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Weak-Interaction Corrections to the Muon Magnetic Moment and to Muonic-Atom Energy Levels*

Roman Jackiw and Steven Weinberg
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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The weak contributions to the anomalous magnetic moment of the muon are calculated in a proposed theory of the weak and electromagnetic interactions. The result is finite and of order $\Delta g_\mu \approx 10^{-8}$, too small to be measured at present. The present agreement between theoretical and experimental values of the muon magnetic moment does not even rule out the possibility that the scalar meson required by this theory has a very small mass. If this mass were sufficiently small, then the scalar-meson field would produce shifts of the order of a hundred parts per million in muonic-atom energy levels.

If weak interactions are mediated by intermediate vector bosons, then the virtual emission and absorption of these bosons should contribute to the magnetic moment of the muon. There have been a number of calculations of this effect,[1] but in the absence of any systematic method for dealing with ultraviolet divergences in higher-order weak processes, it was not possible to put much confidence in the results. This note will reconsider the problem in the context of a proposed theory[2] of the weak and electromagnetic interactions, which appears to be free of the divergence difficulties of conventional theories.[3] The results could conceivably provide an experimental test of the proposed theory, and in any case will serve as one more check of its renormalizability.

In conventional theories, the only diagram of first order in the Fermi coupling constant G which contributes to the muon magnetic moment is one in which the photon is coupled to a virtual charged intermediate boson W_λ. [See Fig. 1(a).] In the literature[1] the weak contribution to the gyromagnetic moment of the muon is given in this case as (in our notation)

$$(\Delta g_\mu)_W = \frac{G m_\mu^2}{3\pi^2\sqrt{2}}\left(\frac{5}{2} + 3(g_W - 2)\ln\frac{\Lambda}{m_W}\right), \qquad (1)$$

where g_W is the gyromagnetic ratio of the W, and Λ is an ultraviolet cutoff. In the proposed theory,[2] the W-lepton and W-photon interactions are the same as in conventional theories, but with a W gyromagnetic ratio $g_W = 2$. Hence this diagram is automatically finite here.

The proposed theory also contains a neutral intermediate boson Z_λ, whose coupling to the muons is of the form[2]

$$\mathcal{L}'_{Z\mu} = i(g^2+g'^2)^{-1/2} Z_\lambda \times \bar{\mu}\gamma^\lambda[\tfrac{1}{2}(1-\gamma_5)g'^2 + \tfrac{1}{4}(1+\gamma_5)(g'^2-g^2)]\mu,$$

where g and g' are a pair of independent coupling constants related to the masses of Z and W by

$$g^2 = 8m_W^2 G/\sqrt{2}, \quad g^2+g'^2 = 8m_Z^2 G/\sqrt{2}.$$

Thus the magnetic moment also receives a contribution from a diagram in which the photon couples to a muon in the presence of a virtual Z_λ. [See Fig. 1(b).] This diagram contains both quadratic and logarithmic divergences, but the divergent terms in the vertex function Γ^λ turn out to be proportional to γ^λ or $\gamma^\lambda\gamma_5$, and presumably are removed by lepton field and charge renormalization. The magnetic form factor is *finite*:

$$[F(k^2)]_Z = \frac{m_\mu^2(g^2+g'^2)}{128\pi^2}\int_0^1 dx\int_0^x dy[m_\mu^2x^2 + k^2 y(x-y) + m_Z^2(1-x)]^{-1} \times\left(\frac{(3g'^2-g^2)^2}{(g'^2+g^2)^2}x(1-x) - (4-x)(1-x) - \frac{2m_\mu^2x^2}{m_Z^2}\right).$$

The first and second terms in the large round parentheses arise from the $g_{\mu\nu}$ part of the Z-meson propagator; the first being the contribution of the vector coupling and the second of the pseudovector coupling. The third term comes from the part of the Z-meson propagator proportional to $q^\mu q^\nu$; here only the pseudovector coupling contributes.[4] Although this quantity is finite, it is not uniquely

