Asymptotic Algebraic Realization of SU(3), Baryon Classification, g_A (0), and D/F Ratio*

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In the framework of the chiral SU(3)⊗SU(3) charge algebra and asymptotic SU(3), a scheme of asymptotic algebraic realization of SU(3) is proposed. For the axial-vector couplings, it yields $D/F=\frac{3}{2}$ but $g_A(0)=\frac{5}{3}\sqrt{f}$, f being the universal fractional contribution (≈50%) to the sum rules coming from the ground-state baryons. Neither the assumption of saturation by low-lying states nor the introduction of large configuration mixing is necessary.

We propose a possible scheme of algebraic realization of SU(3) in the framework of the chiral SU(3)⊗SU(3) charge algebra and present a new derivation of the D/F ratio of hyperon axial-vector semileptonic couplings and $g_A(0)$. Our proposed scheme may provide new insight in hadron physics. Some years ago, Gerstein[1] observed that, by saturating the algebra *solely* by the $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet, the exact SU(6) value[2] of D/F can be reproduced without assuming SU(6) or SU(6) current-commutation relations. However, though certainly remarkable, Gerstein's result cannot be taken too seriously for the following reasons: (i) The saturation argument used is hardly justified. Actually the calculation yields, at the same time, a bad value $g_A(0)=\frac{5}{3}$, as is also the case with *exact* SU(6).[2] On the contrary, the Adler-Weisberger calculation,[3] which is based on the same algebra but does not use the saturation argument, gives a correct value of $g_A(0)$. (ii) Exact SU(3) is assumed for the matrix elements involved. There is no *a priori* justification for this assumption.

In this paper, we point out that there is a way out of these difficulties. Our new points are as follows: (a) The saturation argument of Gerstein can be replaced by our scheme of asymptotic algebraic realization of SU(3). (b) Exact SU(3) used in Ref. 1 can be simply replaced by our asymptotic SU(3) proposed before.[4] The degree of accuracy of our asymptotic SU(3) can be best seen as follows.[4] If the *basic* (not effective) SU(3)-breaking Hamiltonian belongs to an octet, in the framework of asymptotic SU(3) the Gell-Mann–Okuko mass formulas (including the effect of particle mixing) become exact mass formulas (rather than first-order formulas[5]).

We have already applied our proposed scheme and asymptotic SU(3) to bosons and found an encouraging result.[6] There, the generalized boson nonet coupling scheme has emerged as a consequence which reduces to the ideal nonet scheme [also obtained[2] from exact SU(6)], whenever one of the constraints of ideal nonet (such as $m_\omega \simeq m_\rho$) is realized.

We denote the baryons by $B_{\alpha,s}$. α stands for the (physical) SU(3) multiplets, i.e., $(N_s, \Lambda_s, \Sigma_s, \Xi_s)$ for octet and $(\Delta_s, \Sigma_s^*, \Xi_s^*, \Omega_s)$ for decuplet. s denotes the J^P and other quantum numbers. We consider the algebra sandwiched between the one-particle baryon state with *infinite* momentum. The algebra $[V_i, V_j]=if_{ijk}V_k$ will then be automatically satisfied by our asymptotic SU(3). The algebra $[V_i, A_j]=if_{ijk}A_k$ yields the following important information when combined with our asymptotic SU(3): Although SU(3) is broken, the matrix elements of the form $\langle B_{\alpha,s}(\vec{q})|A_i|B_{\beta,t}(\vec{q})\rangle$ with $\vec{q}\to\infty$, do allow[7] exact SU(3) parametrization (including the usual particle mixing which is defined[4] only in the limit $\vec{q}\to\infty$). The implication of this fact in the $\bar{B}_\alpha B_\beta P$ couplings was discussed in detail before.[7]

Now, the most interesting commutation relation which presumably contains the most dynamical information is[8] $[A_i, A_j]=if_{ijk}V_k$. Although we can proceed as in Ref. 1 without specifying the A_i's, we pick out the commutator ($A_+ \equiv A_1+iA_2$, $V_3\equiv I_3$, etc.)

