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[12]An analogous situation exists in the study of chiral dynamics where phenomenological fields are very convenient for formulating the chiral constraint equations for the scattering amplitudes.

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[17]If the interaction Hamiltonian is taken to be $V(k) = V_+(k)V_-(k)$, where $V_\pm(k) = :e^{ik\cdot Y_\pm(0)}:$, one obtains the Virasoro-Shapiro amplitude. M. A. Virasoro, Phys. Rev. 177, 2309 (1969). J. Shapiro, Phys. Letters 33B, 361 (1970). See also G. Domokos, S. Kovesi-Domokos, and E. Schonberg, Phys. Rev. D 2, 1026 (1970).

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Generalized Goldstone Theorems in an Effective-Lagrangian Framework

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A set of sum rules among coupling tensors of *different* rank is derived for spontaneously broken continuous symmetries in an effective-Lagrangian framework. The sum rules, among which is the Goldstone theorem, constitute in this framework the physical content of that part of the spontaneously broken symmetry not contained in the conventionally realized subgroup which leaves the vacuum invariant, the latter subsymmetry manifesting itself in Wigner-Eckart relations among coupling tensors of the *same* rank.

In a spontaneously broken[1] symmetry (SBS) group G of a Hamiltonian H, the vacuum state is invariant only against a lower symmetry G', a subgroup of G, and hence zero-mass excitations or Goldstone-Nambu particles[1,2] (GNP's) are required to support the noninvariance of the vacuum against group operations in G but not in G' which leave H invariant. The single-particle states form irreducible G' multiplets, the mass matrix is invariant against G' but not G, and group operations in G but not in G' take single-particle states into multiparticle states which have zero-mass particles in them and take the vacuum into zero-mass particle states.[1,2] Multiplets irreducible under G exist[3] but do not correspond to particle states with fixed particle number.[1,2] Wigner-Eckart selection rules arising from the G' symmetry result in the usual relations among scattering amplitudes for fixed n-particle processes.

It is interesting to inquire into the nature and physical content of the residual symmetry of H under operations not in G', that is, Wigner-Eckart selection rules involving these operations. If experience with broken chiral symmetry is a guide, and if the soft-meson limits endemic in chiral-symmetry calculations[4] take one to an underlying spontaneously broken theory with a multiplet of mesons playing the role of GNP's,[5] then the invariance of the theory in this limit against group operations not in G' results not in the usual Wigner-Eckart relations among n-particle amplitudes but in theorems for the emission or absorption of GNP's,[4] that is, relations among amplitudes having different numbers of particles in external states. This is not surprising since the fixed-particle-number states[6] cannot diagonalize the conserved charges which are generators of group operations in G but not in G'. That the results of an SBS can be extracted by Wigner-Eckart relations among states not having fixed-particle number, but diagonal in the spontaneously broken generators, has been shown for the particular case of theories invariant against c-number translations of scalar boson fields.[7] This method, while complete and straightforward, is also awkward and tedious, involving first the diagonalization process and then the restatement of the resultant Wigner-Eckart relation in terms of fixed-particle-number states. More importantly, the method does not easily generalize to the physical case in which the vacuum degeneracy is lifted by the addition of explicit symmetry-breaking terms which remove the vacuum degeneracy and give the GNP's a nonzero mass.

Apart from the practical purpose of establishing *all* physical consequences of SBS over and above what is well known[1] [namely (i) conventional invariance under G', the symmetry of the vacuum, and (ii) the appearance of GNP's as required by the Goldstone theorem] the question of full consequence of the SBS is interesting from a formal point of view because it is related to the converse problem: Given a theory invariant under G' with a vacuum also invariant under G', with a spectrum which includes a multiplet of zero-mass mesons, under what conditions is it possible to extend the symmetry G' to a higher symmetry G which is spontaneously broken?

