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CP Violation and $K_{S,L}\to\mu\bar{\mu}$ Decays*

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A discussion of *CP* violation in $K_{S,L}\to\mu\bar{\mu}$ is presented. It is argued on the basis of order-of-magnitude estimates for the absorptive and dispersive parts that a class of *CP*-violating theories is inconsistent with the present experimental information. Okubo's theory of *CP* violation, which lies outside this class, is also discussed.

Recently there has been a great deal of interest in the analysis of $K_{S,L}\to\mu\bar{\mu}$ decays. If *CP* is conserved in these decays, the experimental upper bound[1] on the rate of $K_L\to\mu\bar{\mu}$ contradicts the unitarity limit.[2] If, however, *CP* conservation is given up, Christ and Lee[3] have shown that the resulting inequalities are consistent with the present experimental results. More recently Gaillard[4] has shown that the superweak theory of *CP* violation predicts a branching ratio for $K_S\to 2\gamma$ decay which is much larger than the presently known experimental limit.

The purpose of the present note is to examine the role of some other types of *CP*-violating theories in the context of $K_{S,L}\to\mu\bar{\mu}$ decays. Unfortunately, however, for any specific model of *CP* violation other than the simplest superweak theory, the explicit calculations of the amplitudes in $K_{S,L}\to\mu\bar{\mu}$ are in general intractable, except for what essentially amounts to order-of-magnitude estimates. It is of interest, nevertheless, to see how even such admittedly crude estimates can provide guidelines for the type of *CP*-violating theories that could, in principle, lead to consistency with presently available experimental information.

Following Christ and Lee,[3] we define the K^0 and $\bar{K}^0$ decay matrix elements in terms of four complex numbers $a_\pm$ and $b_\pm$:

$$M(K^0\to(\mu\bar{\mu})_\pm)=b_\pm+ia_\pm\,,\qquad M(\bar{K}^0\to(\mu\bar{\mu})_\pm)=\pm(b_\pm^*+ia_\pm^*)\,,\tag{1}$$

where the subscript ± refers to *CP*-even and -odd states. Note that as in Ref. 3 $a_\pm$ and $b_\pm$ are real if *CP* is conserved. Furthermore, the matrix elements are defined to include appropriate phase-space factors, so that their absolute squares directly give the transition rates. Using the states

$$|K_{S,L}\rangle=[2(1+|\epsilon|^2)]^{-1/2}[(1+\epsilon)|K^0\rangle\pm(1-\epsilon)|\bar{K}^0\rangle]\tag{2}$$

defined in terms of the usual *CP*-violating parameter ϵ ($\approx 2\times10^{-3}e^{i\pi/4}$), the matrix elements for $K_{S,L}\to\mu\bar{\mu}$ decays can be written as

$$\begin{aligned}
T_{S\mu+}&\equiv M(K_S\to(\mu\bar{\mu})_+)\\
&=\sqrt{2}\,[(\mathrm{Re}b_+-\epsilon\,\mathrm{Im}a_+)+i(\mathrm{Re}a_++\epsilon\,\mathrm{Im}b_+)],\\
T_{S\mu-}&\equiv M(K_S\to(\mu\bar{\mu})_-)\\
&=\sqrt{2}\,[(-\mathrm{Im}a_-+\epsilon\,\mathrm{Re}b_-)+i(\mathrm{Im}b_-+\epsilon\,\mathrm{Re}a_-)],\\
T_{L\mu+}&\equiv M(K_L\to(\mu\bar{\mu})_+)\\
&=\sqrt{2}\,[(-\mathrm{Im}a_++\epsilon\,\mathrm{Re}b_+)+i(\mathrm{Im}b_++\epsilon\,\mathrm{Re}a_+)],\\
T_{L\mu-}&\equiv M(K_L\to(\mu\bar{\mu})_-)\\
&=\sqrt{2}\,[(\mathrm{Re}b_--\epsilon\,\mathrm{Im}a_-)+i(\mathrm{Re}a_-+\epsilon\,\mathrm{Im}b_-)],
\end{aligned}\tag{3}$$

where terms of order $|\epsilon|^2$ have been neglected.