$$[A_+, A_-]=2V_3 \tag{1}$$

for illustration. Sandwich this commutator between the states $\langle B_{\alpha,s}(\vec{q}')|$ and $|B_{\alpha,s}(\vec{q})\rangle$. We then obtain for $\vec{q}\to\infty$

$$\tfrac{1}{2}(I_3)_{\alpha,s}{}^{-1}\sum_{n,m}[\langle B_{\alpha,s}(\vec{q}\,')|A_+|n_{\beta,t}\rangle\langle n_{\beta,t}|A_-|B_{\alpha,s}(\vec{q})\rangle-\langle B_{\alpha,s}(\vec{q}\,')|A_-|m_{\beta,t}\rangle\langle m_{\beta,t}|A_+|B_{\alpha,s}(\vec{q})\rangle]=1\times(2\pi)^3\delta^3(\vec{q}-\vec{q}\,'). \quad (2)$$

Here $(I_3)_{\alpha,s}$ denotes the (nonzero) eigenvalue of I_3 of the baryon $B_{\alpha,s}$. $n_{\beta,t}$ and $m_{\beta,t}$ represents one of the presumably infinite set of intermediate states with appropriate quantum numbers (β,t). Note that we have normalized the right-hand side of Eq. (2) to unity up to the trivial factor $(2\pi)^3\times\delta^3(\vec{q}-\vec{q}\,')$. In Eq. (2) by varying the SU(3) indices α for given s, one can study the SU(3) implications of the algebra in the limit $\vec{q}\to\infty$. An intriguing question is: How does the fractional amount of contribution coming from the (fixed) state (β,t) change when the SU(3) indices α's are varied for given s?

Let us now choose $B_{\alpha,s}$ to be the $\frac{1}{2}^+$ baryon (N,Λ,Σ,Ξ) and write down Eq. (2) for the choice $\alpha=p$, Σ^+, and Ξ^0. For the intermediate states (β,t), only the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons will be written explicitly,[9]

$$\langle p|A_+|n\rangle\langle n|A_-|p\rangle+\langle p|A_+|\Delta^0\rangle\langle\Delta^0|A_-|p\rangle-\langle p|A_-|\Delta^{++}\rangle\langle\Delta^{++}|A_+|p\rangle-\cdots=1,$$

$$\tfrac{1}{2}(\langle\Sigma^+|A_+|\Lambda\rangle\langle\Lambda|A_-|\Sigma^+\rangle+\langle\Sigma^+|A_+|\Sigma^0\rangle\langle\Sigma^0|A_-|\Sigma^+\rangle+\langle\Sigma^+|A_+|\Sigma^{*0}\rangle\langle\Sigma^{*0}|A_-|\Sigma^+\rangle+\cdots)=1,$$

$$\langle\Xi^0|A_+|\Xi^-\rangle\langle\Xi^-|A_-|\Xi^0\rangle+\langle\Xi^0|A_-|\Xi^{*-}\rangle\langle\Xi^{*-}|A_+|\Xi^0\rangle+\cdots=1.$$

Since we have taken the limit $\vec{q}\to\infty$, all the matrix elements involved allow exact SU(3) parametrization[7] in the framework of asymptotic SU(3). Thus, we already have overcome one of the difficulties of Ref. 1. We use the usual parametrization[9]:

$$\langle p(\vec{q})|A_+|n\rangle=G\sqrt{2}\,(D+F),$$

$$\langle\Sigma^+(\vec{q})|A_+|\Lambda^0\rangle=G(2/\sqrt{3})D,$$

$$\langle\Sigma^+(\vec{q})|A_+|\Sigma^0\rangle=G(-2F),$$

etc., with $\vec{q}\to\infty$ and $D+F=1$. By choosing the helicity state of $\frac{1}{2}^+$ baryons, for example,

$$u_\alpha^*(\vec{q})=(q_0+m_\alpha)^{1/2}(2m_\alpha)^{-1}(1\,0\,0\,|\vec{q}|(q_0+m_\alpha)^{-1}),$$

we observe[9]