A simple SBS framework in which these questions can be precisely posed and one of some practical importance and current interest is the effective-Lagrangian[8] or tree-approximation approach[9] in which the Lagrangian $\mathfrak{L}(\phi)$, a polynomial or entire function on a real set of scalar or pseudoscalar fields ϕ_i, with $\mathfrak{L}$ symmetric against a group of continuous orthogonal[10] transformations G on the fields,[2] has a minimal constant field strength $\tilde{\phi}_i \neq 0$, with $\tilde{\phi}_i$ (Ref. 11) a solution of

$$\frac{\partial \mathfrak{L}}{\partial \phi_i}=0, \tag{1}$$

where

$$\mathfrak{L} \equiv \tfrac{1}{2}\partial^\mu \phi_i \partial_\mu \phi_i + \sum_n \frac{1}{n!} \frac{\partial^n \mathfrak{L}}{\partial\phi_i \partial\phi_j \partial\phi_k \cdots}\bigg|_0 \phi_i\phi_j\phi_k \cdots$$

$$\equiv \tfrac{1}{2}\partial^\mu \phi_i \partial_\mu \phi_i + \sum_n \frac{1}{n!} M^{(n)}_{ijk\cdots}(0)\phi_i\phi_j\phi_k\cdots . \tag{2}$$

$\mathfrak{L}$ may also be expanded about $\tilde{\phi}_i$,

$$\begin{aligned}\mathfrak{L}(\phi) &= \tfrac{1}{2}\partial^\mu \phi_i \partial_\mu \phi_i \\ &\quad + \sum_n \frac{1}{n!} M^{(n)}_{ijk\cdots}(\tilde{\phi})(\phi_i-\tilde{\phi}_i)(\phi_j-\tilde{\phi}_j)(\phi_k-\tilde{\phi}_k)\cdots \\ &= \tfrac{1}{2}\partial^\mu \phi'_i \partial_\mu \phi'_i + \sum_n \frac{1}{n!} M^{(n)}_{ijk\cdots}(\tilde{\phi})\phi'_i\phi'_j\phi'_k\cdots \\ &\equiv \mathfrak{L}'(\phi'),\end{aligned} \tag{3}$$

where $\phi'_i \equiv \phi_i - \tilde{\phi}_i$ are displaced fields with $\tilde{\phi}'_i \equiv 0$ a solution of $\partial\mathfrak{L}'(\phi')/\partial\phi'_i=0$. If $\tilde{\phi}_i$ is identified[2,8,9,12] with $\langle 0|\phi_i|0\rangle$ and ϕ'_i with $\langle 0|\phi'_i|0\rangle$ then ϕ'_i are physical fields with zero vacuum expectation values. $\mathfrak{L}'(\phi')$, the Lagrangian in terms of physical fields, is not G-invariant but is invariant against the set of transformations $\{g'\}$ which leave $\tilde{\phi}$ invariant,

$$g'\tilde{\phi}=\tilde{\phi}, \tag{4}$$

where $\{g'\}$ form the little group $G' \equiv G_{\tilde{\phi}}$, a subgroup of G, with $G_{\tilde{\phi}}$ the vacuum symmetry.

It is clear that the $G_{\tilde{\phi}}$ invariance is realized in the usual way, that is, by the $G_{\tilde{\phi}}$ irreducible multiplets, and a $G_{\tilde{\phi}}$-invariant mass matrix and coupling tensors $M^{(n)}$.[2] Apart from this, deMottoni and Fabri[2] have shown that to operations in G but not in $G_{\tilde{\phi}}$, that is, those for which $g\tilde{\phi}\neq\tilde{\phi}$, there corresponds a field $\chi \neq 0$,

$$g\tilde{\phi} \equiv e^{\lambda Q}\tilde{\phi} \equiv \tilde{\phi} + \lambda\chi + O(\lambda^2), \tag{5}$$

which is an eigenvector of the mass matrix $M^{(2)}(\tilde{\phi})$ with eigenvalue zero, and that the number of such GNP fields is the dimension of the orbit of $\tilde{\phi}$.

It is instructive to repeat their proof of the Goldstone theorem in this framework because a generalization produces the sum rules among the coupling tensors: Since $\partial\mathfrak{L}/\partial\phi_i$ transforms like ϕ_i under G, $g\tilde{\phi}$ is also a solution of $\partial\mathfrak{L}/\partial\phi_i=0$ if $\tilde{\phi}$ is and therefore

$$\begin{aligned}0 &= \frac{\partial\mathfrak{L}}{\partial\phi_i}\bigg|_{g\tilde{\phi}} - \frac{\partial\mathfrak{L}}{\partial\phi_i}\bigg|_{\tilde{\phi}} \\ &= \frac{\partial^2\mathfrak{L}}{\partial\phi_i\partial\phi_j}\bigg|_{\tilde{\phi}}(g\tilde{\phi}-\tilde{\phi})_i + O((g\tilde{\phi}-\phi)^2)\end{aligned} \tag{6}$$

implies, for an infinitesimal transformation $g\tilde{\phi} = \tilde{\phi}+\lambda\chi$,

$$0=\lambda M^{(2)}_{ij}(\tilde{\phi})\chi_j + O(\lambda^2), \tag{7}$$

with χ therefore a GNP field, $M^{(2)}(\tilde{\phi})\chi=0$.

