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Nonexistence of Baryon Number for Black Holes. II*

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In a previous paper we showed that a static (nonrotating) black hole cannot be endowed with exterior scalar-meson or massive vector-meson fields. Here we show that the same is true for massive spin-2 meson fields. We also extend the above results to the case of a rotating stationary black hole. We conclude from our results that a black hole in its final (static or stationary) state cannot interact with the exterior world via the strong interactions which are mediated by meson fields such as the π (scalar), ρ (vector), and f (spin-2). A direct consequence of this is the impossibility of determining the baryon number of the black hole by means of exterior measurements alone. This results in the transcendence of the law of baryon-number conservation as originally predicted by Wheeler. All the above conclusions hold both in general relativity and in the Brans-Dicke theory. We also show that the result of Hawking, that stationary black holes are identical in general relativity and in Brans-Dicke theory, holds even when the effects of the strong interactions (of the stellar material out of which the black hole was formed) are taken into account. Our final conclusion is that in both theories the Israel-Carter conjecture that "all stationary single-black-hole exteriors are of the Kerr-Newman type" holds even when the strong interactions are "turned on."

I. INTRODUCTION

A rather well-established result in black-hole physics is the Israel-Carter conjecture[1]: The exterior of a single black hole in a stationary state is always completely described by one of the Kerr-Newman solutions of Einstein's equations, with mass, charge, and angular momentum as the only parameters.[2] Support for the conjecture comes from theorems of Israel,[3] Carter,[4] Hawking,[5] and Wald.[6] These theorems are mainly concerned with the effects of gravitation and electromagnetism. The effects of a massless scalar field on the problem were examined by Chase,[7] Price,[8] and Fackerell and Ipser.[9] Their conclusion was that a nonsingular stationary field, other than a constant, cannot exist in Schwarzschild (Price), general static (Chase), or Kerr (Fackerell and Ipser) backgrounds which extend down to their respective event horizons. This conclusion is consistent with the conjecture.

The effects of the weak interactions in black-hole physics have been considered by Hartle[10] and by Teitelboim.[11] Hartle concludes that Kerr black holes cannot interact with the exterior world through weak-interaction forces. Teitelboim has shown, for the special case of spherical symmetry, that the lepton number of a black hole cannot be measured from the exterior by means of the weak interactions. Both of these results are in agreement with the conjecture.

The possible effects of the strong interactions in black-hole physics were considered by the author in a previous paper[12] which will be referred to as I from now on. It was shown there that a generic static black hole cannot be endowed with exterior massive scalar-meson or massive vector-meson fields, regardless of whether the fields are charged or neutral. The proof given was of a classical nature, but a simple argument suggesting that similar results are to be expected from a full quantum treatment was also given. Since then, Teitelboim[13] has considered the problem of a point source (baryon) of scalar- and vector-meson fields which is being lowered into a Schwarzschild black hole. He finds, in agreement with our results, that the effects of the meson fields in the exterior vanish as the source nears the horizon.

The significance of the above results is twofold. First, the author's results in I, when combined with Israel's theorems,[3] show that a static black-hole exterior is necessarily of the Schwarzschild or Reissner-Nordström types, even if one allows for the effects of the strong interactions (of the material out of which the black hole was formed) by *a priori* allowing exterior scalar- and vector-meson fields such as the π and ρ.[12] On the other hand, both Teitelboim's and the author's results support the prediction of Wheeler that the law of baryon-number conservation ought to be transcended in black-hole physics.[14] For suppose that a baryon is dropped, or lowered into a Schwarzschild black hole. When the exterior of the black hole has become static again,[15] there cannot be left in it any of the meson fields characteristically associated with baryons. It is thus impossible to establish, from measurements made exclusively in the exterior, that the black hole's baryon number has increased by one in the process. It is in this sense that the law of baryon-number conservation is transcended. One can also say that the black hole has no well-defined baryon number.[12]

Our purpose in this paper is to extend the results of I in three directions. In Sec. II we extend the conclusions of I to the case of massive (neutral) spin-2 meson fields. With this generalization all the meson fields known at present (scalar, vector, and spin-2) are excluded from a static black-hole exterior. In Secs. III–VI we show that all the mentioned meson fields (neutral) are also excluded from the exteriors of stationary (rotating) black holes. Our work is considerably simplified by the theorem of Hawking which states that rotating stationary black holes are always axisymmetric.[5] We conclude in Sec. VII that a black hole in its final state does not have a well-defined baryon number. All the previous conclusions are equally valid in general relativity and in the Brans-Dicke theory.

Hawking,[16] and independently the author,[17] have shown that the exterior of a stationary black hole is identical in general relativity and in the Brans-Dicke theory, a result first conjectured by Penrose.[18] We show in Sec. VIII that the identity of stationary black holes in both theories holds even when the effects of the weak and strong interactions are taken into account. From the exclusion of exterior meson fields, and from the results of other workers previously mentioned,[3-11] we draw the conclusion that the exterior of a single black hole in its final state is necessarily of the Kerr-Newman type (Israel-Carter conjecture), even when the strong and weak interactions are "turned on." Finally, in Sec. IX we show that all our conclusions retain their validity when the assumption of minimal gravitational coupling for meson fields is relaxed, and certain direct couplings to the curvature are allowed.

