Phase Problem and the Signature Rule in Multiple-Scattering Regge Cuts*

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It is often stated that a double-Regge-pole-exchange amplitude has definite signature which is given by the product of the signatures of the exchanged poles. This signature rule places severe restrictions on any model which generates Regge cuts from the nonlinear direct-channel unitarity relation. With the intermediate-state particles treated as composite rather than elementary, however, the signature rule no longer follows. The phase of a double-exchange amplitude is found to be the same as in the simplified treatment (i.e., a simple function of the exchanged trajectories) while the s dependence follows from the details of the spectra and coupling of the intermediate-state particles. It is suggested that multiple-scattering models should be considered as unitarity-generated approximations in which s-u crossing is viewed as a constraint on the couplings and spectra of the intermediate states and not as a constraint on the signatures of the exchanged Regge poles. Phenomenological consequences are discussed.

I. INTRODUCTION

It is often stated[1] that *a double-Regge-pole-exchange cut has definite signature which is given by the product of the signatures of the exchanged poles* (and similarly for higher-order exchanges). The question of the validity of this "signature rule" is of importance not only because of the considerable current interest in Regge cuts,[2] but more specifically because it places severe restrictions on any model which generates Regge cuts from Regge-pole Born terms through imposition of the nonlinear unitarity condition.[3] A simple example is provided by πN charge-exchange reactions. This reaction is odd under s-u crossing and the above rule would therefore forbid any contribution from, say, a $\rho\rho$ cut while allowing contributions from, say, ρf. This has been referred to as the "phase problem" since the signature and phase of an amplitude are related. The present paper deals with the phase and energy dependence of Regge cuts and their implications for the signature rule. While we are principally concerned with crossing relations, we should note that some related arguments[4,5] motivated a (direct-channel) model for the Pomeranchukon.

There are currently several quite different types of models in which multiple Regge-pole exchange arises. Roughly grouping them into three types, we shall refer to them in the present paper as (1) *field-theoretic* (FT) models, (2) *multiple-scattering* (MS) models, and (3) *dual-resonance* models. The former (FT) type of cut model arises from the observation that the high-energy behavior of sums of Feynman diagrams can give rise to moving cuts in the j plane.[6] This type of model is the most fundamental in a theoretical sense, deriving many of the properties of Regge poles and cuts from field-theoretic considerations. Whether a field theory can "eikonalize" in a natural way and produce multiple-scattering-type cuts is still a very open question with pro and con papers appearing regularly. Such FT models are of heuristic value but at the present time do not provide practical models with which to confront experiment. The second (MS) model is the type most commonly seen in phenomenological applications. Models which utilize the nonlinear unitarity condition to "correct" a Regge-pole input are of this type (e.g., K-matrix, eikonal, and absorption models).[7] The relation between the MS and FT types of cuts is at best heuristic, the connection being even more obscure than that between a Regge pole and a sum of Feynman "ladder" graphs in the t channel. The Veneziano model in its operatorial form[8] produces cuts via the "loop" diagrams but, like the FT models, is not yet an acceptable vehicle for phenomenology. Arguments for the signature rule have been given for both FT and MS types of cuts and we now turn our attention to these arguments.

For FT cuts, the signature rule has been proved by Branson[9] for a certain class of Mandelstam-type Feynman diagrams. For MS cuts, the argument proceeds as follows. If $A(s, t)$ (omitting spin) is the amplitude for the reaction $ab \to cd$, then the amplitude $A^*(s e^{i\pi}, t)$ corresponds to the reaction $\bar{c}b \to \bar{a}d$ at the same energy and momentum transfer. Consider an amplitude corresponding to the exchange of two Regge poles of signatures τ_1 and τ_2,

$$A_c(s, t) = \frac{i}{16\pi^2 s} \int dt_1 \int dt_2 R_1(s, t_1) \times R_2(s, t_2)\theta(K)K^{-1}, \tag{1}$$

where

$$K \equiv -(t_1^2 + t_2^2 + t^2) + 2(t_1 t_2 + t_2 t + t t_1)$$

and R_1 and R_2 are the Regge amplitudes. Equation (1) is represented schematically in Fig. 1(a). It is easy to verify that

