of which are charged, for which the contribution of an ω pole does not enter.

[19]This is expected to be a reasonable approximation in the energy range of interest. More exact numerical estimates could be made using exact phase space; however, the approximations made in our matrix element are of similar order to those made in estimating phase space. We are working only to lowest order in $T/4m_\pi$ here; $\sigma(e^+e^- \to \pi^+\pi^+\pi^-\pi^-)$ will contain corrections of the form $1 - T/3m_\pi + O((T/4m_\pi)^2)$.

[20]We have not considered the amplitude for production of an odd number of pions by two photons, which is anomalous in the framework of the current algebra and requires the use of additional dynamical assumptions. These processes are discussed by R. Aviv, N. D. Hari Dass, and R. F. Sawyer, Phys. Rev. Letters 26, 591 (1971); 26, 1290(E) (1971); S. Adler, B. W. Lee, S. B. Treiman, and A. Zee, Phys. Rev. D 4, 3497 (1971), and other authors.

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Fixed Pole and Shielding Cut in πN Charge Exchange*

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A model for the relationship of a fixed pole to a shielding cut is constructed. In order to remove the constraints imposed by unitarity we required the following to hold in the limit $l \to l_0$: (1) The branch point of the shielding cut coincides with the elastic threshold. (2) The discontinuity of the amplitude on the elastic cut is singular and finite at the end point $l = \alpha_c(t)$ of the cut in the angular momentum plane. (3) This discontinuity contains the fixed pole. We find that the fixed pole does not contribute to the asymptotic behavior in the crossed channel as it has been asserted, but the shielding cut gives a negative nonvanishing contribution at a wrong-signature point. We discuss phenomenological implications in connection with dip-bump structure in the differential cross section, and the dependence of the polarization upon both energy and momentum transfer in πN charge exchange. We find that our results agree with recent experimental results.

I. INTRODUCTION

Previously, the assumption of Regge-pole dominance was motivated by the observed peripheral character of scattering angular distributions and by the practical consideration of simplicity. In fact, pole models have been remarkably successful in describing many two-body processes and in correlating scattering data with the discrete particle spectrum. In particular, dips observed in the momentum-transfer distribution of several reactions have been rather unambiguously correlated with the zeros of Regge-pole terms at points where the trajectory passes through an integer of the wrong signature in the "nonsense" region. Nonetheless, serious embarrassments for single-pole model have cropped up such as the experimental polarization in πN charge exchange.

On the other hand, we know that there are fixed poles at nonsense-wrong-signature points. Such poles in the amplitude are themselves in conflict with the analytically continued unitarity condition. The residues of these poles are related to moments of the third double-spectral function, and for a small third double-spectral function the residue of the fixed pole is weak. Such a fixed pole induces a singularity in the Regge-pole residue, which causes a slight displacement of the dip.

In this paper we assume that the effects of the third double-spectral function are important, and therefore in order to preserve unitarity we assume that the fixed poles are shielded by moving branch points in the angular momentum plane. We discuss the properties of Regge cuts and fixed poles and their phenomenological implications in πN charge exchange. The plan of the paper is as follows: In Sec. II we first review the properties of fixed poles and shielding cuts and we construct a model for the fixed-pole–shielding-cut relationship. In Sec. III we discuss the phenomenological implications of the model.

II. FIXED POLES AND SHIELDING CUTS

Let us consider for example π-π scattering. The Froissart-Gribov representation for the partial-wave amplitude which is free of kinematical singularities at the threshold is given by

$$B_l^{\pm}(t)=\frac{1}{\pi(q^2)^l}\int_{z_0}^{\infty} Q_l(Z_t)\,\mathrm{Im}A_{su}^{\pm}(Z_t,t)\,dZ_t, \tag{1}$$

where Z_t is the cosine of the scattering angle in the c.m. of the t channel. This integral representation defines a holomorphic domain in the l plane for $\mathrm{Re}\,l>N$, where N is given by the asymptotic behavior of the amplitude. However, we cannot define an analytic continuation to negative integer values of l where the Legendre function has poles. Therefore we consider instead of (1) its left-hand cut discontinuity in the t plane which consists essentially of two terms; one of them is holomorphic in l and the other one is given by a finite integral

$$\mathrm{Im}_L B_l^{\pm}(t)=\frac{1}{\pi(q^2)^l}\int_{a(t)}^{b(t)} \rho_{su}(z,t)(1\mp e^{-i\pi l})Q_l(Z_t)\,dZ_t, \tag{2}$$