Following Gaillard,[4] we assume now that (i) only the *CP*-odd two-photon state $(\gamma\gamma)_-$ contributes to the absorptive part of $K^0,\bar{K}^0\to(\mu\bar{\mu})_-$ and $K^0,\bar{K}^0\to(\gamma\gamma)_-$ has no absorptive part, (ii) only the *CP*-even $(\gamma\gamma)_+$ intermediate state contributes to the absorptive part of $K_2\to(\mu\bar{\mu})_+$ and $K_2\to(\gamma\gamma)_+$ has no absorptive part. For justification of these assumptions, one may refer to Gaillard's paper.[4] Unitarity and *CPT* invariance then imply[4]

$$\mathrm{Re}\,T_{1\gamma-} = \mathrm{Im}\,T_{2\gamma-} = \mathrm{Re}\,T_{2\gamma+} = 0\,,$$
$$\mathrm{Im}\,T_{2\mu-} \equiv \sqrt{2}\,\mathrm{Re}\,a_- = (\phi_-)^{1/2} T_{2\gamma-}\,,$$
$$\mathrm{Re}\,T_{1\mu-} \equiv -\sqrt{2}\,\mathrm{Im}\,a_- = i(\phi_-)^{1/2} T_{1\gamma-}\,, \tag{4}$$
$$\mathrm{Re}\,T_{2\mu+} \equiv -\sqrt{2}\,\mathrm{Im}\,a_+ = i(\phi_+)^{1/2} T_{2\gamma+}\,,$$

where 1 and 2 refer to the usual CPT-even and -odd eigenstates K_1^0 and K_2^0, and $(\phi_\pm)^{1/2}$ denote the $2\gamma \to \mu\bar{\mu}$ amplitudes in the $CP=\pm 1$ channels. Explicit calculations give[2]

$$\phi_- \simeq 1.2\times 10^{-5}\,,$$
$$\phi_+ \simeq 0.8\phi_-\,. \tag{5}$$

From Eq. (4), the matrix elements for $K_{L,S} \to 2\gamma$ decays can be constructed as follows:

$$T_{S\gamma+} = T_{1\gamma+} + i\left(\frac{2}{\phi_+}\right)^{1/2} \epsilon\,\mathrm{Im}\,a_+\,,$$
$$T_{S\gamma-} = \left(\frac{2}{\phi_-}\right)^{1/2} (i\,\mathrm{Im}\,a_- + \epsilon\,\mathrm{Re}\,a_-)\,,$$
$$T_{L\gamma+} = i\left(\frac{2}{\phi_+}\right)^{1/2} \mathrm{Im}\,a_+ + \epsilon T_{1\gamma+}\,, \tag{6}$$
$$T_{L\gamma-} = \left(\frac{2}{\phi_-}\right)^{1/2} (\mathrm{Re}\,a_- + i\epsilon\,\mathrm{Im}\,a_-)\,.$$

It is well known that in the usual perturbation theory for weak and electromagnetic interactions, the contribution to $K_{L,S} \to \mu\bar{\mu}$ can come from (a) lowest-order weak and fourth-order electromagnetic interactions and (b) second-order weak effects alone. Now, a general class of CP-violating weak interactions can be described by the Hamiltonian

$$H_w = H_w^{(+)} + H_w^{(-)}\,, \tag{7}$$

where $\pm$ refer to CP-even and -odd parts. If the CP-conserving part is characterized by the usual weak coupling constant G, and the CP-violating part by G', experiments suggest $G'/G \approx 10^{-3}$. It is also well known that the absorptive part $a_\pm, a_\pm^*$ in the matrix elements (1) can receive contributions only from (a) and not (b). Furthermore, in the weak-electromagnetic contributions of the type (a), clearly $H_w^{(+)}$ would contribute to $\mathrm{Re}\,a_\pm$ and $H_w^{(-)}$ to $\mathrm{Im}\,a_\pm$. Assuming CP is conserved in electromagnetic interactions, one then expects

$$\frac{\mathrm{Im}\,a_+}{\mathrm{Re}\,a_+} \sim \frac{\mathrm{Im}\,a_-}{\mathrm{Re}\,a_-} \sim O(G'/G)\,. \tag{8}$$

