Weak and Electroproduction of Nucleon Resonances in the Harmonic-Oscillator Quark Model*

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Using the model that the baryons are composed of quarks with harmonic interactions, we compute cross sections for high-energy neutrinos and electrons to excite the nucleon to the resonance states with masses below 1.9 GeV. The frame dependence of the nonrelativistic quark-model results is emphasized and discrepancies between some previous model calculations of electroproduction form factors and data are shown to be due to the frame chosen. For neutrino production, the model predicts that a significant excitation of the second and third resonance regions will be seen from neutrons, but with proton targets only the $\Delta(1236)$ excitation will be observed. An interesting result of the electroproduction calculation is the prediction that the $P_{11}(1470)$, while suppressed at small t (and hence in photoproduction), will dominate over the $S_{11}(1550)$ and $D_{13}(1520)$ at large $t \gtrsim 1.5$ (GeV/c)^2.

I. INTRODUCTION

The symmetric quark model of hadrons with harmonic interaction has been discussed extensively in the literature of recent years. The model has had continued success in classifying the observed resonance states[1]; many of the states that the model predicts to exist have been discovered in phase-shift analyses while states with quantum numbers that cannot be accommodated in the conventional formalism have not had their existences confirmed.[1,2] Dynamical calculations have been made of both strong and electromagnetic resonance widths using both nonrelativistic[3-7] and relativistic[8,9] formulations of the model. A particular success of the model has been in the selection rules that it predicts, some of these following from the $SU(6)_W \otimes O(3)$ symmetry[10,11] whereas others seem to depend critically upon the harmonic-oscillator formulation.[7,12]

A survey of Refs. 3–12 will show the variety of "quark models" that exist. In Refs. 3–7 the interactions which lead to the transition from the ground-state nucleon to the resonant three-quark state were written in nonrelativistic form in direct analogy with nuclear-physics calculations, the implicit assumption being that on the one hand the quarks were very massive (so that a nonrelativistic calculation was relevant) while on the other hand they were quasifree.[13] It is not at all clear that such a strange picture should have any relevance to the real world, and yet the real world seems in many cases to behave in the manner that such a model would predict.[14] An attempt to understand why such a model works motivated the relativistic approach of Ref. 8.

Granted that the model of Ref. 7 works well for photoproduction, then one should proceed to test it as extensively as possible. Thornber[5] made computations of the electroproduction transition form factors for various resonances and in comparing the ratios of the excitation and elastic form factors, found marked disagreement with the data for $|t| > 1$ (GeV/c)^2 (where t is the invariant mass squared of the exchanged virtual photon). However, we shall see that the disagreements at large t were due to the particular choice of frame that she employed (a similar conclusion to this has been independently reached in Ref. 6). When account is taken of the frame dependence and the nonrelativistic nature of the model, the agreement with the data is considerably improved. Electroproduction has been considered in a relativistic quark model by Ravndal and also by Copley *et al.*[8]

The main purpose of this paper is to use the nonrelativistic model[4,5,7] to compute the cross sections and form factors for the production of resonances by neutrinos interacting with nucleons. The reasons for this computation are twofold. First, it enables the model to be tested further than has been done to date; in particular the axial-vector excitation matrix elements are probed in this case, a feature not present in photoproduction and electroproduction. Secondly, neutrino physics is about to enter a new and exciting era, with resonance excitation being measured for the first time from hydrogen[15] and excitation from nuclei, using very high-energy neutrinos, being in prospect with the advent of the National Accelerator Laboratory (NAL).[16] Very few calculations or model expectations for resonance excitation rates with neutrinos exist[16-18] and so it is of interest to see what features of resonance production the model predicts will be seen. We compute the excitation rates to the $\Delta(1236)$, and to the negative-parity resonances in the mass range 1.50–1.55 GeV (S_{11}, D_{13}), and 1.65–1.70 GeV ($S_{11}, D_{13}, D_{15}, S_{31}, D_{33}$). In the mass range 1.70 to 2 GeV is a complicated band of positive-parity resonances, and we consider the excitation of the most dominant of these in the third resonance region [$F_{15}(1688)$]. We shall not discuss excitation of the many other resonances in this energy band whose quark-model assignments are in many cases uncertain. We also consider possible excitation of the Roper resonance [$P_{11}(1470)$], there being considerable debate as to whether this resonance does or does not couple significantly in photoproduction.[14,19]

Due to the nonrelativistic nature of our model we shall show how the results depend upon the choice of frame in which the calculations are performed. This enables us to specialize to particular frames and compare our results with those of other authors. In the case of weak production we predict that from neutrons significant excitation of $D_{13}(1520)$ and $S_{11}(1550)$ will take place and will dominate the second resonance region at small t (momentum transfer squared), whereas at large t we expect the $P_{11}(1470)$ to become significant and even dominant.[20] In photoproduction and small-t electroproduction the D_{13} and S_{11} are again predicted to dominate, but for $|t|>1$ (GeV/c)^2 the $P_{11}(1470)$ is expected to become significant. For weak production from neutrons and also for electroproduction, the third resonance region will be dominated at large t by $F_{15}(1688)$ excitation. At small t there is expected to be a significant contribution from $D_{33}(1670)$ excitation.[21] The dominance of F_{15} at large t is due to its assignment $L=2$ in the oscillator spectrum while the D_{33} is only $L=1$; this behavior is analogous to the behavior of the $P_{11}(1470)$ in the second resonance region. With proton targets only $I=\frac{3}{2}$ resonance states can be excited with neutrino beams and so no second resonance will be seen. In this case we also expect the third resonance to be suppressed, and so the total excitation from proton targets will be dominated by the $P_{33}(1236)$.

These conclusions for the weak production agree with those of Ravndal[18] but are in contradiction with those of Albright *et al.* These latter authors claim that the negative parity 70plet of SU(6) will not be excited; however, the fact that these resonances are known to be excited by the vector current (photoproduction and electroproduction) and also by the axial-vector current ($\pi N \to N^*$) makes it appear that these authors are in error.

In Sec. II we discuss our kinematics and the form of the current-current interaction. In Sec. III the problem of satisfying the conserved-vector-current hypothesis is discussed. Matrix elements and cross sections are treated in Secs. IV and V, while parameters and frame dependences are considered in Sec. VI.

The reader whose interests lie primarily in the excitation-rate predictions rather than in the formalism of the calculations is recommended to proceed directly to the figures which summarize our results in a digestible fashion. The frame dependence of the nonrelativistic quark-model predictions is highlighted in Fig. 3 (below). The insensitivity to the choice of parameters is seen in the comparison of the model predictions for the ratios of transition to the elastic electroproduction form factors (Fig. 4, below) for a wide range of the quark's mass and g factor.

II. CURRENT-CURRENT INTERACTION

All semileptonic weak interactions observed up to now can be described by the phenomenological Lagrangian[22]

$$\mathcal{L} = \frac{G}{\sqrt{2}}\left[J_\lambda^\dagger(j_{(e)}^\lambda + j_{(\mu)}^\lambda) + \text{H.c.}\right], \tag{2.1}$$

where G is the Fermi constant, $G^{-1} \sim 10^5 \times M_p^2$ (M_p = proton mass). The muon and electron weak currents are given by

$$j_{(\mu,e)}^\lambda = \overline{\psi}_{(\mu,e)}(x)\gamma^\lambda(1-\gamma_5)\psi_{(\nu_\mu,\nu_e)}(x), \tag{2.2}$$

where this form implicitly assumes that the neutrino is massless, has left-handed helicity, and that the electron and muon number are separately conserved. The weak hadron current $J_\lambda^\dagger$ is assumed to consist of vector and axial-vector parts, and between states A and B is written

$$J_\lambda^\dagger = \langle B | V_\lambda + A_\lambda | A\rangle, \tag{2.3}$$

where in this form the state B has one unit more electrical charge than does the state A. We shall assume that the hadronic current consists only of first-class currents[23] [i.e., the vector (axial-vector) parts are even (odd) under G parity], second-class currents being absent, or negligible.