which occurs only in relativistic theory. This integration contour is taken along a line in the region where the third double-spectral function ρ_{su} is nonzero and positive. This expression has a meaning for any complex value of l, since it is determined by integrals over a finite region of an analytic function, so that we can perform an analytic continuation of this expression over the whole angular momentum plane. The two terms within the bracket of the above expression add when l is wrong-signature and cancel when l is right-signature. For small values of t, the third spectral function is known exactly and the above integral does not vanish. In (1) there is no cancellation between the poles on the left-hand cut and those on the right-hand cut, because this cancellation would imply a very strong constraint on the integrals of the three spectral functions. We can therefore conclude that the whole scattering amplitude will have these poles. These poles subsist even if there are moving cuts in the l plane. We cannot restate here why we need cuts; this was done before Mandelstam,[1,2] who proved that cuts are the result of inelastic contributions to the unitarity relation and are particular to the relativistic problem. These cuts arise from complicated diagrams (Fig. 1). The essential features of these graphs are right- and left-hand portions (i.e., the crosses) which, when considered by themselves, exhibit a third double-spectral function. Therefore the diagram of Fig. 1 will have a Gribov-Pomeranchuk singularity at $l=\sigma-1$, where l is the angular momentum in the t channel and σ is the spin of the bound state which is lying on the Regge trajectory. Mandelstam was then able to show by a number of ingenious tricks that the singularity can be made to disappear by moving the cut past the point $l=\sigma-1$, and that the discontinuity across the cut can obviously not cancel. In the angular momentum plane the Sommerfeld-Watson transformation for the amplitude will have, in addition of the usual Regge singularities (poles and cuts), poles at $\alpha(M_i^2)=0,1,2,$ where M_i are the masses of the physical bound states or resonances lying on the Regge trajectories. The Sommerfeld-Watson contour will be pinched between the moving branch point and the above-mentioned poles when $t_i=(m+M_i)^2$. This gives the position of the singularities in t of $A(s,t)$ which arise from bound states and resonances lying on the Regge trajectory. They are in fact the two-body normal threshold branch points, and for $l=-1$, t_i will coincide with the elastic threshold. We have seen that for $\mathrm{Re}\,l>N$ the Froissart-Gribov representation defines an holomorphic domain for the partial-wave amplitude. In the t plane, this means that the branch point $t_c(l)$ must be located on an unphysical sheet reached by continuing through a physical sheet cut. (See Fig. 2.) As l decreases, the moving branch points will emerge from the inelastic threshold. In particular, when $-1<l<0$ the branch point $t_c(l)$ moves into the physical sheet between the threshold $t=4m^2$ and the first inelastic threshold, so as l approaches -1 the elastic unitarity cut becomes completely blanketed and the unitarity equation in the form $\mathrm{Im}B=\rho(t)|B|^2$ cannot therefore be used. On the other hand, when l is reduced from large real values, Eq. (2) is still valid; therefore we can now continue analytically in l and $\mathrm{Im}B_l^{\pm}$ will continue to be given by (2) which is an analytic function of l. We see that the moving cut does not overlap the left-hand cut as l is varied from a large real value, so that the amplitude has the fixed pole. Now we have shown that for nega-

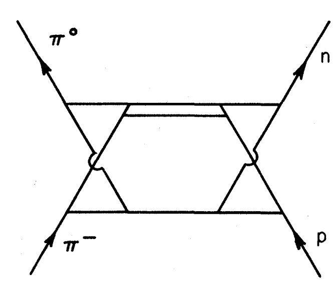

FIG. 1. The Mandelstam diagram with the third double-spectral function.

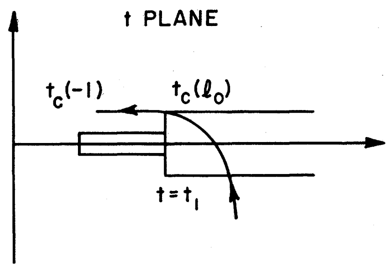

FIG. 2. Moving branch point in the energy plane.

tive value of t the imaginary part of the amplitude will have a fixed pole (among the others) associated with angular momentum of wrong signature; it cannot be cancelled by a fixed pole which can occur for positive value of t, or overlapped by moving cuts.

Let us consider now the trouble with the analytically continued unitarity relation.[3] Let us assume that the partial wave has a fixed pole on the first sheet; we can easily find its value at the corresponding point on the second sheet by unitarity. Then we have an amplitude which is bounded by the phase-space factor[4] on the second sheet and which is singular on the other; hence the pole would be lost as we continue $B(l, t)$ through the elastic cut into the second sheet. Such a disappearance of an isolated singularity is not compatible with the continuity theorem.[5] In order to reconcile the Gribov-Pomeranchuk singularity with the unitarity relation, we consider small effects of the third double-spectral function. We take only account of the first-order effect of ρ_{su} and ignore the cuts which come from the second order. By expanding in terms of the strength of ρ_{su}, the partial-wave amplitude, in the t channel, may be expressed as

$$B^{\pm}(t_+, l) = B_0^{\pm}(t_+, l) + B_1^{\pm}(t_+, l),$$

$$B^{\pm}(t_-, l) = B_0^{\pm}(t_-, l) + B_1^{\pm}(t_-, l).$$

The t-channel unitarity relation can also be rewritten in zeroth order as

$$B_0^{\pm}(t_+, l) - B_0^{\pm}(t_-, l) = 2i\rho(t) B_0^{\pm}(t_+, l) B_0^{\pm}(t_-, l)$$

and in the first order as

$$B_1^{\pm}(t_+, l) - B_1^{\pm}(t_-, l) = 2i\rho(t)\left[B_0^{\pm}(t_+, l)B_1^{\pm}(t_-, l) + B_1^{\pm}(t_+, l)B_0^{\pm}(t_-, l)\right].$$

