Comments and Addenda

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Improved Simulation of Extensive Air Showers with a Medium-Strong Interaction*

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(Received 25 June 1971; revised manuscript received 12 January 1972)

An improved version of a previous simulation of extensive air showers above 10^{13} eV is discussed. In the simulation, the central assumption of a medium-strong contact interaction between protons is retained, but a model is used in which secondary pions are produced during pp collisions rather than through isobar decay. The influence of several different expressions for the multiplicity of pions and for the magnetic form factor of the proton is examined. While these expressions have noticeable quantitative effects on the behavior of the showers, the previous qualitative observation that the medium-strong interaction leads to a gradual increase in a measure of transverse momentum above a primary energy of 10^{14} eV is unchanged.

In a previous paper,[1] denoted here by (A), it was suggested that observations of high-energy extensive air showers with unusually large transverse components of momenta could be explained in part if the shrinkage of the diffraction peak in proton-proton elastic scattering were to stop above some characteristic energy. Although the interpretation of the effect in (A) was that the shrinkage eventually uncovered in the differential cross section $d\sigma/dt$ an s-independent "core" produced by a medium-strong interaction, there have been other models[2] leading to a similar asymptotic form for $d\sigma/dt$. A numerical simulation of extensive air showers may not be able to distinguish between the different models, but it can test the suggestion that the behavior of $d\sigma/dt$ and the behavior of the momentum distribution of secondary particles are connected. The computation reported in (A) showed that the transverse momenta associated with showers of primary energy above about 10^{14} eV increased with energy, as in the experimental results of McCusker, Peak, and Rathgeber,[3] but contrary to the constant behavior which has been seen at lower energies. However, the method of generating secondary pions in (A), in reactions separate from the proton-proton collisions, was somewhat artificial. The question of whether the simulated showers behave as they do because of the assumed form of the initial pp interaction or because of the extra assumptions about the pion-producing mechanism requires attention, since there are now different high-energy measurements from different groups in the field.[3,4] A recent experimental paper summarizes the situation.[5] This note describes the major features of a further simulation of extensive air showers at high energies, with a model in which pions are produced during the pp interaction. It is in no sense a competitor for the detailed papers[5,6] which discuss simulations as ends in themselves. Instead, it examines the effect of the introduction of a medium-strong interaction on the broad outline of a prototype simulation. Like (A), it is prompted by the qualitative idea that the apparent behavior of transverse momenta in high-energy events fits what one expects naively from the uncovering of a nonshrinking part of the diffraction peak in pp scattering. As the medium-strong interaction here has all the characteristics of the elusive SU(3)-breaking interaction,[7] it is of interest to investigate quantitative evidence for its existence, even if this evidence turns out to be rather indirect.

The basic method follows a proposal of Abarbanel, Drell, and Gilman[8] which has arisen from the observed approximate dependence of $d\sigma/dt$ for the reaction $p(p_1)+p(p_2)\to p(p_3)+p(p_4)$ on $G^4(t)$ for large values of $-t=-(p_1-p_3)^2$, where $G(t)$ is the magnetic form factor of the proton. In this scheme,[8,9] the T matrix is

$$g^2G^2(t)\bar{u}(p_4)\gamma_\mu u(p_2)\bar{u}(p_3)\gamma^\mu u(p_1)+B(s,t), \qquad (1)$$

where g is the coupling constant for the medium-strong interaction, M is the proton mass, $s=(p_1+p_2)^2$, and $B(s, t)$ contains all the information about the usual strong-interaction effects. Further analysis of the T matrix, which leads to the differential cross section through

$$\frac{d\sigma(s,t)}{dt}=|T(s,t)|^2,$$

is carried on with the Fourier-Bessel-transformed representation of $T(s, t)$, in exactly the same way as in (A). While it is not possible to write a closed analytic expression for the differential cross section with this treatment, values of $f(t)$, a, s_0, and $\alpha(t)$ can be found so that the equation

$$\frac{d\sigma(s,t)}{dt}=(aG^4(t)+f(t)\exp\{2[\alpha(t)-1]\ln(s/s_0)\})\times\frac{d\sigma(s,0)}{dt} \qquad (2)$$

gives a very approximate picture of what is happening during elastic pp scattering. The low-energy inputs to the computation, of which the most important is $g^2/4\pi\simeq 5.1$ for the medium-strong coupling constant,[9] are the same as those in Sec. II of (A).