We first consider the lepton current. Using the spinor normalization

$$u^{(s)\dagger}(p)u^{(r)}(p) = \delta_{rs}, \tag{2.4}$$

we evaluate

$$W^\lambda(\rho) = \overline{u}(p_\mu)\gamma^\lambda(1-\gamma_5)u(p_\nu) \tag{2.5}$$

in a coordinate system where the muon and neutri-

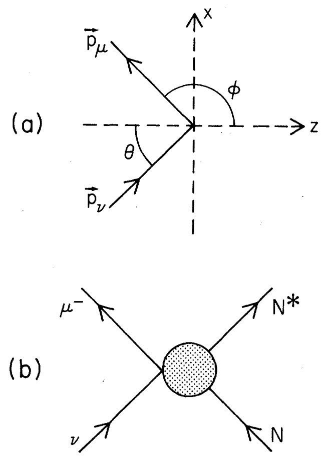

FIG. 1. Definition of the scattering angles for the lepton current in the x-z plane (a) in the current-current interaction, (b) for $\nu N \to N^*\mu^-$.

no momenta define the x-z plane and the three-momentum transfer k is in the z direction (Fig. 1). With the neutrino momentum in the direction $(\theta, 0)$ with respect to the z axis and the muon momentum in a direction $(\phi, 0)$ we obtain

$$W^\lambda(+)=\frac{E_\mu+m-|\vec{p}_\mu|}{[E_\mu(E_\mu+m)]^{1/2}}\left(\sin\left(\frac{\phi-\theta}{2}\right), \cos\left(\frac{\phi+\theta}{2}\right), -i\cos\left(\frac{\theta-\phi}{2}\right), -\sin\left(\frac{\theta+\phi}{2}\right)\right) \quad (\lambda=0,1,2,3) \qquad (2.6)$$

and

$$W^\lambda(-)=\frac{E_\mu+m+|\vec{p}_\mu|}{[E_\mu(E_\mu+m)]^{1/2}}\left(\cos\left(\frac{\theta-\phi}{2}\right), -\sin\left(\frac{\theta+\phi}{2}\right), i\sin\left(\frac{\phi-\theta}{2}\right), -\cos\left(\frac{\theta+\phi}{2}\right)\right), \qquad (2.7)$$

where $W^\lambda(+-)$ are the expressions for the lepton current when the left-handed neutrino produces a muon with positive helicity ($\rho=+$) or negative helicity ($\rho=-$), respectively, energy E_μ, mass m, and momentum magnitude $|\vec{p}_\mu|$. The helicity and momentum transfer from the lepton to hadron system is then represented by

$$W^\lambda(\rho)\exp(i\vec{k}\cdot\vec{x}) \qquad (2.8)$$

where $\vec{k}=\vec{p}_\nu-\vec{p}_\mu$ with $\vec{p}_{\nu,\mu}$ the neutrino and muon momenta.

In order to compute transition amplitudes, and hence excitation rates to the various resonance states B, we must contract the above expression (2.8) in the form of (2.7) or (2.6) into the hadronic charge-raising current (2.3) where A is a nucleon. Once the form of $J_\lambda^\dagger$ is specified then it is a laborious but otherwise straightforward procedure to compute the rates for exciting the various resonances. Therefore we must consider what form the hadronic current will have within the confines of our model assumptions. Following Refs. 3–7 we assume that the quarks which constitute the nucleon resonances, A or B, are quasifree insofar as the weak and electromagnetic interactions are concerned and that the lepton weak current couples locally to the weak current of each quark. This is analogous to the electromagnetic case where the photon field is assumed to couple locally to the electromagnetic current of each quark. This is the additivity assumption, where the weak (electromagnetic) interaction of the baryon is written as the sum of the weak (electromagnetic) interactions of the individual constituent quarks. The quark structure of the nucleon resonances is then taken into account through the use of appropriate initial- and final-state quark wave functions. In this manner, once the form of the individual quark weak or electromagnetic current is specified, then the weak and electromagnetic transition amplitudes between any two nucleon resonance states may be computed.

The quark's weak-interaction current, which transforms as an isospin-raising operator, connects the quark ($\mathfrak{N}$) with the isospin and strangeness of the neutron to the quark ($\mathcal{P}$) with isospin and strangeness of the proton. Assuming that the quarks are quasifree and that we may neglect second-class currents at the quark level, then the general form of the weak quark current is

$$\langle \mathcal{P}|V_\lambda+A_\lambda|\mathfrak{N}\rangle =\bar{\psi}(p')(a\gamma_\lambda+ib\sigma_{\lambda\nu}k^\nu-c\gamma_\lambda\gamma_5-dk_\lambda\gamma_5)\psi(p), \qquad (2.9)$$

TABLE I. Nonrelativistic reduction of terms in the quark's vector and axial-vector currents.

Term	Nonrelativistic reduction
$\bar{u}(p')\gamma_0 u(p)$	$\chi_f^\dagger\chi_i$
$\bar{u}(p')\gamma_j u(p)$	$\chi_f^\dagger\left[\frac{\vec{p}+\vec{p}'}{2M_q}+\frac{i(\vec{\sigma}\times\vec{k})}{2M_q}\right]\chi_i$
$\bar{u}(p')\sigma_{ij}k^i u(p)$	$\chi_f^\dagger[\vec{\sigma}\times\vec{k}]\chi_i$
$\bar{u}(p')\gamma_j\gamma_5 u(p)$	$\chi_f^\dagger\vec{\sigma}\chi_i$
$\bar{u}(p')\gamma_0\gamma_5 u(p)$	$\chi_f^\dagger\left[\frac{\vec{\sigma}\cdot(\vec{p}+\vec{p}')}{2M_q}\right]\chi_i$
$\bar{u}(p')\gamma_5 k\, u(p)$	$\chi_f^\dagger\left[\frac{-\vec{\sigma}\cdot\vec{k}}{2M_q}k\right]\chi_i$

where a, b, c, d are functions of the invariant four-momentum transfer squared, $t = k^\nu k_\nu$.

In order to reproduce an SU(6) structure of the baryon spectrum the quarks must be described by two-component spinors and so, in analogy with Ref. 7, we perform a nonrelativistic reduction of the quark's weak current to first order in the inverse quark mass. (The result will include the familiar electromagnetic form in its vector part.) In Table I we give the nonrelativistic reduction of the various terms in the quark's current. We find to the order $1/M_q$ (with M_q the quark mass) that

$$(2\pi)^3 W^\lambda \langle \mathcal{P} | V_\lambda + A_\lambda | \mathfrak{N} \rangle e^{i\vec{k}\cdot\vec{x}} = e^{i\vec{k}\cdot\vec{x}} \chi_f^\dagger \left[aW_0 - \frac{a\vec{W}\cdot(\vec{p}+\vec{p}\,')}{2M_q} - \frac{ai\vec{W}\cdot(\vec{\sigma}\times\vec{k})}{2M_q} - ib\vec{W}\cdot\vec{\sigma}\times\vec{k} + c\vec{W}\cdot\vec{\sigma} - \frac{cW_0\vec{\sigma}\cdot(\vec{p}+\vec{p}\,')}{2M_q} + \frac{d\vec{\sigma}\cdot\vec{k}}{2M_q}(W_0 k_0 - \vec{W}\cdot\vec{k}) \right] \chi_i e^{i(\vec{p}\,'-\vec{p})\cdot\vec{x}}, \qquad (2.10)$$

where $\chi_{i,f}$ are two-component spinors. The term in the square brackets above represents the interaction operator between free-quark states. Using this operator we replace the free-quark wave functions $(2\pi)^{-3/2}e^{ip'\cdot x}$ and $(2\pi)^{-3/2}e^{-ip\cdot x}$ by the quark-model wave functions appropriate to the initial and final baryon states. Summing this single-quark interaction operator over the quarks which constitute the baryon states and integrating over the spatial coordinates we obtain the hadronic current matrix element describing transition between these states. In our calculation we shall assume the quark form factors to be constant such that $a(t) \equiv a(0) \equiv a$ and similarly for $b(t)$, $c(t)$, and $d(t)$.[24]

If now we write $c/a = R$ and $b/a = (g-1)/2M_q$ we obtain the interaction operator in the form

$$\left\{ a\left[W_0 - \frac{\vec{W}\cdot(\vec{p}+\vec{p}\,')}{2M_q} - \frac{ig}{2M_q}\vec{W}\cdot(\vec{\sigma}\times\vec{k}) + R\vec{\sigma}\cdot\vec{W} - \frac{RW_0}{2M_q}\vec{\sigma}\cdot(\vec{p}+\vec{p}\,') \right] + \frac{d\vec{\sigma}\cdot\vec{k}}{2M_q}(W_0 k_0 - \vec{W}\cdot\vec{k}) \right\} e^{i\vec{k}\cdot\vec{x}}. \qquad (2.11)$$

Our formalism has defined the z direction to be the direction of the momentum transfer $\vec{k} = \vec{p} - \vec{p}\,'$, from the leptonic to the hadronic system. This leaves us with the choice of which frame to employ in the evaluation of the transition amplitudes in our nonrelativistic approximation. We consider the following frames: (i) laboratory, (ii) isobar rest frame where the final-state nucleon resonance is produced at rest, (iii) Breit frame where the initial- and final-state baryon three-momenta are equal and opposite. The isobaric frame, where no energy is transferred, is not considered by us as it clearly has a nonsensical threshold dependence in t (i.e., at $t = 0$ all excitations would vanish by kinematics). In Fig. 2 we illustrate the kinematics for the three cases we consider. All of these frames are connected by a simple Lorentz boost in the z direction.