In order to exhibit the relevant poles in $B_0^{\pm}(t_{\pm}, l)$, $B_1^{\pm}(t_{\pm}, l)$ we write

$$B_0^{\pm}(t_+, l) \simeq \frac{\beta^{\pm}(t)}{l - \alpha(t)}, \quad B_1^{\pm}(t_+, l) \simeq \frac{\gamma_1^{\pm}(t)}{l - l_0},$$

$$B_0^{\pm}(t_-, l) \simeq C^{\pm}(t), \qquad B_1^{\pm}(t_-, l) \simeq \frac{\gamma_2^{\pm}}{l - l_0}.$$

Note that in the zeroth-order expression the fixed pole effect is not involved. In the first order, on the other hand, the most singular term is

$$\operatorname{disc}[B_1^{\pm}(t, l)] \simeq \frac{\gamma_2^{\pm} B^{\pm}(t)}{[l - \alpha(t)](l - l_0)}.$$

We can now draw the following conclusions:

(1) In the first order of the third double-spectral function, the fixed pole is present both above and under the unitarity cut, consequently there is no difficulty with the continuity theorem, and we do not need any shielding branch points in this approximation.

(2) There is however a nonsense-wrong-signature pole at $l = l_0$ in the Regge residue. The phenomenological implications of this treatment have been considered by Chiu and Matsuda.[6] These authors, in particular, can explain only the dip in πN charge-exchange scattering, but not the polarization effect which has been observed experimentally.[7,8]

In this paper we suppose that the contribution of the third double-spectral function is large. In this case the contribution of the cut will be very large, in agreement with phenomenological considerations,[9] so we cannot use the previous arguments in order to reconcile fixed pole and unitarity. On the other hand, the existence of a moving cut with the trajectory given by $l = \alpha_c(t)$ and having the properties that $\alpha_c(t_0) = l_0$, where t_0 is the elastic-channel threshold and where l_0 is the wrong-signature fixed pole, is not enough to reconcile a fixed pole and unitarity. If we deform the cut in the l plane to cross the fixed pole, the discontinuity across the cut can obviously not cancel. The Gribov-Pomeranchuk singularity loses its character of an essential singularity, so that the fixed pole is all that remains of the Gribov-Pomeranchuk essential singularity when the deformed cut is present.[10]

Let us now give some consequences which remove the contradiction of a fixed pole with the unitarity relation. In the limit $l \to l_0$ (l_0 integer):

(1) The branch point of the moving cut $t_c(l)$ coincides with the elastic threshold.

(2) The unitarity cut is completely overlapped by the shielding cut and consequently the latter must have the same singularity character as the unitarity cut.

(3) The fixed pole is in the partial-wave amplitude, on the unphysical sheet, when the moving cut is present.

(4) The discontinuity of the shielding cut contains the fixed pole.

Now, we deduce from the above requirements the structure of the discontinuity in l of $B_c^{\pm}(l, t)$. We find that $B_c^{\pm}(l, t)$ with the discontinuity

$$\mathrm{disc}[B_c^{\pm}(l,t)] = \beta_c^{\pm}(t)\frac{[t-t_c(l)]^{l_0+1/2}}{l-l_0}$$

satisfies all the conditions. However, we have to assume that $\beta_c^{\pm}(t)$ is a real analytic function of t in the physical region of the s channel ($t<0$, $s\to\infty$).

A similar amplitude, with such a discontinuity, which removes the constraints imposed by the unitarity relation has been obtained by Bronzan and Jones.[11] The latter consider a positive-signature amplitude with a wrong-signature negative integer angular momentum $l=-1$ in the spinless case. They construct a Regge-cut amplitude which masks the elastic unitarity cut at $l=-1$ and removes the Gribov-Pomeranchuk essential singularity and satisfies the unitarity relation. They obtain a discontinuity in l, which also contains the fixed pole in the angular momentum plane. They also show that the residue of the discontinuity which contains the fixed pole has a logarithmic branch point at the elastic threshold. At this point (which is in the unphysical region) the residue of the discontinuity may vanish.

In this paper, we assume that a wrong-signature fixed pole is present in the $B^-(z,t)$ spin-flip amplitude of the π-N charge-exchange scattering. This fixed pole is at a wrong-signature nonsense-sense point $l=0$. This singularity does appear like a pole of the Legendre function $Q_j^1(z,t)$ which is present in the Froissart-Gribov spin-flip amplitude analytically continued in the angular momentum plane. At the same time the fixed pole at $l=0$ has been observed in the spin-flip amplitude by using finite-energy sum rules.[12] This pole is in conflict with the analytically continued unitarity relation, which is written down for the spin-flip amplitude. We assume a singularity structure where π-π is a relevant threshold[4,5]; therefore the fixed pole at $l=0$ in $\pi\pi\to N\bar{N}$ requires a moving shielding cut connected with the π-π threshold with the condition $\alpha_c(4\mu^2)=0$. In accordance with our previous conclusions, we find that a Regge cut with a discontinuity of the form

$$\beta_c(t)[t-t_c(l)]^{1/2}/(l-l_0)$$

at $l_0=0$, in the spin-flip amplitude, overlaps perfectly the unitarity cut and removes the trouble with unitarity relation. We note that the point $l=0$ is a sense-sense point for the non-spin-flip amplitude. Furthermore it does not correspond to a pole of the Legendre function of the Froissart-Gribov for the non-spin-flip amplitude; therefore we do not need any shielding cut for this amplitude. Our purpose is to investigate, in the following part, the phenomenological implications of our assumptions.