$$A_c^*(s\, e^{i\pi}, t) = \tau_1 \tau_2 A_c(s, t). \tag{2}$$

Another approach is to consider the fixed-t asymptotic behavior as $s \to \infty$. The amplitude $A_c(s, t)$ for fixed $t < 0$ satisfies the boundedness, analyticity, and asymptotic criteria for application of the Phragmén-Lindelöff theorem[10] in the half-plane $\mathrm{Im}\, s > 0$. As $s \to \infty + i$ at fixed t,

$$\pm \frac{A_c(s, t)}{s^{\alpha_c}} \propto \begin{cases} e^{-i\pi\alpha_c/2}, & \tau_1\tau_2 = +1 \\ e^{-i\pi(\alpha_c - 1)/2}, & \tau_1\tau_2 = -1, \end{cases} \tag{3}$$

where

$$\alpha_c(t) \equiv \alpha_1(\tfrac{1}{4}t) + \alpha_2(\tfrac{1}{4}t) - 1.$$

This means that the leading behavior of $A_c(s, t)$ is entirely of signature $\tau_1\tau_2$. This latter argument pertains only to the leading behavior at large s. Throughout this paper we shall primarily be concerned with the cut amplitudes and signature rule at large energies.

II. SIGNATURE RULE REEXAMINED

A. The Role of Intermediate States

There are two observations to be made about the signature rule for FT cuts. Firstly, the rule has only been shown to hold for a limited class of Feynman graphs.[9] Secondly, it has been noted[11] that the s dependence (j-plane position) of perturbation-theoretic cuts is sensitive to details of the dynamics and is not determined solely from the asymptotic behavior of the Jacobians in Feynman integrals. We shall arrive at this latter conclusion via other arguments and discuss its implications for the signature rule below. The importance of composite intermediate states in FT cut models has recently been stressed by Risk[12] and Quigg.[13]

The MS cuts are usually generated in some manner from the unitarity condition in the direct channel with the Regge-pole amplitudes as input. Such an expansion into a multiple-scattering series may be a reasonable approximation for some region of physical s and t in the direct channel and be quite meaningless when extrapolated to the u channel, i.e., a good approximation for a certain region in the s channel need not analytically continue to a good approximation of the u-channel amplitude. The use of the direct-channel unitarity condition to generate the "corrections" would suggest that this is a reasonable viewpoint. It is the physical amplitudes which (presumably) satisfy exact crossing symmetry and not necessarily the individual terms of an approximation. Therefore the question of the s-u crossing properties of MS amplitudes generated in this fashion may very well be a meaningless one. While this must be kept in mind, we shall for the rest of this section assume that the MS expansions in the two channels are s-u crossing-symmetric at high $|s-u|$ and small $|t|$ (even though an entirely satisfactory expansion has yet to be found), and therefore with this assumption the crossing properties of individual terms has meaning.

In connection with unitarity-generated MS expansions, we make the following two observations. (1) Intermediate-state particles in Fig. 1(a) lie on Regge trajectories and this should be taken into account, thus incorporating some aspects of the compositeness of these particles. (2) It has been pointed out[4] that duality may provide the connection between the usual unitarity sum over many-particle on-mass-shell intermediate states and a sum over quasi-two-particle intermediate states. We should therefore be dealing with diagrams such as the one shown in Fig. 1(b)[14] rather than that of Fig. 1(a); the intermediate-state wavy lines denote leading Regge trajectories and also possible daughters.

We represent the full double Regge-pole exchange in terms of a sum over intermediate states, which is shown graphically in Fig. 1(c). Following Freund[4] we let the resonance mass spectra be given by

$$m_{A_{jl}}{}^2 = m_{A_j}{}^2 = m_{A_0}{}^2 + \frac{1}{\alpha'} j, \quad l = 1, \ldots, a_j$$
$$m_{B_{kn}}{}^2 = m_{B_k}{}^2 = m_{B_0}{}^2 + \frac{1}{\alpha'} k, \quad n = 1, \ldots, b_k. \tag{4}$$

The index j (k) represents the position on a linear trajectory (spin) while the a_j (b_k) index degenerates A_{jl}'s (B_{kn}'s) of various spins. We thus write the double-Regge-pole-exchange amplitude as

$$A_c(s, t) = \frac{i}{16\pi^2 s} \sum_{jl\;kn} \int\!\!\int dt'\, dt''\, \frac{\theta(K)}{\sqrt{K}} \times A^{R_1}_{AB \to A_{jl}B_{kn}}(s, t') A^{R_2}_{A_{jl}B_{kn} \to CD}(s, t''), \tag{5}$$

where A^{R_1} and A^{R_2} are the Regge-pole-exchange amplitudes.