$$\langle p(\vec{q})|A_+|n\rangle=G\sqrt{2}\equiv g_A(0),$$

etc., for $\vec{q}\to\infty$, i.e., baryon masses do not appear in the above equation.[7] This implies that in the framework of asymptotic SU(3), exact SU(3) parametrization is permitted for the *physical* axial-vector semileptonic couplings (but only in the zero-four-momentum transfer limit). We emphasize that this is by no means trivial. In general, asymptotic SU(3) does *not* allow exact SU(3) parametrization for physical couplings even in the zero four-momentum transfer limit.[7] Thus the experimental indication that the hyperon semileptonic decays seem to permit exact SU(3) parametrization for the g's supports our asymptotic SU(3). We normalize the matrix elements $\langle\frac{1}{2}^+|A_i|\frac{3}{2}^+\rangle$, which also allow exact SU(3) parametrization at $\vec{q}\to\infty$, by writing[9]

$$\langle p(\vec{q})|A_+|\Delta^0\rangle\langle\Delta^0|A_-|p\rangle\equiv G^{*2}\ \text{ for }\vec{q}\to\infty.$$

Then the above three equations become

$$2G^2(D+F)^2-2G^{*2}+\cdots=1, \quad (3)$$

$$\tfrac{2}{3}G^2D^2+2G^2F^2+\tfrac{1}{4}G^{*2}+\cdots=1, \quad (4)$$

$$2G^2(D-F)^2+G^{*2}+\cdots=1. \quad (5)$$

Let us look at the fractional contributions coming from, for example, the $\frac{3}{2}^+$ states (i.e., G^* terms) in Eqs. (3)–(5). They are in the ratio $-2:\frac{1}{4}:1$ compared with the (normalized) SU(3) ratio of the right-hand sides of Eqs. (3)–(5), $1:1:1$. Therefore, the $\frac{3}{2}^+$ intermediate states *alone* cannot realize the SU(3) contents of the algebra. However, this might suggest an alternative. Our present conjecture is as follows.

The baryon states (with infinite momenta) will make a certain grouping, $R_0, R_1, R_2, \ldots$ and the contribution from *each* R_i will now realize the (asymptotic) SU(3) contents of the algebra. This possibility is probably related to the possible existence of an asymptotic higher symmetry. We use the quark model as a guide, recalling that the powerful commutation relation, Eq. (1), comes also from this model.[8] The simplest grouping will be the orbital angular momentum classification $(L=0,1,2,\ldots)$ with the possible introduction of principal quantum numbers $n=0,1,2,\ldots$. Therefore, we may have $R_0(n=L=0)$, $R_1(n=0, L=1)$, ... In the simple (qqq) model the octet and decuplet baryons[10] with $J^P=\frac{1}{2}^+$ or $\frac{3}{2}^+$ are possible for the ground state R_0. Experimentally, we found indeed the octet $\frac{1}{2}^+$ and decuplet $\frac{3}{2}^+$ baryons for low-lying baryons. Thus, the R_0 will consist of the (N,Λ,Σ,Ξ) and $(\Delta,\Sigma^*,\Xi^*,\Omega)$. Then, according to our conjecture applied for the *ground* state R_0, we obtain from Eqs. (3)–(5)

$$2G^2(D+F)^2-2G^{*2}=\tfrac{2}{3}G^2D^2+2G^2F^2+\tfrac{1}{4}G^{*2}=2G^2(D-F)^2+G^{*2}=f. \quad (6)$$

f denotes the fractional contribution coming from R_0, Eq. (6) gives (with $G^*\neq 0$)

$$\frac{D}{F}=\frac{3}{2},\quad |G^*|=|\tfrac{4}{5}G|=|(4/5\sqrt{2})g_A(0)|, \quad (7)$$