In this paper Greek indices run from 0 to 3, Latin ones from 1 to 3; our signature convention is $(-+++)$. Commas will denote partial derivatives, semicolons covariant derivatives. We collect here some results from I which we shall require. We describe a meson field with components Φ_k (k takes on one value for a scalar field, four for a vector field, etc.) by a Lagrangian density $\mathcal{L}(\Phi_k, \Phi_{k,\mu})$ which satisfies the Euler-Lagrange equations arising from the variational principle $\delta\int \mathcal{L}(-g)^{1/2}d^4x=0$:

$$(-g)^{-1/2}\left[(-g)^{1/2}\frac{\partial\mathcal{L}}{\partial\Phi_{k,\mu}}\right]_{,\mu}-\frac{\partial\mathcal{L}}{\partial\Phi_k}=0. \tag{1}$$

By multiplication of (1) by Φ_k, summation over k, integration over the entire black-hole exterior, and use of Gauss's theorem we arrive at

$$-\int b^\mu dS_\mu+\sum_k\int\left[\Phi_{k,\mu}\frac{\partial\mathcal{L}}{\partial\Phi_{k,\mu}}+\Phi_k\frac{\partial\mathcal{L}}{\partial\Phi_k}\right](-g)^{1/2}d^4x=0, \tag{2}$$

where

$$b^\mu=\sum_k\Phi_k\frac{\partial\mathcal{L}}{\partial\Phi_{k,\mu}}. \tag{3}$$

In (2) dS_μ is the element of hypersurface (the horizon, spatial infinity, and past and future timelike infinity) which bounds the black-hole exterior.

The event horizon is (by definition) a closed, null, and nonsingular hypersurface. Its normal n_μ satisfies $n_\mu n^\mu=0$. Consider dS_μ at the horizon, and recall the definition of dS_μ in terms of the determinant (Det) of three coordinate intervals spanning the surface element[19]:

$$dS_\mu=\tfrac{1}{6}(-g)^{1/2}e_{\mu\alpha\beta\gamma}\,\mathrm{Det}(dx_1^\alpha, dx_2^\beta, dx_3^\gamma),$$

where $e_{\mu\alpha\beta\gamma}$ is the totally antisymmetric symbol with values $\pm1, 0$. In regular (Kruskal-like) coordinates, which must exist by the requirement that the event horizon be nonsingular, the dS_μ are clearly nonsingular. Let us write $dS_\mu=d\sigma n_\mu$ where $d\sigma$ is an invariant. This can always be done. Since the dS_μ are nonsingular, the $d\sigma$ must also be nonsingular. From the invariant character of $d\sigma$ it follows that

$$dS_\mu dS^\mu = d\sigma^2 n_\mu n^\mu = 0 \text{ on horizon} \tag{4}$$

in all coordinate systems. In particular, we shall work with asymptotically flat coordinates. Since we shall confine our attention to stationary situations, we choose the metric so that $g_{\mu\nu,0}=0$.

II. EXCLUSION OF SPIN-2 MESON FIELDS IN STATIC CASE

The spin-2 field, first studied by Fierz and Pauli,[20] is described by a symmetric tensor field $h_{\mu\nu}$ which in flat space obeys the field equation

$$h_{\mu\nu,\alpha,}{}^{\alpha} - m^2 h_{\mu\nu} = 0, \tag{5}$$

and the supplementary conditions

$$h^\mu{}_\mu = 0, \tag{6}$$

and

$$h^{\mu\nu}{}_{,\nu} = 0. \tag{7}$$

Here m^{-1} is the Compton wavelength of the field. We shall only consider the case $m \neq 0$. In curved space we shall replace (5) by

$$h_{\mu\nu;\alpha;}{}^{\alpha} - m^2 h_{\mu\nu} = 0, \tag{8}$$

which is derivable from the Lagrangian density

$$\mathfrak{L} = -\tfrac{1}{2}(h_{\mu\nu;\alpha} h^{\mu\nu;\alpha} + m^2 h_{\mu\nu} h^{\mu\nu}). \tag{9}$$

[In Sec. IX we shall consider the possibility of modifying (8) and (9) by the addition of extra terms involving the curvature tensor.] We take over the condition (6) unchanged; but the straightforward generalization of (7) to curved space is inconsistent with (8) because of the noncommutativity of covariant derivatives. Since we do not require the correct condition for our proof, we shall not try to discover it.

We consider the field in a static black-hole exterior. The metric can always be chosen as

$$ds^2 = g_{00}(dx^0)^2 + g_{ij} dx^i dx^j . \tag{10}$$

In I we showed that, in the black-hole exterior, and on the horizon, $g_{00} \leq 0$ and the g_{ij} is a positive definite metric. We have not only $g_{\mu\nu,0}=0$, but also $h_{\mu\nu,0}=0$. After all, the field equation (8) has no gauge invariance; the $h_{\mu\nu}$ are in this sense physical quantities. They appear directly in the stress-energy tensor, etc. Therefore, they cannot depend on x^0 in a static situation and in static coordinates.

According to (3) we have

$$b^\mu = -h_{\alpha\beta} h^{\alpha\beta;\mu} = -\tfrac{1}{2}(h_{\alpha\beta} h^{\alpha\beta})_{,}{}^{\mu}. \tag{11}$$

Clearly from what we have just said

$$b^0 = 0. \tag{12}$$

Using this we proceed to show that the boundary integral over timelike infinity in (2) vanishes. Because $g^{00} = (g_{00})^{-1} < 0$ in the exterior, the boundary at timelike infinity may be chosen to have $n_i = 0$ and $dS_i = 0$; the normal n_μ is then timelike as required. Then $b^\mu dS_\mu = b^0 dS_0 = 0$ by (12), and the boundary integral over timelike infinity vanishes.

It is also simple to show that the boundary term in (2) receives no contributions from spacelike infinity either. The static solution of (8) which is bounded asymptotically has the behavior (recall that asymptotically the metric becomes flat) $h_{\mu\nu} \sim r^{-1} e^{-mr}$ where r has the familiar meaning. Thus for $m \neq 0$ the b^i fall off exponentially, and the boundary at spatial infinity contributes nothing to (2).