In order that the $A_{jl}B_{kn}$ intermediate state contribute appreciably, this state must be produced with small squared momentum transfer[4] $|t|_{\min}$. For large s,

$$|t|_{\min} \approx m_{A_j}{}^2 m_{B_k}{}^2 / s, \tag{6}$$

and we may roughly account for this kinematical

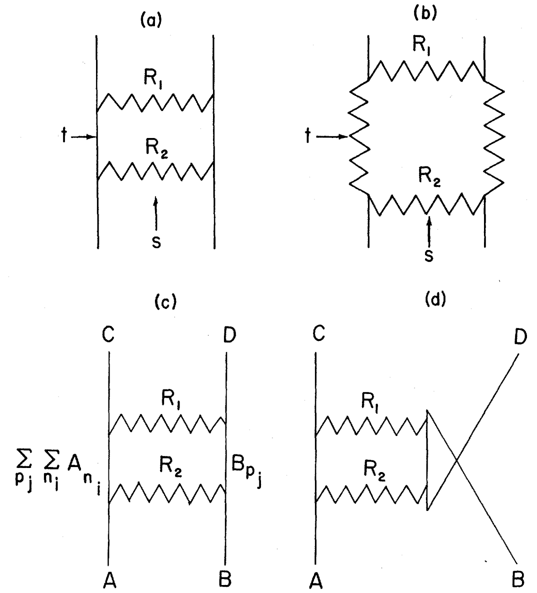

FIG. 1. (a) Double-Regge-exchange "box" diagram schematically representing Eq. (1). (b) Box diagram representing Eq. (5) with s-channel intermediate-state wavy lines denoting Reggeized particles, daughters, etc. (c) Representation of Eq. (5) with the intermediate-state sum over the resonant-state spectrum of Eq. (4) explicitly shown. (d) Crossed diagram representing the analytic continuation of a u-channel double scattering to the s channel.

constraint by requiring

$$m_{A_j}^2 m_{B_k}^2 \lesssim \mu^2 s\,, \tag{7}$$

where μ is a small mass constant. Kinematic arguments leading to Eqs. (6) and (7) are included in Secs. III B and III C.

The point to note here is that the number of terms in the sums over p_j and n_i is not fixed but *increases with* s. This introduces an additional s dependence over that given by Eq. (1) even though the same Regge poles, R_1 and R_2, are exchanged. Asymptotically, the s dependence of the amplitude of Eq. (1) is given by $s^{\alpha_c(t)}$ [where $\alpha_c(t)$ is given after Eq. (3)] while that of Eq. (5) depends on the details of the spectrum of intermediate states and their couplings.

B. Kinematics and the Phase of the Cut Amplitude

For simplicity, let us consider a process $ab \to cd$ in which these particles are spinless mesons. In calculating the amplitude of Eq. (5), we shall consider the exchange of two Regge poles, both of trajectory $\alpha(t)$, and intermediate states lying on a Regge trajectory which we will take to have the same slope α'.

We can write the amplitude for one of the single-Regge-pole-exchange processes (omitting spin indices, as usual) as

$$R(s,t) = \beta(t)[P_{\alpha(t)}(-\cos\theta_t) + \tau P_{\alpha(t)}(\cos\theta_t)]\,. \tag{8}$$

The expression for $\cos\theta_t$ is

$$\cos\theta_t = \frac{t^2 + t(2s - 2m^2 - \Sigma)}{|(s^2 - 4sm^2)(s^2 + \Delta^2 - 2s\Sigma)|^{1/2}}\,, \tag{9}$$

where $\Delta \equiv M_j^2 - M_k^2$, $\Sigma \equiv M_j^2 + M_k^2$. We are interested in the leading behavior in s for fixed $t \neq 0$. Also, we consider couplings which do not vary rapidly with intermediate-state spins j and k. We may therefore approximate $\cos\theta_t$ as

$$\cos\theta_t \sim 1 + \frac{2ts}{M_j^2 M_k^2} \sim \frac{2ts}{M_j^2 M_k^2}\,. \tag{10}$$