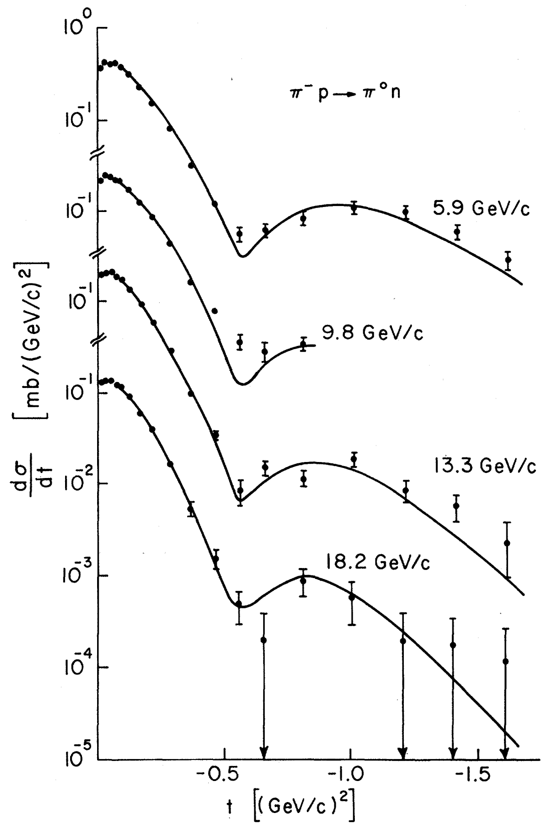

FIG. 3. Differential cross section for πN charge exchange; data from Ref. 24.

III. PHENOMENOLOGICAL IMPLICATIONS

The high-energy πN charge-exchange differential cross section near the forward direction has been explained[13] (Fig. 3) in terms of the t-channel ($\pi\pi\to N\bar{N}$) ρ trajectory exchange. In this case the phase of the helicity-flip and -nonflip amplitudes should be about the same and therefore the charge-exchange polarization, which occurs when the phases of the helicity flip and nonflip differ, should be zero. Polarization[7,8] has been observed in $\pi^-p\to\pi^0n$ [Figs. 4(a) and 4(b)]. The main features of these experimental results are:

(1) The polarization decreases with increasing energy.

(2) In a large momentum-transfer interval the polarization is positive and exhibits a large positive spike near the point where the ρ trajectory crosses the zero.

Various possibilities within the Regge model have been proposed: complex Regge poles,[14] absorption Regge cuts,[15] interference between the ρ trajectory and another trajectory,[16] or direct-channel resonances[17] or cuts.[18]

The essential feature of our model is that it explains the polarization in πN charge-exchange

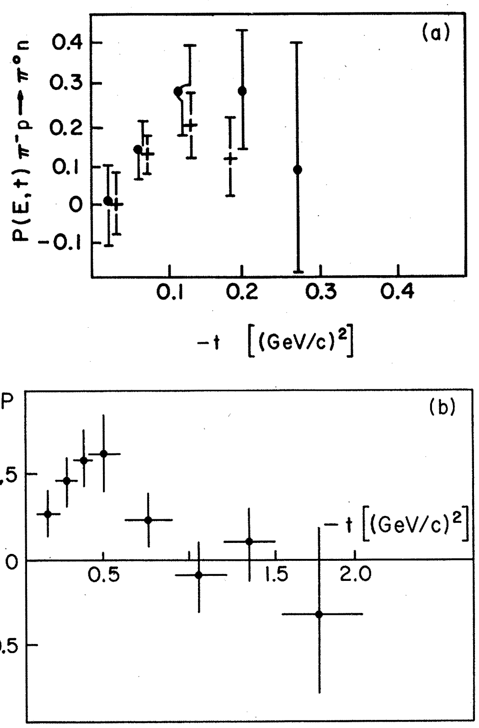

FIG. 4. (a) π^-p charge-exchange polarization at (i) 11 GeV/c (crossed lines); (ii) 5.9 GeV/c (closed circles). (b) π^-p charge-exchange polarization at 5 GeV/c. Data from Refs. 7 and 8.

