton model to think of a particle as a core of partons and antipartons with the quantum numbers of the vacuum plus some valence partons and antipartons which carry the quantum numbers of the particle. A common choice is to suppose that the number of valence partons is fixed and finite, and that states of higher total occupation number N have just more partons in the core.[18] If proton and neutron structure functions approach each other for small x $[M\nu/(-q^2)$ large$]$, then the leading term in the distribution function $f(x)$ near $x=0$ comes from the core partons and antipartons.

Assume that the charged partons consist of an isodoublet $(\mathcal{P},\mathfrak{N})$ and an isosinglet (λ). The parton wave function of a neutron is obtained from that of a proton by replacing $\mathcal{P}$ partons by $\mathfrak{N}$ partons and $\bar{\mathfrak{N}}$ partons by $\bar{\mathcal{P}}$ partons one at a time. If we assume that the valence particles for a proton are partons only and not antipartons, and we assume that the proton-neutron mass difference is dominated by the tadpole contribution, then we conclude that

$$\delta M_{\text{proton}}-\delta M_{\text{neutron}}\propto Q_{\mathcal{P}}^2-(Q_{\mathcal{P}}-1)^2$$

and

$$M_{\text{neutron}}>M_{\text{proton}}\text{ only if }Q_{\mathcal{P}}<\tfrac{1}{2}.$$

For the usual quark triplet model, of course, $Q_{\mathcal{P}}=\frac{2}{3}$, $Q_{\mathfrak{N}}=Q_\lambda=-\frac{1}{3}$. Therefore, if the physical proton-neutron mass difference is to be ascribed to the singular contribution of the quark partons, there must be some admixture of antiquarks in the *valence* part of the nucleon wave function. Similar discussions can be given for multitriplet models.

(6) It would seem possible to turn this model into a general tadpole scheme for octet dominance in SU(3). There are some problems, however. For electromagnetic masses, assume that there is a single quark-parton triplet, neglect SU(3) breaking in the parton wave functions of a physical multiplet, and neglect mass differences among the quarks. Then $\delta M^{2(s)}$ transforms as a sum of an SU(3) singlet operator and an SU(3) octet operator with the quantum numbers of electric charge. In the models of comment (5), the presence of a nonvanishing d/f ratio implies that there are antiquarks in the valence part of the wave function.

Suzuki and Sugawara have shown from partial conservation of axial-vector current (PCAC) and current algebra that hyperon s-wave nonleptonic decays are determined from matrix elements like

$$\langle B'|H_w(0)|B\rangle .$$

H_w is the effective nonleptonic weak Hamiltonian; B and B' are baryons. If we assume for H_w a current-current interaction mediated by an intermediate boson, then we can analyze the singular part of the weak spurion vertex above and get a result analogous to (13). A tadpole associated with this (quadratically) divergent term will have the same d/f ratio as the electromagnetic mass splitting. However, if we can apply the parton model directly to the complete energy-momentum conserving matrix element describing the decay

$$\langle B'M|H_w(0)|B\rangle ,$$

then the analysis of Nussinov and Preparata[19] predicts that the quadratic and logarithmic divergences vanish for both a direct current-current and a boson-mediated form for $H_w(0)$.

It is tempting to assume that SU(3)-breaking mass differences come mainly from the mass differences between the $(\mathfrak{N}, \mathcal{P})$ quarks and the λ quark. Using the operator $s_8(0)=\bar{q}\lambda_8 q$ evaluated between SU(3)-symmetric wave functions, we will get a result analogous to (14), which will give a matrix element with the same d/f ratio as our electromagnetic mass shifts. However, it will be the *squared masses* which are described by this operator and which are supposed to satisfy a Gell-Mann–Okubo formula for all particles. For the baryon octet a squared-mass formula works to within ~2%, but it fails significantly for the $\frac{3}{2}^+$ decimet. It is, of course, a dynamical assumption that ψ_B should be considered SU(3) symmetric. If, instead, we make the *ad hoc* assumption that $\psi_B/\sqrt{M_B}$ is symmetric, we will get a Gell-Mann–Okubo formula linear in the baryon masses.

III. SOME FURTHER SUPPORT FOR THE RESULT

The basic result, Eq. (14), can be made more plausible by some further calculations. For spin-$\frac{1}{2}$ partons, the original method of Bjorken[6] for analyzing logarithmic divergences in the calculated electromagnetic self-energy can be adapted to our model. For spin-0 partons, a perturbation calculation to second order in the strong coupling constant agrees with (14).