As for the dispersive part $b_\pm, b_\pm^*$ of the matrix elements (1), both (a) and (b) can contribute. One knows, however, that the explicit calculation for the 2γ intermediate state in contributions of type (a) is logarithmically divergent,[2] but for the higher-order weak process (b) the degree of divergence is quadratic.[5] As usual, we shall assume the existence of appropriate cutoffs. We then expect $(\mathrm{Re}\,b_\pm)_a \sim O(G\alpha^2)$, $(\mathrm{Re}\,b_\pm)_b \sim O(G^2)$ and $(\mathrm{Im}\,b_\pm)_a \sim O(G'\alpha^2)$, $(\mathrm{Im}\,b_\pm)_b \sim O(GG')$, where α is the fine-structure constant, so that as an order-of-magnitude estimate,

$$\frac{\mathrm{Im}\,b_+}{\mathrm{Re}\,b_+} \sim \frac{\mathrm{Im}\,b_-}{\mathrm{Re}\,b_-} \sim O(G'/G)\,. \tag{9}$$

An accidental almost total cancellation between the two contributions (a) and (b) can upset this estimate. If this happens for $\mathrm{Re}\,b_\pm$, (9) could be a gross underestimate. However, in view of the different degrees of divergence in the two contributions, we regard such a near cancellation to be highly unlikely. The estimate (9) is clearly less reliable than (8). However, it seems safe to exclude the possibility that $|\mathrm{Im}\,b_\pm|$ may become comparable to or larger than $|\mathrm{Re}\,b_\pm|$. Note that in the superweak theory, CP violation appears only through the mixing parameter ϵ in Eq. (2), and $\mathrm{Im}\,a_\pm = \mathrm{Im}\,b_\pm = 0$.

Since both ϵ and G'/G are small parameters, we may in the following ignore the quadratic terms ϵ^2, $\epsilon G'/G$, and $(G'/G)^2$ compared with terms of order unity. From Eqs. (3), using Eqs. (8) and (9), we obtain in this approximation

$$R(K_L \to \mu\bar{\mu}) \equiv |T_{L\mu+}|^2 + |T_{L\mu-}|^2 \simeq 2[(\mathrm{Re}\,a_-)^2 + (\mathrm{Re}\,b_-)^2]\,. \tag{10}$$

In the same manner, Eqs. (6) lead to

$$R(K_L \to \gamma\gamma) \equiv |T_{L\gamma+}|^2 + |T_{L\gamma-}|^2 \simeq \frac{2}{\phi_-}(\mathrm{Re}\,a_-)^2\,. \tag{11}$$

From Eqs. (10) and (11), we then readily obtain the inequality

$$\frac{R(K_L \to \mu\bar{\mu})}{R(K_L \to \gamma\gamma)} \geqslant \phi_- = 1.2\times 10^{-5}\,. \tag{12}$$

Even if $\mathrm{Im}\,b_\pm \sim \mathrm{Re}\,b_\pm$, to the extent that ϵ is small, one still has the approximate relation

$$R(K_L \to \mu\bar{\mu}) \simeq 2[(\mathrm{Re}\,a_-)^2 + (\mathrm{Re}\,b_-)^2 + (\mathrm{Im}\,b_+)^2] \tag{13}$$

instead of Eq. (9). From Eqs. (13) and (11), the inequality (12) follows again. Experimentally, using the result[6]

$$\frac{R(K_L \to \gamma\gamma)}{R(K_L \to \mathrm{all})} = (4.7 \pm 0.6)\times 10^{-4}$$

and the upper bound[1]

$$\frac{R(K_L \to \mu\bar{\mu})}{R(K_L \to \mathrm{all})} \leqslant 1.8\times 10^{-9}\,,$$

we get

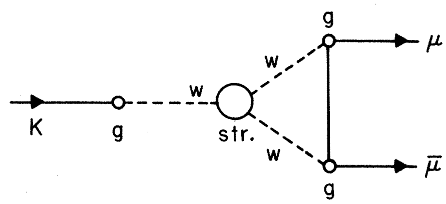

FIG. 1. CP-violating contribution to $b_\pm$ in model (15).

$$\frac{R(K_L \to \mu\bar{\mu})}{R(K_L \to \gamma\gamma)} \leq 0.4 \times 10^{-5} \qquad (14)$$

which contradicts the result (12).