scattering (CEX) by a fixed pole in the πN spin-flip amplitude. It reflects the influence on the amplitude of the shielding cut associated with the two-pion threshold in the crossed channel $\pi\pi \to p\bar{p}$. We can write the t-channel $N\bar{N} \to \pi\pi$ helicity amplitudes contained in the variable s and t to the s-channel physical region in terms of scalar amplitudes $A^\pm$ and $B^\pm$. Following Singh[19] we define the non-spin-flip amplitude

$$f_0^\pm(s,t) = (t-4M^2)^{-1/2}[(4M^2-t)A^\pm(s,t) + M(s-u)B^\pm(s,t)] \tag{3a}$$

and the spin-flip amplitude

$$f_1^\pm(s,t) = 2(|t|)^{1/2} p\cdot k \frac{\sin\theta_t}{(t-4M^2)^{1/2}} B^\pm(s,t), \tag{3b}$$

where p and k are the momenta of π and N in the center-of-mass system. The kinematics give

$$t = 4(k^2+\mu^2) = 4(p^2+M^2),$$

$$z_t = \cos\theta_t = -\frac{s-u}{4pk}.$$

The differential cross section is given by

$$\frac{d\sigma}{dt} = \frac{1}{16\pi E^2}(|f_0|^2 + |f_1|^2) \tag{4}$$

and the polarization of the recoiling nucleon is

$$P = \frac{\mathrm{Im}(f_0 f_1^*)}{|f_0|^2 + |f_1|^2}. \tag{5}$$

E is the energy in the laboratory frame. We have to include in the helicity flip amplitude, in addition to the usual Regge trajectory, the shielding cut contribution. Taking the high-energy part of (3a) and (3b) we have for $\mathrm{Re}\,\alpha > -\frac{1}{2}$

$$f_1^\pm(s,t) \underset{s\to\infty}{\sim} \frac{\sqrt{t}}{(t-4M^2)^{1/2}(-2M\cos\theta)} \times \left[\alpha(t)\gamma_1^\pm(t)R_\rho^\pm(s,t) + \beta_c^\pm(t)\frac{R_c^\pm(s,t)}{[\ln(E/E_0) - i\pi/2]^{3/2}}\right], \tag{6a}$$

$$f_0^\pm(s,t) \underset{s\to\infty}{\sim} \frac{\gamma_0^\pm(t)}{(t-4M^2)^{1/2}} R_\rho^\pm(s,t), \tag{6b}$$

where

$$R_\rho^\pm(s,t) = \frac{1}{\Gamma(\alpha_\rho+1)}\,\frac{1 \pm e^{-i\pi\alpha_\rho}}{\sin\pi\alpha_\rho}\left(\frac{s-u}{2s_0}\right)^{\alpha_\rho},$$

$$R_c^\pm(s,t) = \left(\tan\frac{\pi\alpha_c}{2} \pm i\right)\frac{1}{\Gamma(\alpha_c+1)}\left(\frac{s-u}{2s_0}\right)^{\alpha_c}.$$

At $t=0$, the helicity flip $f_1^\pm(s,t)$ does not contribute to the differential cross section. At $t \simeq -\frac{1}{2}$, we see that, if the ρ trajectory chooses nonsense, only the shielding cut contributes; but, if the ρ trajectory chooses sense, the Regge cut contributes to the nonflip amplitudes, and we have a displacement of the dip.

From (5), (6a), and (6b) we obtain the energy dependence of the polarization, which is given by

$$\frac{d\sigma}{dt}P \simeq \frac{\sqrt{-t}}{16\pi M}\left(\frac{E}{E_0}\right)^{\alpha_c+\alpha_\rho-2}\frac{\sin\frac{1}{2}\pi(\alpha_\rho - \alpha_c + 3\theta/\pi)}{\cos\frac{1}{2}\pi\alpha_\rho\cos\frac{1}{2}\pi\alpha_c} \times \gamma_0'(t)\beta_c'(t)\frac{1}{[\ln^2(E/E_0) + (\frac{1}{2}\pi)^2]^{3/4}}, \tag{7}$$

with $\tan\theta = \pi/2\ln E$. The polarization is zero in the forward direction and will vanish for values of momentum transfer for which

$$\alpha_\rho(t) - \alpha_c(t) + 3\theta/\pi = n \quad (n = 0, 2, 4, \ldots),$$

so that the location of the zeros depends on the energy. The energy-dependent zero in the polarization reflects the fact that there exists a physical value of t for which the sine is zero, and here the polarization has change of sign. If we suppose that for some values of t the cut and the pole will get closer together, then the polarization will be essentially energy dependent and controlled by

$$\frac{\sin\frac{3}{2}\theta}{[\ln^2(E/E_0) + (\frac{1}{2}\pi)^2]^{3/2}}.$$

In this region the polarization will decrease for increasing E. At large t the contribution of the

TABLE I. Results of fits to the charge-exchange differential cross sections.