The boundary term over the horizon also vanishes. The horizon is static; thus $n_0 = 0$ and $dS_0 = 0$ on it. Using (12) we have[21]

$$\begin{aligned}(b^\mu dS_\mu)^2 &= (g_{ij} b^i dS^j)^2 \\ &\leq (g_{ij} b^i b^j)(g_{lk} dS^l dS^k) \\ &= b^\mu b_\mu dS^\nu dS_\nu ,\end{aligned} \tag{13}$$

where we have used Schwarz's inequality (applied to the positive definite metric g_{ij}) in the second step.[12] If we substitute (4) into (13) we see that $b^\mu dS_\mu = 0$ on the horizon provided $b^\mu b_\mu$ is bounded there. But b^μ is constructed out of $h_{\mu\nu}$ and $h_{\mu\nu;\alpha}$, both physical quantities in the sense that they are not subject to any gauge transformations, and as a result appear explicitly in the stress-energy tensor for the field. Thus $b^\mu b_\mu$ is a physical scalar, and as discussed in I, a physical scalar must be bounded on a (nonsingular) horizon. Therefore, $b^\mu dS_\mu$ vanishes on the horizon. We have thus shown that the boundary term in (2) vanishes. The remaining integral in (2) is just

$$\int (h_{\mu\nu;\alpha} h^{\mu\nu;\alpha} + m^2 h_{\mu\nu} h^{\mu\nu})(-g)^{1/2} d^4x = 0. \tag{14}$$

We proceed to show that the integrand above is positive definite. First, we note that in the static case h_{0i} must vanish identically. One way to see this is to realize that under time reversal ($x^0 \to -x^0$) $h_{0i} \to -h_{0i}$ just by the transformation properties of tensors. But since h_{0i} is physical, it cannot change under time reversal in a *static* situation; hence, $h_{0i} = 0$. It follows from this that

$$h_{\mu\nu;0} = 0 \quad \text{and} \quad h_{0i;j} = 0. \tag{15}$$

Remembering this let us consider the scalar $h_{\mu\nu} h^{\mu\nu}$ appearing in (14). Since it is a scalar, we can evaluate it in any coordinates. Thus we make a transformation of the space coordinates x^i only which has the property of bringing g_{ij} to its diagonal form $\lambda_i \delta^j{}_i$ *at a given point P*. We denote the quantities in the new coordinates by bars. Thus we have, according to the rules for transforming tensors, that $\bar{h}_{00}=h_{00}$, $\bar{g}_{00}=g_{00}$, $\bar{h}_{0i}=h_{0i}=0$, and $\bar{g}_{0i}=g_{0i}=0$. We now recall that because g_{ij} is a positive-definite metric in the black-hole exterior, its eigenvalues at P, the λ_i, must be positive. Thus

$$h_{\mu\nu}h^{\mu\nu}=(\bar{g}_{00}\bar{h}^{00})^2+\sum_{ij}\lambda_i\lambda_j(\bar{h}^{ij})^2\geq 0. \tag{16}$$

A very similar argument, taking (15) into account, leads to

$$h_{\mu\nu;\alpha}h^{\mu\nu;\alpha}\geq 0. \tag{17}$$

The point P was perfectly arbitrary; thus, (16) and (17) must hold everywhere in the exterior. Therefore, the integrand of (14) is positive definite everywhere, and the integral can vanish only if all the $h_{\mu\nu}=0$ identically. We thus conclude that *a massive spin-2 field, for example, the f-meson field, must vanish identically in a static black-hole exterior*

The proof we have given breaks down in the case $m=0$ because then (8) is gauge-invariant under $h_{\mu\nu}\to h_{\mu\nu}+eg_{\mu\nu}$ where e is a constant. Thus $h_{\mu\nu}$ is no longer physical (like the A_μ of electromagnetism) and we cannot argue that $b^\mu b_\mu$ is a physical scalar, and is therefore bounded on the horizon; the proof fails. Therefore, we cannot exclude a massless spin-2 field of the type (8) from a static black-hole exterior. However, no such field is known in nature, so we need not worry about this possibility. (Gravitation is a self-coupled, nonlinear, spin-2 field; it clearly does not belong to the class of fields discussed above.)

III. SOME PROPERTIES OF ROTATING STATIONARY BLACK HOLES

From now on we confine our attention to rotating stationary black holes. Fortunately, we need not consider situations with arbitrary symmetry. A theorem of Hawking[5] assures us that a rotating stationary black hole is necessarily axisymmetric, and in addition, its event horizon must have the topology of a sphere. The basic conditions for this theorem to hold are that any fields present in the exterior obey well behaved (with respect to the Cauchy problem) hyperbolic equations, and that the fields satisfy the (weak) positive-energy condition. Both of the above requirements hold for the fields we consider in this paper, so from now on we restrict our attention to black holes with topologically spherical horizons, and axisymmetric exteriors.

Such a black-hole exterior may be covered by a coordinate system for which the line element takes the form[4]

$$\begin{aligned}ds^2&=g_{tt}dt^2+2g_{t\varphi}dtd\varphi\\&\quad+g_{\varphi\varphi}d\varphi^2+W(d\rho^2+dz^2),\end{aligned} \tag{18}$$

where t is the time, φ the axial angular variable, and $g_{\mu\nu,t}=g_{\mu\nu,\varphi}=0$. We may arrange it so that the asymptotic form of (18) is

$$ds^2=-dt^2+\rho^2d\varphi^2+d\rho^2+dz^2, \tag{19}$$

the familiar one for flat space in cylindrical coordinates.

