Current Algebra and Nonleptonic Hyperon Decays - A Fresh Approach

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It is shown how to obtain consequences of current algebra for nonleptonic hyperon decays in the limit $q^2=0$, with the baryons kept on the mass shell. The p waves are model-independent, and an excellent fit determines the parity-conserving spurion. When a K^* pole is included for s waves, a rough fit is achieved.

It is generally recognized that a unified theory of s-wave (parity violating) and p-wave (parity conserving) hyperon decays does not exist. The accepted s-wave theory of Suzuki[1] and Sugawara[2] based on soft pions and current algebra implies a different d/f ratio of the weak spurion than that implied by the p-wave pole model.[3] What is worse, the ratios of theoretical p to s waves are smaller than the experimental ratios by factors of 2 to 3.[4] The resolution of these inconsistencies is usually taken to be that the s-wave current-algebra analysis is correct, but the p-wave analysis is ambiguous because the soft extrapolation in pion four-momentum destroys the SU(3) baryon mass breaking and leads to large unknown background terms. The problem is how to take the pion soft while still preserving momentum conservation at the *three*-particle vertex.

In this paper we point out that one can obtain a unified approach to s and p waves in the limit $q^2 \to 0$, rather than in the soft-pion limit $q\to 0$. Using the axial-vector Ward identity in conjunction with the usual SU(3) assumptions, current algebra leads to the p-wave solutions of Itzykson and Jacob[5] and of Kumar and Pati[6] to lowest order in SU(3) baryon mass breaking as $q^2\to 0$. The s-wave results of Suzuki-Sugawara are also recovered in the $q^2=0$ limit; and we find it advantageous to include the K^* pole as well. The $q^2=0$ limit effectively removes any ambiguity in the soft-pion limit.

We note that the p waves are not susceptible to "SU(3) anomalies," and that the Ward identity for the p waves appears less model-dependent than for the s waves. Hence we regard the former to be on firmer footing than the latter, and we fit the data accordingly.

We begin by defining the axial-vector amplitude M_μ for the process $B^i(p)\to B^f(p')+A^j_\mu(q)$ by[7]

$$M_\mu = i\int e^{iq\cdot x}\theta(x_0)\langle B^f|[A^j_\mu(x), H_w]|B^i\rangle d^4x. \qquad (1)$$

Computing the divergence of (1) and taking the pions to be massless according to Nambu's version of partial conservation of axial-vector current (PCAC),[8] we obtain by Gauss's theorem

$$q\cdot M = -\langle B^f|[J^j_5(\vec{q}), H_w]|B^i\rangle, \qquad (2)$$

where

$$J^j_5(\vec{q}) = \int d^3x\, e^{-i\vec{q}\cdot\vec{x}} A^j_0(\vec{x}, 0), \qquad (3)$$

and the commutator in (2) is at time $t=0$. We isolate the pion-pole contribution to M_μ, giving the pion amplitude M_π for $B^i(p)\to B^f(p')+\pi^j(q)$ as

$$q\cdot M = if_\pi M_\pi + q\cdot M^x + q\cdot \bar{M}, \qquad (4)$$

where M^x_μ corresponds to the baryon pole diagrams with axial-vector coupling $\frac{1}{2}g^{fi}_A i\gamma_\mu\gamma_5$, and $\bar{M}_\mu$ is the unknown background amplitude. Separating the weak Hamiltonian density as $H_w = H^{\text{pv}}_w + H^{\text{pc}}_w$, we have (suppressing the baryon spinors)

$$M_\pi(s \text{ wave}) = -\langle B^f\pi^j|H^{\text{pv}}_w|B^i\rangle = iA, \qquad (5)$$

$$M_\pi(p \text{ wave}) = -\langle B^f\pi^j|H^{\text{pc}}_w|B^i\rangle = B\gamma_5, \qquad (6)$$

with A and B relatively real ($\gamma_5{}^2=-1$) by T invariance (neglecting final-state interactions). Assuming a current-current form for H_w, with $J=V-A$,

$$H_w \sim J^\dagger_\mu J^\mu + J^\mu J^\dagger_\mu, \qquad (7)$$

the commutator in (2) contains $\delta^3(\vec{x})$ from the algebra of currents,[9,10] or from Feynman diagrams.[11] Thus the dependence of (3) upon $\vec{q}$ can be eliminated and (2) can be evaluated exactly by replacing $J_5(\vec{q})$ with the axial charge $J^j_5 = J^j_5(\vec{q}=0)$. The axial-vector Ward identities for s and p waves can then be written as

$$q\cdot M(s \text{ wave}) = -\langle B^f|[J^j_5, H^{\text{pv}}_w]|B^i\rangle, \qquad (8)$$

$$q\cdot M(p \text{ wave}) = -\langle B^f|[J^j_5, H^{\text{pc}}_w]|B^i\rangle. \qquad (9)$$

We should stress that (8) and (9) are "exact" Ward identities, valid for any q as opposed to the "soft" Ward identities used when $q\to 0$.[1-3,5,6]

Finally we state the SU(3) assumptions that we shall use:

(i) SU(3) is conserved at the strong and weak vertices, but the hadronic masses are broken and kept physical.