	α_ρ	α_c	$\gamma_0'(t)=a_0e^{a_1t}$	$\gamma_1'(t)=b_0e^{b_1t}$	$\beta_c'(t)=b_0'e^{b_1't}$
Case 1	$t+0.58$	$0.25t-0.02$	$2.20e^{1.2t}$	$38.5e^{0.06t}$	$-12.80e^{0.1t}$
Case 2	$t+0.58$	$0.25t-0.02$	$3.9\,e^{0.1t}$	$7.6e^{0.1t}$	$-9.6e^{0.02t}$

phase of the cut is bigger than that of the pole and the polarization will increase with energy. So far we have not discussed the question of where the shielding cut comes from. We assume that the relevant shielding cut is a Reggeon-Reggeon cut. We approximate it by a linear structure of the form

$$\alpha_c(t)=\alpha_c' t+\alpha_c(0)\,,$$

with

$$\alpha_c'=\frac{\alpha_1'\alpha_2'}{\alpha_1'+\alpha_2'} \quad\text{and}\quad \alpha_c(0)=\alpha_1(0)+\alpha_2(0)-1;$$

α_1', $\alpha_1(0)$ and α_2', $\alpha_2(0)$ are the parameters of the linear trajectories exchanged which cause it. The threshold condition gives us $\alpha_c(0)=-0.08\alpha_c'$.

In our numerical analysis we fit[20] the charge-exchange differential cross section in the region $0<|t|\leqslant 2.5$ $(\mathrm{GeV}/c)^2$, $3<E<18.2$ GeV and compute the polarization for different values of the energy. We consider two different parametrizations for the ρ amplitudes, depending upon whether the ρ trajectory chooses sense (case 1) or nonsense (case 2) at $\alpha_\rho=0$. The fit of the charge exchange differential cross section gives, from a total of 56 data points, the results shown in Table I.

From the fit of the data we can now discuss the question of where the shielding cut comes from. The fit gives us $\alpha_c(0)\simeq 0$; this numerical result suggests us that this cut can be identified with a $\rho p'$ cut.[21]

If we omit the dip region at about $t=-0.58$ $(\mathrm{GeV}/c)^2$, then models with ρ choosing sense and ρ choosing nonsense give essentially indistinguishable fits; it is a matter of parameter adjustment. At the dip point, the model in which ρ chooses nonsense gives a value which is too low by about an order of magnitude (both the pole and the pole-cut interference amplitudes vanish; only the cut contribution is nonzero). On the other hand, the model in which ρ chooses sense fills in the dip and is the preferred solution. In neither model does the cut significantly alter the energy dependence of the cross section. This is because the ρ still dominates and the spin-flip amplitude, which is the more important, has an α_ρ factor in both models so that the basic features are similar.

The predictions of charge-exchange polarization in a larger interval for different energies are il-

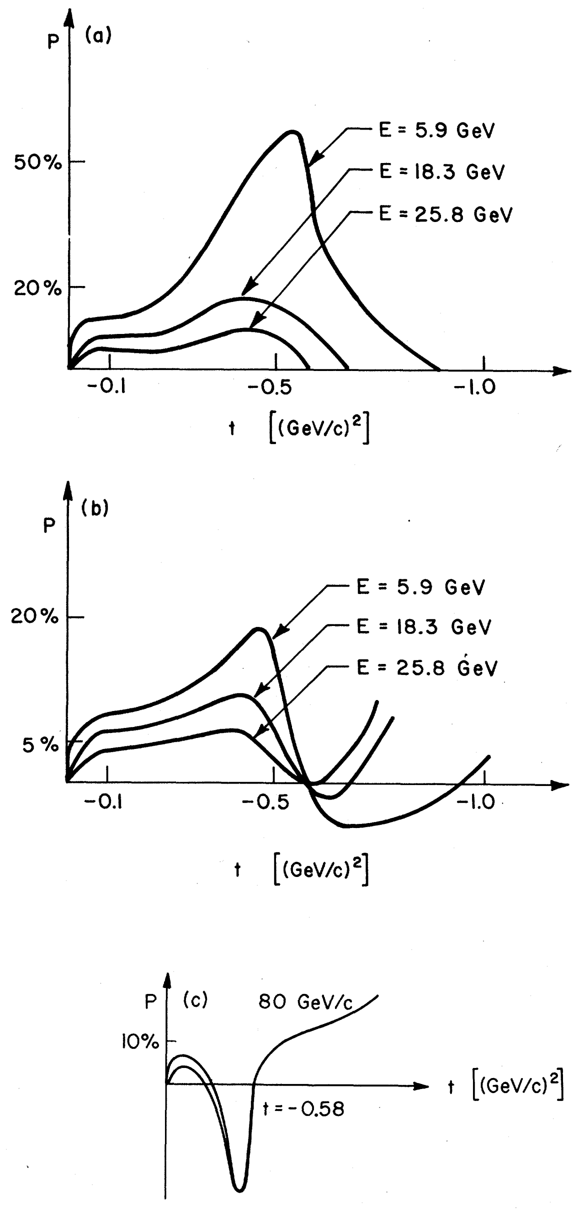

FIG. 5. $\pi^- p$ charge exchange polarization: (a) case 1, choosing sense; (b) case 2, choosing nonsense; (c) case 2, choosing nonsense.

TABLE II. Results of our calculation compared with experiment.