The determinant of the metric of (18) must be nonpositive. Thus

$$g_{tt}g_{\varphi\varphi}-(g_{t\varphi})^2\leq 0. \tag{20}$$

We assume here that causality holds everywhere in the black-hole exterior. We therefore must have

$$g_{\varphi\varphi}>0 \tag{21}$$

and

$$W>0. \tag{22}$$

For suppose that $g_{\varphi\varphi}$ were negative or zero at some point (and, by the axial symmetry, also on a circle). Then the curve of constant t, ρ, and z at that point, and with φ running from 0 to 2π, would be a closed timelike or null curve. This would imply a violation of causality. Hence (21) must hold everywhere in the exterior. Similarly, if W were negative somewhere, then a closed curve with fixed t and φ, and with ρ and z confined to the region in which $W<0$, would be a closed timelike curve. And if W were zero on more than just an isolated circle (running in the φ direction), one could likewise construct a closed null curve. Therefore, (22) must hold everywhere in the exterior, except possibly on isolated circles. Conditions (20)–(22) all hold in the exteriors of Kerr-Newman black holes.[2]

Later on (in Secs. V and VI) we shall find it useful to refer components of tensors to orthonormal frames. Therefore, we shall define here such a system of frames. We represent the frames by the orthonormal one-forms

$$\begin{aligned}\omega^0&=[(g_{t\varphi})^2/g_{\varphi\varphi}-g_{tt}]^{1/2}dt,\\\omega^1&=W^{1/2}d\rho,\\\omega^2&=W^{1/2}dz,\end{aligned} \tag{23}$$

and

$$\omega^3=(g_{\varphi\varphi})^{1/2}(d\varphi+g_{t\varphi}dt/g_{\varphi\varphi}),$$

where $ds^2=\eta_{\mu\nu}\omega^\mu\omega^\nu$, and $\eta_{\mu\nu}$ is the Lorentz metric. These one-forms are, according to (20), (21), and (22), well defined everywhere in the exterior, except possibly on isolated circles (sets of measure zero).

IV. EXCLUSION OF SCALAR-MESON FIELDS IN ROTATING STATIONARY CASE

As in I we describe a neutral spinless meson field by a scalar field ψ whose Lagrangian density is

$$\mathfrak{L} = -\tfrac{1}{2}(\psi_{,\mu}\psi_{,}{}^{\mu} + m^2\psi^2). \tag{24}$$

Here again m^{-1} is the Compton wavelength of the field. From (24) follows the Klein-Gordon equation for ψ in curved space:

$$\psi_{,\mu;}{}^{\mu} - m^2\psi = 0. \tag{25}$$

We consider this scalar field in the exterior of a rotating stationary black hole. According to the discussion of Sec. III we take the metric to be that of (18). In equilibrium the field must share the symmetries of the geometry, so that

$$\psi_{,t} = \psi_{,\varphi} = 0. \tag{26}$$

According to the definition of b^μ, Eq. (3), we have

$$b^\mu = -\psi\psi_{,}{}^{\mu}. \tag{27}$$

It follows from this and (26) that

$$b^t = b^\varphi = 0. \tag{28}$$

We now proceed to show that the boundary integral in (2) vanishes. As in the static case, the boundary at timelike infinity may be chosen so that on it $n_i = 0$ and $dS_i = 0$. The reason is that according to (20) and (21),

$$g^{tt} = g_{\varphi\varphi}[g_{tt}g_{\varphi\varphi} - (g_{t\varphi})^2]^{-1} < 0 \tag{29}$$

so that the normal $n_\mu = (n_t, 0, 0, 0)$ is timelike as required. We then have that $b^\mu dS_\mu = b^t dS_t = 0$ by (28); thus, the boundary integral over timelike infinity vanishes.

The asymptotically bounded solution of the field equation (25), which has the symmetries (26), has the asymptotic behavior [recall the asymptotic form of the metric (19)] $\psi \sim r^{-1}e^{-mr}$, where $r = (\rho^2 + z^2)^{1/2}$. Therefore, for $m \neq 0$, b^ρ and b^z vanish exponentially. For $m = 0$ the solution of (25) is determined only up to an additive constant (besides the familiar multiplicative constant). We work with the solution which vanishes asymptotically. Then (25) shows that $\psi \sim r^{-1}$ so that b^ρ and b^z vanish as r^{-3}. Thus for both $m \neq 0$ and $m = 0$ the boundary integral in (2) receives no contributions from spacelike infinity.

We now show that the contribution to (2) of the boundary at the horizon also vanishes. The horizon must be stationary and axisymmetric; thus on it $n_t = n_\varphi = 0$, and so $dS_t = dS_\varphi = 0$. If we now take (28) into account, we have[21]

$$\begin{aligned}(b^\mu dS_\mu)^2 &= W^2(b^\rho dS^\rho + b^z dS^z)^2 \\ &\leq W^2[(b^\rho)^2 + (b^z)^2][(dS^\rho)^2 + (dS^z)^2] \\ &= b^\mu b_\mu dS^\nu dS_\nu,\end{aligned} \tag{30}$$

where we have used Schwarz's inequality in the second step. It follows from (30) and (4) that $b^\mu dS_\mu = 0$ on the horizon provided $b^\mu b_\mu$ is bounded there. In I we showed that $b^\mu b_\mu = \psi^2\psi_{,\mu}\psi_{,}{}^{\mu}$ can be expressed entirely in terms of the contractions of the stress-energy tensor, $T^\mu{}_\mu$ and $T_{\mu\nu}T^{\mu\nu}$, which are physical scalars. Thus $b^\mu b_\mu$ is a physical scalar, and must be bounded on a (nonsingular) horizon. The preceding discussion is strictly true only for $m \neq 0$. But for $m = 0$ one must also *assume* that ψ is bounded at the horizon (a reasonable boundary condition) in order to establish that $b^\mu b_\mu$ is bounded.[12] We thus see that, both when $m \neq 0$ and when $m = 0$, $b^\mu dS_\mu$ vanishes on the horizon. We have thus established that the boundary integral in (2) vanishes for all non-negative m.