III. CONSERVED VECTOR CURRENT

The conserved-vector-current hypothesis (CVC) identifies the weak hadronic vector current with a conserved isospin current, the I_3 component of which is the isovector part of the electromagnetic current. Thus CVC implies that the isovector part of the electromagnetic current is related to the weak vector current by a rotation in isotopic-spin space and the weak vector form factors can be determined from electron-nucleon scattering (once the data from both neutron and proton targets are accurate enough to enable the isovector separation).

The vector current is conserved at the quark level where V^λ has the form

$$\bar{u}(p')(a\gamma^\lambda + ib\sigma^{\lambda\nu}k_\nu)u(p) \qquad (3.1)$$

and $k_\lambda V^\lambda = 0$.

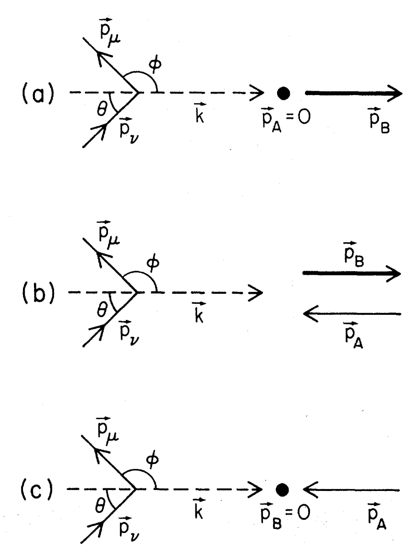

FIG. 2. Kinematics of $\nu A \to \mu B$ in (a) laboratory, (b) Breit, and (c) isobar rest frames.

With our nonrelativistic reduction we obtain for the vector current

$$V_0=\chi_f^\dagger a\chi_i\, e^{ikz},$$
$$\vec{V}=\chi_f^\dagger a\left[\frac{\vec{p}+\vec{p}\,'}{2M_q}+\frac{ig(\vec{\sigma}\times\vec{k})}{2M_q}\right]\chi_i\, e^{ikz} \tag{3.2}$$

(which agrees with the form for the electromagnetic current used in Refs. 4–7) and

$$\begin{aligned} k_\lambda V^\lambda &= (k_0V_0-\vec{k}\cdot\vec{V}) \\ &= \chi_f^\dagger a e^{ikz}\left(k_0-\frac{\vec{k}\cdot(\vec{p}+\vec{p}\,')}{2M_q}\right)\chi_i, \end{aligned} \tag{3.3}$$

where $k_0=E_B-E_A$ is the energy transfer to the hadronic system.

Matrix elements of the nonrelativistic form of the operator $k_\lambda V^\lambda$ between appropriate quark-model wave functions give a nonzero result. Thus the CVC condition is no longer satisfied in the nonrelativistic limit.

Adler[25] has shown that if the vector current is conserved then the forward production of N^* resonances in the process

$$\nu+N\rightarrow N^*+\mu^-$$

arises entirely from the axial-vector current in the limit that the muon mass vanishes. Thus the CVC condition has important consequences for the near-forward production of N^* resonances with which we are here concerned. We are thus led to modify the nonrelativistic form of the vector current so that the CVC condition is satisfied explicitly. This we do following Dalitz and Yennie[26] by defining $J_z^V=J_0^V k_0/k_z$ and we obtain the following form for the vector current:

$$J_0^V=\chi_f^\dagger a e^{ikz}\chi_i,$$

$$\vec{J}^V=\chi_f^\dagger a e^{ikz}\left[\frac{(\vec{p}+\vec{p}\,')}{2M_q}+\frac{ig}{2M_q}(\vec{\sigma}\times\vec{k}) -\left(\frac{(\vec{p}+\vec{p}\,')\cdot\vec{k}}{2M_qk^2}-\frac{k_0}{k^2}\right)\vec{k}\right]\chi_i,$$

where $k^2\equiv|k_z^{\,2}|$. In this form the vector current is divergenceless and may be used to compute photoproduction and electroproduction rates. With the additional contribution of the axial-vector current to the interaction operator we have the final form of the weak interaction, which we write as

$$\begin{aligned} a\sum_{j=1}^{3}\tau_j^+e^{ik\cdot r_j}\Bigg(&W_0-W_z\frac{k_0}{k}-\frac{1}{2M_q}[(W_x+iW_y)(p_x^{(j)}-ip_y^{(j)})+(W_x-iW_y)(p_x^{(j)}+ip_y^{(j)})] \\ &-\frac{gk}{2M_q}[\sigma_{(j)}^+(W_x-iW_y)-\sigma_{(j)}^-(W_x+iW_y)]+RW_z\sigma_z^{(j)}+R(W_x+iW_y)\sigma_{(j)}^-+R(W_x-iW_y)\sigma_{(j)}^+-RW_0\sigma_z^{(j)}\frac{k}{2M_q} \\ &-\frac{RW_0}{M_q}[\sigma_zp_z+\sigma^+(p_x-ip_y)+\sigma^-(p_x+ip_y)]+\frac{d}{a}\frac{k\sigma_z}{2M_q}(W_0k_0-W_zk)\Bigg)\,. \end{aligned} \tag{3.4}$$

IV. EVALUATION OF MATRIX ELEMENTS

Using the interaction operator (3.4) and the quark-model wave functions given by Faiman and Hendry,[3] we can determine the matrix element of this operator for the weak excitation of various resonances. The interaction operator is summed over the quark index (j) $(j=1,2,3)$ and τ^+ is the isospin-raising operator such that $\tau^+(n)=(p)$. The operator $p(j)$ is taken to be the differential operator $-i\nabla(j)$ operating on the quark-model wave functions. In evaluating the spatial integrals as given in Table II, we have used

$$e^{ikz}=\sum_L i^L(2L+1)^{1/2}j_L(kr)(4\pi)^{1/2}Y_{L0}(\hat{r})$$

and (4.1)

$$\int dr\, r^{L+2}e^{-\alpha^2r^2}j_L(kr)=(k/2\alpha^2)^L\left(\frac{\sqrt{\pi}}{4\alpha^3}\right)e^{-k^2/4\alpha^2}\,.$$

The center-of-mass motion has been factored out in these spatial integrals.

We can determine the parameters a and R by considering the process $\nu+n\rightarrow p+\mu^-$ in the limit

TABLE II. Spatial integrals. Wave functions are written ψ_{LM} for quark orbital state L. The radial excitation for the Roper resonance is ψ_{00}^*.

$\langle\psi_{00}\vert e^{ikz}\vert\psi_{00}\rangle=e^{-k^2/6\alpha^2}$
$\langle\psi_{00}\vert e^{ikz}p_z\vert\psi_{00}\rangle=-\frac{1}{3}k\,e^{-k^2/6\alpha^2}$
$\langle\psi_{10}^x\vert e^{ikz}\vert\psi_{00}\rangle=-\frac{ik}{\alpha\sqrt{3}}e^{-k^2/6\alpha^2}$
$\langle\psi_{10}^x\vert e^{ikz}p_z\vert\psi_{00}\rangle=-i\alpha(\frac{1}{3})^{1/2}\left(1-\frac{k^2}{3\alpha^2}\right)e^{-k^2/6\alpha^2}$
$\langle\psi_{11}^x\vert e^{ikz}(p_x+ip_y)\vert\psi_{00}\rangle=i\alpha(\frac{2}{3})^{1/2}e^{-k^2/6\alpha^2}$
$\langle\psi_{20}\vert e^{ikz}\vert\psi_{00}\rangle=-\frac{1}{6}(\frac{2}{3})^{1/2}\left(\frac{k^2}{\alpha^2}\right)e^{-k^2/6\alpha^2}$
$\langle\psi_{21}\vert e^{ikz}(p_x+ip_y)\vert\psi_{00}\rangle=\frac{1}{3}k\,e^{-k^2/6\alpha^2}$
$\langle\psi_{20}\vert e^{ikz}p_z\vert\psi_{00}\rangle=-\frac{1}{3}(\frac{2}{3})^{1/2}k\left(1-\frac{k^2}{6\alpha^2}\right)e^{-k^2/6\alpha^2}$
$\langle\psi_{00}^*\vert e^{ikz}\vert\psi_{00}\rangle=\frac{1}{\sqrt{3}}\frac{k^2}{6\alpha^2}e^{-k^2/6\alpha^2}$
$\langle\psi_{00}^*\vert e^{ikz}p_z\vert\psi_{00}\rangle=\frac{k}{3\sqrt{3}}\left(1-\frac{k^2}{6\alpha^2}\right)e^{-k^2/6\alpha^2}$

$\vec{k}$ and k_0 tending to zero ("static limit"). The hadronic current matrix element is of the form

$$J_\lambda(0)=\bar{u}(p)[\gamma_\lambda F_1(q^2)+i\sigma_{\lambda\nu}k^\nu F_2(q^2) -\gamma_\lambda\gamma_5 F_A(q^2)-k_\lambda\gamma_5 F_p(q^2)]u(n) \quad (4.2)$$

for the neutron-proton transition. In this static limit we obtain for $W^\lambda J_\lambda$

$$W_0\chi_f^\dagger\chi_i F_1(0)+\vec{W}\cdot\chi_f^\dagger\vec{\sigma}\chi_i F_A(0), \quad (4.3)$$

where χ_f and χ_i are the two-component spin functions for the proton and neutron, respectively. Taking quark wave functions for the proton and neutron and using the quark-model interaction operator in the same limit we obtain for $W^\lambda J_\lambda$

$$aW_0\chi_f^\dagger\chi_i+\tfrac{5}{3}R\vec{W}\cdot\chi_f^\dagger\vec{\sigma}\chi_i. \quad (4.4)$$

We are thus led to identify $a=F_1(0)$ and $R=\frac{3}{5}F_A(0)\simeq 0.7$.