Thus the spontaneous breakdown imposes a strong physical constraint on the mass matrix $M^{(2)}(\tilde{\phi})$, a residual effect of the higher symmetry of $\mathfrak{L}(\phi)$. Generalizing Eq. (6) to second-rank couplings,

$$\begin{aligned}M^{(2)}_{ij}(g\tilde{\phi}) - M^{(2)}_{ij}(\tilde{\phi}) &= \frac{\partial^2\mathfrak{L}}{\partial\phi_i\partial\phi_j}\bigg|_{g\tilde{\phi}} - \frac{\partial^2\mathfrak{L}}{\partial\phi_i\partial\phi_j}\bigg|_{\tilde{\phi}} \\ &= \frac{\partial^3\mathfrak{L}}{\partial\phi_k\partial\phi_i\partial\phi_j}\bigg|_{\tilde{\phi}}(g\tilde{\phi}-\tilde{\phi})_k + O(\lambda^2) \\ &= \lambda M^{(3)}_{ijk}(\tilde{\phi})\chi_k + O(\lambda^2).\end{aligned} \tag{8}$$

However, since $\partial^2\mathfrak{L}/\partial\phi_i\partial\phi_j$ transforms like a second-rank tensor under G,

$$\begin{aligned}M^{(2)}_{ij}(g\tilde{\phi}) &= g_{il}g_{jm}M^{(2)}_{lm}(\tilde{\phi}) \\ &= (1+\lambda Q)_{il}(1+\lambda Q)_{jm}M^{(2)}_{lm}(\tilde{\phi}) + O(\lambda^2) \\ &= M^{(2)}_{ij}(\tilde{\phi}) + \lambda[Q_{il}M^{(2)}_{lj}(\tilde{\phi}) + Q_{jm}M^{(2)}_{im}(\tilde{\phi})] + O(\lambda^2).\end{aligned} \tag{9}$$

With ϕ real and g orthogonal, Q is antisymmetric[10] and we have

$$M^{(2)}_{ij}(g\tilde{\phi}) - M^{(2)}_{ij}(\tilde{\phi}) = \lambda[Q, M]_{ij} + O(\lambda^2), \tag{10}$$

whence, combining with Eq. (8),

$$M^{(3)}_{ijk}(\tilde{\phi})\chi_k = [Q, M^{(2)}(\tilde{\phi})]_{ij}, \tag{11}$$

which is a relation between third- and second-rank tensors with nontrivial physical content. The coupling between χ and physical fields $\psi^{(1)}$ and $\psi^{(2)}$ (eigenvectors of $M^{(2)}$) for example is given by

$$M^{(3)}_{ijk}\psi_i^{(1)}\psi_j^{(2)}\chi_k = \psi_i^{(1)}[Q, M^{(2)}]_{ij}\psi_j^{(2)}$$
$$= (m_2{}^2 - m_1{}^2)\psi_i^{(1)} Q_{ij}\psi_j^{(2)}. \qquad (12)$$

Thus trilinear couplings invloving at least one GNP are given by two-particle matrix elements of the spontaneously broken charge Q associated with the GNP. Equation (12) states, among other things, that GNP's do not couple trilinearly to particles with the same mass, that the couplings of two GNP's to a third particle is proportional to the third particle's mass, and that the mutual coupling of three GNP's always vanishes. These results depend on the absence, apart from kinetic-energy terms, of derivative couplings, but if derivative couplings are added the rules persist among the nonderivative couplings. The restriction to scalar or pseudoscalar fields is therefore a fundamental one because, even though, for example, vector fields are easily incorporated (with $\tilde{A}_\mu = 0$) these must couple via derivatives to two pseudoscalars. Correlations induced by spontaneous breakdown among derivative couplings similar to Eq. (12) are under study, but one cannot conclude at this stage the unlikely result $g_{NN\pi} = 0$ or $g_{\rho\pi\pi} \sim m_\rho{}^2$. In Eq. (12), of course,