A. Bjorken Method

The mass shift can be written as

$$\delta M^2=\frac{-i\alpha}{8\pi^3}\int\frac{d^4q}{(q^2)_R}\,T_\mu{}^\mu(P,q)\,,$$
$$\alpha=\frac{e^2}{4\pi}\cong\frac{1}{137}, \tag{15}$$
$$T_{\mu\nu}=2Ei\int d^4r\,e^{iq\cdot r}\langle P|\,T^*(J_\mu(r)J_\nu(0))|P\rangle\,.$$

It will be convenient to have a well-defined expression for the singular mass shifts which we can obtain by regulating the photon propagator. For fermion self-energies we replace $1/(q^2)_R$ by $-\Lambda^2/q^2(q^2-\Lambda^2)$. The singular term is identified with the leading term in the limit $\Lambda^2\to\infty$. For bosons $1/(q^2)_R$ will be replaced by $\Lambda^4/q^2(q^2-\Lambda^2)^2$.

If there are no contact terms in the Compton amplitude, the large-q^2 behavior of T is found by taking the limit $q_0\to i\infty$, $\vec{q}=0$, which gives

$$\lim_{\substack{q_0\to i\infty\\ \vec{q}=0}} T_{\mu\nu}=\frac{2E}{q_0^2}\sum_n\int d^3r[\langle P|J_\mu(\vec{r},0)|n\rangle\langle n|J_\nu(0)|P\rangle(E_n-E_p)+\langle P|J_\nu(0)|n\rangle\langle n|J_\mu(\vec{r},0)|P\rangle(E_n-E_p)]\,. \tag{16}$$

We can evaluate the coefficient of $1/q_0^2$ for E_p very large so that the states $|P\rangle$ and $|n\rangle$ contain only forward moving partons:

$$\lim_{\substack{q_0\to i\infty\\ \vec{q}=0}} T_{\mu\nu}=\frac{1}{q_0^2}\lim_{P\to\infty}2E\sum\int d^3r[\langle UP|j_\mu(\vec{r},0)|Un\rangle\langle Un|j_\nu(0)|UP\rangle(E_n-E_p)$$

$$+\langle UP|j_\nu(0)|Un\rangle\langle Un|j_\mu(\vec{r},0)|Up\rangle(E_n-E_p)], \tag{17}$$

where $j_\mu(r)$ is the undressed electromagnetic current in the infinite-momentum frame. The sum of the energies of the free partons in the states $|UP\rangle$ and $|Un\rangle$ are equal to the energies of the physical states $|P\rangle$ and $|n\rangle$, respectively:

$$H_0|UP\rangle=E_p|UP\rangle\,,\quad H_0|Un\rangle=E_n|Un\rangle\,,$$

where H_0 is the free Hamiltonian of the partons. Then closure allows us to write (17) as

$$\lim_{\substack{q_0\to i\infty\\ \vec{q}=0}} T_{\mu\nu}=\frac{1}{q_0^{\,2}}\left(\lim_{E\to\infty}2E\int d^3r\,\langle UP|\,[\,[j_\mu(\vec{r},0),H_0],j_\nu(0)]\,|UP\rangle\right). \tag{18}$$

For spin-$\frac{1}{2}$ partons, the double commutator is bilinear in the free field operators, and $T_{\mu\nu}$ can be expressed as a sum of parton contributions

$$\lim_{\substack{q_0\to i\infty\\ \vec{q}=0}} T_{\mu\nu}=\frac{1}{q_0^{\,2}}\lim_{E\to\infty}\sum_{\{N_j\}}\sum_i N_i\int_0^1\frac{E}{E_i}f_i^{\{N_j\}}(x,p_t)dx\,d^2p_t$$
$$\times\int d^3r\,2E_i\langle xP,p_t,i|\,[\,[j_\mu(\vec{r},0),H_0],j_\nu(0)]\,|xP,p_t,i\rangle\,. \tag{19}$$

The second-order mass shift of a free fermion is one case in which the Bjorken limit is known to give the correct logarithmically divergent contribution. Using $E/E_i=1/x$, we recover the result (14) with[20]

$$\delta M_i=\frac{3\alpha}{4\pi}Q_i^{\,2}M_i\ln\left(\frac{\Lambda^2}{M_i^{\,2}}\right).$$

B. Perturbation Calculation for Spin-Zero Particles

For spin-0 fields, the Compton amplitude has a contact term and the Bjorken method may not be applicable. In this case we turn to a model of a charged scalar field Ψ and a neutral scalar field Φ interacting via a trilinear interaction, $g\bar{\Psi}\Psi\Phi$.