In view of the above the volume integral in (2) reduces to

$$\int \{W[(\psi_{,}{}^{\rho})^2 + (\psi_{,}{}^{z})^2] + m^2\psi^2\}(-g)^{1/2}d^4x = 0. \tag{31}$$

We recall that W is non-negative. For $m \neq 0$ the integral above, having a positive definite integrand, can vanish only if ψ vanishes identically. For $m = 0$ (31) only implies that all the derivatives of ψ vanish identically so that ψ is a constant. But we chose ψ to be asymptotically vanishing; hence ψ vanishes identically in this case also.

We conclude that *a massive real scalar field, the π-meson field, for example, must vanish everywhere in the exterior of a rotating stationary black hole.* On the other hand, *a massless real scalar field must be a constant everywhere in the exterior* (this is the additive constant mentioned earlier). A special case of this last result (with the Kerr geometry as background) was proven by Fackerell and Ipser.[9]

V. EXCLUSION OF MASSIVE VECTOR-MESON FIELDS IN ROTATING STATIONARY CASE

We now consider a neutral, massive spin-1 meson field. As in I we describe such a field by a 4-vector field B_μ, and an antisymmetric tensor field $H_{\mu\nu}$ defined by

$$H_{\mu\nu} = B_{\nu,\mu} - B_{\mu,\nu}. \tag{32}$$

The field equations are derived from the Lagrangian density

$$\mathfrak{L} = -(H_{\mu\nu}H^{\mu\nu} + 2m^2B_\mu B^\mu)/16\pi. \tag{33}$$

They are

$$H^{\mu\nu}{}_{;\nu} + m^2 B^\mu = 0, \tag{34}$$

the familiar Proca equations in a form appropriate to curved space. Here again m^{-1} is the Compton wavelength of the field; we consider the case $m \neq 0$ only. The $H^{\mu\nu}$ is entirely analogous to the electromagnetic $F^{\mu\nu}$. But unlike the electromagnetic A_μ, B_μ is not subject to gauge changes. B_μ is a physical field; in fact, it is uniquely determined by (34) if $H^{\mu\nu}$ is known.

We consider the vector field in the exterior of a rotating stationary black hole; the metric is (18). In equilibrium the field must share the symmetries of the geometry; thus B_μ and $H^{\mu\nu}$ are independent of t and φ. Furthermore,

$$B_\rho = B_z = H_{z\rho} = H_{t\varphi} = 0. \tag{35}$$

This follows by analogy with the electromagnetic case, or from the requirement that the components $T_{t\rho}$ and T_{tz} of the stress-energy tensor derived from (33) vanish (they represent energy fluxes not consistent with the symmetries of the problem).

From (3) we have

$$b^\mu = -H^{\mu\nu} B_\nu / 4\pi, \tag{36}$$

and from (35) it follows that

$$b^t = b^\varphi = 0. \tag{37}$$

These conditions are formally identical to those for the scalar field, (28). Therefore, the proof that the contribution from timelike infinity to the boundary integral in (2) vanishes is identical to that given in Sec. IV. The asymptotically bounded solutions of (34) fall off exponentially, so the contribution from the boundary at spatial infinity also vanishes. Finally, the proof that the boundary integral over the horizon vanishes is again identical to that given for the scalar field. The necessary boundedness of $b^\mu b_\mu$ on the horizon is established by noting that this quantity is constructed entirely from $H^{\mu\nu}$ and B_ν, so that it is a physical scalar, and must therefore be bounded on a (nonsingular) horizon. Thus the boundary integral in (2) vanishes.

The remaining integral (multiplied by 4π)

$$-\tfrac{1}{2}\int [H_{\mu\nu}H^{\mu\nu} + 2m^2 B_\mu B^\mu](-g)^{1/2} d^4x = 0 \tag{38}$$

does not, unfortunately, have a positive definite integrand. Thus we shall have to proceed in a different way than in our earlier proofs. We now find it more useful to work, not with the components of the fields in the coordinate system (18), but with components in the orthonormal frames defined by (23). In obvious notation (38) takes the form (lengths measured in units of m^{-1})

$$\int [(H_{01})^2 + (H_{02})^2 + (B_0)^2](-g)^{1/2} d^4x = \int [(H_{31})^2 + (H_{32})^2 + (B_3)^2](-g)^{1/2} d^4x. \tag{39}$$

Both of the integrands above are positive definite, but this by itself is no great help in our search for a proof that all field components vanish. But if we make the assumption that the fields and the metric are analytic in a certain parameter α, which we shall specify presently, then it becomes possible to show that the fields vanish identically. The parameter α is to be a measure of the *degree and sense* of the rotation of the black hole. The value $\alpha = 0$ will correspond to a static black hole. Two black holes for which α has opposite signs will differ in that they will be rotating in opposite senses about the same axis. An example of a suitable α is the angular momentum of the black hole along its symmetry axis.

The dependence of the fields and metric on α can be learned in part from time-reversal arguments. We know that under time reversal $(t \to -t)$ the $g_{\mu\nu}$ in (18) are unchanged with the sole exception of $g_{t\varphi}$ which simply changes sign. The time-reversal invariance of the field equation (34) requires that B_t, $H_{t\rho}$, and H_{tz} remain unchanged while B_φ, $H_{\varphi\rho}$, and $H_{\varphi z}$ change sign under time reversal. [Actually the opposite scheme is also consistent with the invariance of the sourceless Eq. (34), but would not work if sources were present; in any case the argument that follows can be modified to suit the opposite scheme.] For the field components in the orthonormal frames we find that

$$\begin{aligned} &B_0 \to B_0, \quad B_3 \to -B_3, \quad H_{01} \to H_{01}, \\ &H_{31} \to -H_{31}, \quad H_{02} \to H_{02}, \quad H_{32} \to -H_{32} \end{aligned} \tag{40}$$

while all other components are zero. If we think of time reversal as an active transformation (in contrast with a coordinate transformation) then it is clear that α changes sign under time reversal. Thus B_0, H_{01}, and H_{02} are even in α while B_3, H_{31}, and H_{32} are odd. Similarly $g_{t\varphi}$ is odd in α, but the rest of the $g_{\mu\nu}$ are even; consequently the determinant g is even in α.