There exists another class of CP-violating theories where this contradiction can be avoided. A typical example is Okubo's model.[7] Okubo assumes the existence of intermediate vector bosons (charged and neutral) which have strong interactions among themselves but are coupled weakly (or electromagnetically) to all other particles. The weak Hamiltonian is wholly odd under CP,

$$H_w = igJ_\mu W_\mu + \text{H.c.}, \qquad (15)$$

where g is real and describes the semiweak coupling constant ($g^2/m_w{}^2 = G/\sqrt{2}$), and J_μ contains both charged and neutral hadronic currents, as well as the usual leptonic currents. The detailed structure of J_μ and H_w is not important for our purposes here. It is easy to see that the lowest-order CP-conserving interaction arises in the second-order g^2, while the CP-violating effects manifest themselves only in the third-order g^3. In terms of the earlier notation, $G'/G = g$. It is easy to check that in this theory $\text{Re}a_\pm \sim O(g^2\alpha^2)$ and $\text{Im}a_\pm \sim O(g^3\alpha^2)$, so that the estimate (8) is again valid. However, now Eq. (9) is incorrect. To see this, note that as before $(\text{Re}b_\pm)_a \sim O(g^2\alpha^2)$, $(\text{Re}b_\pm)_b \sim O(g^4)$ and $(\text{Im}b_\pm)_a \sim O(g^3\alpha^2)$, but this time one expects in general $(\text{Im}b_\pm)_b \sim O(g^3)$ and *not* $O(g^5)$. This is the gross CP-violation effect discussed by Marshak *et al.*,[8] and arises from the Feynman diagram displayed in Fig. 1. If numerically g and α are of the same order of magnitude, $\text{Im}b_\pm$ will in general dominate over $\text{Re}b_\pm$. Thus in this case results (9), (10), and (12) can be avoided.

We would also like to mention that if CP violation is indeed responsible for the discrepancy between the unitarity lower bound and the experimental upper bound for $K_L \to \mu\bar{\mu}$, in general one must have[3,4]

$$\frac{R(K_S \to \mu\bar{\mu})}{R(K_L \to \mu\bar{\mu})} > 5.5 \times 10^4 .$$

With the order-of-magnitude arguments based on the class of theories (7), this result seems hard to understand. For the model (15), however, Okubo[9] has shown recently that the third-order CP-violating interaction in the local limit is effectively of the current-current type formed with neutral currents. The hadronic neutral current has $CP = -1$, and the leptonic $\mu\bar{\mu}$ current is of the $V - A$ form. This would then lead[10] to a vanishing contribution to the dispersive part of the $K_2 \to \mu\bar{\mu}$ matrix element, so that one could, in principle, understand a large $K_S \to \mu\bar{\mu}$ rate compared with $K_L \to \mu\bar{\mu}$. Note that in the local limit, whereas $\text{Im}b_+$ is small, $\text{Im}b_-$ is still large, so that the arguments leading to the inequality (12) break down once again.

It is worthwhile to note that from our point of view Nishijima's theory[11] of CP violation, which has some features in common with Okubo's theory, however, falls into the category (7) and not (15). This is because in Nishijima's theory, CP violation is confined only to weak all hadronic processes, so that there are no CP-violating contributions of the type (b). This theory then leads to the estimates (8) and (9), as before.