Drell and Yan[21] have considered this model because it is superrenormalizable and gives structure functions which satisfy scaling in the Bjorken limit. They calculated the structure function for the Ψ particle of the two-particle state to order g^2 and found

$$\lim_{\text{Bj}}2M\nu W_2(q^2,\nu)=\frac{8\pi M^2g^2}{(4\pi)^2}\,\frac{x^2(1-x)}{M^2(1-x)^2+\mu^2x}, \tag{20}$$

where M is the mass of the Ψ field and μ is the mass of the Φ field. Comparing with (11) the parton distribution function to order g^2 is

$$f_\psi^{\{1,1\}}=\frac{g^2}{(4\pi)^2}\,\frac{x(1-x)}{M^2(1-x)^2+\mu^2x}. \tag{21}$$

The corresponding contribution to the electromagnetic mass shift is given by Feynman diagrams of Fig. 1.

To separate the leading mass-shift contribution from the wave-function renormalization, let $(p+k)^2=M^2$ in the electromagnetic self-energy part. The sum of the Feynman diagrams is

$$\delta M^2(g^2)=ig^2\int\frac{d^4k}{(2\pi)^4}(k^2-\mu^2)^{-1}(k^2+p\cdot k)^{-2}\delta M^2(0)\,, \tag{22}$$

where

$$\delta M^2(0)=-ie^2\int\frac{d^4q}{(2\pi)^4}\,\frac{\Lambda^4}{q^2(q^2-\Lambda^2)^2}\left[\frac{q^2(q^2-4M^4)}{q^4-4[q\cdot(p+k)]^2}-2\right]\cong\frac{3\alpha}{4\pi}\Lambda^2\,.$$

The k integration in (22) can be performed after combining denominators by the standard Feynman parametrization. The result is

$$\delta M^2(g^2)=\frac{g^2}{(4\pi)^2}\int_0^1\frac{dx(1-x)}{M^2(1-x)^2+\mu^2x}\,\delta M^2(0)$$

$$=\int_0^1 \frac{dx}{x} f_{\psi}^{\{1,1\}}(x)\delta M^2(0). \tag{23}$$

In this case $f(x)$ vanishes linearly as $x \to 0$ and there is no problem with wee partons.

IV. COTTINGHAM'S FORMULA AND THE APPROACH TO SCALING

Scaling in deep-inelastic scattering has been shown to imply divergences in electromagnetic mass shifts and differences calculated from Cottingham's dispersion-relation method.[7,9] However, the correspondence between divergences in the Cottingham approach and Eq. (14) is not always apparent in previous calculations. We will make this relation explicit. Much of the following material is review and is presented for coherence and completeness. The principal new results are formula (33) for σ_S in spinor parton models and the sum rule (34) for the average charge squared times mass squared per parton.

We start from

$$\delta M^2 = \frac{-i\alpha}{8\pi^3}\int \frac{d^4q}{(q^2-i\epsilon)_R} T_\mu{}^\mu(P,q),$$
$$T_{\mu\nu} = i\int d^4r\, e^{iq\cdot r}(2E)\langle P|\, T^*(J_\mu(r)J_\nu(0))|P\rangle . \tag{15}$$

$T_{\mu\nu}$ is expanded in kinematic-singularity-free amplitudes

$$T_{\mu\nu} = (q_\mu q_\nu - q^2 g_{\mu\nu})t_1(q^2,\nu) + \left(\frac{q^2}{M^2}P_\mu P_\nu - \frac{\nu}{M}(P_\mu q_\nu + P_\mu q_\nu) + \nu^2 g_{\mu\nu}\right)t_2(q^2,\nu). \tag{24}$$

A Wick rotation of the q_0 contour leads to

$$\delta M^2 = \frac{\alpha}{4\pi^2}\int_0^\infty \frac{dq^2}{(q^2)_R}\int_{-q}^{+q} d\nu (q^2-\nu^2)^{1/2}[3q^2 t_1(-q^2, i\nu) - (q^2+2\nu^2)t_2(-q^2,i\nu)]. \tag{25}$$

