present calculation] which is based on the assumption of a residual contact interaction between the protons, there is good reason to believe that an eventual increase of y with energy is at least partly the consequence of the existence of such an interaction.

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Comments on Duality and the Final-State-Interaction Approach to $N\bar{N}\to 3\pi$†

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Duality suggests that it is more informative to factorize an amplitude for $N\bar{N}\to 3\pi$ at the exchanged nucleon poles than at a direct-channel singularity. Experimental verification of t-channel factorization and the implications for the final-state-interaction approach to $N\bar{N}$ annihilations are discussed.

Analysis of $N\bar{N}\to 3\pi$ has usually been based on an assumption that the main features of the data are due to a final-state interaction in the pion system. Lovelace's[1] interpretation of the data on $\bar{p}n\to \pi^+\pi^-\pi^-$ annihilations at rest as a continuation in an external mass leg of the Veneziano $\pi\pi$ scattering amplitude is an example of this kind of simple final-state-interaction approach. The impact of Lovelace's suggestion and the widespread acceptance of his basic assumption can be measured by the number of his imitators. Fits to $\bar{p}n\to \pi^+\pi^-\pi^-$ with the same basic model for the amplitude but with more free parameters have proliferated.[2-5] Fits to other annihilation reactions[6,7] with sums of Veneziano four-point functions have also emerged.

Of these fits, the most careful treatment of the problems inherent in the *ad hoc* "unitarization" of a Veneziano amplitude for phenomenological purposes is by Pokorski, Raitio, and Thomas.[5] They expand the Veneziano functions simultaneously in terms of the poles in the s and t channels. This expansion is convergent within the Dalitz plot[8] and enables them to "unitarize" by giving each resonance an appropriate total width while the partial widths are determined by the coefficients of the Veneziano functions. Cruder treatments of the unitarization problem forced the total widths of all resonances within a given tower to be the same.

Treated in this way, the use of the Veneziano model in the sense of a sum over $\Gamma\Gamma/\Gamma$ terms is a convenience for reproducing a general final-state $\pi\pi$ interaction with a reasonable spectrum of resonances. As a comparison, the rising-phase-shift model of Gleeson, Meggs, and Parkinson[9] does not mention Γ functions, but the fit it gives to $\bar{p}n \to \pi^+\pi^-\pi^-$ at rest produces almost the same resonances and resonance parameters as the fit of Pokorski, Raitio, and Thomas, except that the more flexible parametrization of Gleeson *et al.* allows mass shifts between the resonances in a tower as well as different total widths. All these fits have in common the fact that they can achieve only a fair description of the data. Even allowing more resonances, more flexibility in the parametrization in terms of the masses and widths, these approaches reach a level at which they can do no more.

The limitation seems to be due to the fact that the final-state-interaction approach to $N\bar{N}$ annihilation reactions is only partially valid. The fact is that duality, in the general sense in which quantum numbers determine the basic singularity structure of amplitudes, gives us a very definite indication of the deficiencies of the final-state-interaction approach in this case. To see this we have to consider the process as a $2 \to 3$ reaction. The duality diagrams for $\bar{p}n \to \pi^+\pi^-\pi^-$ shown in Fig. 1 illustrate a very simple point which must be considered in any reasonable model for this process. Two of the diagrams, A and B, correspond to functions with poles in the $\bar{p}n$ channel. If the reaction proceeded in terms of a prominent isolated resonance in this channel, these two terms would dominate. A B_5 model expanded in terms of the poles in this channel would have residues equal to a sum of terms of the form $\Gamma\Gamma/\Gamma$,[10] and the dominance of these diagrams in a dual model is necessary to justify the fits of Refs. 1–5.[11]

On the other hand, all four diagrams in Fig. 1 have an exchanged nucleon pole. These diagrams indicate that while it would be dangerous to "factorize" a dual model in the $N\bar{N}$ channel, it does make sense to factorize at the exchanged nucleon pole. The diagrams with poles in the $N\bar{N}$ channel and those without combine to give the proper signature to the exchanged Reggeon. As emphasized by Berger,[2,12] the nucleon pole is quite close to the physical region for these processes. For $\bar{N}N \to 3\pi$ annihilations at rest

$$t_1 = t_{\pi_1 N} = t_{\pi_1 \bar{N}} = \tfrac{1}{2}(s_{23} + m_\pi^2 - 2M_N^2)\,. \quad (1)$$