We now assume that the field components in the orthonormal frames, and $(-g)^{1/2}$, are analytic in α. A little thought shows that this is equivalent to assuming analyticity for the field components in the coordinate system (18) and for $g_{\mu\nu}$. This assumption is not as restrictive as it sounds. If it is not satisfied for a given choice of α, it may be satisfied for another choice, for example, any odd function of the original α. Even the assumption of analyticity is not strictly required. The proof to be given will also work if the fields are analytic functions of α multiplied by a common factor, a nonanalytic function of α and any other parameters of the black-hole solution (but not coordinates). This common factor can be canceled out from (39) and is of no consequence in the proof that follows.

Imagine now that the fields and $(-g)^{1/2}$ are expanded in α. The expansions for B_0, H_{01}, and H_{02} as well as that for $(-g)^{1/2}$ contain only even powers of α; those for B_3, H_{31}, and H_{32} only odd powers. We substitute the expansions in (39) and collect terms with the same power of α. The terms independent of α all appear in the left-hand side of (39). The corresponding vanishing integral has an integrand which is a sum of squares of field quantities times the part of $(-g)^{1/2}$ which is independent of α. Clearly this last quantity is just the $(-g)^{1/2}$ for the static case, that is, a positive quantity. Thus the terms of the fields which are independent of α must vanish identically. Next, the terms in α^2 appear only in the right-hand side of (39) in the form of a vanishing integral whose integrand is again a sum of squares of the field quantities linear in α, all multiplied by the same part of $(-g)^{1/2}$ as above. Thus the terms linear in α of the expansions of the fields must vanish identically. This procedure can be iterated; at each step the result will be similar because the lower powers of α vanish. We conclude that all terms in the expansions for the fields vanish. By transforming back to the original coordinates we see that the B_μ and $H^{\mu\nu}$ all vanish everywhere in the black-hole exterior.

The proof given above breaks down in the case $m=0$ because then the field equation (34) is invariant under $B_\mu \to B_\mu + \Lambda_{,\mu}$ so that B_μ is no longer physical, and neither is b^μ in (36). Thus $b^\mu b_\mu$ need not be bounded on the horizon, and the proof fails. We conclude that *a rotating stationary black hole cannot be endowed with an exterior massive vector field, such as the* ρ*-meson field for example, but it can be endowed with a massless vector field.* The only example known of such a massless field is the electromagnetic field, and black holes with such an exterior field are possible, for example, the Kerr-Newman ones.[2]

VI. EXCLUSION OF MASSIVE SPIN-2 FIELDS IN ROTATING STATIONARY CASE

Here we take up again the spin-2 field introduced in Sec. II, but now in the exterior of a rotating stationary black hole. The metric is again that of (18). The symmetries of the geometry must be mirrored by the fields in the equilibrium state, so that $h_{\mu\nu,t}=h_{\mu\nu,\varphi}=0$. It then follows from (11) that

$$b^t = b^\varphi = 0. \tag{41}$$

From here on the proof that the boundary integral in (2) vanishes is so similar to those given before that we omit it. The remaining integral in (2) is simplified by the fact that $h_{\mu\nu;t}=h_{\mu\nu;\varphi}=0$ as a result of the symmetries. In terms of components in the orthonormal frames (23) the integral takes the form (lengths measured in units of m^{-1})

$$\int\left\{\sum_k\left[(h_{00;k})^2+(h_{33;k})^2+\sum_j(h_{jj;k})^2\right]+(h_{00})^2+(h_{33})^2+\sum_k(h_{kk})^2\right\}(-g)^{1/2}d^4x = 2\int\left[\sum_k(h_{03;k})^2+(h_{03})^2\right](-g)^{1/2}d^4x, \tag{42}$$

where k and j run over 1 and 2 only. All the components not appearing explicitly in (42) vanish because of the symmetries.

We can repeat the procedure given in the case of the vector field. We find in like manner that h_{03} and $h_{03;k}$ change sign under time reversal so that their expansions will contain only odd powers of α. All the other $h_{\mu\nu}$ and their covariant derivatives are unchanged under time reversal so that their expansions will only contain even powers of α. The proof then goes through exactly as before. The conclusion is that *a rotating stationary black hole cannot be endowed with exterior massive spin-2 fields, for example, the f-meson field.* As discussed in Sec. II, the massless case is very different, and the massless field cannot be ruled out by our proof.

VII. CONCLUSIONS: NO BARYON NUMBER FOR BLACK HOLES

We summarize the results of I and of the present paper as follows: *A black hole in its final (static or stationary) state cannot be endowed with exterior scalar, vector, or spin-2 massive meson fields.* Examples of the fields excluded are the π, K, ρ, ω, f, and other meson fields. In addition *a massless scalar field other than a constant is also ruled out.* Most proofs given assumed that the fields were real and thus electrically neutral, but we showed in I how the proofs may be extended to charged fields (which may even interact with the black hole's electromagnetic field). In view of this we see that all known meson fields (scalar, vector, spin-2) are excluded.

Our proofs were classical in nature. What conclusions are to be expected from a full quantum treatment of the problem? Such a treatment could be based on a decomposition of the fields into transverse and longitudinal components, such as is accomplished by the ADM procedure in the case of the electromagnetic field.[22] The transverse components are the dynamical ones, and are the only ones that should be quantized. *The longitudinal components can still be described classically.*[23] The statement that the black-hole exterior is in its final state (or ground state) means that there are no excitations of the meson fields (no waves in classical language; no real mesons in quantum language). This means that the transverse components are zero, except for the inevitable random vacuum fluctuations. Our classical proofs then show that the longitudinal components (analogous to the Coulomb field) vanish identically in the black-hole exterior.