$$g_A(0) = \tfrac{5}{3}\sqrt{f}\ . \tag{8}$$

Therefore, we have reporduced the results of Gerstein[1] or of exact[2] SU(6) [Eq. (7)], assuming neither the concept of exact symmetry nor the saturation of the algebra by low-lying baryons. However, the value of $g_A(0)$ is not $\frac{5}{3}$ [see Eq. (8)]. The value $f \simeq 50\%$ yields the experimental value $g_A(0) \simeq 1.2$. The corresponding value of G^* [from Eq. (7)] enables us to evaluate $\Gamma(\Delta \to N\pi)$, if the partially conserved axial-vector current hypothesis is used.[11] We obtain a reasonable value $\Gamma(\Delta \to N\pi) \simeq 150$ MeV. We need to study also the case $B_{\alpha,s} = \frac{3}{2}^+$ in Eq. (2) by taking $\alpha = \Delta^+$, Σ^{*+}, and Ξ^{*0}. In this case, the contributions coming from the states belonging to R_0 automatically satisfy our requirement. Namely, both the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryon intermediate states *separately* realize the SU(3) contents of the algebra, a simpler situation which supports our simple conjecture. Exact[2] SU(6) or the saturation of the algebra by low-lying baryons[1] are certainly inaccurate as exemplified by the bad result, $g_A(0) = \frac{5}{3}$. We have shown, however, that in deriving the ratios D/F and $G^*/g_A(0)$, the proposed asymptotic algebraic realization of SU(3) in the algebra is sufficient. There is no need to bring in a large configuration mixing with higher states.[12] Our conjecture could be applied also to the computation of other physical quantities.

In this paper our discussion has been confined to the ground state R_0. The most interesting question is: What will be the correct classification of baryon resonances compatible with our scheme? $SU(6) \otimes O(3)$ classification is a candidate, judging from the present result. We may assume that our R_0 belongs to the $(56, n = L = 0)$ representation. [For bosons, the $SU(4) \otimes O(3)$ classification may be a candidate.[6]] For baryons belonging to R_1, $R_2, \ldots$, complications arise from the particle mixing possibility since there are many octet (or singlet) and decuplet baryons with the same J^P within the comparable mass values for higher states. Furthermore, physical coupling constants do *not* allow exact SU(3) parametrization.[7]

Finally, in our framework, the correction to our result, Eqs. (7) and (8), comes from the possible mixing between the ground-state baryons and the higher-lying ones with the *same* J^P [such as, for example, the member of the $(56, n=2, L=0)$ representation] which will modify the exact SU(3) parametrization used in this paper. That is, we need to consider only the small configuration mixing of this type. Our Gell-Mann–Okubo mass formulas neglecting such an effect contain some errors, as exemplified by the quadratic decuplet mass formulas,[4] $m_\Omega^2 - m_{\Xi^*}^2 = m_{\Xi^*}^2 - m_{\Sigma^*}^2 = m_{\Sigma^*}^2 - m_\Delta^2$, which read $0.46 \simeq 0.42 \simeq 0.39$ in GeV^2. The inclusion of such an effect could change the values of D/F, $g_A(0)$, and $g_{\Delta N\pi}$ obtained by several percent.[13]

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[5]Thus the so-called octet dominance comes out automatically if our asymptotic SU(3) is correct.

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[7]S. Oneda and Seisaku Matsuda, Phys. Rev. D 2, 887 (1970).

[8]M. Gell-Mann, Physics 1, 63 (1964).

[9]The factor $(2\pi)^3\delta(\vec{p} - \vec{p}\,')$ will be omitted throughout the paper.

[10]For the ground state, the unitary singlet may not appear because of statistics.

[11]The $\Delta^{++} \to p + \pi^+$ coupling constant $g_{\Delta N\pi}$ is then related to G^* by $G^{*2} = \frac{1}{18}(m_\Delta + m_N)^2 (m_\Delta)^{-2} (F_\pi g_{\Delta N\pi})^2$.

[12]For this approach see, for example, F. Buccella, H. Kleinert, C. A. Savoy, E. Celeghini, and E. Sorace, Nuovo Cimento 59A, 133 (1970), and earlier papers cited there.

[13]The $\Gamma(\Delta \to N\pi)$ obtained is a little larger than the experimental value $\simeq 125$ MeV. However, this should be compared with the improvement achieved, i.e., from $g_A^2(0) = (\frac{5}{3})^2$ to $(1.2)^2$.