(ii) The $\Delta I = \frac{1}{2}$ sum rules $\Lambda_-^0 = -\sqrt{2}\,\Lambda_0^0$, $\Xi_-^- = -\sqrt{2}\,\Xi_0^0$, and $\Sigma_+^+ - \Sigma_-^- = \sqrt{2}\,\Sigma_0^+$, as well as the Lee-Sugawara[12] (LS) sum rule $\Lambda_-^0 + 2\Xi_-^- = \sqrt{3}\,\Sigma_0^+$ must be obeyed for both the s and p waves. According to Suzuki[1] and Sugawara,[2] this is valid in the current-algebra analysis of s waves if the $\underline{27}$ representation of H_w is suppressed:

$$\langle B^f | H_w(\underline{27}) | B^i \rangle = 0\,, \tag{10}$$

which implies $\Sigma_+^+(s \text{ wave}) = 0$, agreeing with experiment.

(iii) The C parities[1] of the commutators in (8) and (9) are +1 and −1, respectively. By CP invariance, the octet part of H_w given by (7) transforms as λ_6 and therefore has $C = +1$ for H_w^{pc} and $C = -1$ for H_w^{pv}. The same is true of the $\underline{27}$ part of H_w.[1] Application of the algebra of currents transmits this C parity to the commutator but with opposite sign. Clearly this property does not depend upon the over-all scaling of current algebra, and we take it as a general constraint upon H_w. Models such as the quark density model[4] do not obey (7) nor have the C-parity assignment as stated above, and we do not consider them. Because the $B\overline{B}$ states which couple to a neutral pion have $C = +1$, we can write

$$\langle B^f | H_w^{\mathrm{pv}} | B^i \rangle = 0\,, \tag{11}$$

$$\langle B^f | H_w^{\mathrm{pc}} | B^i \rangle = h(d' d^{6fi} - f' i f^{6fi})\,, \tag{12}$$

and from (9),

$$q \cdot M(p \text{ wave}) = 0\,, \tag{13}$$

valid for any q. Such a current-conservation statement as (13) is the basis of many of the results of current algebra,[13] in particular the Goldberger-Treiman relation, and we consider if far more natural than any other possible p-wave Ward identity.

The p waves. We are now in a position to combine (13) with (4) at $q^2 = 0$ to obtain a model-independent p-wave result. The conservation equation, (13), is reminiscent of the conservation of the parity-conserving (on-shell) axial-vector vertex

$$\tfrac{1}{2} i (g_A^{fi} \gamma_\mu \gamma_5 - h_A^{fi} q_\mu \gamma_5)\,,$$

which gives

$$(m_f + m_i) g_A^{fi}(q^2) + q^2 h_A^{fi}(q^2) = 0$$

for any value of q. Applying pole dominance by a massless pion near $q^2 = 0$ to h_A then leads to the generalized Goldberger-Treiman relation[14]

$$\tfrac{1}{2}(m_f + m_i) g_A^{fi} = f_\pi g^{fi}\,. \tag{14}$$

This technique of first separating the invariants from the covariants and then taking the $q^2 = 0$ limit can be directly applied to the hyperon decays. More specifically, by writing the unknown background for p waves as

$$\overline{M}_\mu(p \text{ wave}) = i\overline{G}_A \gamma_\mu \gamma_5 + i\overline{H}_A q_\mu \gamma_5 + \overline{L}_A [\gamma_\mu, \gamma \cdot q] \gamma_5\,, \tag{15}$$

we find

$$q \cdot \overline{M} = -i(m_f + m_i) \overline{G}_A \gamma_5 \text{ at } q^2 = 0\,.$$