For annihilations in flight, the kinematic correlation between t and the longitudinal and transverse momentum of the pion for the missing-mass reaction $N\bar{N} \to \pi(\mathrm{MM})$ is illustrated in Fig. 2. In both cases

$$t^{\max} = (M_N - m_\pi)^2 = 0.64\ \mathrm{GeV}^2 \quad (2)$$

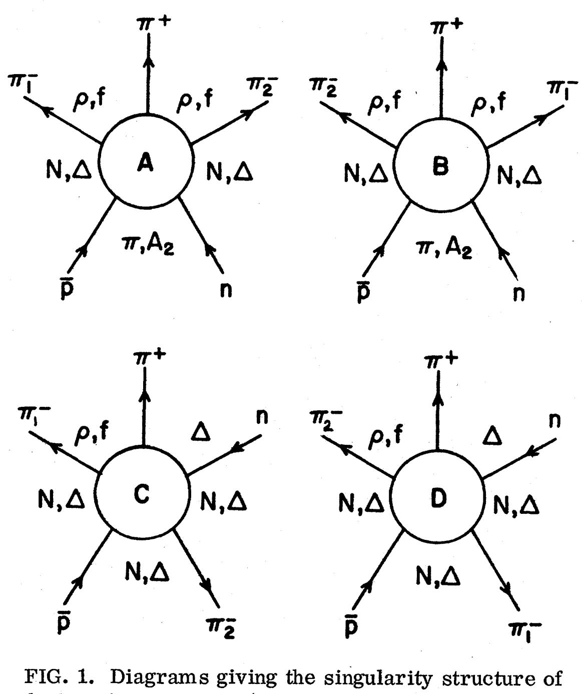

FIG. 1. Diagrams giving the singularity structure of a dual model for $\bar{p}n \to \pi^+\pi_1^-\pi_2^-$. Poles occur in channels defined by adjacent particles.

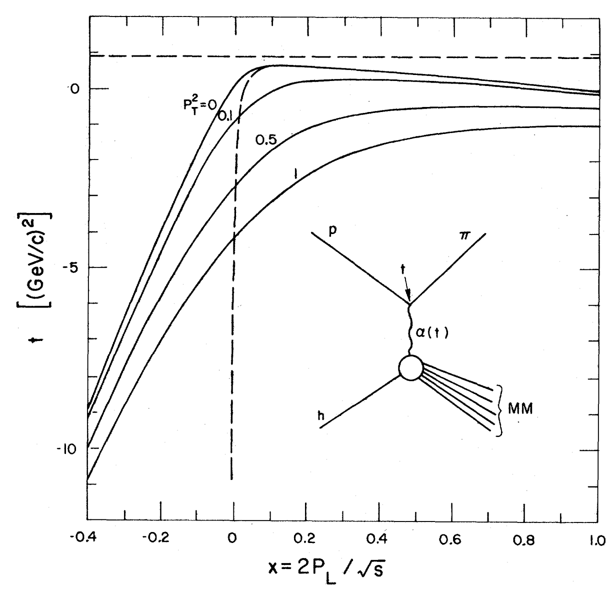

FIG. 2. Connection between t and the Feynman scaling variable $x = 2P_L/\sqrt{s}$ for different values of $P_\perp^2$ in the inclusive reaction $N\bar{N} \to \pi +$ missing mass. The considerations concerning the closeness of the nucleon pole to an important segment of the physical region can be seen valid for all annihilation reactions. The solid curves are for $P_{\mathrm{lab}} = 12.5$ GeV/c and the dashed curve for $P_\perp^2 = 0$ and $P_{\mathrm{lab}} = 500$ GeV/c. For lower values of P_{lab}, the maximum of the curves is broader and extends further to negative x. Figure is from Berger, Salin, and Thomas, Ref. 12.

compared with

$$M_N^2 = 0.88\,\text{GeV}^2\,. \qquad (3)$$

In the $\pi^+\pi^-\pi^-$ final state, closeness to the nucleon pole is correlated kinematically with large $s_{\pi^+\pi^-}$ so that a nucleon-exchange mechanism should imply an enhancement at two of the corners of the $\pi^+\pi^-\pi^-$ Dalitz plot. Such an enhancement is seen even in the data for annihilations at rest, and cannot be reproduced naturally by sums over $\Gamma\Gamma/\Gamma$ terms. In the rising-phase-shift model of Gleeson, Meggs, and Parkinson, this enhancement is parametrized by a form factor attributed to the nucleon pole. In allowing this additional flexibility in their parametrization, these authors have inserted more dynamics than is present in simple Veneziano-type expansions.

The purpose of this discussion based on the diagrams in Fig. 1 is not to suggest a detailed fit to the data on annihilation processes into three mesons in terms of B_5 functions, although the B_5 approach offers a consistent way to combine the t-channel-exchange picture and the final-state-interaction picture. Pending a solution of some difficult theoretical problems of general dual models, B_5 phenomenology is more promising as a qualitative guide to data than as a consistent detailed phenomenological scheme.[12] The point is that since any resonances in the $N\bar{N}$ channel can be expected to be quite broad, Dolen-Horn-Schmid duality[13] would imply that treating the five-particle amplitude in terms of nucleon exchange could be expected to be approximately valid right down to threshold in the $N\bar{N}$ channel, and that the exchange picture is more appropriate, and since it can use signature to incorporate crossing, more complete than the final-state-interaction picture based on direct-channel resonances.