The expression for the s-channel scattering angle is

$$\cos\theta_s = \frac{s^2 + s(2t - 2m^2 - \Sigma)}{|t^2 + m^4 + M_j^4 - 2t(m^2 + M_j^2) - 2m^2 M_j^2|^{1/2}\,|t^2 + m^4 + M_k^4 - 2t(m^2 + M_k^2) - 2m^2 M_k^2|^{1/2}}\,, \tag{11}$$

from which we can derive the smallest squared momentum transfer kinematically allowed as

$$-t_0 \cong \frac{M_j^2 M_k^2}{s - \Sigma}\,. \tag{12}$$

As pointed out in Ref. (3), this squared momentum transfer must be small in order for the (j,k) amplitude to contribute appreciably. If we let $s_T(j,k)$ denote the threshold for production of the (j,k) state, i.e.,

$$s_T(j,k) \equiv (M_j + M_k)^2\,, \tag{13}$$

we get from Eq. (12) that the (j,k) state contributes appreciably only if

$$s \gg s_T(j,k)\,, \tag{14}$$

since $M_j^2 + M_k^2 > 2M_j M_k$ and $s \gg M_j^2 + M_k^2$. For large s, we therefore have the condition that *intermediate states in Eq. (7) contribute appreciably only far above their production thresholds*. From this observation it immediately follows that all such terms contribute the *same phase* (up to a sign) to Eq. (7); this phase is given by Eq. (3) of Sec. II. *The phase of Eq. (7) is therefore fixed by the Regge trajectories exchanged and is equal (up to a sign) to that of the simpler Eq. (1).*

C. Example of a Signature-Rule-Violating Double-Exchange Amplitude

In this section, we give an example of a "signature-rule"-violating cut amplitude. We calculate a double-exchange amplitude according to Eq. (5) corresponding to Fig. 1(c) and show explicitly that the exchange of two even-signatured poles can yield, for example, an odd-signatured cut amplitude for particular choices of intermediate-state couplings. In the example we construct below, we shall, for simplicity, consider only intermediate states lying on a single leading trajectory and omit degeneracies and daughter states [i.e., $a_j = b_k = 1$ in Eq. (4)].

In terms of factorized residues, we can write the Regge amplitude Eq. (8) in the form

$$R_1(s, -x^2) = g''_{1j}(x^2) g''_{1k}(x^2) \times \left[\left(\frac{2x^2 s}{M_j^2 M_k^2}\right)^{\alpha(-x^2)} + \tau_1 \left(\frac{2x^2 s}{M_j^2 M_k^2}\right)^{\alpha(-x^2)}\right], \tag{15}$$

where we have put $x \equiv \sqrt{-t}$. Assuming the resonances j, k to lie on infinitely rising linear trajectories of slope α', and extracting the signature factor, we rewrite Eq. (15) for an even-signatured pole as

$$R_1(s, -x^2) = g'_{1j}(x^2) g'_{1k}(x^2)\, e^{i\frac{1}{2}\alpha(-x^2)} \times \left(\frac{2x^2\alpha'}{j^2k^2}\right)^{\alpha(-x^2)} (\alpha' s)^{\alpha(-x^2)}, \tag{16}$$

where g'' and g' are related by

$$g'_{1j}\, g'_{1k} = 2 g''_{1j}\, g''_{1k} \cos[\tfrac{1}{2}\alpha(-x^2)]. \tag{17}$$

We choose both R_1 and R_2 to be of even signature for our example.

We now make a specific choice of coupling constants (residues):

$$g'_{nj}(x^2) g'_{nk}(x^2) = f^{1/2}(k) g_n \left(\frac{2x^2\alpha'}{j^2k^2}\right)^{-\alpha'(-x^2)}, \tag{18}$$

where g_n are real constants, $n = 1, 2$, and $f(k)$ is any well-behaved real function satisfying

$$\int_1^s \frac{f(k)}{k} dk \to C = \text{const} \tag{19}$$

which converges to C faster than $O(1/s)$. Equation (18) is not singular in the forward direction because of the kinematic condition (12) (i.e., x^2 is always greater than zero). The (j, k) term of Eq. (5) is now straightforward to calculate.

$$A_{jk}(s, t) = \frac{iC}{s} \frac{g_1 g_2}{4\beta} \exp[G(t)\beta(s)], \tag{20}$$

where

$$\beta(s) \equiv \ln(s\alpha') - i\tfrac{1}{2}\pi, \quad G(t) \equiv 2\alpha(0) + \tfrac{1}{2}\alpha' t\,.$$