Then the axial-vector pole diagrams

$$M_\mu^x(p \text{ wave}) = -\tfrac{1}{2} i \sum_n [g_A^{fn} \gamma_\mu \gamma_5 (\gamma \cdot p_n - m_n)^{-1} H_w^{ni} + H_w^{fn} (\gamma \cdot p_n - m_n)^{-1} \gamma_\mu \gamma_5 g_A^{ni}] \tag{16}$$

evaluated between on-shell baryon spinors allow us to separate the invariant amplitude B from the covariant γ_5 in the p-wave part of (4). Using (13) and (14) we find at $q^2 = 0$

$$B^{fi} = (m_f + m_i) \sum_n \left(\frac{g^{fn} H_w^{ni}}{(m_f + m_n)(m_n - m_i)} + \frac{H_w^{fn} g^{ni}}{(m_n - m_f)(m_n + m_i)} \right) + (m_f + m_i) \frac{g}{m g_A} \overline{G}_A\,. \tag{17}$$

We regard (17) as a power series in SU(3) breaking, so that neglect of the last term leads to an expected 10% accuracy; since the baryons have been kept on the mass shell, this expansion is unambiguous. Additional support for our neglect of $\overline{G}_A$ comes from an argument of Brandt and Preparata.[15] They first generalize the decay to a four-particle interaction involving a scalar spurion, then use high-energy Regge and low-energy resonance considerations to establish the smallness of what in our language is $\overline{G}_A$.

Compare our approach to the $q \to 0$ limit. First, one separates M_π^{pole} with $g\gamma_5$ coupling from the background $\overline{M}_\pi$, and determines the latter from (4) as $q \to 0$. One obtains

$$B^{\mathrm{pole}} = \sum_n \left(\frac{g^{fn} H_w^{ni}}{m_n - m_i} + \frac{H_w^{fn} g^{ni}}{m_n - m_f} \right), \tag{18}$$

which is the pole-model result of Brown and Sommerfield,[3] and

$$\overline{B} = -\sum_n \left(\frac{g^{fn} H_w^{ni}}{m_f + m_n} + \frac{H_w^{fn} g^{ni}}{m_n + m_i} \right) + O(1)\,. \tag{19}$$

Both terms in (19) are of $O(1)$ and so one usually neglects all of (19). However, Itzykson and Jacob[5] and Kumar and Pati[6] kept both terms and obtained a better fit to the data. If one actually combines the mass factors in (18) and (19) the $O(1)$ pole terms cancel and one obtains our $q^2 \to 0$ statement, Eq. (17).

Another approach to the p waves is the SU(3) pole model for the pion amplitude, which leads to (18) with a kaon pole added. Since $\langle K|H_w^{\rm pc}|\pi\rangle$ vanishes by current algebra and SU(3), Eq. (18), taken alone, can be regarded as a consequence of SU(3) and current algebra. The difference between this and our result, Eq. (19), would seem to arise from the opposite order in which SU(3) and current algebra are applied. Notice that since the kaon pole does not contribute to the axial-vector covariant $\gamma_\mu\gamma_5$, it never enters the SU(3) analysis of our Eq. (17). Note too that our unambiguous procedure leading to nonvanishing p waves strengthens the assertion[16] that current algebra is inconsistent with universality.[17]

We have fitted (17) to the p-wave data, and determined the parameters in (12) to be $d'/f' \cong -0.86$ and $h \cong 16.2$ eV. The fit is displayed in Table I.

The s waves. In this case, the right-hand side of (8) does not vanish and explicitly depends on the structure of the algebra of currents, which implies

$$q\cdot M(s\text{ wave}) = if^{fjb}\langle B^b|H_w^{\rm pc}|B^i\rangle - if^{jia}\langle B^f|H_w^{\rm pc}|B^a\rangle\,. \tag{20}$$

Because (20) is valid for any q, we again take the $q^2=0$ limit. Writing the background in (4) as

$$\overline{M}_\mu(s\text{ wave}) = \overline{F}_1\gamma_\mu + \overline{F}_2[\gamma_\mu, \gamma\cdot q] + \overline{F}_3 q_\mu\,, \tag{21}$$

and noting that M_μ^x does not contribute to s waves by (11), we obtain from (4) and (20) at $q^2=0$

$$A = (1/f_\pi)X + (\Delta m/m)(g/g_A)\overline{F}_1\,, \tag{22}$$

TABLE I. p waves.

Decay	10^6B Experiment[a]	Theory[b]
Λ_-^0	2.28 ± 0.07	2.39
Σ_0^+	$2.92^{+0.25}_{-0.19}$	2.89
Σ_+^+	$4.17^{+0.08}_{-0.21}$	4.17
Σ_-^-	-0.21 ± 0.02	0.10
Ξ_-^-	1.55 ± 0.12	1.47

[a] Experimental values were calculated from those for α and ϕ [Particle Data Group, Rev. Mod. Phys. 43, S1 (1971)] by assuming T invariance and incorporating known information about final-state phase shifts.