The model in (A) for the production of secondary pions and the resulting electromagnetic cascades observed on the ground used the following sequence of reactions:

$$\begin{aligned}&p+p\rightarrow p+p,\\&p+p\rightarrow p+N^{*+}(1470),\\&N^{*+}(1470)\rightarrow p+\pi^++\pi^-\text{ or }p+\pi^0,\\&\pi^0\rightarrow\text{"electromagnetic cascade."}\end{aligned} \qquad (3)$$

Besides implying that the pp collisions are not directly responsible for pion production, (3) selects a very specific type of high-energy isobar production. Any alternative to (3) which assumes that pions are produced directly at the time of the pp collision not only avoids having the computer program choose one of the first two reactions in (3) at each pp interaction on the basis of relative sizes of cross sections, but also models a situation which appears better than isobar production for a description of all the existing data.[10] Several previous calculations[11-13] have used approaches in which the amplitude for the reaction $pp\rightarrow pp+n\pi$ factorizes into an elastic-scattering part times a part for the production of n pions from the vacuum. Provided that the total energy of the pions is much less than the energy of the incoming proton, which is a constraint which can be imposed on the simulation, the differential cross section

$$\frac{d\sigma}{dt}(pp\rightarrow pp+n\pi)=\frac{d\sigma}{dt}(pp\rightarrow pp)e^{-3x}J_0(2ix)\frac{x^{n_0}}{n_0!} \qquad (4)$$

derived from such a starting point by Kastrup[12] is most suitable for this computation. Here, n_0 of the n pions produced are neutral while a summation is made over the remaining $n-n_0$ particles. J is a Bessel function, and the average number of charged pions in the process is $-2ixJ_1(2ix)/J_0(2ix)$, or approximately $2x$. The exact relation defines x.

The simulation calls on (4) repeatedly for information about particles p and π^0 in the development of a shower. For the cascades, it is simply assumed that a π^0 of a particular energy which decays produces on the ground a collection of electrons and positrons whose energies add up to that value, and whose center of distribution is found at the point where the trajectory of the pion, when extended, meets the ground. In the present computation, there is no need for any other information about the internal details of the cascade.

At this point the chief visible unknown is n_0, which is connected with the pion multiplicity. Firstly, it is assumed in the program (to reduce computing time) that $3n_0=n$, which is actually in line with (A) and with some cosmic-ray observations by Avakian and Pleshko,[14] and which seems to generate results which are not greatly different from those in a limited simulation of the case $2n_0=n$. For the multiplicity n itself, there is an open choice of substitutions, with at least two widely differing forms available. One, which is quoted in conjunction with a differential cross section[10] similar to (4), is expressed most simply as

$$n=[p^*/0.4\ (\text{GeV}/c)]\sin\theta^*, \qquad (5)$$

where p and θ are the three-momentum and scattering angle for the incident proton, and the asterisk denotes center-of-mass quantities. The other view is that n is largely energy-independent (apart from the dependence hidden in $p_\perp$), such as

$$n=\left(\frac{1}{p_\perp{}^2+2M^2(2-2K+K^2)}\right)^{1/3}\left(\frac{2\pi M^2K^2}{m_\pi}\right)^{2/3}\left(\frac{g^2}{4\pi}\right)^{1/3}, \qquad (6)$$

in which K is the fraction of momentum which the incident proton loses in a collision, m_π is a pion mass, $g^2/4\pi\simeq 5.1$, and $p_\perp$ is the proton's component of momentum transverse to its original path.

Equation (6) is adapted from an infinite-energy limit in a paper by Białas and Ruijgrok.[13] In the program it is truncated downwards to make n_0 an integer, in order to compensate for a possible 10% error[13] when it is applied to finite energies. Each simulation is performed once with (5) and once with (6).