$$M^{(3)}_{ijk} = M^{(3)}_{ijk}(\tilde{\phi}) = \left.\frac{\partial^3 \mathcal{L}}{\partial\phi_i\partial\phi_j\partial\phi_k}\right|_{\tilde{\phi}}$$

are the physical couplings [see Eq. (3)] among the physical fields $\phi_i' = \phi_i - \tilde{\phi}_i$ in the original basis; $\psi^{(1)}$, $\psi^{(2)}$ are arbitrary fields which diagonalize $M^{(2)}(\tilde{\phi})$, and $\chi_k = (Q\tilde{\phi})_k$. Since there is clearly a basis which diagonalizes $M^{(2)}(\tilde{\phi})$, let us assume we are in this physical and diagonal basis, and then Eq. (12) reads

$$M^{(3)}_{ijk}(Q\tilde{\phi})_k = (m_j{}^2 - m_i{}^2)Q_{ij}, \qquad (12a)$$

where Q is any broken generator. It is clear from (12a) that a GNP does not couple to two identical particles, or two particles of the same mass. At this point one might question whether Eq. (12a) has any content beyond that implied by the $G_{\tilde{\phi}}$. Equation (17) below addresses that question, but for illustration consider the simple SU(2) scalar-field model (i = 1, 2, 3)

$$\mathcal{L} = \tfrac{1}{2}\partial^\mu\phi_i\partial_\mu\phi_i + (\phi^2 - a^2)^2,$$

with SBS solution $\tilde{\phi}_i = a\delta_{i3}$. With $\phi_i' \equiv \phi_i - \tilde{\phi}_i$,

$$\mathcal{L} = \tfrac{1}{2}\partial^\mu\phi_i'\partial_\mu\phi_i' + (\phi'^2)^2 + 2a\phi_3'\phi'^2 + 4\phi_3'^2 a^2,$$

$$M^{(2)}_{ij}(\tilde{\phi}) = 8a^2\delta_{i3}\delta_{j3} = m_3{}^2\delta_{i3}\delta_{j3},$$

$$M^{(3)}_{ijk}(\tilde{\phi}) = 8a(\delta_{k3}\delta_{ij} + \delta_{ik}\delta_{j3} + \delta_{jk}\delta_{i3}),$$

where $\phi_1 = \phi_1'$ and $\phi_2 = \phi_2'$ are GNP fields. Equation (12a) requires (with $Q_{ij} = \epsilon_{1ij}$ or ϵ_{2ij})

$$M^{(3)}_{ijk}Q_{kl}a\delta_{l3} = (m_j{}^2 - m_i{}^2)Q_{ij},$$

$$M^{(3)}_{ij2} = \frac{m_j{}^2 - m_i{}^2}{a}\epsilon_{1ij},$$

$$M^{(3)}_{ij1} = \frac{m_i{}^2 - m_j{}^2}{a}\epsilon_{2ij},$$

which requires not only $M^{(3)}_{ii2} = 0 = M^{(3)}_{ii1}$ (implied already by charge conservation) but also

$$M_{322} = \frac{m_3{}^2}{a} = \frac{m_3{}^2}{\tilde{\phi}_3} = \frac{m_3{}^2}{\langle\phi_3\rangle} = M_{311},$$

which is not implied by $G_{\tilde{\phi}}$ and of course is consistent with the couplings calculated directly from $\mathcal{L}(\phi')$. In the more physical case of chiral symmetry broken to SU(3), with ϕ_i a $(3, 3^*)\oplus(3^*, 3)$ doublet nonet (u_i, v_i) of scalar and pseudoscalar fields with $\langle u_0\rangle = \tilde{u}_0 \neq 0$ and the v_i octet a GNP multiplet, the mutual coupling of 3 GNP's is forbidden already by parity but the mass conditions are not obvious, i.e., we have then, from (12a), $g_{\pi K\kappa} \sim m_\kappa{}^2/\langle u_0\rangle$ and $g_{\pi\sigma\pi} \sim m_\sigma{}^2/\langle u_0\rangle$ in the $m_\pi{}^2 = m_K{}^2 = 0$ SBS limit.