For the induced pseudoscalar we first define $f_p=d/c$, where in general $f_p=f_p(q^2)$, but we here assume it to be independent of q^2 [see Sec. II, immediately following (2.10)]. Using the Goldberger-Treiman relation

$$F_p(t)=2M_N F_A(0)/(m_\pi{}^2-t)$$

we estimate that $f_p\sim 10/m$ (with m the muon mass in GeV). From experiments on radiative muon capture (Ref. 27, p. 366) one estimates

$$f_p(t=m^2)=\frac{1}{m}(13\pm 2.8),$$

while in Ref. 28 is estimated the value

$$f_p(t=m^2)=7.2/m.$$

TABLE III. Resonance production rates relative to $P_{33}(1236)$ in "forward" [$t=-0.015$ (GeV/c)^2] $\nu N^0\to N^{*+}\mu^-$ when $E_\nu\gtrsim 3$ GeV.

Resonance	Quark-model assignment	Relative production rate Laboratory	Breit	Isobar
$P_{33}(1236)$	$^4(10)$	100	100	100
$D_{13}(1520)$	$^2(8)$	14.8	14.7	13.5
$S_{11}(1550)$	$^2(8)$	32.3	45.1	49.6
$P_{11}(1470)$	$^2(8)$	14.1	9.9	7.5
$S_{31}(1650)$	$^2(10)$	0.9	1.8	2.1
$D_{33}(1670)$	$^2(10)$	0.5	1.0	1.3
$D_{15}(1670)$	$^4(8)$	1.0	1.8	1.7
$F_{15}(1688)$	$^2(8)$	5.3	4.5	2.8
$S_{11}(1700)$	$^4(8)$	2.7	5.7	6.9
$D_{13}(1700)$	$^4(8)$	0.4	0.6	0.5

In Fig. 9 below, we show the effect of including the induced pseudoscalar term with $f_p=10/m$. The importance of this term can then be seen by comparison with the curves where f_p is set $=0$.

In the Appendix we give the square of the matrix element, summed and averaged over final and initial spins for the various resonances. We also consider possible configuration mixing of the form

$$S_{11}(a)=\cos\theta_s S_{11}\{^48\}+\sin\theta_s S_{11}\{^28\},$$
$$S_{11}(b)=-\sin\theta_s S_{11}\{^48\}+\cos\theta_s S_{11}\{^28\}$$

and

$$D_{13}(a)=\cos\theta_d D_{13}\{^48\}+\sin\theta_d D_{13}\{^28\},$$
$$D_{13}(b)=-\sin\theta_d D_{13}\{^48\}+\cos\theta_d D_{13}\{^28\},$$

which allows for the possibility that the observed states S_{11}(1550 and 1710) may be mixtures of the quark-model eigenstates $S_{11}\{^48\}$ and $S_{11}\{^28\}$ (similarly for D_{13}). The results quoted in Table III assume $\theta_{s,d}=0$ (which is suggested by the mass spectra). The results for arbitrary θ may be straightforwardly constructed.[29] Table III shows the expressions for the lepton current $W_\lambda(\rho)$ appearing in the squared matrix elements of Eq. (3.4).

In the forward direction ($\theta=\phi=0$) the lepton current vanishes in all but the following cases (see Table IV):

$$|W_x(+)+iW_y(+)|=2C(+),$$
$$W_0(-)=W_z(-)=C(-).$$

Furthermore, in the limit of the vanishing muon mass, $t=0$ in the forward direction which implies that the quantity $kW_0(-)-k_0W_z(-)$ occurring in the Appendix should be zero.

In general the production rate to positive-helicity

TABLE IV. Expressions for the leptonic current $W_\lambda(\pm)$ with
$$C(+)\equiv\frac{E_\mu+M-|\vec{p}_\mu|}{[E_\mu(E_\mu+m)]^{1/2}} \text{ and } C(-)\equiv\frac{E_\mu+m+|\vec{p}_\mu|}{[E_\mu(E_\mu+m)]^{1/2}}.$$

Positive-helicity muons
$W_0(+)=\sin[\frac{1}{2}(\phi-\theta)]C(+)$
$W_z(+)=\sin[\frac{1}{2}(\phi+\theta)]C(+)$
$\lvert W_x(+)+iW_y(+)\rvert=2\cos\frac{1}{2}\theta\cos\frac{1}{2}\phi\, C(+)$
$\lvert W_x(+)-iW_y(+)\rvert=2\sin\frac{1}{2}\theta\sin\frac{1}{2}\phi\, C(+)$
Negative-helicity muons
$W_0(-)=\cos[\frac{1}{2}(\phi-\theta)]C(-)$
$W_z(-)=\cos[\frac{1}{2}(\phi+\theta)]C(-)$
$\lvert W_x(-)+iW_y(-)\rvert=2\sin\frac{1}{2}\phi\cos\frac{1}{2}\theta\, C(-)$
$\lvert W_x(-)-iW_y(-)\rvert=2\cos\frac{1}{2}\phi\sin\frac{1}{2}\theta\, C(-)$

muons is suppressed with respect to that for negative-helicity muons due to the behavior of $C(+)$ which vanishes as the muon mass tends to zero (or equivalently as the neutrino energy increases). In this limit of being able to neglect the muon mass, the only contribution to $|M|^2$ for the forward direction comes from the term BR^2. This is a purely axial-vector contribution and we thus satisfy the Adler condition on the vanishing of the vector amplitude in this forward configuration. We also note that the term A_3 represents the vector–axial-vector interference term and vanishes in the above limit consistent with the Adler condition.

V. DIFFERENTIAL CROSS SECTIONS

With matrix elements of the interaction operator between quark-model wave functions denoted by M, the transition matrix element is given by

$$S_{fi}=(2\pi)^{-3}\left(\frac{m_A m_B}{E_A E_B}\right)^{1/2}(2\pi)^4 \times\delta^4(p_\mu+p_B-p_\nu-p_A)MG/\sqrt{2}$$

and the cross section by

$$d\sigma=(2\pi)^4\frac{m_A m_B}{E_A E_B}|M|^2\frac{d^3p_\mu}{(2\pi)^3}\frac{d^3p_B}{(2\pi)^3}\frac{G^2}{2} \times\delta^4(p_\mu+p_B-p_\nu-p_A)\frac{E_\nu E_A}{p_\nu\cdot p_A}\,. \qquad (5.1)$$

For the process $\nu A\to\mu B$ we can write

$$d\sigma=\frac{G^2}{8\pi^2}E_\nu E_\mu|M|^2\frac{m_A m_B}{p_\nu\cdot p_A}\frac{d^3p_\mu}{E_\mu}\frac{d^3p_B}{E_B} \times\delta^4(p_\mu+p_B-p_\nu-p_A)\,. \qquad (5.2)$$

The expression for the cross section should be a Lorentz invariant. The second factor in the above expression is explicitly Lorentz-invariant and implies that the factor $E_\nu E_\mu|M|^2$ should be the same. We therefore evaluate $E_\nu E_\mu|M|^2$ in the frame of our choice and express it in terms of Lorentz invariants. This then allows us to calculate the differential cross section in any frame. In the laboratory frame the differential cross section takes the form

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{lab}}=\frac{G^2}{8\pi^2}(E_\nu E_\mu|M|^2)_F \times\left\{\frac{m_B|\vec{p}_\mu|^2}{E_\nu|\vec{p}_\mu|(E_\nu+m_A)-E_\nu E_\mu\cos\theta_L}\right\}, \qquad (5.3)$$

where the term in curly brackets is evaluated in the laboratory frame and θ_L is the scattering angle in this frame. The subscript F on the term $(E_\nu E_\mu|M|^2)_F$ denotes that we can calculate this term in the frame of our choice.

We have thus put our nonrelativistic calculation into a relativistic form but it still remains, of course, a nonrelativistic calculation. Our results depend upon the frame which we choose as we see in the discussion of the next section.