By virtue of the preceding argument we expect, in advance of a full quantum treatment, that *a stationary black hole cannot interact with the exterior world by means of the strong interactions.* This is because it cannot be endowed with the longitudinal (Yukawa-like) meson fields that would be required for such an interaction to take place. Measurements carried out in the exterior of the black hole cannot yield any information about the presence or absence of strongly interacting material in the interior. This fact leads to the transcendence of the law of baryon number conservation first predicted by Wheeler.[14]

Suppose that a baryon is dropped into a stationary black hole. As it approaches the horizon, it "disappears" from view because of the redshift effect. Furthermore, the baryon leaves no "footprints" – once the exterior returns to a stationary state, no meson fields may be left in it. An exterior observer sees the baryon disappear, and is unable to verify its continued existence by observing the meson fields characteristically associated with a baryon. Thus the law of conservation of baryon number is transcended. Furthermore, *the exterior observer cannot measure the baryon number of the black hole* by means of the strong interactions. Thus from *his* point of view, the black hole's baryon number is not well defined. Of course, the baryon number is defined on a complete spacelike hypersurface, but the observer cannot determine it.

VIII. CONCLUSIONS: GRAVITATIONAL FIELD NOT INFLUENCED BY STRONG INTERACTIONS

In our proofs we made no explicit use of Einstein's equations. Hawking's theorem, on whose strength we restricted our attention to axisymmetric rotating black holes, does not depend on them either. Thus, all our conclusions are valid both in general relativity (GR) and in the Brans-Dicke theory (BDT). One can say even more than this. It was conjectured by Penrose,[18] and established by Hawking,[16] and independently by the author,[17] that the exterior of a stationary black hole is identical in both theories. Here we wish to show that this identity still holds when the strong interactions are "turned on."

We recall the Brans-Dicke equations[24]

$$G_{\mu\nu} = 8\pi\Phi^{-1}T_{\mu\nu} + \omega\Phi^{-2}(\Phi_{,\mu}\Phi_{,\nu} - \tfrac{1}{2}g_{\mu\nu}\Phi_{,\alpha}\Phi_{,}{}^{\alpha}) + \Phi^{-1}(\Phi_{,\mu;\nu} - g_{\mu\nu}\Phi_{,\alpha;}{}^{\alpha}) \tag{43}$$

and

$$\Phi_{,\alpha;}{}^{\alpha} = 8\pi(2\omega+3)^{-1}T^{\alpha}{}_{\alpha}\,, \tag{44}$$

where we use units for which $c=1$. The ω is the Brans-Dicke coupling constant, and Φ is a scalar field whose interpretation is (roughly) that of the reciprocal of the local gravitational constant. The $T_{\mu\nu}$ is the stress-energy tensor for matter and nongravitational fields, and $G_{\mu\nu}$ is the Einstein tensor.

Consider a stationary black-hole exterior in GR. The geometry obeys Einstein's equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$. It will also obey the Brans-Dicke equations if Φ is a constant, G^{-1}, *and* if $T^{\alpha}{}_{\alpha}=0$ in the black-hole exterior. In a stationary exterior there may exist no matter, but there could be electromagnetic fields, neutrino fields (manifestation of weak interactions) and, *a priori,* meson fields (manifestation of strong interactions). It is well known that for electromagnetic and neutrino fields $T^{\alpha}{}_{\alpha}=0$ is an identity. But this is not true for meson fields. However, we have shown that all meson fields must vanish identically in the exterior. Thus $T^{\alpha}{}_{\alpha}=0$, and we see that even in the presence of electromagnetic, weak, and strong interactions, a stationary black-hole exterior in GR is also allowed in BDT.

We now prove the converse. Consider a stationary black-hole exterior in BDT. As we have just pointed out, $T^{\alpha}{}_{\alpha}=0$ even in the presence of all interactions. Therefore,

$$\Phi_{,\alpha;}{}^{\alpha} = 0, \tag{45}$$

which is the familiar massless scalar equation. The results we obtained in I and in Sec. IV of the present paper are applicable to it, but the proofs require two changes. We recall that the proofs apply, not directly to Φ, but to $\psi = \Phi - \Phi_0$ where Φ_0 is the asymptotic limit of Φ. We cannot establish that $b^{\mu}b_{\mu} = \psi^2\Phi_{,\mu}\Phi_{,}{}^{\mu}$ is bounded at the horizon in the usual way. Instead we note that ψ must be bounded, for otherwise the local gravitational constant would vanish, a very singular occurrence which is not compatible with the nonsingular character of the horizon. The boundedness of $\Phi_{,\mu}\Phi_{,}^{\ \mu}$ follows from the boundedness of $R^\mu{}_\mu$ and $T^\mu{}_\mu$ (nonsingular geometry and nonsingular physics) if we take the trace of (43) and simplify it with (44). Thus $b^\mu b_\mu$ is bounded.

The second change has to do with the fact that we cannot extend the volume integral in (2) all the way to spatial infinity without doing violence to the philosophy of the BDT which envisages the main contribution to Φ as coming from the distant matter in the universe (Mach's principle). Therefore, instead of taking part of the boundary of the domain of integration at spatial infinity, we take it to be a large sphere in the almost flat region far from the black hole, but not at cosmological distances. In this manner none of the distant matter is present in the domain of integration, and (45) holds everywhere in it. The boundary term thus introduced is easily shown[17] to have the same sign as the volume integral in (2). Thus the proofs go through with little change, and the result is that Φ can only be a constant in a stationary black-hole exterior.