Momentum-transfer values		−0.1	−0.12	−0.50	−0.58	−0.64	−0.93
Results from our calculations							
Case 1	(a) 5.8 GeV/c	11.5%			50%		0
	(b) 18.2 GeV/c	5%			21%	0	
Case 2	(a) 5.8 GeV/c	5.5%		18%	0		0
	(b) 80 GeV/c	4%	0	−65%	0		
Experimental results from Ref. 7							
	(a) 5.9 GeV/c	15%					
	(b) 11 GeV/c	14%					
from Ref. 8							
	(a) 5 GeV/c	23%			60%	30%	0

lustrated in Fig. 5. If the ρ trajectory chooses sense (case 1) in the forward direction the polarization is zero, while for $0.01 \leq |t| \leq 0.58$ $(\mathrm{GeV}/c)^2$ the polarization is dominated by the factor

$$\frac{\sin\frac{1}{2}\pi(\alpha_c - \alpha_\rho + 3\theta/\pi)}{[\ln^2 E + (\frac{1}{2}\pi)^2]^{3/4}} E^{\alpha_c - \alpha_\rho},$$

which decreases with increasing energy when E varies from 3 to 18.2 GeV/c. Near $t \simeq -0.58$ $(\mathrm{GeV}/c)^2$ the α_ρ trajectory crosses the zero; therefore, the $B_\rho B_c$ interference term vanishes at this point, and only the A_ρ and the B_c amplitudes contribute to the differential cross section. At this point, the differential cross section is minimum; therefore, from Eq. (7) we see that the polarization is maximum. This gives at $t = -0.58$ $(\mathrm{GeV}/c)^2$ a large positive spike in the polarization. After this spike the polarization falls quickly [Fig. 5(a)]. This spike has been experimentally observed [Fig. 4(b)].

If the ρ trajectory chooses nonsense (case 2) the polarization will have, in addition to the moving zero, a fixed zero at the position of the dip. We therefore distinguish two cases:

(i) The moving zero is on the right of the Regge zero. In this case [Fig. 5(b)], the polarization is zero at the point where the ρ trajectory crosses the zero.

(ii) The moving zero is on the left of the Regge zero. In this case the polarization [Fig. 5(c)] shows a large negative spike between these two zeros.

In both cases in the interval $0 \leq |t| \leq 0.58$ $(\mathrm{GeV}/c)^2$ the polarization decreases with increasing energy. In case 1, the numerical results agree with experimental values; in case 2, the cut parameters fill the dip, and we find a polarization which is too small. Clearly, the model in which the ρ chooses sense is closer in magnitude to the data. The variations of the predicted polarization with parameter changes are rather different below or above about $t \simeq -0.58$ $(\mathrm{GeV}/c)^2$. In Table II, we compare the numerical results of our calculations with experimental results from Refs. 7 and 8. It should be remarked that the large negative spike in the charge-exchange polarization in case 2 only appears for $E > 26$ GeV.

Let us now compare our results with the absorption-model calculation for πN charge exchange. The Michigan model[22] with ρ choosing sense and with a ρ-P cut (this is also the model of Cohen-Tannoudji *et al.*[23]) has no $\alpha_\rho(t)$ factor in β_ρ. The cuts are magnified through multiplication with coefficients of the order 1.5 and B_c is then large enough to cancel β_ρ near $\alpha_\rho(t) = 0$. In the small-momentum-transfer region they are in agreement with experimental values.[7] The polarization changes sign at $t = -0.4$ $(\mathrm{GeV}/c)^2$ for $E = 9.9$ GeV and has a sharp negative spike near $t = -0.55$

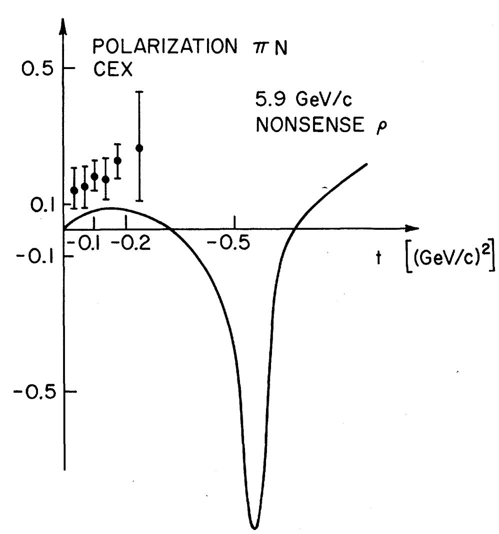

FIG. 6. πN charge-exchange polarization in the Michigan model. Data from Ref. 22.

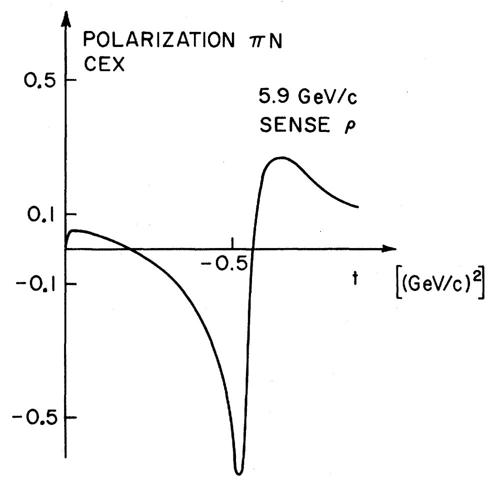

FIG. 7. πN charge-exchange polarization in the Argonne model. Data from Ref. 24.