The total amplitude of Eq. (5) is given by

$$A(s, t) = \sum_{jk} A_{jk}(s, t), \tag{21}$$

where the sum runs over all states j, k not suppressed by the kinematic condition that $|t_0|$ be small in Eq. (12). While there are several ways to impose this condition (e.g., an exponential cutoff), we have chosen to follow Ref. (4) and choose a small constant $-t_0 = \mu^2$ and include only those terms in the j, k sum for which the Reggeon momentum transfer is less than this value, i.e.,

$$\frac{M_j^2 M_k^2}{s} \simeq \frac{jk}{(\alpha')^2 s} < \mu^2\,. \tag{22}$$

This cutoff produces an s-dependent upper limit on the j, k sum. Converting the sum to a double integral, it is easily evaluated.

$$A(s, t) = \frac{i}{s} \frac{g_1 g_2}{4\beta} \exp[G(t)\beta(s)] \times \int_1^s dk \int_1^{(\alpha')^2 \mu^2 s k^{-1}} dj\, f(k), \tag{23}$$

$$A(s, t) = i\gamma \frac{\ln s}{\beta} \exp[G(t)\beta(s)], \tag{24}$$

where $\gamma \equiv \frac{1}{4} g_1 g_2 (\alpha')^2 \mu^2 C$ is a constant, and as $s \to \infty$, $(\ln s)/\beta \to 1$.

The Regge poles exchanged were both chosen to have positive signature; the signature rule states that the double-exchange amplitude must therefore be of *even* signature. The amplitude $A(s, t)$ of Eq. (24) is asymptotically of odd signature. This is a consequence of the sum over intermediate states in Eq. (5) which, in our example, raises the energy dependence compared to Eq. (1) by one power of s in the amplitude without changing the phase. One could just as easily have chosen couplings which lower the s dependence by one power [resulting again in an odd-signature $A(s, t)$] or which produce an $A(s, t)$ of even or of mixed signature. This example is purely illustrative, and the couplings in Eq. (18) were chosen to produce a maximal violation of the "rule" and are in no way intended to represent actual physical couplings, about which we know very little.

D. The Phragmén-Lindelöff Theorem

The situation, then, is the following. The asymptotic phase of the "new" amplitude [Eq. (5)] is the same as that of Eq. (1) whereas the s dependence of the new form of the amplitude may be shifted from that of Eq. (1) by the sum over higher-mass (and spin) intermediate states. It follows that the signature rule of Sec. I cannot, in general, be derived for the amplitude of Eq. (5).

Unless the s dependence of Eq. (7) is shifted from that of Eq. (1) by a factor s^{2N}, where $N = \ldots, -2, -1, 0, 1, 2, \ldots$, the amplitude of Eq. (5) is not asymptotically of signature $\tau_1\tau_2$. This is a consequence of the strong phase–s-dependence–signature connection provided by the Phragmén-Lindelöff (PL) theorem.[15] If the amplitude oscillates as $s \to \infty$, the PL theorem is not applicable and the signature rule is still not derivable in this way.

III. REACTIONS OF DEFINITE SIGNATURE

Because of the signature rule, it is sometimes stated that double-Regge-pole exchange violates crossing symmetry.[16] Implicit in this assertion are the assumptions that the expansions in the two channels must analytically continue to one another and that Eq. (1) gives the (second-order) cut amplitude; as pointed out in Sec. II, neither assumption is a necessary requirement of a realistic model. The arguments leading to Eq. (5) [or Eq. (1) for that matter] indicate that the multiple-scattering series consists entirely of MS "ladder" diagrams and does not explicitly include the crossed diagrams such as the one shown in Fig. 1(d). Let us consider reactions with definite symmetry under crossing (e.g., $\pi^- p \to \pi^0 n$, $\pi^0\pi^0 \to \pi^0\pi^0$, etc.). It might be expected that trouble with crossing symmetry would show itself most glaringly in this class of reactions since such reactions obey the stringent requirement that they are equal up to a sign,

$$A^{\rm I}(E, \theta) = \tau A^{\rm II}(E, \theta), \tag{25}$$

where I and II denote the two crossed reactions, $\tau = \pm 1$ is the signature, and the spin indices have been omitted for simplicity.