This procedure easily generalizes to coupling tensors of arbitrary rank. With the notation

$$M^{(n)}(\psi^{(1)}, \psi^{(2)}, \psi^{(3)}, \ldots, \psi^{(n)}) \equiv M^{(n)}_{ijk\ldots p}\psi_i^{(1)}\psi_j^{(2)}\psi_k^{(3)}\cdots\psi_p^{(n)}, \qquad (13)$$

we have

$$M^{(n+1)}(\psi^{(1)}, \psi^{(2)}, \psi^{(3)}, \ldots, \psi^{(n)}, \chi) = \sum_{\text{perm}} M^{(n)}(Q\psi^{(1)}, \psi^{(2)}, \psi^{(3)}, \ldots, \psi^{(n)}), \qquad (14)$$

where $\sum_{\text{perm}}$ means the sum of all possible distributions of Q among the fields $\psi^{(1)}, \ldots, \psi^{(n)}$. The sum rules relate the $(n+1)$-rank coupling of n arbitrary fields to the n-rank coupling of the n fields $Q\psi^{(1)}, \psi^{(2)}, \ldots, \psi^{(n)}$ (plus permutations). For $n = 1$, Eq. (14) reduces to the Goldstone theorem and with $n > 1$ we have its generalization. If one of the fields $\psi^{(1)}, \ldots, \psi^{(n)}$ is itself a GNP [not necessarily the same as the χ in Eq. (14)] then at least all but one of the $M^{(n)}$ terms may be further reduced to $M^{(n-1)}$ couplings, and so on. If all the $\psi^{(1)}, \ldots, \psi^{(n)}$ are multiples of $\tilde{\phi}$, the vacuum direction, then

$$M^{(n+1)}(\tilde{\phi}, \tilde{\phi}, \ldots, \tilde{\phi}, \chi) = nM^{(n)}(\tilde{\phi}, \tilde{\phi}, \ldots, \tilde{\phi}, \chi)$$
$$= n!M^{(2)}(\tilde{\phi}, \chi)$$
$$= 0. \qquad (15)$$

That is, fields with $\tilde{\phi}$ quantum numbers do not couple to a single GNP.

Had we applied the process leading to Eq. (14)

to a transformation in $G_{\tilde{\phi}}$, with generator Q, $Q\tilde{\phi}=0$, we would have obtained

$$M^{(n)}(\psi^{(1)}, \psi^{(2)}, \ldots, Q\tilde{\phi})=0 = \sum_{\text{perm}} M^{(n)}(Q\psi^{(1)}, \psi^{(2)}, \ldots, \psi^{(n)}), \tag{16}$$

which simply states the $G_{\tilde{\phi}}$ invariance of the $M^{(n)}$ couplings, implying the usual Wigner-Eckart relations for the conventionally realized symmetry $G_{\tilde{\phi}}$; for example, with $n=2$, Eq. (16) implies

$$[Q, M^{(2)}]=0, \quad Q \text{ a generator of } G_{\tilde{\phi}}. \tag{17}$$

If $\mathfrak{L}(\phi)$ is a polynomial of degree N, then $M^{(N+1)}=0 \Rightarrow \sum_{\text{perm}} M^{(n)}(Q\psi^{(1)}, \psi^{(2)}, \ldots, \psi^{(n)})=0$, implying the full G invariance of the highest-rank coupling.

Consider now the case in which explicit symmetry-breaking terms appear in $\mathfrak{L}(\phi)$: If to $\mathfrak{L}(\phi)$ is added a G-symmetry breaking but $G_{\tilde{\phi}}$-invariant term, $-\delta\tilde{\mathfrak{L}}(\phi)$, $\mathfrak{L}'(\phi)=\mathfrak{L}(\phi)-\delta\tilde{\mathfrak{L}}(\phi)$, with new vacuum $\tilde{\phi}$ a solution of

$$0=\frac{\partial \mathfrak{L}'}{\partial \phi_i}\bigg|_{\tilde{\phi}} = \frac{\partial \mathfrak{L}}{\partial \phi_i}\bigg|_{\tilde{\phi}} - \delta\frac{\partial \tilde{\mathfrak{L}}}{\partial \phi_i}\bigg|_{\tilde{\phi}}, \tag{18}$$

and if $\tilde{\mathfrak{L}}(\phi)$ is a polynomial of degree N', then the sum rules

$$M'^{(n+1)}(\psi^{(1)}, \psi^{(2)}, \ldots, \psi^{(n)}, \chi) = \sum_{\text{perm}} M'^{(n)}(Q\psi^{(1)}, \psi^{(2)}, \ldots, \psi^{(n)}) \tag{19}$$