Finally, we might mention that in models such as (15), since the largest contribution to $K_S \to \mu\bar{\mu}$ decay comes from the $\text{Im}b_-$ term, one observes from Eqs. (3) that $|T_{S\mu-}| \gg |T_{S\mu+}|$, so that the $\mu\bar{\mu}$ pair would be predominantly in an s state. On the other hand, for models such as (7), if the real parts of $a_\pm$ and $b_\pm$ dominate over the imaginary parts as given in Eqs. (8) and (9), $K_S \to \mu\bar{\mu}$ would be mostly a p-wave decay. Needless to say, such a test has to first await the detection of the $K_S \to \mu\bar{\mu}$ decay mode itself.

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Comments on Single-π^+ Electroproduction and the Vector-Meson-Dominance Model*

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The problem of mass extrapolation in the vector-meson-dominance predictions of single-π^+ electroproduction is examined within the framework of the electric Born model.

Recent data on electroproduction of π^+[1,2] were compared to predictions of vector-meson dominance (VMD).[3,4] The data were taken at the pion-nucleon center-of-mass energy W from 2.0 to 2.45 GeV at DESY[1] and from 1.85 to 2.5 GeV at CEA,[2] and the square of the virtual-photon mass K^2 ($K^2>0$ for space-like four-vectors in our metric) was taken from 0.1 to 0.9 $(\mathrm{GeV}/c)^2$ at DESY and from 0.18 to 1.2 $(\mathrm{GeV}/c)^2$ at CEA. The comparison of these data with the VMD prediction was found to be quite good,[3,4] with the possible exception of the longitudinal-transverse interference term.[4] In fact, it is almost surprising that the VMD prediction should work so well for the electroproduction of π^+ in the range of W and K^2 measured when we know that the VMD prediction for inelastic e-p scattering fails in the same kinematic region.[5,6] Also, when W^2 is not much larger than K^2 there are ambiguities in applying VMD to predict the cross section for longitudinally polarized virtual photons.[4] In view of these ambiguities, and in order to get a better idea about the effect of going from $K^2>0$ to $K^2=-m_\rho{}^2$ in the VMD prediction of π^+ electroproduction, we feel that it is worthwhile to use the electric Born model (one-pion exchange plus nucleon pole term with a γ_μ-type $\gamma N\bar{N}$ coupling) to study this mass-extrapolation problem in electroproduction of π^+. We have used the electric Born model to study the mass-extrapolation problem in the application of VMD to π^+ photoproduction before,[7] but there we confined ourselves to the transversely polarized photon amplitudes. We now extend our analysis to the longitudinally polarized amplitudes when we have virtual photons.

For convenience, we use the same notation as in Ref. 4. The electroproduction of π^+ cross section can be expressed as

$$\frac{d\sigma}{dE'd\Omega_e dt\,d\phi}=\Gamma\frac{d\sigma^\gamma}{dt\,d\phi}=\frac{1}{2\pi}\Gamma\{\sigma_U+\epsilon\cos2\phi\,\sigma_T+\epsilon\sigma_L+[2\epsilon(\epsilon+1)]^{1/2}\cos\phi\,\sigma_I\}. \tag{1}$$

The first two terms in the bracket correspond to virtual photoproduction by transversely polarized photons; σ_L is the cross section for longitudinal photons; and σ_I corresponds to transverse-longitudinal interference.

Using VMD, we can relate the π^+ electroproduction cross section to the ρ-production density-matrix elements and cross section[8] as follows:

$$2\pi\frac{d\sigma^\gamma}{dt\,d\phi}=\left(\frac{e}{f_\rho}\right)^2\left(\frac{m_\rho{}^2}{m_\rho{}^2+K^2}\right)^2\{\rho_{11}-\epsilon\cos2\phi\,\rho_{1-1}+\epsilon c^2\rho_{00}+2[\epsilon(\epsilon+1)]^{1/2}\cos\phi\,c\,\mathrm{Re}\rho_{10}\}\,\frac{q^2}{k}\frac{2W}{W^2-m^2}\frac{d\sigma}{dt}(\pi^-p\to\rho^0n). \tag{2}$$

q and k are, respectively, the pion and virtual-photon momenta in the pion-nucleon center-of-mass system. c describes the variation of longitudinal amplitudes with K^2. As we mentioned before, there are several different proposals concerning the value of c in the application of VMD. As in Ref. 4, we give