VI. CHOICE OF PARAMETERS AND FRAME

The parameters g/M_q are determined by requiring that the matrix elements of the magnetic moment operator

$$\vec{M}=\sum_{i=1}^{3}q_i\vec{\sigma}$$

(q_i = the charge of the ith quark) between quark-model wave functions for the proton yield the proton's magnetic moment. We obtain $\mu_p=\mu_q$, where $\mu_p=2.79e/2M_p$ is the proton magnetic moment with M_p the proton mass and $\mu_q=ge/2M_q$. Therefore $g/M_q\sim3\ \text{GeV}^{-1}$.

The spacing between the adjacent levels in the oscillator potential is given by α^2/M_q and from examination of the observed baryon spectrum the separation of the bands is approximately 400 MeV.[30] Thus we take $\alpha^2/M_q=0.4$ GeV. Following Copley *et al.*,[7] we shall assume $g=1$ and so $M_q=0.333$ GeV with $\alpha^2=0.14\ \text{GeV}^2$. The values of our parameters are similar to those used in Refs. 3–7 and our final results are not significantly altered if we changed these parameters to the particular values of any of the various references cited. As examples we show in Fig. 4 below that the electroproduction form-factor ratios predicted by the model are almost independent of the particular values of the parameters. This highlights the fact that it is the *choice of frame* which is the most critical feature of the nonrelativistic calculation.

By virtue of the harmonic interaction between the quarks in the nucleon the resulting prediction for the elastic electromagnetic form factor is of Gaussian form, whereas empirically the proton's form factor has a much less dramatic behavior, the data being well approximated by a function of the form $(1-t/0.71)^{-2}$ for $0<-t<25\ (\text{GeV}/c)^2$, where t is the four-momentum transfer squared. For values of $-t>0.5\ (\text{GeV}/c)^2$ the Gaussian form and the above behaviors diverge, the Gaussian having a much faster falloff than do the data for increasing $|t|$. In order to obtain predictions which are not sensitive to the nature of the form factors, we consider the ratio of resonance to elastic differential cross sections:

$$\frac{d\sigma}{d\Omega}(eN\to eN^*)\Big/\frac{d\sigma}{d\Omega}(eN\to eN)$$

and also

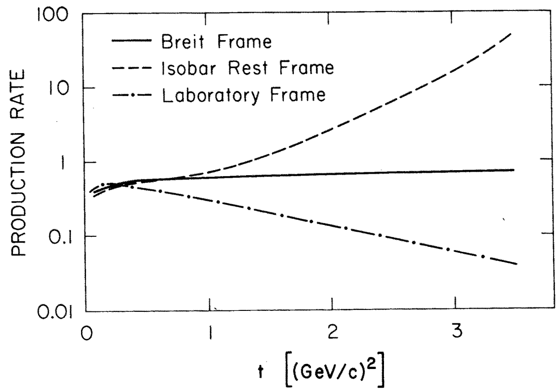

FIG. 3. Frame dependence of the nonrelativistic quark-model predictions for the ratio of resonance to elastic cross sections for a typical resonance $P_{33}(1236)$. t is the modulus of the four-momentum transfer squared.

$$\frac{d\sigma}{d\Omega}(\nu N \to N^*\mu^-)\Big/\frac{d\sigma}{d\Omega}(\nu N \to p\mu^-)\,.$$

The calculations that we have performed are necessarily nonrelativistic and depend upon the three-momentum transfer in the particular frame in which the calculation is performed. Below we give expressions for the squared three-momentum transfer $\vec{k}^2$ in terms of the invariant squared four-momentum transfer t for the various frames we are considering.

Laboratory frame:

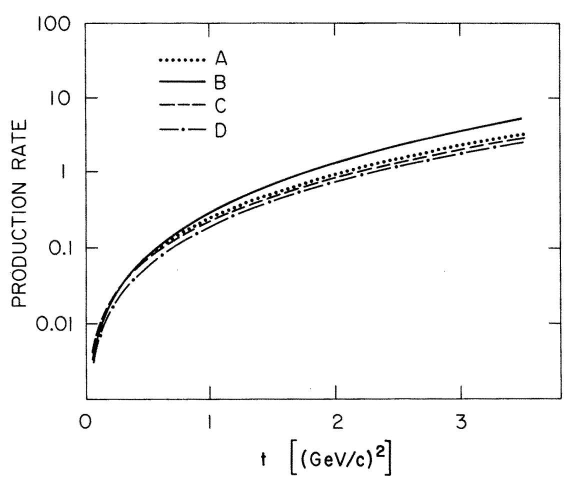

FIG. 4. Parameter dependence of the ratio of resonance to elastic cross sections for the resonance $F_{15}(1688)$. The various parmetrizations are (a) $g=30$, $M_q=10$ GeV, $\alpha^2=0.14$ GeV^2; (b) $g=1$, $M_q=333$ MeV, $\alpha^2=0.06$ GeV^2; (c) $g=2$, $M_q=0.5$ GeV, $\alpha^2=0.14$ GeV^2; (d) $g=1$, $M_q=333$ MeV, $\alpha^2=0.14$ GeV^2. Note that the essential shape of the curves is a function of the $L=2$ assignment of $F_{15}(1688)$ and is only minimally dependent upon the choice of parameters.

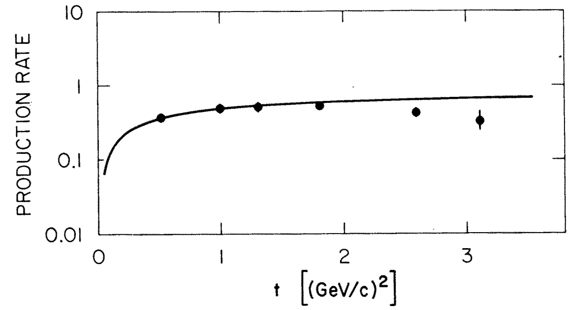

FIG. 5. Ratio of resonance to elastic cross section for electroproduction of $P_{33}^+(1236)$ in the Breit frame. Data are from Ref. 31.

$$\vec{k}^2 = -t + (m_B{}^2 - m_A{}^2 - t)^2/4m_A{}^2\,.$$

Isobar rest frame:

$$\vec{k}^2 = -t + (m_B{}^2 - m_A{}^2 + t)^2/4m_B{}^2\,.$$

Breit frame:

$$\vec{k}^2 = -t + (m_B{}^2 - m_A{}^2)^2/[2(m_A{}^2 + m_B{}^2) - t]\,.$$

Here m_A, m_B are the masses of A, B, respectively, in the process $lA \to lB$ with l a lepton and $-t>0$. For the case of quasielastic scattering, and neglecting the neutron-proton mass difference, these relations reduce to the following.

Laboratory and isobar rest frames:

$$\vec{k}^2 = -t + t^2/4m_A{}^2\,.$$

Breit frame:

$$\vec{k}^2 = -t\,.$$

The ratio

$$[\exp(-\vec{k}^2/3\alpha^2)]_{\text{inel}}/[\exp(-\vec{k}^2/3\alpha^2)]_{\text{el}}$$

then has the following behavior for large values of

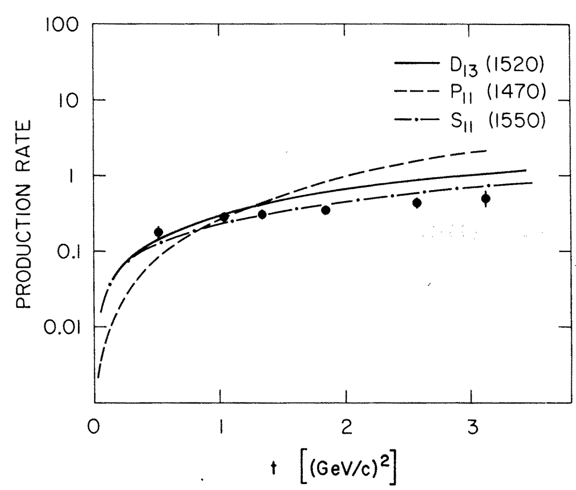

FIG. 6. Same as Fig. 5 but for the second resonance region.

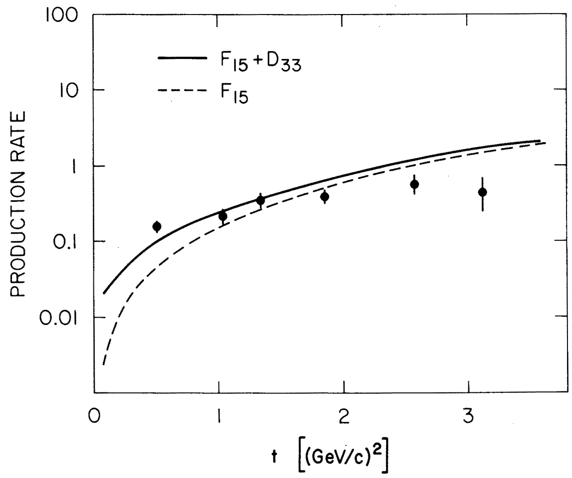

FIG. 7. Same as Fig. 5 but for the third resonance region.