Since Φ must be a constant we see that the Brans-Dicke equations (43) reduce to Einstein's equations. We thus have shown that, *even in the presence of the electromagnetic, weak, and strong interactions, a stationary black-hole exterior which is a solution of BDT is also a solution of GR and vice versa.*

What are the allowed stationary black-hole exteriors? We have shown that the strong interactions can make no contributions to the stress-energy $T_{\mu\nu}$ in the black-hole exterior (no meson fields). The results of Hartle[10] and of Teitelboim[11] suggest that the weak interactions do not contribute either (no neutrino fields). Thus the $T_{\mu\nu}$ is purely electromagnetic in nature. The uniqueness theorems mentioned earlier[3-6] apply in this case, and state (with some technical qualifications) that the stationary black-hole exterior is necessarily of the Kerr-Newman type. (We exclude multiple black-hole solutions from consideration.) Thus *even when the strong (and, possibly, the weak) interactions are "turned on," a stationary black-hole exterior is parametrized only by mass, charge, and angular momentum.* Not even from its gravitational field may one determine the baryon number of a black hole.

IX. NONMINIMAL GRAVITATIONAL COUPLING

All the Lagrangian densities which we have considered were obtained from the corresponding ones for flat space by the use of the minimal coupling rule: the replacement of ordinary by covariant derivatives. How much do the conclusions of this paper depend on this choice of coupling? We shall show that all our conclusions with regard to meson fields remain valid even if we allow a wide range of direct couplings to the curvature (nonminimal couplings) in the meson Lagrangian densities.

We confine our attention to couplings for which the Lagrangian densities are still quadratic in the meson fields. Thus to the Lagrangian density for the scalar field, Eq. (24), we may add

$$\mathcal{L}' = a_1 R\psi^2 + b_1 m^{-2} R\psi_\mu \psi^\mu + c_1 m^{-2} R^{\mu\nu}\psi_{,\mu}\psi_{,\nu}\,, \tag{46}$$

to that for the vector field, Eq. (33), we may add

$$\mathcal{L}' = a_2 RB_\mu B^\mu + b_2 R^{\mu\nu} B_\mu B_\nu + c_2 m^{-2} RH_{\mu\nu}H^{\mu\nu} + d_2 m^{-2} R^{\mu\nu} H_{\mu\alpha} H_\nu{}^\alpha + e_2 m^{-2} R^{\alpha\beta\mu\nu} H_{\alpha\beta} H_{\mu\nu}\,, \tag{47}$$

and to that for the spin-2 field, Eq. (9), we may add

$$\mathcal{L}' = a_3 R h_{\mu\nu} h^{\mu\nu} + b_3 R^{\mu\nu} h_{\mu\alpha} h_\nu{}^\alpha + c_3 R^{\alpha\beta\mu\nu} h_{\alpha\mu} h_{\beta\nu} + d_3 m^{-2} R^{\alpha\beta\mu\nu} h_{\alpha\mu;\gamma} h_{\beta\nu;}{}^\gamma\,. \tag{48}$$

Here the subscripted Latin letters stand for *dimensionless* coupling constants, $R_{\alpha\beta\mu\nu}$ is the Riemann curvature tensor, and $R^{\mu\nu}$ and R its contractions. We have by no means tried to be exhaustive in the above lists of possible terms; we merely wish to illustrate the possibilities.

We can carry out our usual procedure for the modified Lagrangian densities. The proofs that the boundary term in (2) vanishes are always similar to the ones given before; they make use of the symmetries of the problem, and of the fact that a physical scalar (now involving the curvature) must be bounded on a nonsingular horizon. The remaining integral in (2) is just (because the Lagrangian density is quadratic in the fields)

$$2\int (\mathcal{L}+\mathcal{L}')(-g)^{1/2} d^4x = 0. \tag{49}$$

In the integrand the normal terms and the terms involving curvature are scalars separately.

Let us consider the two types of terms *in local inertial frames*. In the black-hole exterior the components of $R_{\alpha\beta\mu\nu}$ will attain their maximum values near the horizon, and these maxima will be of the order L^{-2} where L is a characteristic dimension of the black hole, the Schwarzschild radius for example. The various components of the fields and their derivatives are physical (they explicitly appear in $T_{\mu\nu}$), and therefore bounded in the local inertial frames. For each term in $\mathcal{L}'$

involving the curvature there is a similar one in $\mathfrak{L}$ which does not. It is easy to see that the normal terms will dominate in magnitude the terms with curvatures everywhere in the exterior provided the Compton wavelength m^{-1} times the square root of the appropriate coupling constant is much smaller than L. If this condition is satisfied for all the coupling constants, the integrands $\mathfrak{L}+\mathfrak{L}'$ will have the sign of $\mathfrak{L}$ regardless of what the sign of $\mathfrak{L}'$ is. Our usual arguments will then show that the fields must vanish identically in the exterior. (The above argument is valid only for massive fields.)

As an example consider a black hole of solar mass ($L \simeq 10^5$ cm). The π meson field ($m^{-1} \simeq 10^{-13}$ cm) will be excluded from its exterior when the coupling constants a_1, b_1, and c_1 have magnitudes ranging from zero to 10^{34}–10^{35}.

Note added in proof. The metric (18) may be derived from the requirement that $g_{\mu\nu}$ be invariant under the simultaneous inversion of t and φ together with a coordinate transformation involving only ρ and z. The above discrete symmetry is guaranteed in the stationary and axisymmetric case by a theorem of B. Carter [J. Math. Phys. 10, 70 (1969)] which holds even in the nonvacuum case provided the $T_{\mu\nu}$ is invariant under the inversion of t and φ. This condition is satisfied for all the fields considered here. The author thanks Robert Wald for pointing out the need for a justification of the use of metric (18) in the nonvacuum case.

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