A second large assumption may be concealed in (2). Normally one uses the familiar expression

$$G(t)=\{1-[t/0.71\ (\text{GeV}/c)^2]\}^{-2}, \qquad (7)$$

which is reasonable only if one is dealing with a range of t in which it is agreed to give a fit to experimental results that is superior to other alternatives. Lately, however, forms of $G(t)$ have been put forward which are extracted from Veneziano models and which appear to give a slightly better fit than (7) to the available data. A typical example[15] is

$$G(t)=\frac{\Gamma(\frac{11}{4})\Gamma(\frac{1}{2}-t)}{\Gamma(\frac{1}{2})\Gamma(\frac{11}{4}-t)}\ . \qquad (8)$$

The naive interpretation of a detailed comparison of (8) with (7) is that (8) should either reinforce the behavior of high-energy extensive air showers under discussion here, or start the behavior off at a lower energy than for (7), or both. To test these ideas further, separate simulations are made in the program with both (7) and (8).

Details of the organization of the simulations themselves are given in Sec. III of (A). The quantity of interest is a distribution or plot of rp_L/h (a measure of $p_\perp$) against the primary energy E or p_1^0, where r is the separation of the cores of any pair of π^0-induced cascades at ground level, h is the height of production of the π^0, and p_L is its component of momentum along the original direction of the primary particle. When primaries of energy down to 10^{13} eV are considered, rp_L/h is in all cases fairly constant with E up to about 8×10^{13} eV. The means derived from the simulations for each of the four cases [57], [67], [58], and [68], corresponding to the possible pairs of choices of n and $G(t)$ from Eqs. (5) through (8), are shown in the curves in Fig. 1. There, $y=\log_{10}(rp_L/h)$, and $\epsilon=\log_{10}E$ (p_L in units of GeV/c, E in units of eV). Also on Fig. 1 are plotted some representative observational points from those given by Bakich, McCusker, and Winn.[5]

The curves in Fig. 1 show that the choice of (8) over (7) does not make any important change in the magnitude of the transverse-momentum effect, but that it may cause the effect to set in at a slightly lower value of s with (8) than with (7) on substitution for $G(t)$ in (1).

One's preference for a given form of multiplicity n has a more significant influence on the simu-

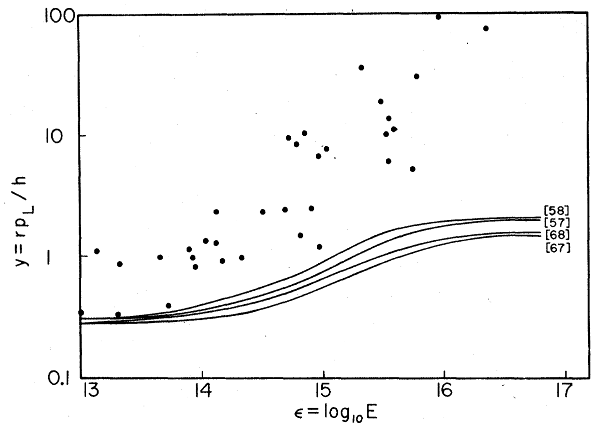

FIG. 1. A plot of mean values of y against ϵ for the simulated showers. The label $[ij]$ on a curve indicates that it is the result of a simulation which uses equations (i) and (j). The small circles show representative observational points.

lation and on Fig. 1. The substitution (5), which is closer than (6) to the spirit of the derivation of the cross section (4), produces the greatest deviation of y from independence of energy. Even when (6) is used, though, the steady increase of y with E when $E>10^{14}$ eV is an effect which persists.

The appearance of $G(t)$ in (2) makes the differential cross section fall off rather more slowly with increasing $-t$ than if only the conventional strong interaction is taken into account. Nevertheless, the medium-strong interaction does not lead to a wholesale shift of pp scattering events into a region of t well away from zero. The maximum of $d\sigma(s,t)/dt$ with t is still at $t=0$, as (2) indicates. Thus, although the curves drawn in Fig. 1 represent means, the occurrence of individual events with values of y well above a mean for given ϵ is not excluded. In the computations, events with values of y up to 3.2 times the mean for any ϵ have been noted. These events, fortunately, fall at the lower edge of the envelope of observational points in Fig. 1. (The observational points were not given originally[3,5] on a scale of energy, but on a scale of shower size. They have been transferred to Fig. 1 with the help of the folklore-based assumption[16] that the total number of particles in the shower at ground level times 2×10^9 gives the primary energy in eV, an assumption which may be in error by a factor of 2 and may thus shift the horizontal scale of Fig. 1 by ±0.3 units.)