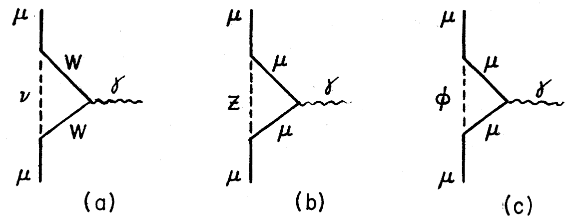

FIG. 1. Feynman diagrams responsible for weak corrections to the muon magnetic moment.

given by the ordinary Feynman rules, due to ambiguities in routing the (integration) momentum of the virtual Z meson.[5] *Our* result follows when this line does not carry any external momenta; however, an additional, finite, and arbitrary term will be present when the external momenta are shared by the Z-meson line. One can justify our routing by the ξ-limiting method,[6] which replaces

$$\frac{q^\mu q^\nu}{m_Z^2}(q^2+m_Z^2)^{-1}$$

by

$$\frac{q^\mu q^\nu}{m_Z^2}[(q^2+m_Z^2)^{-1}-(q^2+M^2)^{-1}].$$

This leads to our answer as $M^2 \to \infty$.

The muon gyromagnetic ratio thus receives another contribution

$$(\Delta g_\mu)_Z = 4[F(0)]_Z = \frac{m_\mu^2 G}{3\pi^2\sqrt{2}}\left(\frac{g'^4-4g'^2g^2-g^4}{(g^2+g'^2)^2}\right)+O\left(\frac{m_\mu^4 G}{m_Z^2}\ln\frac{m_Z}{m_\mu}\right). \tag{2}$$

Adding the contributions (1) and (2) gives the total contribution of Figs. 1(a) and 1(b) to the muon gyromagnetic ratio:

$$(\Delta g_\mu)_{W+Z} = \frac{Gm_\mu^2}{\pi^2\sqrt{2}}f(\theta) = 0.9\times10^{-8}f(\theta), \tag{3}$$

where

$$f(\theta) \equiv \tfrac{5}{12}+\tfrac{4}{3}(\sin^2\theta-\tfrac{1}{4})^2,$$

$$\tan\theta \equiv g'/g.$$

The factor $f(\theta)$ is between $\frac{5}{12}$ and $\frac{7}{6}$ for all θ, so the weak contribution to the muon magnetic moment is of order 10^{-8}, too small to be measured, or to be distinguished from the hadronic corrections in conventional electrodynamics.

In the proposed model, there is one final possible contribution to the muon anomalous magnetic moment, in which the photon couples to the muon in the presence of a virtual scalar meson ϕ. [See Fig. 1(c).] The coupling of the ϕ meson to the muon in this model is

$$\mathcal{L}'_{\phi\mu} = -m_\mu(2G/\sqrt{2})^{1/2}\phi\bar{\mu}\mu. \tag{4}$$

The muon gyromagnetic ratio thus receives the additional contribution

$$(\Delta g_\mu)_\phi = \frac{Gm_\mu^4}{2\pi^2\sqrt{2}}\int_0^1 \frac{y^2(2-y)}{m_\mu^2y^2+m_\phi^2(1-y)}dy. \tag{5}$$

For $m_\phi \gg m_\mu$, this is smaller than the W and Z terms by a factor of order m_μ^2/m_ϕ^2, and hence may be neglected here. For $m_\phi \ll m_\mu$, Eq. (5) gives

$$(\Delta g_\mu)_\phi \simeq \frac{3Gm_\mu^2}{4\pi^2\sqrt{2}}, \tag{6}$$

which is of the same order as the W and Z terms.

In summary, the proposed theory certainly passes the test of giving the muon a finite anomalous magnetic moment. It is a pity that the weak-interaction contributions to this moment are one or two orders of magnitude too small to be measured. The weak contributions to the electron magnetic moment are of course also finite, and even smaller by a factor m_e^2/m_μ^2.