Will a multiple-scattering Regge-cut model, applied separately to reactions I and II, satisfy Eq. (25)? This question has a straightforward answer. *Any multiple-scattering Regge-cut model which satisfies isospin invariance at the vertices obeys Eq. (25) to all orders in the scattering.* (Note: this does not mean that the amplitude for reaction II is the analytic continuation of the amplitude for reaction I; that will depend on the details of the model.) Equation (25) is preserved to each order in the scattering and, further, there is a diagram-by-diagram correspondence where reactions I and II are related by charge symmetry (as well as s-u crossing), whereas otherwise the correspondence is only between sums of diagrams. These properties follow from the observation that reactions of definite signature result only in cases where the crossed reactions are related to one another by isospin. (The proofs of these statements are straightforward and we omit them.)

A specific example will place these remarks in perspective. A $\rho\rho$ amplitude calculated from Eq. (5) for the reaction $\pi^- p \to \pi^0 n$ will automatically be exactly the negative of the corresponding $\rho\rho$ amplitude calculated for $\pi^0 p \to \pi^+ n$, at the same energy and scattering angle in each reaction, and similarly for all other box or ladder diagrams resulting from multiple exchanges contributing to these two processes. This is totally a consequence of isospin conservation at the vertices.

Thus the stringent requirements of Eq. (25) for reactions of definite signature are *automatically* satisfied in MS models which conserve isospin at the vertices. Such MS models may therefore be constructed as approximations to the physical amplitudes without fear of violating Eq. (25), or any isospin relation, even though such a model may not satisfy asymptotic s-u crossing symmetry. If in addition one wishes to construct an MS model with explicit asymptotic crossing symmetry, this additional feature may be included as a constraint on the intermediate-state spectrum and couplings. We have shown that this is possible (at least to second order in the cuts), that it leaves much freedom in the model, and that it is therefore unnecessary to s-u "symmetrize" the model by the artifice of simply adding the crossed (u-channel-diagrams [see Fig. 1(d)]. This latter method of imposing s-u crossing symmetry has the added disadvantage of abandoning the direct connection with unitarity, which was the original motivation for the double exchange. A phenomenological model of this latter type has recently been proposed by Quigg.[17]

IV. CONCLUSIONS AND DISCUSSION

We have, in the present paper, examined the signature rule of Sec. I in the context of unitarity-generated multiple-scattering Regge cuts.[18] We observed that a more realistic handling of the intermediate states leads to the following conclusions (at high s and fixed t): (a) The phase of an MS amplitude is determined by the trajectories of the exchanged poles just as in the simpler treatment of intermediate states. (b) The s dependence is sensitive to the dynamical details such as the spectra and couplings of the intermediate-state particles and is not merely the simple function of the Regge trajectories obtained from the simplified treatment. (c) Taken together, these two observations remove the foundations of the signature rule and leave no "phase problem" for the unitarity-generated cut models. This suggests that s-u crossing in such models should be viewed as a constraint on the couplings and spectra of the intermediate-state particles and not as a constraint on the signature of the exchanged Regge poles.

We have *not* disproved the signature rule for all models. Henyey[19] has shown that in at least one case Mandelstam-type cuts can provide cancelations which might lead to the signature rule, and, as mentioned before, Branson[9] has proved the rule for a certain class of such perturbation-theoretic cuts.

We noted that MS models which conserve isospin at the vertices satisfy all isospin relations to each order in the scattering. Certain regularities between line-reversed reactions of definite signature follow from this without imposing any additional constraints whatsoever. These remarks, and the observations that such cuts are generated from the unitarity condition in the direct channel, suggest that it is also possible, and perhaps preferable, to view MS cut models as direct-channel approximations to the physical amplitudes and not as exact crossing-symmetric representations of them.[20]