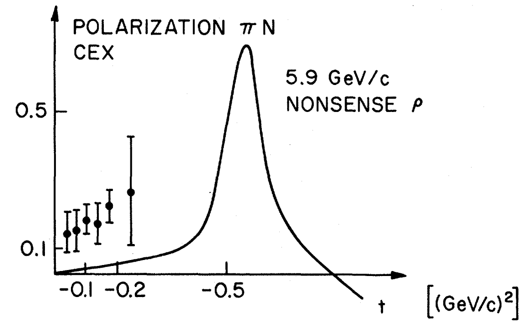

FIG. 8. πN charge-exchange polarization in the Rivers and Saunders model. Data from Ref. 25.

$(\mathrm{GeV}/c)^2$. The typical polarization curve is shown in Fig. 6. This negative spike does not agree with the recent data.[8]

In the Argonne model[24] (ρP and $f^0\rho$ cuts) and ρ and f^0 choosing nonsense, the Regge parametrization is not extended in t sufficiently far to include the $\alpha_f(t)+1$ factor in $A_f'(z,t)$. The polarization follows the trend above with a negative spike at the dip point (Fig. 7). The predictions of this model also do not agree with the recent data. The model of Rivers and Saunders[25] and White[26] gives a polarization of about 8% for small t at 5.9 GeV/c which continues to rise, i.e., there is no negative spike (Fig. 8).

In summary, following the requirements of the unitarity relation, we introduce a cut in the B^- amplitude which shields the fixed pole at $l=0$. The discontinuity of this cut contains the fixed pole, while the latter does not contribute to the asymptotic behavior in the crossed channel. The shielding cut does give a negative nonvanishing contribution to the differential cross section. In contrast to the absorption model, the charge-exchange polarization is due to the interference between B_c' and A_ρ. The charge-exchange polarization is energy- and momentum-dependent - a result which is consistent with the available data. It is further predicted that the polarization must go to zero for some value of t dependent upon the value of the energy. Furthermore, if the trajectory chooses nonsense, there is an additional zero of the polarization which does not depend upon the value of the energy. Clearly, the model in which the ρ chooses sense is closer in magnitude to the data. At the point where the ρ trajectory is zero, the polarization exhibits a large positive spike and at $E=5.9$ GeV, the polarization is 50%. These results are consistent with the recent data.[8] The results of the fit of the data allow us to identify the shielding cut with a $\rho P'$ cut.

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Regge-Pole Model of Pion-Nucleon Scattering with Explicit Quarks

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A U(6)-symmetric Regge-pole model with explicit quark spin is applied to meson and baryon exchange in the πN system. Attention is focused on the general form of the polynomial residues which result from including the required projection operators. Detailed calculations are exhibited for forward charge-exchange scattering within the context of a dual model with fixed cuts. For the case of baryon exchange a Regge residue appropriate to the symmetric-quark-model spectrum is presented and studied.

Recent work by several authors, of whom we can refer to only a few,[1-3] has stimulated a renewed interest in considering a U(6)-symmetric quark picture of hadrons. For our purposes the essential feature of this scheme is the utilization of explicit quark-spin structure to generate the desired particle spectrum. This leads to polynomial Regge residues which exhibit many desirable features. Several of these features are independent of detailed assumptions about the particle spectrum, such as those made in Ref. 3, and we shall present the results of the quark-spin calculation for meson-baryon scattering in a general form so that these properties can be exhibited without further assumptions. Then we shall discuss the results of assuming the detailed structure appropriate to the dual model with fixed cuts presented in Ref. 1.[4] In particular we shall be concerned with ρ exchange in forward πN charge-exchange scattering and Δ exchange in backward $\pi^- p$ scattering.

To define the desired particle spectrum we shall assume that mesons are composed of a quark-antiquark pair and belong to a mass-degenerate $(6, \bar{6}; L)$ representation of the group $U(6)\times U(6) \times O(3)$, i.e., the usual 36 multiplet. Similarly, baryons are taken to be composites of three quarks and to appear in the $(56, \bar{1}; L)$ and $(70, \bar{1}; L)$ representations for the case of meson-baryon scattering. We shall also assume that all couplings occur via $U(6)_W \times O(2)_{L_z}$-invariant vertices.

Once the quarks have been explicitly introduced via the usual external U(6) wave functions, which are given along with other details in the Appendix, the desired quark-model spectrum can be obtained by utilizing projection operators for the individual quarks.[1] The structure introduced by these projection operators is the essential feature to be studied in the present work. These operators serve to prevent the negative-parity components (MacDowell twins) of the spin-$\frac{1}{2}$ quarks from contributing to the resonances. In the case of the mesons the $q\bar{q}$ propagator must include a factor $(1+\not{k}/M)(1-\not{k}/M)$ near the pole, $k^2 \cong M^2$, where M is the resonance mass and the appropriate indices are understood to be present. This will en-