It is interesting that the scalar meson ϕ is coupled so weakly to the muon that the present agreement between theoretical and experimental values of the muon magnetic moment sets *no* constraint whatever on the ϕ meson mass. We are therefore free to speculate on the possible effects of a very light ϕ meson. In general, the existence of a nonvanishing scalar field vacuum expectation value $\lambda=(\sqrt{2}/2G)^{1/2}$ will induce what seems to be an intrinsic chiral-symmetry-breaking term $\delta\mathcal{L}$, so the coupling of the ϕ meson to the hadrons is

$$\mathcal{L}'_{\phi h} = \lambda^{-1}\phi\,\delta\mathcal{L} = (2G/\sqrt{2})^{1/2}\phi\,\delta\mathcal{L}$$

[compare Eq. (4)]. The one-nucleon matrix elements of $\delta\mathcal{L}$ may be determined from the observed baryon and pseudoscalar-meson masses and from the low-energy pion-nucleon and kaon-nucleon phase shifts, but there is a good deal of controversy[7] as to the results. To be impartial, we shall write

$$\langle \bar{N}'|\delta\mathcal{L}|N\rangle = \frac{\xi m_\mu}{(2\pi)^3(4E'E)^{1/2}}.$$

Here $|\xi|$ is a dimensionless number which takes values ranging from $\lesssim 0.1$ to 10 in various estimates of the "σ term."[7] Since this is an isoscalar interaction, its effect in muonic atoms of nucleon number A is to generate an effective muon-nucleus potential proportional to A:

$$\delta V(r) = \frac{2\xi m_\mu^2 GA}{r\sqrt{2}}\exp(-rm_\phi).$$

Hence, as long as we consider only Bohr orbits whose radius $\langle r\rangle = n/2Z\alpha m_\mu$ is much smaller than $1/m_\phi$, the ϕ field simply changes the constant $Z\alpha$ in Coulomb's law to

$$Z\alpha+\frac{2\xi m_\mu^2 GA}{\sqrt{2}},$$

and therefore each energy level is shifted by a fractional amount

$$\frac{\delta E}{|E|}=\frac{4\xi m_\mu^2 GA}{\sqrt{2}\,Z\alpha}=100\xi\,\frac{A}{2Z}\ \text{ppm}.$$

(The level shifts caused by Z exchange are very much smaller, except in S states.) As it happens, Dixit *et al.*[8] have reported a systematic discrepancy between calculated and theoretical energy values for the x rays emitted from μ-mesonic atoms, of the order of a few hundred ppm. Furthermore, this discrepancy is largest for transitions among Bohr orbits of smallest radius (such as the $5g\to 4f$ transition in Pb and the $4f\to 3d$ transition in Ba), as would be expected if $1/m_\phi$ were comparable to the radii of these orbits, i.e., $m_\phi \approx 20$ MeV. However, as Dixit *et al.*[8] themselves point out, additional work will be needed to clarify the nature of this discrepancy. If this discrepancy disappears, and if (as we expect) the $SU(3)\times SU(3)$-breaking parameter ξ turns out to be of order unity, then the agreement between experiment and conventional theory for muonic atoms would require m_ϕ to be greater than about 30 MeV.

Another consequence of the present theory is a modification of hyperfine splittings. Such splittings are insensitive to scalar-particle exchange. The contribution of neutral-vector exchange has been estimated by Adams[9] – the effect is too small to be seen by the present experiments. We have not examined other processes which might also set useful bounds in the ϕ mass. Some aspects of a scalar lepton-nucleon potential have been discussed by Yennie.[10]

In conclusion we wish to emphasize that the present theory *does not require* the charge and magnetic *form factors* to be finite. Only the S matrix is cutoff-independent, while Green's functions, such as the electromagnetic vertex function, need not possess this property. Of course in calculations of S matrix elements, these divergences must disappear and the cancellation is effected by other diagrams which correspond to divergent point interactions.[11] Clearly the charge and magnetic moment must be finite, since they do not lead to point interactions. However, terms of $O(q^2)$ in the form factors, such as the charge radius, give rise to point interactions and can diverge.

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