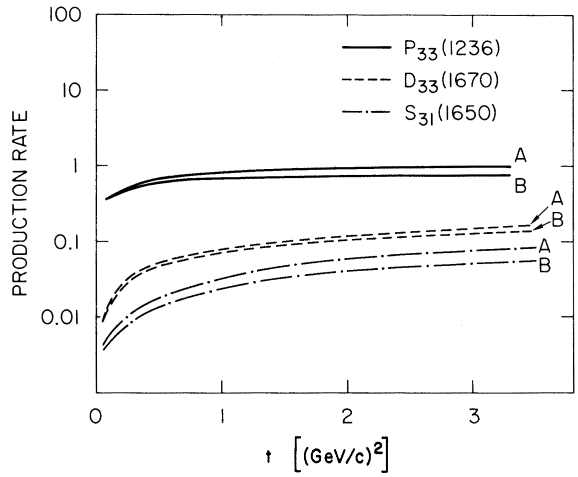

FIG. 9. Weak production from proton targets. Ratio of resonance to quasielastic cross sections for $P_{33}(1236)$, $S_{31}(1650)$, $D_{33}(1670)$. The curves A include an induced pseudoscalar term described in text. Neglect of this term yields the curves B. Neutrino energy is assumed greater than about 3 GeV (see Fig. 12).

the four-momentum transfer $[-t>1\ (\text{GeV}/c)^2]$.

In the laboratory frame there is an exponential decrease with increasing momentum transfer of the form e^{ct}. In the Breit frame there is a weak t dependence and the ratio of form factors is nearly unity. In the isobar rest frame we find an exponential increase with four-momentum transfer of the form dt^2 (c and d are positive numbers depending only upon the masses m_A and m_B). It is these frame-dependent kinematic relations which govern the large-t behaviors of the ratios of resonance to elastic differential cross sections for the various frames that we consider.

In Fig. 3 we show for a typical resonance the effect of frame dependence upon the resulting form-factor predictions. It is clear that frame choice plays an important role, more so than the choice of parameters, as can be seen by examining Fig. 4, where for the $F_{15}(1688)$ we have plotted the resulting form-factor ratio in the Breit frame for a wide range of parametrizations. As the nonrelativistic approximation is best in the Breit frame, we show our results (Figs. 4–12) in this frame.

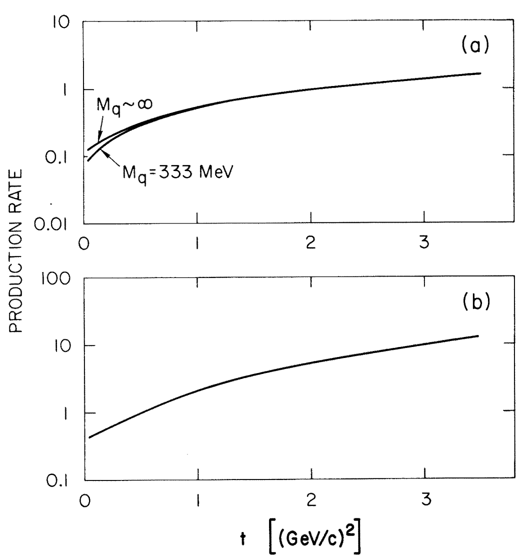

FIG. 8. (a) Ratio of electroproduction resonance cross sections P_{11} to the sum of S_{11} and D_{13} in the second resonance region. The P_{11} is suppressed at low q^2 (and in photoproduction) but is of comparable magnitude to S_{11} and D_{13} for 1 $(\text{GeV}/c)^2$. Two extremes of quark mass are shown; the shape is due to the P_{11} being a second radial excitation whereas S_{11} and D_{13} are $L=1$. (b) As in 8(a) but for F_{15} to D_{33} in the third resonance region. Inclusion of S_{31} does not significantly alter this curve.

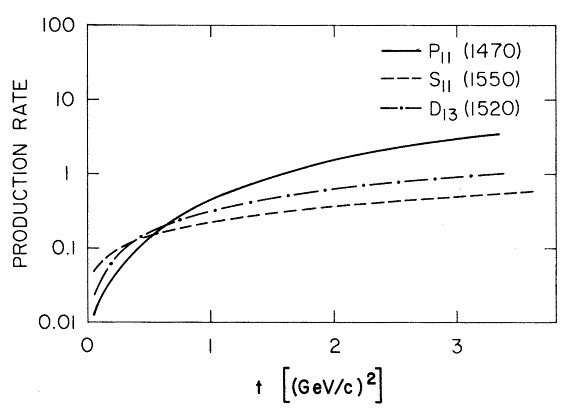

FIG. 10. Weak production of the second resonance region from neutrons. Ratio of resonance to quasielastic cross sections for $P_{11}(1470)$, $D_{13}(1520)$, $S_{11}(1550)$.

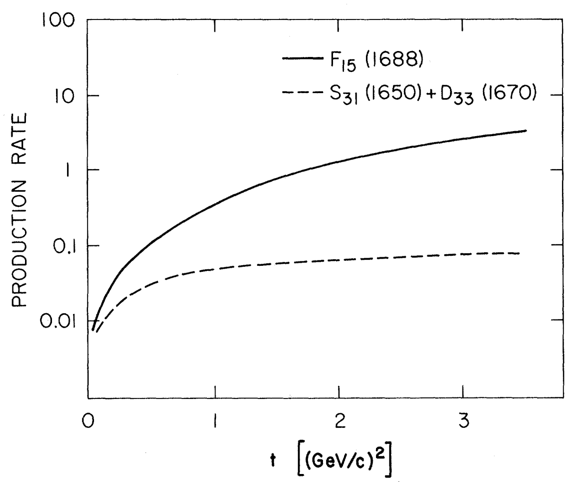

FIG. 11. Same as Fig. 10 but for the third resonance region.

The vector current which we have used is similar to the one used by Thornber[5] in her consideration of the electroproduction of resonances. We have attempted to derive the hadronic current that we have used by a nonrelativistic reduction. Subsequently we use an effective mass, M_q, which is rather light. This brings into question the validity of our nonrelativistic reduction especially as concerns the region away from $t=0$. We do not attempt to justify the use of this hadronic current for nonforward scattering, but we assume its validity in our calculations.

VII. RESULTS

In Figs. 5–7 we exhibit the electroproduction form factors as predicted by the model in the Breit frame and compare with the experimental data[31] in the first, second, and third resonance regions. In Fig. 8 we show an interesting prediction of the model, namely, that the $P_{11}(1470)$ will be suppressed in comparison with the $S_{11}(1550)$ and $D_{13}(1520)$ in photoproduction and low-t electroproduction, but that it will dominate over these resonances for large t. This result is due to the fact that the P_{11} is believed to be a radial excitation at $N(=2n+L)=2$, whereas the S_{11} and D_{13} are orbital excitations with $L=1$. Consequently these latter resonances pick up a factor of k in their form factor near $t=0$ while the P_{11} picks up k^2. That the P_{11} should dominate is a result of the quark assignment; the value of t at which it begins to dominate is proportional to the magnitude of the quark spring constant α^2. (See Table II, where the spatial integrals which yield the form factors may be found.) An analogous behavior is expected for the $F_{15}(1688)$ in the third resonance region due to this resonance being in the $L=2$ level of the oscillator potential while the S_{31} and D_{33} are at $L=1$. This behavior is exhibited in Fig. 8(b).

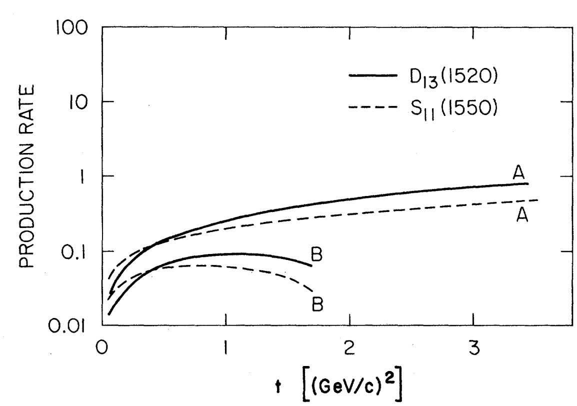

FIG. 12. All previous graphs have assumed the incident lepton to have an energy greater than approximately 3 GeV. The effect of kinematic suppression on the weak production of the resonances $S_{11}(1550)$ and $D_{13}(1520)$ is shown; (A) 5-GeV neutrino (B) 2-GeV neutrino.