For spacelike q^2, the t's satisfy dispersion relations in ν. Assume for the moment that no subtractions are required. We obtain

$$\delta M^2 = \frac{\alpha}{2\pi^3}\int_0^\infty \frac{dq^2}{(q^2)_R}\int_{-q}^{+q} d\nu (q^2-\nu^2)^{1/2}$$
$$\times\left[3q^2\int_{\nu_0}^\infty \frac{\mathrm{Im}t_1(-q^2,\nu')}{\nu'^2+\nu^2}\nu' d\nu' - (q^2+2\nu^2)\int_{\nu_0}^\infty \frac{\mathrm{Im}t_2(-q^2,\nu')}{\nu'^2+\nu^2}\nu' d\nu' + \text{Born terms}\right]. \tag{26}$$

With our normalization conventions,

$$\mathrm{Im}t_1(-q^2,\nu) = \frac{-1}{2q^2}\left[W_1(-q^2,\nu) - \frac{\nu^2}{q^2}W_2(-q^2,\nu)\right],$$
$$\mathrm{Im}t_2(-q^2,\nu) = \frac{-1}{2q^2}W_2(-q^2,\nu). \tag{27}$$

Assume that in evaluating the large-q^2 contribution to the mass shift, the limit $q^2\to\infty$ may be taken inside the dispersion integrals. Then,

$$\delta M^{2(s)} = \frac{3\alpha}{8\pi^2}\int \frac{q^2dq^2}{(q^2)_R}\left[-\int_0^1 \frac{dx}{x}\lim_{\mathrm{Bj}}\left(W_1 - \frac{1}{4M^2}\frac{1}{x}(2M\nu W_2)\right) + \frac{1}{2q^2}\int_0^1 dx \lim_{\mathrm{Bj}}(2M\nu W_2)\right]. \tag{28}$$

If the integrals over x exist, the first integral gives rise to a quadratically divergent self-mass and the second term generates a logarithmic divergence.

For charged scalar partons, $W_1 = 0$,

$$\lim_{\mathrm{Bj}}(2M\nu W_2) = 8\pi M^2 \sum_{\{N_j\}}\sum_i f_i^{\{N_j\}}(x)xN_iQ_i^2,$$

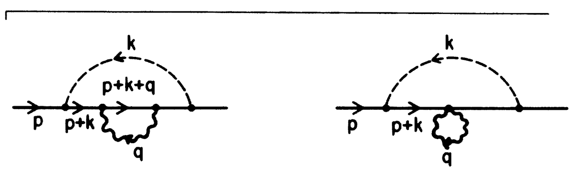

FIG. 1. Feynman diagrams for $\delta M^2(g^2)$ in $\bar\psi\psi\phi$ model.

and we set

$$\frac{1}{(q^2)_R}=\frac{\Lambda^4}{q^2(q^2+\Lambda^2)^2}.$$

The first x integral is dominant, and we get

$$\delta M^{2(s)}=\frac{3\alpha}{4\pi}\Lambda^2\sum_{\{N_j\}}\sum_i N_i Q_i^2\int_0^1\frac{f_i^{\{N_j\}}(x)dx}{x}, \qquad (29)$$

which agrees exactly with our earlier result. Equation (29) has been derived previously by Hall and Osborn, who interpret it slightly differently.[9]

It now appears, as the reader may already have realized, that the possibility of a divergence in the x integral above near $x=0$ is precisely the same as the possible high-energy divergence of the unsubtracted dispersion relations. In Sec. II, our physical picture showed that this is due to the contribution of wee partons and requires a cutoff in the x integral. The small x contribution cannot be simply expressed in terms of the scaling limit of the distribution function $f(x)$ but it gives rise to the same high-q^2 behavior. The same result should hold here.

If

$$\lim_{\text{Bj}}\text{Im}t_1(-q^2,\nu)=\frac{1}{q^2}h(x),$$

then the calculated mass shift diverges quadratically independent of the possible need for a subtraction in the dispersion integral over the energy ν. There does not seem to be any physical reason to expect the degree of divergence of the q^2 integral to depend on whether the intercept of a Regge trajectory lies above or below $J=0$. To make the correspondence with the parton model explicit we could write the Cauchy integral for $t(-q^2, i\nu)$ on a finite contour of radius $|\nu|=R-q^2/2Mx_0$. x_0 is the cutoff of Sec. II and R is some constant greater than ν_0. Then, for large q^2, the integral $\int_0^1 dx$ is replaced by $\int_{x_0}^1 dx$ and corresponds to the contribution from partons described by the scaling distribution $f(x)$, while the integral over the arc corresponds to the effect of wee partons.