[b] $d/f = -0.865$; $h = 16.2$ eV.

where X is the right-hand side of (20),

$$X(\Lambda_-^0) = (-1/\sqrt{2})\langle n|H_w|\Lambda\rangle\,,$$

$$X(\Xi_-^-) = (-1/\sqrt{2})\langle \Lambda|H_w|\Xi^0\rangle\,,$$

$$X(\Sigma_0^+) = \tfrac{1}{2}\langle p|H_w|\Sigma^+\rangle\,,$$

$$X(\Sigma_-^-) = \langle n|H_w|\Sigma^0\rangle\,,$$

$$X(\Sigma_+^+) = -[\langle n|H_w|\Sigma^0\rangle + (1/\sqrt{2})\langle p|H_w|\Sigma^+\rangle]\,.$$

Neglecting the background in (22) because it is $O(\Delta m/m)$ corresponds to the Suzuki[1]-Sugawara[2] solution of the s waves as $q\to 0$. This of course leads to the $\Delta I = \frac{1}{2}$ rule and the LS rule for s waves because of (10). However, the best fit to the s waves gives $d'/f' \sim -0.3$ and $h \sim 70$ eV. If we insist upon the p-wave parameters $d'/f' = -0.86$ and $h = 16.2$ eV, the theoretical s waves for the Suzuki-Sugawara solution in Table II are not good and the B/A ratio is still off by a factor of 2.

Although the second term in (22) is first order in SU(3) breaking, it is not suppressed by the Brandt-Preparata argument[15] applied to s waves. A possible contribution to the axial-vector amplitude M_μ is the K^* pole. In contrast to the K pole for p waves, the K^* pole contributes to the s-wave γ_μ covariant, and leads to[18]

$$A^{K^*} = \alpha(m^f - m^i)d^{6jl}(-f_v if^{lfi} + d_v d^{lfi})\,, \tag{23}$$

where f_v and d_v arise from the K^*-baryon vertex $(f_v + d_v = 1)$. With the aid of a short current-algebra argument relating $\langle\pi|H_w^{\rm pv}|K^*\rangle$ to $\langle 0|A_\mu H_w^{\rm pv}|K^*\rangle$, we find

$$\alpha = h' g_{VBB}/m_{K^*}{}^2\,, \tag{24}$$

where

$$\langle \pi^j(q)|H_w|K_\mu^{*l}\rangle = h' d^{6jl} q_\mu\,, \tag{25}$$

and $g_{VBB}{}^2/4\pi \approx 2$.

For $\Delta Y = 1$ decays, $d^{6-i7,fi} = f^{6-i7,fi} = 0$; using this together with isospin invariance one finds that Eq. (23) exactly parallels the structure of the Suzuki-Sugawara contribution in the limit $m_\Lambda - m_N = m_\Sigma - m_N = m_\Xi - m_\Lambda$. In fact, the K^* pole has been invoked as a model for the entire pion amplitude.[18-21]

We will now argue that as a first-order contributor to A, A^{K^*} is anomalously large.[22] We can relate the parameter α or h' to $K_S \to \pi\pi$ decay by applying pole dominance by the K^* to the latter process.[21] According to a theorem of Gell-Mann and Cabibbo,[23] with our assumptions the $K\to\pi\pi$ matrix element is first order in SU(3); yet it is not correspondingly small. In fact, we find $\alpha = 3.4\times 10^{-6}$ BeV^{-1}, or $h' \sim 600$ eV in Eq. (25). This is to be compared with the strength of $H_w^{\rm pv}$

TABLE II. s waves.

		$10^6 A$		
Decay	Experiment[a]	Current commutator	With K^*, $d^*=0$	With K^*, $d^*/f^*=-0.33$
Λ^0_-	0.335 ± 0.004	0.505	0.24	0.16
Σ^+_0	$-0.308^{+0.020}_{-0.014}$	−0.76	−0.54	−0.33
Σ^+_+	$0.070^{+0.084}_{-0.046}$	0	0	0
Σ^-_-	0.428 ± 0.003	1.08	0.77	0.47
Ξ^-_-	-0.441 ± 0.005	−0.915	−0.62	−0.42

[a] Experimental values were calculated from those for α and ϕ [Particle Data Group, Rev. Mod. Phys. 43, S1 (1971)] by assuming T invariance and incorporating known information about final-state phase shifts. In the case of $A(\Sigma^+_+)$, large cancellations of the $I=\frac{3}{2}$ and $\frac{1}{2}$ parts allow ϕ to be any value. Hence $A(\Sigma^+_+)$ might be rather larger than has often been assumed.

in (12), $h|d'| \sim 100$ eV.