As the methods of this note and the methods of (A) differ completely in their treatments of pion production, but have in common the use of a differential cross section for elastic pp scattering [substituted into the right-hand side of (4) in the present calculation] which is based on the assumption of a residual contact interaction between the protons, there is good reason to believe that an eventual increase of y with energy is at least partly the consequence of the existence of such an interaction.

I thank Dr. M. G. Gundzik for some helpful discussions in the early stages of this work.

*Work supported in part by the National Science Foundation and the U. S. Atomic Energy Commission.

[1]J. A. Campbell, Phys. Rev. 183, 1111 (1969).
[2]A. O. Barut and W. Plywaski, Phys. Rev. D 2, 572 (1970).
[3]C. B. A. McCusker, L. S. Peak, and M. H. Rathgeber, Phys. Rev. 177, 1902 (1969).
[4]M. Samorski, R. Staubert, J. Trümper, E. Böhm, W. Büscher, and R. Fritze, Z. Physik 230, 1 (1970).
[5]A. M. Bakich, C. B. A. McCusker, and M. M. Winn, J. Phys. A 3, 662 (1970).
[6]H. V. Bradt and S. A. Rappaport, Phys. Rev. 164, 1567 (1967).
[7]Y. Ne'eman, Phys. Rev. 172, 1818 (1968).
[8]H. D. I. Abarbanel, S. D. Drell, and F. J. Gilman, Phys. Rev. Letters 20, 280 (1968).
[9]H. D. I. Abarbanel, S. D. Drell, and F. J. Gilman, Phys. Rev. 177, 2458 (1969).
[10]E. W. Anderson and G. B. Collins, Phys. Rev. Letters 19, 201 (1967).
[11]Z. Chylinski, Nucl. Phys. 44, 58 (1963); H. A. Kastrup, Phys. Rev. 147, 1130 (1966); R. C. Arnold and P. E. Heckman, *ibid.* 164, 1822 (1967); M. G. Gundzik, *ibid.* 184, 1537 (1969).
[12]H. A. Kastrup, Nucl. Phys. B1, 309 (1967).
[13]A. Białas and T. Ruijgrok, Nuovo Cimento 39, 1061 (1965).
[14]V. V. Avakian and M. P. Pleshko, Can. J. Phys. 46, S709 (1968).
[15]P. H. Frampton, Phys. Rev. 186, 1419 (1969).
[16]G. B. Yodh, J. R. Wayland, and Y. Pal, Fields and Quanta 1, 233 (1971).

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Comments on Duality and the Final-State-Interaction Approach to $N\bar{N}\to 3\pi$†

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Duality suggests that it is more informative to factorize an amplitude for $N\bar{N}\to 3\pi$ at the exchanged nucleon poles than at a direct-channel singularity. Experimental verification of t-channel factorization and the implications for the final-state-interaction approach to $N\bar{N}$ annihilations are discussed.

Analysis of $N\bar{N}\to 3\pi$ has usually been based on an assumption that the main features of the data are due to a final-state interaction in the pion system. Lovelace's[1] interpretation of the data on $\bar{p}n\to\pi^+\pi^-\pi^-$ annihilations at rest as a continuation in an external mass leg of the Veneziano $\pi\pi$ scattering amplitude is an example of this kind of simple final-state-interaction approach. The impact of Lovelace's suggestion and the widespread acceptance of his basic assumption can be measured by the number of his imitators. Fits to $\bar{p}n\to\pi^+\pi^-\pi^-$ with the same basic model for the amplitude but with more free parameters have proliferated.[2-5] Fits to other annihilation reactions[6,7] with sums of Veneziano four-point functions have also emerged.

Of these fits, the most careful treatment of the problems inherent in the *ad hoc* "unitarization" of a Veneziano amplitude for phenomenological purposes is by Pokorski, Raitio, and Thomas.[5] They expand the Veneziano functions simultaneously in terms of the poles in the s and t channels. This expansion is convergent within the Dalitz plot[8] and enables them to "unitarize" by giving each resonance an appropriate total width while the partial widths are determined by the coefficients of the Veneziano functions. Cruder treatments of the unitarization problem forced the total widths of all resonances within a given tower to be the same.

Treated in this way, the use of the Veneziano model in the sense of a sum over $\Gamma\Gamma/\Gamma$ terms is a convenience for reproducing a general final-state $\pi\pi$ interaction with a reasonable spectrum of resonances. As a comparison, the rising-phase-