For spin-$\frac{1}{2}$ partons, the problem is not so straightforward, but the result is more interesting. Using the results tabulated in (11), we get

$$\lim_{\text{Bj}}\text{Im}t_1(-q^2,\nu)=-\frac{1}{q^2}\left(F_1(x)-\frac{1}{4M^2}\frac{1}{x}F_2(x)\right)$$
$$=0, \qquad (30)$$
$$\lim_{\text{Bj}}\text{Im}t_2(-q^2,\nu)=-\frac{1}{2q^4}xF_2(x).$$

The first term is the coefficient of the quadratic divergence and it is expected to vanish. The second term gives a logarithmically divergent expression, but it is easy to see it does not agree with our earlier result. They can be reconciled if the corrections to $\text{Im}t_1$ are $O(1/q^4)$. More precisely, to reproduce (14) it is sufficient that

$$\lim_{\text{Bj}}\left(W_1(-q^2,\nu)-\frac{1}{4M^2}\frac{1}{x}[2M\nu W_2(-q^2,\nu)]\right)=\frac{1}{q^2}g(x), \qquad (31)$$

with

$$g(x)=4\pi\sum_{\{N_j\}}\sum_i N_i Q_i^2(x^2M^2-M_i^2)f_i^{\{N_j\}}(x).$$

This new information about a nonleading term in the Bjorken limit is deduced from demanding consistency between our direct parton model result of Sec. II and the dispersion theoretic formulation of Cottingham. Again, if $f(x)$ does not vanish as x approaches zero, the dispersion relation must be subtracted and wee partons contribute to the singular mass shift or difference. We expect, however, that (31) should hold independent of the behavior of the distribution function near $x=0$.

Our deduction can be expressed in terms of other physical quantities. W_1 and W_2 can be written in terms of the total cross section for the photoabsorption by a particle at rest of virtual photons with transverse and scalar polarizations, σ_t and σ_s, respectively.

With our normalization[1]

$$\sigma_t=\frac{2\pi\alpha}{2M\nu-q^2}W_1,$$
$$\sigma_s=\frac{2\pi\alpha}{2M\nu-q^2}\left[\left(1+\frac{\nu^2}{q^2}\right)W_2-W_1\right]. \qquad (32)$$

For spin-$\frac{1}{2}$ charged partons, the scaling behavior can be expressed as

$$\lim_{\text{Bj}}\sigma_t=\frac{4\pi^2}{q^2}\sum_{\{N_j\}}\sum_i\frac{x}{1-x}f_i^{\{N_j\}}(x)N_iQ_i^2,$$
$$\lim_{\text{Bj}}\sigma_s/\sigma_t=0. \qquad (33a)$$

Equation (31) says

$$\lim_{\text{Bj}}\sigma_s=\frac{8\pi^2\alpha}{q^4}\sum_{\{N_j\}}\sum_i\frac{x}{1-x}f_i^{\{N_j\}}(x)$$
$$\times N_iQ_i^2(x^2M^2+M_i^2). \qquad (33b)$$

The longitudinal cross section vanishes relative to the transverse as $1/q^2$. For high energy $(x\to 0)$, σ_s has the same power behavior as the transverse cross section. The term proportional to M_i^2 dominates here.

If σ_s were measured and turned out to have general behavior implied by (33b), then we could write a sum rule for $\langle Q_i^2M_i^2\rangle$, the average charge squared times mass squared per parton. For nonsymmetric ψ^n's this is a weighted average:

$$\langle Q_i^2 M_i^2\rangle = \frac{1}{8\pi^2\alpha}\int_0^1 dx(1-x)(q^4\sigma_s - q^2M^2x^2\sigma_t)\,. \tag{34}$$

This has been written in complete analogy to the Bjorken-Paschos sum rule for $\langle Q_i^2\rangle$. If (34) could be measured, it could be combined with the Bjorken-Paschos calculation of $\langle Q_i^2\rangle$ to estimate the order of magnitude of the effective mass of the so-called partons.