In Table II we display the s-wave fit for the above value of h' and two values of d_v/f_v. Universal vector-meson coupling requires $d_v=0$ at $q^2=0$, whereas Regge estimates[24] from meson-baryon scattering give $d_v/f_v \sim -0.2$ to -0.5. The value $d_v/f_v=-0.33$ gives close-to-the-best fit obtainable with Eq. (23) added to the current-commutator term. Significant improvement results with the K^*, but the fit is still not completely satisfactory.[25]

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Asymptotic Bounds on the Absorptive Parts of the Elastic Scattering Amplitudes*

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We establish exact bounds on the absorptive parts $A(s,t)$ of an elastic scattering amplitude (spinless case) and evaluate them for positive t values lying within the Lehmann-Martin ellipse [the major axis $=2(1+t_0/2k^2)$]. These bounds are used to derive a number of asymptotic results; e.g., (i) the "diffraction-peak width" W is larger than $W_{\min}\sim 4t_0/(1+\lambda)^2 \times(1-\frac{1}{2}\sigma)(\ln s)^2$ (for $s\to\infty$); (ii) the leading Regge trajectory for $t_0>t>0$ lies below $[1+(t/t_0)^{1/2}] -\lambda[1-(t/t_0)^{1/2}]$; (iii) there are no complex zeros of $A(s,t)$ for $|t|<4t_0/(1+\lambda)^2e^2(\ln s)^2$ (for $s\to\infty$) and no real zeros for $t_0>t>-W_{\min}$, where $\lambda=\lim_{s\to\infty}\ln\sigma_{\text{tot}}(s)/\ln s$ and $\sigma=\lim_{s\to\infty}t_0\sigma_{\text{tot}}(s)/4\pi(\ln s)^2$.

I. INTRODUCTION

The investigation of bounds on scattering amplitudes, and in particular on absorptive parts, following from the general principles of analyticity, unitarity, crossing, etc., has proved to be quite fruitful in the study of the strong interactions.[1] The restrictions on the absorptive part, for a given total cross section $\sigma_{\text{tot}}(s)$ ($\sqrt{s}$ = c.m. energy), were first studied by Martin.[2] The exact solution to this problem, as well as to the one with both the total and elastic cross sections given, was given by Singh and Roy, who succeeded in constructing the correct Fresnel-plate solution.[3] A comparison of the unitarity upper bound, involving the total and elastic cross section, with the experimental data in the diffraction-peak region showed that the bound was almost achieved.[3,4] The purpose of the present paper is to show how one can establish bounds on the absorptive parts which, apart from unitarity constraints, also take into account their polynomial boundedness. These bounds are evaluated in the positive-momentum-transfer region, lying within the Lehmann-Martin ellipse and have important consequences for the "diffraction-peak-width," Regge behavior, and zeros of the amplitude.

In order to derive these new bounds we make use of only (i) unitarity, (ii) analyticity within the Lehmann-Martin ellipse [the major axis $=2(1+t_0/2k^2)$, k = c.m. momentum], and (iii) the Jin-Martin upper bound[5]

$$A(s,t_0)\underset{s\to\infty}{<}(s/s_0)^2, \tag{1.1}$$

where $A(s,t)$ is the absorptive part of the elastic scattering amplitude (we restrict ourselves to the spinless problem for simplicity) and s and t are, respectively, the squared c.m. energy and momentum-transfer variables. No assumption shall be made about the high-energy behavior of the total cross section $\sigma_{\text{tot}}(s)$.

II. BASIC THEOREMS

The absorptive part $A(s,t)$ of the elastic scattering amplitude has the following partial-wave expansion, valid within the Lehmann-Martin ellipse:

$$A(s,t)=\frac{\sqrt{s}}{k}\sum_{l=0}^{\infty}(2l+1)\,\mathrm{Im}\,a_l(s)P_l(z), \tag{2.1}$$

where $t=-2k^2(1-z)$. We also have the following unitarity restrictions on partial-wave amplitudes:

$$\frac{k}{\sqrt{s}}A(s,0)=\frac{k^2}{4\pi}\sigma_{\text{tot}}(s) =\sum_{l=0}^{\infty}(2l+1)\,\mathrm{Im}\,a_l(s) \tag{2.2}$$