In Figs. 9–11 we show the predictions for the ratios of weak-production form factors to the elastic form factor. For $I=\frac{3}{2}$ resonances the results shown are for proton targets in Fig. 9. The excitations of the second and third resonances from neutrons are shown in Figs. 10 and 11, respectively. The inclusion of an induced pseudoscalar term with magnitude $f_p = 10/m$ (see Sec. IV) is shown in the curves labeled A, while neglect of the term yields the curves B.

Table III shows the weak excitation rates of the various resonances compared to the $P_{33}(1236)$ rate. Weak excitation from proton targets can only take place for $I=\frac{3}{2}$ resonances. Our results show that the $S_{31}(1650)$ and $D_{33}(1670)$ are very weakly excited. The $P_{33}(1236)$ resonance will therefore dominate any resonance excitation from proton targets, the second and third resonance regions being markedly suppressed.

From neutron targets we expect that significant excitation of the $D_{13}(1520)$ and $S_{11}(1550)$ will be seen, and with the prominent excitation of $P_{11}(1470)$ also expected at large t there will be a clearly visible second-resonance enhancement. Similarly we expect to see a prominent third resonance with $F_{15}(1688)$ the dominating resonance, especially away from the forward direction.

It has been claimed by Albright *et al.*[18] that the weak excitation of resonances in the (70, $L=1-$) supermultiplet will vanish. This is contrary to the results we have described above and also to the results of Ravndal.[18] This latter author examined both electromagnetic and weak excitation of resonances in a relativistic formulation of the model and his results are very similar to those that we have obtained in this nonrelativistic approach. He implicitly is comparing the excitation form factors to the elastic form factor, as we do here, and also finds the result that the Roper resonance could be important at large t (examination of his Fig. 5 in his electroproduction paper shows that the P_{11} transverse photoabsorption cross section is falling much less rapidly with increasing t than those of the S_{11} and D_{13}). Detailed phenomenological analyses of coincidence electroproduction in the second resonance region will be required to see whether this prediction is borne out by the data; these we await with interest.

ACKNOWLEDGMENTS

We are indebted to Professor R. H. Dalitz for his continued interest and advice. A substantial part of this work was performed at the Theoretical Physics Department, Oxford University, and we are indebted to Professor Sir Rudolf Peierls for his hospitality. One of us (F.E.C.) wishes to thank R. Lipes, F. Ravndal, and F. Gilman for discussions on electroproduction, and the authors of Ref. 6 for informing us of their work. We thank the Science Research Council of London for administering a postdoctoral NATO fellowship (F.E.C.) and research studentship (T.A.); we also thank G. Karl for useful advice in the early stages of this work.

APPENDIX

We collect here the expressions for the spin-averaged square of the matrix element $|M|^2$. The essential structure of $|M|^2$ is

$$|M|^2 = e^{-k^2/3\alpha^2}[A_1(kW_0 - k_0W_z)^2 + A_2(|W_x + iW_y|^2 + |W_x - iW_y|^2) + A_3(|W_x + iW_y|^2 - |W_x - iW_y|^2) + BR^2].$$

All that remains is to list the resulting expressions for A_1, A_2, A_3, and B for the various resonances considered. We present here the results for the process $\nu N^0 \to N^{*+}\mu^-$. The electroproduction results can be derived by setting $R=0$ (i.e., removing the axial-vector matrix elements) and making necessary adjustments in the isospin Clebsch-Gordan coefficients. We do not list these explicit expressions here. One can also select out those terms depending upon R (axial-vector terms), and making necessary phase-space adjustments one can then immediately read off the $N^* \to N\pi$ widths. We have checked these and find that they contain the results of Faiman and Hendry (Ref. 3).

$S_{11}\{^2 8\}$:

$$A_1 = \frac{2}{9}\left(\frac{1}{\alpha}\right)^2,$$

$$A_2 = \frac{1}{9}\left[\left(\frac{\alpha}{M_q} + \frac{gk}{2M_q}\frac{2k}{3\alpha}\right)^2 + R^2\left(\frac{2k}{3\alpha}\right)^2\right],$$

$$A_3 = \frac{1}{9}\left[\frac{\alpha}{M_q} + \frac{gk}{2M_q}\frac{2k}{3\alpha}\right]\frac{4k}{3\alpha}R,$$

$$B = \frac{8}{81}\left\{\frac{k}{\alpha}W_z\left(1 - \frac{f_pk^2}{2M_q}\right) - W_0\left[\frac{3\alpha}{M_q} + \frac{k}{\alpha}\left(\frac{k}{6M_q} - \frac{f_pkk_0}{2M_q}\right)\right]\right\}^2.$$

$D_{13}\{^2 8\}$:

$$A_1 = \frac{4}{9}\left(\frac{1}{\alpha}\right)^2,$$

$$A_2 = \frac{2}{9}\left\{\frac{4k^2}{9\alpha^2}\left[\left(\frac{gk}{2M_q}\right)^2 + R^2\right] - \frac{2k}{3M_q}\left(\frac{gk}{2M_q}\right) + \frac{\alpha^2}{M_q^2}\right\},$$

$$A_3 = \frac{2}{9}\left[\frac{8k^2}{9\alpha^2}R\left(\frac{gk}{2M_q}\right) - \frac{2k}{3M_q}R\right],$$

$$B = \frac{16}{81}\left[\frac{k}{\alpha}W_z\left(1 - \frac{f_pk^2}{2M_q}\right) - W_0\frac{k}{\alpha}\left(\frac{k}{6M_q} - \frac{f_pkk_0}{2M_q}\right)\right]^2.$$

$S_{11}\{^4 8\}$:

$$A_1 = 0,$$

$$A_2 = \frac{1}{9}\left\{\left(\frac{k}{6\alpha}\right)^2\left[\left(\frac{gk}{2M_q}\right)^2 + R^2\right]\right\},$$

$$A_3 = \frac{2}{9}\left[\left(\frac{k}{6\alpha}\right)^2 R\left(\frac{gk}{2M_q}\right)\right],$$

$$B = \frac{2}{81}\left\{\frac{k}{\alpha}W_z\left(1 - \frac{f_pk^2}{2M_q}\right) - W_0\left[\frac{3\alpha}{M_q} + \frac{k}{\alpha}\left(\frac{k}{6M_q} - \frac{f_pkk_0}{2M_q}\right)\right]\right\}^2.$$

$D_{13}\{^4 8\}$:

$$A_1 = 0,$$

$$A_2 = \frac{7}{405}\frac{k^2}{\alpha^2}\left[\left(\frac{gk}{2M_q}\right)^2 + R^2\right],$$

$$A_3 = \frac{7}{405}\frac{k^2}{\alpha^2}\left(\frac{gk}{2M_q}\right)2R,$$

$$B = \frac{2}{405}\left[\frac{k}{\alpha}W_z\left(1 - \frac{f_pk^2}{2M_q}\right) - \frac{k}{\alpha}W_0\left(\frac{k}{6M_q} - \frac{f_pk_0k}{2M_q}\right)\right]^2.$$

$D_{15}\{^4 8\}$:

$$A_1=0\,,$$

$$A_2=\frac{1}{60}\,\frac{k^2}{\alpha^2}\left[\left(\frac{gk}{2M_q}\right)^2+R^2\right]\,,$$

$$A_3=\frac{1}{60}\,\frac{k^2}{\alpha^2}\left(\frac{gk}{2M_q}\right)2R\,,$$

$$B=\frac{2}{45}\left[\frac{k}{\alpha}\,W_z\left(1-\frac{f_pk^2}{2M_q}\right)-\frac{k}{\alpha}\,W_0\left(\frac{k}{6M_q}-\frac{f_pk_0k}{2M_q}\right)\right]^2.$$

$S_{31}\{^2 10\}$, $\nu N^+\to N^{*++}+\mu^-$:

$$A_1=\frac{1}{6}\left(\frac{1}{\alpha}\right)^2,$$

$$A_2=\frac{1}{12}\left\{\left[\frac{k}{3\alpha}\left(\frac{gk}{2M_q}\right)-\frac{\alpha}{M_q}\right]^2+R^2\,\frac{k^2}{9\alpha^2}\right\}\,,$$

$$A_3=\frac{1}{12}\left[\frac{k}{3\alpha}\left(\frac{gk}{2M_q}\right)-\frac{\alpha}{M_q}\right]\frac{2k}{3\alpha}R\,,$$

$$B=\frac{1}{54}\left\{\frac{k}{\alpha}\,W_z\left(1-\frac{f_pk^2}{2M_q}\right)-W_0\left[\frac{3\alpha}{M_q}+\frac{k}{\alpha}\left(\frac{k}{6M_q}-\frac{f_pk_0k}{2M_q}\right)\right]\right\}^2.$$

$S_{31}\{^2 10\}$, $(\nu N^0\to N^{*+}+\mu^-)=\frac{1}{3}(\nu N^+\to N^{*++}+\mu^-)$.