According to a model of spin-$\frac{1}{2}$ partons, not only the integral of (34) but the integrand at each value of x must be positive. It can vanish only if all the charged partons have zero mass. This is just the condition for mass shifts or differences to be finite. If this condition is satisfied, the high-energy longitudinal cross section falls by two powers of the energy relative to the transverse cross-section at least for large q^2. To be precise

$$\lim_{\rm Bj}\frac{\sigma_s}{\sigma_t} = \frac{2x^2M^2}{q^2} = \frac{q^2}{2\nu^2}\,. \tag{35}$$

This is not in contradiction with the observed cross sections for protons. For the differences in cross sections which are relevant for the neutron-proton mass difference, nothing is known yet about σ_s.

Condition (35) is sufficient to guarantee finite mass differences independent of its origin in the parton model. It was stated first by Hall and Osborn.[9] It is useful to define

$$W_s(-q^2,\nu) = W_2(1+\nu^2/q^2) - W_1\,. \tag{36}$$

Then, the Cottingham formula can be written

$$\delta M_c^2 = \delta M^2 - \delta M_{\rm Born}^2 = \frac{\alpha}{4\pi^3}\int_0^\infty dq^2\int_{-1}^1 dz(1-z^2)^{1/2}\left(-3\int\frac{\nu'}{\nu'^2+q^2z^2}[W_2(-q^2,\nu') - W_s(-q^2,\nu')]d\nu' + (1+2z^2)\int\frac{\nu'}{\nu'^2+q^2z^2}W_2(-q^2,\nu')d\nu'\right). \tag{37}$$

If νW_2 scales in the Bjorken limit, the coefficient of the logarithmically divergent term in the final q^2 integration vanishes if

$$W_s = \tfrac{1}{2}W_2 \tag{38}$$

in the scaling limit.[22] From (32) and (36) this implies the relation between σ_s and σ_t given in (35).[23]

Bloom and Gilman[24] have indicated that the scaling limit of W_2 for protons is a good average representation of the structure function even in the low-q^2 region where prominent resonances can be seen. If we assume that this is also true for W_s, valid for both protons and neutrons, and that (38) holds for all q^2 in the continuum, then the preliminary data on the neutron structure functions can be used to estimate the continuum contribution to the neutron-proton mass difference. With these assumptions, $t_1(q^2,\nu)$ satisfies an unsubtracted dispersion relation in contradiction with the Regge analysis.[13] The result is

$$(M_{\rm neutron} - M_{\rm proton})_{\rm cont.} \approx 10^{-2}\ {\rm MeV}\,,$$

which has the correct sign, but is too small by two orders of magnitude to cancel the Born term and explain the observed mass difference.

If the scaling of νW_2 in the Bjorken limit is a true fact of nature, it seems that the only remaining way to explain the neutron-proton mass difference by a convergent calculation in conventional electrodynamics is through some unexpectedly large contribution from W_s in the small-q^2 region.

*Work supported in part by the AEC under Contract No. AT(11-1) 3472B.

[1]For a recent survey of experimental results, see H. W. Kendall, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971,* edited by N. B. Mistry (Cornell Univ. Press, Ithaca, N. Y., 1972).

[2]R. A. Brandt, Phys. Rev. Letters 23, 1260 (1969); Phys. Rev. D 1, 2808 (1970); Y. Frishman, Phys. Rev. Letters 25, 966 (1970); R. A. Brandt and G. Preparata, *ibid.* 25, 1530 (1970). Scaling of the inelastic structure functions was related to the existence of "almost-equal-time commutators" by J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[3]B. L. Joffe, Phys. Letters 30B, 123 (1969).

[4]R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969); J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).

[5]K. Wilson, Phys. Rev. 179, 1499 (1969).

[6]J. D. Bjorken, Phys. Rev. 148, 1467 (1966).

[7]See, e.g., H. Pagels, Phys. Rev. 185, 1990 (1969); D. J. Gross and H. Pagels, *ibid.* 172, 1381 (1968); R. Jackiw, R. Van Royen, and G. B. West, Phys. Rev. D 2, 2473 (1970). H. Goldberg and Y. N. Srivastava,

ibid. 4, 1426 (1971).

[8]W. N. Cottingham, Ann. Phys. (N.Y.) 25, 424 (1963).