The experimental support for the absence of resonance effects in the $N\bar{N}$ channel is rapidly accumulating. The energy-dependent structure in backward $\bar{p}p$ elastic scattering originally interpreted[14] as evidence for narrow $\bar{p}p$ resonances has been shown to be more easily understood in terms of the Odorico zero trajectories than in terms of isolated poles.[15] In addition, it has been shown by an experimental examination of $\rho\omega$ interference in $\bar{p}p$ that the production amplitudes for $\bar{p}p \to \rho^0\pi^+\pi^-$ and $p\bar{p} \to \omega^0\pi^+\pi^-$ are largely coherent and maintain approximately the same relative phase over a wide range of lab momenta.[16] Since the $\rho\pi\pi$ and $\omega\pi\pi$ final states have opposite G parity, any sort of smooth behavior of the relative phase would be possible only if the resonances in the $N\bar{N}$ channel were broad and overlapping suggesting, via Dolen-Horn-Schmid duality, a t-channel approach. The fact that the variation is small and the fact that the ratio of ω to ρ production remains about 1.3 is consistent with the assumption that both reactions are dominated by nucleon exchange down to low energies. In addition, Bender and Rothe[17] have shown in the context of a B_5 model that relating the annihilation channel to the amplitudes for $\pi N \to \pi\pi N$ can be quantitatively correct only if $\Gamma M \gtrsim 1\ \text{GeV}^2$ for any meson resonances which couple strongly near the $N\bar{N}$ threshold. This is the same sort of lower limit on the total width of resonances which emerges from the experimental analyses.

Further evidence for the exchange picture is found in the Dalitz plot of Bettini *et al.*[18] shown in Fig. 3. As pointed out by Odorico,[15] these data cannot be understood in terms of the Veneziano model of Refs. 1–5. If we attempt to retain the final-state-interaction approach, to explain the zero structure implied by these data indicates we must use a $\pi\pi$ amplitude which is odd under the crossing $\pi_1^- \leftrightarrow \pi_2^-$ in violation of Bose statistics. Treating the amplitude as a nucleon-exchange process, however, the dip structure in these data still arises from the coherent interference of overlapping resonances, but now the local direction of the zero trajectories is determined, for example, by the relative sign and magnitude of the $N\epsilon\bar{N}$ and $N\rho'\bar{N}$ vertices.[19]

Extending the assumption of nucleon pole dominance to other final states, t-channel factorization can be used to relate quasi-two-body final states coupling to different numbers of mesons.[20] Experimental tests of the t-channel factorization predictions have been made and the predictions have been verified within the limits of statistics.[16]

Occasionally, the final-state-interaction approach is defended for annihilations at rest on the

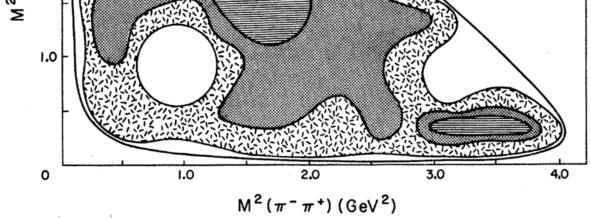

FIG. 3. The Dalitz plot for $\bar{p}n \to \pi^+\pi^-\pi^-$ as given by Bettini *et al.*, Ref. 18.

basis of S-state capture of the $\bar{p}$ while it is admitted that the approach is inappropriate for annihilations in flight.[21] This argument is delicate because the P-wave attraction of the $N\bar{N}$ system is strong and could result in annihilations between Stark collisions.[22] The experimental evidence for S-state capture has evaporated since it has been found that suppression of the $K^0_S K^0_S$ annihilation channel also occurs for $p\bar{p}$ annihilations in flight in regions where the orbital angular momentum of the $p\bar{p}$ system is known to be quite large.[23] By a careful measurement of the rate of annihilations into $2\pi^0$ it is possible to conclude[24] that at rest

$$R[(p\bar{p})_{L\text{odd}}\to 2\pi]/R[(p\bar{p})_{\text{all}}\to 2\pi]=0.39\pm 0.05 \qquad (4)$$

is not much different from the ratio in flight. Considering interference effects, it is apparent that a mixture of P wave in the annihilation channel is sufficient to obscure the predictions of a simple final-state-interaction model, while it is difficult to explain the enhancement at large $\pi^+\pi^-$ mass seen in the data at rest without considering t-channel singularities.

It seems safe to conclude from both the experimental and theoretical situations that the final-state-interaction model cannot be expected to provide a complete description of annihilation processes.[9] Keeping in mind the fact that the direct-channel and exchange pictures are complementary, it is valid to use this approach for extracting resonances parameters from the data, but care must be taken in relating these parameters to $\pi\pi$ scattering phase shifts. From this point of view, the program of using more subtle unitarization schemes and complicated sums of $\Gamma\Gamma/\Gamma$ terms to describe three-meson annihilation processes seems to have limited potential for extracting tractable phenomenological output.

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