Finkelstein[21] has recently proposed a "selection rule" which predicts small or null contributions from certain double Regge-pole exchanges. The selection rule is based on perturbation-theoretic arguments combined with duality-diagram applications to determine which double exchanges are allowed. In the present paper, we have generally avoided perturbation-theoretic arguments in our discussion of MS corrections. We do not, therefore, speculate on the validity of this selection rule for the type of two-Reggeon corrections considered by Finkelstein. We do, however, wish to point out a basic difference between the two types of corrections which suggests that this selection rule should probably not be applied to MS corrections. In the Finkelstein derivation, the double Regge-pole exchange arises from iterating a particle-Reggeon amplitude via t-channel unitarity. (The argument is that both particle-Reggeon amplitudes must then have planar s-u duality diagrams for the cut to exist.) In an MS model, the corrections are generated from the s-channel unitarity relation. This difference in the mechanisms producing the corrections is fundamental and it is doubtful the Finkelstein's selection rule should be applied as it stands to MS models. Another feature of this selection rule which we note in passing is that it forbids for many elastic reactions the diagram proposed by Freund and Rivers[22] as a representation of the Pomeranchukon, which subsequent dual-resonance calculations[23] showed to be sizeable. It is interesting to note that this diagram is, in effect, an s-channel iteration of a single-twist duality diagram and not a t-channel iteration.

Thus far, our arguments concerning the signature rule have dealt with the construction of multiple-Regge-exchange models. We would like to conclude our discussion by addressing the following question: How can the signature rule be assessed experimentally? This is clearly not an easy question to answer because of the model dependence involved and because double-Regge-pole exchange is expected to be a secondary effect in most reactions. Double-charge-exchange (DCEX) reactions[24] might be expected to provide the most sensitive physical tests of the signature rule and the phase-energy-dependence arguments of this paper since double Regge-pole exchange is expected to provide the leading behavior in such cases.[25] A direct test of the signature rule is difficult. The reaction $K^-p\to K^+\Xi^-$ might appear to be the best candidate for such a test because of its definite signature. This reaction is even under s-u crossing, and the leading behavior in the near-forward direction would presumably be given by double (K^*-K^{**}) exchange. Unfortunately, a difficulty is immediately encountered; the strong exchange-degeneracy condition on K^*-K^{**} is sufficient to ensure the cancellation of the $t_1t_2=-1$ contributions to this reaction, quite apart from the signature rule. This simplest model therefore rules out a "test" of the signature rule and underlines the model dependence of any such test. Other accessible double-charge-exchange reactions[24] such as $K^-p\to\pi^+\Sigma^-$ and $\pi^-p\to K^+\Sigma^-$ are of mixed signature, and the construction of simple double-exchange models for this pair of reactions, especially when higher-energy data become available, seems to be the best phenomenological approach to the signature-rule question.

Finally, quite apart from the validity of the signature rule, we would like to mention as an aside an interesting feature of Eq. (2). If it happens that a power-law dependence in s is produced from Eq. (2) for the spin-nonflip $K^-p\to K^+\Xi^-$ amplitude [and of course this does not necessarily follow from Eq. (2)], then the differential cross section at intermediate and high energies in the forward direction $d\sigma/dt(t=t_{\min}\approx 0)$ is proportional to $s^{2\alpha_c-2}$. The *phase* of the amplitude at $t\approx 0$ from Eq. (3) is

$$\exp\{-i\tfrac{1}{2}\pi[\alpha_c(0)-2N]\},$$

where

$$\alpha_c(0)=2\alpha_{K^*}(0)-1\,.$$

The phase-energy-dependence relation then requires that, for $K^-p\to K^+\Xi^-$,

$$\frac{d\sigma}{dt}(t=0)=s^{2\alpha_c(0)-4N-2}\equiv s^x\,, \tag{26}$$

where

$$x=4[\alpha_{K^*}(0)-N-1]\,. \tag{27}$$

Since N is an integer, and $\alpha_{K^*}(0)\simeq\frac{1}{4}$, x is permitted to have *only* one of the values $x\simeq -3, -7, -11, \ldots$, etc.[26] If the phase is required to be unchanged from Eq. (1) by the sum in Eq. (5), then N must be even and not odd, and x is then restricted to one of the values $x\simeq -3, -11, -19, \ldots$. Better statistics on $d\sigma/dt$ $(t=0)$ as a function of energy are needed to test this relation. There is also some uncertainty in the value of $\alpha_{K^*}(0)$. It is interesting to note that $d\sigma/dt \propto s^{-11}$ (with large errors) roughly describes the data on this reaction.[24]

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[26]Equation (1) predicts only $x\simeq -3$, while Eq. (5) permits the other values. Reference (24) shows that $x\simeq -3$ gives too small a decrease in $d\sigma/dt$ $(t=0)$. In a year or two, much more precise data on $K^-p\rightarrow K^+\Xi^-$ and other double-charge-exchange reactions will be available (J. Russ, private communication).