[9]T. Muta, Progr. Theoret. Phys. (Kyoto) 44, 1022 (1970), has attempted to calculate the real part of the virtual Compton amplitude in the deep-inelastic scattering region from a parton model and use this in the Cottingham formula. The Cottingham formula, however, does not directly probe the Compton amplitude in the deep-inelastic region and Muta's results differ from those presented in Sec. IV of this paper.

G. S. Hall and H. Osborn, Nucl. Phys. B34, 445 (1971), have used the deep-inelastic structure functions of the parton model to evaluate the contribution of the scaling region in the dispersion integrals of Cottingham's formula. As we show in Sec. IV, their result is the same as ours for spin-0 charged bosons but is different for spin $\frac{1}{2}$.

[10]S. Coleman and S. Glashow, Phys. Rev. 134, 1307 (1904).

[11]While this manuscript was being prepared, there appeared a report by G. B. West (unpublished), who uses commutator algebra to reach general conclusions about the approach to scaling and divergences in electromagnetic mass calculations. West claims that pion mass differences will, in general, diverge in a model with quark algebra, which does not agree with the conclusions of this paper.

[12]A recent review of the proton-neutron mass difference problem with many references to this and related problems is A. Zee, Phys. Reports (to be published).

[13]H. Harari, Phys. Rev. Letters 17, 1303 (1966). See also M. Elitzur and H. Harari, Ann. Phys. (N.Y.) 56, 81 (1970).

[14]H. Sugawara, Phys. Rev. Letters 15, 870 (1965); 15, 997(E) (1965); H. Suzuki, *ibid.* 15, 986 (1965).

[15]S. D. Drell, D. J. Levy, and T.-M. Yan, Phys. Rev. Letters 22, 744 (1969); Phys. Rev. 187, 2159 (1969).

[16]See the article by Bjorken and Paschos in Ref. 4. These results are derived in our formalism with all the correct factors of x, which is a further check on our normalization conventions.

[17]The author is indebted to Dr. G. Preparata for a discussion which helped to clarify this point as well as the related material in Sec. IV. The result (14) is consistent with the quark-model result derived by Preparata:

$$S(0) \sim \frac{3\alpha}{4\pi}\int \frac{dq^2}{q^2}\, \bar{\psi}\,\{Q^2, M\}\,\psi,$$

where Q is the charge matrix and M is the mass matrix of the quarks. See Z. Grossman and G. Preparata, Phys. Rev. D 5, 2069 (1972).

[18]Such a model has been investigated by J. Kuti and V. F. Weisskopf, Phys. Rev. D 4, 3418 (1971).

[19]S. Nussinov and G. Preparata, Phys. Rev. 175, 2180 (1968). See also C. Bouchiat, J. Iliopoulos, and J. Prentki, Nuovo Cimento 56A, 1150 (1968).

[20]In the model of Lee and Wick, this substitution (14) gives the leading finite contribution to the mass shift where Λ is the mass of the heavy indefinite-metric particle. T. D. Lee, invited talk at Amsterdam Conference on Elementary Particles, 1971 [Columbia University Report No. NYO-1932(2)-200 (unpublished)].

[21]S. D. Drell and T.-M. Yan, Ann. Phys. (N.Y.) 60, 578 (1971).

[22]Goldberg and Srivastava (Ref. 7) considered the implications of a quark-parton model directly in the Cottingham formalism. They claim to prove $\Delta I = 2$ mass differences are finite, but they appear to use

$$\lim_{\mathrm{Bj}} q^2 \nu W_S^{I=2}\,(q^2, \nu) \equiv 0$$

without showing that it can be expressed as a sum of one-particle operators. For $I = 1$ mass differences their results coincide with those of Pagels (Ref. 7).

[23]To eliminate the divergences in mass differences, it is sufficient to require cancellation among the appropriate integrals over the structure functions and possible subtraction constants. See papers listed in Ref. 7. H. Pagels, Phys. Rev. D 3, 610 (1971), in Ref. 9, gives the integral condition for no divergences when the dispersion relations are unsubtracted. The local condition on the structure functions seems natural in the context of parton models since it corresponds to massless partons and guarantees that all calculated electromagnetic mass shifts and radiative corrections will be finite.

[24]E. D. Bloom and F. J. Gilman, Phys. Rev. Letters 25, 1140 (1970).