$D_{33}\{^2 10\}$, $\nu N^+\to N^{*++}+\mu^-$:

$$A_1=\frac{1}{3}\left(\frac{1}{\alpha}\right)^2,$$

$$A_2=\frac{1}{6}\left\{\frac{k^2}{9\alpha^2}\left[\left(\frac{gk}{2M_q}\right)^2+R^2\right]+\left(\frac{gk}{2M_q}\right)\frac{k}{3M_q}+\frac{\alpha^2}{M_q^2}\right\},$$

$$A_3=\frac{1}{6}\left[\frac{k^2}{9\alpha^2}\left(\frac{gk}{2M_q}\right)2R+\frac{k}{3M_q}R\right],$$

$$B=\frac{1}{27}\left[\frac{k}{\alpha}\,W_z\left(1-\frac{f_pk^2}{2M_q}\right)-W_0\,\frac{k}{\alpha}\left(\frac{k}{6M_q}-\frac{f_pk_0k}{2M_q}\right)\right]^2.$$

$D_{33}\{^2 10\}$, $(\nu N^0\to N^{*+}+\mu^-)=\frac{1}{3}(\nu N^+\to N^{*++}+\mu^-)$.

$P_{33}\{^4 10\}$, $(\nu N^+\to N^{*++}+\mu^-)=3(\nu N^0\to N^{*+}+\mu^-)$:

$$A_1=0\,,$$

$$A_2=\frac{4}{3}\left[\left(\frac{gk}{2M_q}\right)^2+R^2\right]\,,$$

$$A_3=\frac{8}{3}\left(\frac{gk}{2M_q}\right)R\,,$$

$$B=\frac{8}{3}\left[W_z\left(1-\frac{f_pk^2}{2M_q}\right)^2-W_0\left(\frac{k}{6M_q}-\frac{f_pk_0k}{M_q}\right)\right]^2.$$

$P_{11}\{^2 8\}$:

$$A_1=\left(\frac{1}{k}\right)^2,$$

$$A_2=\frac{25}{18}\left[\left(\frac{gk}{2M_q}\right)^2+R^2\right]\,,$$

$$A_3=\frac{25}{9}\left(\frac{gk}{2M_q}\right)R\,,$$

$$B=\frac{25}{9}\left[W_z\left(1-\frac{f_pk^2}{2M_q}\right)-W_0\left(\frac{k}{6M_q}-\frac{f_pk_0k}{2M_q}\right)\right]^2.$$

$F_{15}\{^2 8\}$:

$$A_1=\frac{1}{90}\left(\frac{k}{\alpha}\right)^2\left(\frac{1}{\alpha}\right)^2,$$

$$A_2=\frac{1}{5}\left(\frac{1}{18}\right)^2\left\{\left[\left(\frac{gk}{2M_q}\right)\frac{5k^2}{\alpha^2}-\frac{3k}{M_q}\right]^2+\left(5R\,\frac{k^2}{\alpha^2}\right)^2+18\left(\frac{k}{M_q}\right)^2\right\},$$

$$A_3=\frac{1}{5}\left(\frac{1}{18}\right)^2\left\{\left[\left(\frac{gk}{2M_q}\right)\frac{5k^2}{\alpha^2}-\frac{3k}{M_q}\right]10R\,\frac{k^2}{\alpha^2}\right\}\,,$$

$$B=\frac{10}{(18)^2}\left(\frac{k}{\alpha}\right)^2\left[\frac{k}{\alpha}\,W_z\left(1-\frac{f_pk^2}{2M_q}\right)-W_0\,\frac{k}{\alpha}\left(\frac{k}{6M_q}-\frac{f_pkk_0}{2M_q}\right)\right]^2.$$

$P_{11}(1470)$:

$$A_1=\frac{1}{108}\left(\frac{k}{\alpha}\right)^2\left(\frac{1}{\alpha}\right)^2,$$

$$A_2=\frac{1}{108}\,\frac{25}{18}\left(\frac{k^2}{\alpha^2}\right)^2\left[\left(\frac{gk}{2M_q}\right)^2+R^2\right]\,,$$

$$A_3=\frac{1}{108}\,\frac{25}{9}\left(\frac{k^2}{\alpha^2}\right)^2R\left(\frac{gk}{2M_q}\right)\,,$$

$$B=\frac{1}{108}\,\frac{25}{9}\left(\frac{k^2}{\alpha^2}\right)^2\left\{W_z\left(1-\frac{f_pk^2}{2M_q}\right)-W_0\left[\frac{k}{6M_q}\left(1+\frac{12\alpha^2}{k^2}\right)-\frac{f_pk_0k}{2M_q}\right]\right\}^2.$$

In all these expressions, k_0 is the energy transfer to the hadronic system and $k_0=(t+k^2)^{1/2}$ where t is the invariant four-momentum transfer squared. In the Breit frame (which is the frame for which our figures are drawn) we find

$$k_0^{\,2}=\frac{(m_B^{\,2}-m_A^{\,2})^2}{2(m_B^{\,2}+m_A^{\,2})-t}\,.$$

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[21]In particular we expect a significant D_{33} in photoproduction at the third resonance. This prediction has also been made by Ravndal (Ref. 8). Previous phenomenological analyses of the third resonance region did not allow for the possibility that this resonance is present [Ref. 19 and also W. A. Rankin, Ph.D. thesis, Glasgow University (unpublished)]. We are informed [F. Ravndal (private communication)] that an analysis is in progress by R. L. Walker which will test for this resonance.

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Soft-Pion Production in Electron-Positron Collisions*

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The processes $\gamma\to\pi^+\pi^-$, $\gamma\to\pi^+\pi^+\pi^-\pi^-$, $\gamma\gamma\to\pi^+\pi^-$, and $\gamma\gamma\to\pi^+\pi^+\pi^-\pi^-$ can now be studied in e^+-e^- colliding-beam experiments. Amplitudes for these processes are considered in the low-energy region where simple analyticity may be used to extrapolate from the zero-energy values determined by gauge invariance and current algebra. It is argued that in the S-wave two-pion state a broad σ resonance will appear as an enhancement in $\gamma\gamma\to\pi^+\pi^-$ at an energy much lower than the σ mass. For the process $\gamma\to\pi^+\pi^+\pi^-\pi^-$ an approximation is made (which amounts to an extreme interpretation of ρ dominance) to predict a cross section of about 1.6×10^{-34} cm^2 at energies just above the ρ mass. A similar approximation is used to estimate the nonresonant cross section for $\gamma\gamma\to\pi^+\pi^+\pi^-\pi^-$, in substantial accord with a recent result by Terazawa. We find this cross section to be too small $[O(10^{-36}\text{ cm}^2)]$ to be of present experimental interest.

I. INTRODUCTION

The small mass of the pion gives it a privileged position in strong-interaction theory. Low-energy amplitudes for purely pionic processes are expected to be dominated by singularities coming from intermediate states which in turn contain only pions. Furthermore, the pion pole should dominate low-energy matrix elements of the divergence of the axial-vector current, and this allows low-energy pionic amplitudes to be constrained by the current algebra.

Unfortunately, there are no corresponding experimental advantages to purely pionic systems, and up to now our information on these systems has been perforce indirect. With the recent construction and operation of colliding electron-positron beams, however, amplitudes for the production of pions by photons have become measurable. The most obvious and important example is the pion form factor in the timelike region which has already been extensively studied.[1] The colliding beams at Frascati also produce large numbers of multipion events.[1] In addition, Brodsky, Kinoshita, and Terazawa[2] have pointed out that the cross section for $e^+e^-\to e^+e^-+$pions, involving the production of a pair of nearly real photons which then form pions, will be at least as large as $\sigma(e^+e^-\to\gamma\to$ pointlike pions) when electron beam energies exceed about 1 GeV. This means that pionic states of positive charge conjugation will also be available for study in forthcoming experiments at Frascati, CEA, SLAC, and DESY.

In this paper we consider amplitudes for the production of two and four charged pions by photons in the low-energy region. This is the regime of validity of (i) current algebra, (ii) analyticity with simple intermediate states. The basic plan is to find, using gauge invariance and current algebra, the zero-energy limits for these amplitudes, then, using two-particle unitarity, to extrapolate these amplitudes past threshold up to energies of roughly the ρ mass.

This program has already been carried out by Gounaris and Sakurai for the important case of the timelike-pion form factor.[3] For orientation, we review and discuss briefly in Sec. II their results. In subsequent sections we consider, respectively, the processes $\gamma\gamma\to\pi^+\pi^-$ (Sec. III), $\gamma\to\pi^+\pi^+\pi^-\pi^-$ (Sec. IV), and $\gamma\gamma\to\pi^+\pi^+\pi^-\pi^-$ (Sec. V). Appendix A discusses the ambiguity in the unitarization of the $\gamma\gamma\to\pi\pi$ amplitude and the possible application of a concept of σ dominance. Appendix B deals with $\gamma\gamma\to\mu^+\mu^-$, useful for normalization