persist, even in the presence of explicit symmetry breaking, $\tilde{\mathfrak{L}}(\phi)$, for $n>N'$. The sum rules with $n\leq N'$ are modified by terms arising from the $\tilde{M}^{(n)}$, $n\leq N'$, which do not transform like $M^{(n)}$. For example, if to $\mathfrak{L}(\phi)$ is added a linear breaking term, popular[12] in some spontaneously broken effective Lagrangians,

$$\mathfrak{L}'(\phi)\equiv \mathfrak{L}(\phi)-\delta\tilde{\phi}_i\phi_i, \tag{20}$$

with $\delta\tilde{\phi}_i\phi_i$ the $G_{\tilde{\phi}}$-invariant term linear in the fields ϕ, then Eq. (14) persists for $n>1$ whereas for $n=1$ we have, instead of the Goldstone theorem,

$$\frac{\partial \mathfrak{L}'}{\partial \phi_i}\bigg|_{g\tilde{\phi}} = \lambda M'^{(2)}_{ij}\chi_j + O(\lambda^2), \tag{21}$$

with

$$\begin{aligned}\frac{\partial \mathfrak{L}'}{\partial \phi_i}\bigg|_{g\tilde{\phi}} &= \frac{\partial \mathfrak{L}}{\partial \phi_i}\bigg|_{g\tilde{\phi}} - \delta\tilde{\phi}_i \\ &= g_{ij}\frac{\partial \mathfrak{L}}{\partial \phi_j}\bigg|_{\tilde{\phi}} - \delta\tilde{\phi}_i \\ &= \delta g_{ij}\tilde{\phi}_j - \delta\tilde{\phi}_i \\ &= \delta\tilde{\phi}_i + \delta\lambda(Q\tilde{\phi})_i - \delta\tilde{\phi}_i + O(\lambda^2) \\ &= \delta\lambda\chi_i + O(\lambda^2),\end{aligned} \tag{22}$$

and thus

$$M'^{(2)}_{ij}(\tilde{\phi})\chi_j = \delta\chi_j, \tag{23}$$

with $Q\tilde{\phi}=\chi$ still an eigenvector of the mass matrix but now with eigenvalue $\delta=m_\chi^2$.

A host of other sum rules may be derived by considering G transformations to $O(\lambda^m)$ of $M^{(n)}$, resulting in sum rules among tensors of rank $(n+m)$, $(n+m-1), \ldots, n$, involving the couplings of m GNP's to n other particles; these are, however, not independent of Eq. (14) but merely the result of successive reduction of these sum rules.

It is noteworthy that all the sum rules are independent of the specific form of $\mathfrak{L}(\phi)$ (which is an arbitrary entire function of the G-invariant forms) and therefore do not suffer from the arbitrariness of many explicit effective-Lagrangian models. Since the rules are entirely, in this framework, the result of the G-nonvariant vacuum and the transformation properties dictated by the higher symmetry, it is possible that they can in some form be extended, without the embarrassment of a Lagrangian, to a more sophisticated axiomatic or S-matrix approach to SBS.[1]

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[1]For a review see G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, in *Advances in Particle Physics*, edited by R. L. Cool and R. E. Marshak (Interscience, New York, 1968), Vol. II.

[2]For discussion relevant to an effective-Lagrangian framework see P. deMottoni and E. Fabri, Nuovo Cimento 54A, 42 (1968).

[3]These are the field multiplets.

[4]Stephen L. Adler and Roger F. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968), and references quoted therein.

[5]R. F. Dashen, Phys. Rev. 183, 1245 (1969); Phys. Rev. D 3, 1879 (1971), and references quoted therein.

[6]Take the vacuum, for example.

[7]G. Kramer and W. F. Palmer, Phys. Rev. 182, 1492 (1969).

[8]For a review see S. Gasiorowicz and D. Geffen, Rev. Mod. Phys. 41, 531 (1969).

[9]See references in Douglas W. McKay and William F. Palmer, Phys. Rev. D 5, 858 (1972).

[10]The unnecessary restriction to real fields and orthogonal representation, rather than complex fields and unitary representation, is purely a convenience to avoid covariant and contravariant indices.

[11]Parity invariance requires $\tilde{\phi}_i=0$ for *pseudo*scalar components.

[12]P. Carruthers and Richard W. Haymaker, Phys. Rev. D 4, 1808 (1971); Phys. Rev. Letters 27, 445 (1971).